Let $G(x) := \{1/x\}$ be the Gauss map. By $g_n(x) = \frac{1}{x+n}$ we denote its continuous/real analytic inverse branches. We define iterated function system (IFS) G_n by limiting the collection of functions g_k, $k \in \mathbb{N}$, to the first n elements, meaning that $G_n = \{g_k\}_{k=1}^n$. The limit set J_n of this IFS is a conformal repeller, and it is well known that the Hausdorff measure of J_n, evaluated at its Hausdorff dimension h_n is positive and finite.

A famous result of Hensley describes the asymptotics od h_n:

$$\lim_{n\to\infty}(1-h_n)\cdot n = \frac{6}{\pi^2}.$$

We study the value of the Hausdorff measure H_{h_n} of the limit set J_n and prove its exact asymptotics:

$$\lim_{n\to\infty}\frac{1-H_{h_n}(J_n)}{(1-h_n)\ln n} = 1$$

and equivalently, due to Hensley's result,

$$\lim_{n\to\infty}\frac{n(1-H_{h_n}(J_n))}{\ln n} = \frac{6}{\pi^2},$$

where J_n is the limit set of the system G_n, i.e. the set consisting of irrational numbers in $[0,1]$ that continued fraction expansion with entries not exceeding n.

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