# Boundedness and Moduli Problems in Birational Geometry and Foliation Theory

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### **Book of Abstracts**

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## On K3 surfaces with non-elementary hyperbolic automorphism group

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My talk is based on my joint work with Professor Koji Fujiwara (Kyoto University) and Professor Xun Yu (Tianjing University).

Main result of my talk is the finiteness of the N\'eron-Severi lattices of complex projective K3 surfaces whose automorphism groups are non-elementary hyperbolic under the assumption that the Picard number greater than or equal to 6 which is optimal to ensure the finiteness. In this talk, after recalling the notion of hyperbolicity of group due to Gromov and its importance in mathematics, I would like to explain why the non-elementary hyperbolicity of K3 surface automorphism group is the problem of the N\'eron-Severi lattices and how one can deduce the above-mentioned finiteness, via the study of genus one fibrations on K3 surfaces and recent work of Kikuta and Takatsu on geometrically finiteness.

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#### Blowups, Gale duality and moduli spaces.

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In this talk, we discuss the birational geometry of blowups of projective spaces at points in general position. For that, we explore Gale duality – a correspondence between sets of n=r+s+2 points in projective spaces  $\mathbb{P}^s$  and  $\mathbb{P}^r$ . For small values of s, this duality has a remarkable geometric manifestation: the blowup of  $\mathbb{P}^r$  at n points can be realized as a moduli space of vector bundles on the blowup of  $\mathbb{P}^s$  at the Gale dual points. This perspective allows us, in particular, to partially describe the birational geometry of the blowup of  $\mathbb{P}^n$  at n+4 points in general position. This is a joint work with Ana-Maria Castravet, Inder Kaur and Diletta Martinelli.

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## Fano 4-folds with large Picard number and blow-ups of cubic 4-folds

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We will present some classification results for (smooth, complex) Fano 4-folds X with Picard number rho(X)>6. First of all, if rho(X)>9, then X is a product of del Pezzo surfaces; this is sharp, since we know one family of Fano 4-folds with rho(X)=9 that is not a product of surfaces. In the range rho(X)=7,8,9, we will explain some partial classification results, based on a detailed and explicit study of the geometry of X using birational geometry in the framework of the MMP. In particular, if rho(X)>6 and X has no small contractions, then either X is a product of surfaces, or rho(X)=7,8,9 and X is a blow-up of a cubic 4-fold along rho(X)-1 planes that intersect pairwise at a point.

#### The cone conjecture for various finite quotients

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The cone conjecture predicts the shape of the nef and movable cones of certain projective varieties, modulo automorphisms. It is notably known for K3 surfaces, abelian varieties, hyperkähler manifolds, and in a few sporadic cases. Together with standard MMP conjectures, it implies the finiteness of minimal models, which can be used to improve birational boundedness to boundedness for certain classes of varieties.

In this talk, we present a descent result under finite group actions for the cone conjecture. Our result applies to arbitrary finite quotients of products involving abelian varieties and hyperkähler manifolds, as well as to Galois descent over perfect fields for those products. Along the way, we highlight a few properties of convex cones that behave themselves better than the cone conjecture, and that are satisfied by many examples coming from complex algebraic geometry.

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#### **Primitive Enriques varieties**

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The notion of an Enriques manifold was introduced by Oguiso-Schröer as a complex manifold which is not simply connected and whose universal covering is an irreducible symplectic manifold. From the viewpoint of birational geometry, we want to understand its behaviour under Minimal Model Program (MMP) operations. Based on the result of Lehn-Pacienza showing that any MMP starting from a primitive symplectic variety terminates, it is natural to introduce the singular analog of Enriques manifolds as finite quasi-étale quotients of primitive symplectic varieties by nonsymplectic group actions, which are called primitive Enriques varieties.

In this talk, we will give examples of primitive Enriques varieties and outline some basic properties of them. We will then focus on their behaviour under MMP and small deformations. In particular, we will show that the class of primitive Enriques varieties is stable under MMP and that we have a local Torelli theorem for such varieties. This talk is based on joint works with Francesco Denisi, Ángel David Ríos Ortiz and Nikolaos Tsakanikas.

#### Moduli of surface foliations

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I will report on some recent progress on the construction of a moduli space of surface foliations of general type. Time permitting I will also discuss some aspects of this moduli problem in general, as well as applications to the construction of moduli of fibred varieties. This talk will partly cover joint work with S. Velazquez and R. Svaldi.

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#### A tale of three GIT problems.

