

Index Stability for Conformally Invariant Problems

Francesca Da Lio*

Abstract

In a joint paper with Matilde Gianocca and Tristan Rivière, a new method have been developed to prove the upper-semi-continuity of the extended Morse index –the number of negative or null eigenvalues– associated to conformally invariant variational problems in 2-D, which include the case of harmonic maps. This new theory have turned to be very efficient in several other geometrical analysis settings (biharmonic maps, Willmore surfaces, Yang-Mills connections, Ginzburg-Landau energies and of constant mean curvature surfaces).

Conservation laws play a central object in the *bubble tree analysis* of critical points of conformally invariant Lagrangians from an arbitrary closed surface into an arbitrary closed smooth manifold. In this talk we are going to explain how they can be used to tackle problems related to energy quantization, necklessness property and Morse index stability. We also mention how to extend the Morse Index analysis to the case of sequences of maps from sequences of domains degenerating to a punctured Riemann surface, assuming that the lengths of the images by the maps of the collars associated to this degeneration stay below some critical length.

*Department of Mathematics, ETH Zürich, Rämistrasse 101, 8092 Zürich, Switzerland.