Local and nonlocal methods in geometric analysis

Report of Contributions

https://indico.sns.it/e/124

Contribution ID: **1** Type: **not specified**

(1st lesson) Regularity for area-minimizing and almost-minimizing currents

Thursday, 6 November 2025 09:00 (1h 30m)

The Plateau problem concerns the surfaces of least m-dimensional area spanning a given (m-1)-dimensional boundary. To guarantee existence of minimizers and desirable compactness properties for sequences of surfaces, one must consider a weak notion of surface, which allows area-minimizing "surfaces"to have singularities. Two particularly natural frameworks for this problem in arbitrary dimension and codimension are integral currents and mod(q) currents, which have both been studied in great depth since the 1950s, pioneered by works of De Giorgi, Federer & Fleming, Almgren, Taylor and White, and built upon by many others. The former framework allows for surfaces to have integer multiplicities, while the latter allows for multiplicities modulo a fixed integer q. I will give an overview of the history of the problem and some recent breakthroughs in the regularity theory for each framework, as well as regularity (and failure thereof) for almost area-minimizers, with particular focus on branch point singularities.

Primary author: SKOROBOGATOVA, Anna (ETH Zurich)

Presenter: SKOROBOGATOVA, Anna (ETH Zurich)

Contribution ID: **2** Type: **not specified**

Ambrosio-Tortorelli approach to topological singularities and connections with jump minimizing liftings

Thursday, 6 November 2025 15:00 (1 hour)

We study the Gamma-convergence of Ambrosio-Tortorelli-type functionals, for maps u defined on an open bounded set Ω ⊂ R^n and taking values in the unit circle S^1 ⊂ R^2. Depending on the domain of the functional, two different Gamma-limits are possible, one of which is nonlocal, and related to the notion of jump minimizing lifting, i.e., a lifting of a map u whose measure of the jump set is minimal. The latter requires ad hoc compactness results for sequences of liftings which, besides being interesting by themselves, also allow to deduce existence of a jump minimizing lifting. This is based on a joint work with Giovanni Bellettini and Riccardo Scala.

Primary author: MARZIANI, Roberta (Università degli studi di Siena)

Presenter: MARZIANI, Roberta (Università degli studi di Siena)

Contribution ID: **3** Type: **not specified**

Diffuse decay of excess in codimension two

Wednesday, 5 November 2025 16:30 (1 hour)

Entire critical points of the Abelian-Higgs functional are known to blow down to (generalized) minimal submanifolds of codimension 2. We focus on the multiplicity-one regime, where the blow-down consists of a single sheet. In this talk, I will present results from three papers that together provide the analytical tools to prove that entire critical points in this regime exhibit an improvement of flatness, leading to the uniqueness of blow-downs.
We further show that stationary points are flat in low dimensions, using an Allard-type argument, while local minimizers are flat in all dimensions via a De Giorgi-type method. This can be viewed as a large-scale regularity theorem in the spirit of geometric measure theory.
As part of this analytical framework, I will briefly discuss weighted inequalities on two-manifolds, as well as a quantitative stability theorem for the Abelian-Higgs model in dimension 2. This is a joint work with Guido De Philippis and Alessandro Pigati.

Primary author: HALAVATI, Aria

Presenter: HALAVATI, Aria

Contribution ID: **4** Type: **not specified**

About some approximation problems for Sobolev maps to manifolds.

Wednesday, 5 November 2025 15:00 (1 hour)

In a striking contrast with the classical case of real-valued Sobolev function, a Sobolev map with values into a given compact manifold N need not be approximable with smooth N-valued maps. This observation, initially due to Schoen and Uhlenbeck (1983), gave rise to a whole area of research concerned with questions related to approximability properties of Sobolev mappings with values into a compact manifold.
In this talk, I will give a broad overview of this research direction, its history, the main problems it is concerned with, important known results, as well as some recent contributions.

Primary author: DETAILLE, Antoine

Presenter: DETAILLE, Antoine

Contribution ID: **5** Type: **not specified**

Gamma-convergence of the p-Dirichlet energy for manifold-valued maps

Wednesday, 5 November 2025 14:00 (1 hour)

In this talk, we will consider the p-Dirichlet energy of maps with values in a closed manifold. We will discuss a Gamma-convergence result in the limit as the exponent p approaches a certain critical value k, which is defined in terms of the topology of the target manifold. This particular limit is associated with the emergence of topological singularities of codimension k, which can be described in measure-theoretic terms using the language of flat chains with coefficients in normed Abelian groups. This presentation is meant to serve as an introduction to the topic; I will try to avoid technicalities as much as possible. The talk is based on joint work with Van Phu Cuong Le (Universität Heidelberg), Ramon Oliver-Bonafoux (Università di Verona), and Giandomenico Orlandi (Università di Verona).

Primary author: CANEVARI, Giacomo (Università degli Studi di Verona)

Presenter: CANEVARI, Giacomo (Università degli Studi di Verona)

Contribution ID: **6** Type: **not specified**

Regularity for stable phase transitions

Thursday, 6 November 2025 16:30 (1 hour)

We will discuss recent progress in the regularity for stable solutions to the Allen-Cahn equation, with a focus on the connections with minimal surface theory and two influential conjectures of De Giorgi and Yau. Part of this talk is based on recent joint work with Joaquim Serra which classifies all stable solutions to Allen-Cahn with bounded energy density in four dimensions.

Primary author: FLORIT-SIMON, Enric (ETH Zürich)

Presenter: FLORIT-SIMON, Enric (ETH Zürich)

Contribution ID: **7** Type: **not specified**

(Non)local minimal surfaces and phase coexistence models.(1st lesson)

Thursday, 6 November 2025 11:00 (1h 30m)

We present some aspects of the theory of nonlocal minimal surfaces, with special attention to regularity, sheeting, maximum principle, and stickiness. We also link this theory to the study of long-range phase transition and to the analysis of its symmetry properties. Similarities and differences with the classical cases will be outlined.

Primary author: VALDINOCI, Enrico (University of Western Australia)

Presenter: VALDINOCI, Enrico (University of Western Australia)

Contribution ID: **8** Type: **not specified**

A strict maximum principle for nonlocal minimal surfaces

Thursday, 6 November 2025 14:00 (1 hour)

Suppose that two nonlocal minimal surfaces are included one into the other and touch at a point. Then, they must coincide. But this is perhaps less obvious than what it seems at first glance.

Primary author: DIPIERRO, Serena (University of Western Australia)

Presenter: DIPIERRO, Serena (University of Western Australia)

Contribution ID: **9** Type: **not specified**

(2nd lesson) Regularity for area-minimizing and almost-minimizing currents

Friday, 7 November 2025 11:00 (1h 30m)

Primary author: SKOROBOGATOVA, Anna (ETH Zurich)

Presenter: SKOROBOGATOVA, Anna (ETH Zurich)

Contribution ID: **10** Type: **not specified**

(Non)local minimal surfaces and phase coexistence models.(2nd lesson)

Friday, 7 November 2025 09:00 (1h 30m)

Primary author: VALDINOCI, Enrico (University of Western Australia)

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