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In this talk, I will report on work in progress, joint with M. Hattori and T. Papazachariou, concerning a classical cycle of correspondences among (particular) nets of quadrics in P3, nets of cubics in P2, and (smooth) plane curves ofdegree four. I will explain how, by extending these correspondences, one can obtain precise links among the three corresponding GIT problems.

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#### On the birational geometry of algebraically integrable foliations

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I will review recent progress on extending the Minimal Model Program to foliations. Building on techniques from both birational geometry and the theory of foliations, we obtain new results on the structure of foliated pairs. I will focus on applications such as the boundedness of algebraically integrable Fano foliations. Joint work with Han, Liu, Meng, Spicer, Svaldi, and Xie.

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## On the boundedness of algebraically integrable foliations of general type

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I shall discuss recent ongoing work on the boundedness of algebraically integrable foliations of general type based on the theory of adjoint foliated structures. In particular, I will discuss a birational boundedness theorem that builds on the proof of McKernan's ACC conjecture for interpolated log canonical thresholds for algebraically integrable foliations.

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#### A proof of the b-semiampleness conjecture

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A family of Calabi–Yau varieties (or, more generally, log Calabi–Yau pairs) f:X->Y naturally induces a moduli divisor M, measuring the variation of the family f. Based on earlier works of the Japanese school, Prokhorov and Shokurov conjectured that M is semiample. In this talk, we discuss a proof of this conjecture and, time permitting, some immediate applications in birational geometry. In this talk, we will assume the inputs from o-minimality and Hodge theory, discussed in the first talk by Mauri. In particular, we will focus on the birational methods involved: study of log canonical centers, P1-linking, and pluricanonical representations.

This is the second of a series of two talks on the paper "Baily–Borel compactifications of period images and the b-semiampleness conjecture", a joint work with Benjamin Bakker, Mirko Mauri, and Jacob Tsimerman.

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### Boundedness of Polarized Log Calabi–Yau Fibrations with Bounded Bases

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In this talk, we discuss the boundedness problem for log Calabi–Yau fibrations whose bases and general fibers are bounded. We show that, after fixing certain natural invariants, the total spaces of such fibrations are bounded in codimension one. Furthermore, we prove that the total spaces themselves are bounded when the general fibers have vanishing irregularity. As an application, we obtain boundedness results for stable minimal models and fibered Calabi–Yau varieties. This is based on joint work with Junpeng Jiao and Minzhe Zhu.

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#### Semiampleness of line bundles arising from Hodge theory

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Semiampleness criteria are subtle foundational statements in algebraic geometry. In this talk, we present a new semiampleness result for the Hodge bundle of certain Calabi–Yau variations of Hodge structure. This result is a key ingredient in the proof of two long-standing conjectures: Griffiths' conjecture on functorial compactifications of images of period maps, and the b-semiampleness conjecture of Prokhorov and Shokurov. The proof crucially relies on o-minimal GAGA, marking the first use of o-minimality techniques in birational geometry.

This is the first of a series of two talks on the paper "Baily–Borel Compactifications of Period Images and the b-Semiampleness Conjecture", a joint work with Benjamin Bakker, Stefano Filipazzi, and Jacob Tsimerman.

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## Rational curves on very general complete intersections of high degree.

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I will report on a joint work-in-progress with F. Bastianelli. It is well known thanks to Ein that very general complete intersections of multidegree  $(d_1, \dots, d_c)$  in the projective n-space do not contain rational curves as soon as  $d_1+\dots+d_c>2n-c-1$ . This result has been sharpened in the case of hypersurfaces thanks to a method introduced by Voisin that inspired further work by Clemens, Ran and myself. Despite more recent work by Coskun, Riedl, Yang, Abe and others in the hypersurface case, Ein's result has not been improved in the case of higher codimension. The goal of the project is to do so. I will illustrate the main steps of the proof with particular emphasis on a key intermediate result consisting in the proof of the integrability of a certain distribution that naturally comes into the picture.

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#### When Kupka Meets Reeb

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Reeb's local and global theorems are classical results in the theory of smooth foliations, giving conditions under which a foliation is locally or globally the pullback of a foliation by points. These results depend on assumptions such as compactness and the finiteness of the fundamental group or holonomy of a leaf. In contrast, the Kupka Theorem concerns the local study of singular foliations of dimension greater than one and provides sufficient conditions for a germ of a foliation to be the pullback of a germ of a foliation by curves. In this talk, we will present results that may be viewed as fiber products of Reeb's and Kupka's theorems. Based on joint work with Gabriel Michels.

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