The structure of normal currents and regularity of transport operators

Normal currents in R^n can be viewed as a compactification of polyhedral chains with real coefficients, while integral currents are a compactification of polyhedral chains with integral coefficients. For several reasons it would be useful to represent normal currents as superposition (integral convex combination) of integral currents, and this can be done to some extent when the dimension k of the current is either 1 or $n-1$.
For the remaining dimensions not such decomposition holds, even under very weak requirements, because there exist a normal k-current supported on 2-purely unrectifiable set (as shown by Andrea Schioppa). In particular the measure μ underlying such a current cannot be decomposed in terms of (measures supported on) rectifiable sets.
However we can show that this measure μ can be decomposed in terms of (measures supported on) graphs of Hölder functions provided there exists a transport operator T in dimension d=n-k which is Hölder regular.
More precisely, T should be a map that to every probability measure μ in the unit cube Q of R^d associate a map T_μ from Q to Q that takes the Lebesgue measure on Q onto μ (in stating the regularity of T, the space of probability measures on Q is endowed with the the 1-Wasserstein distance, while the space of maps from Q to Q is endowed with the L^1 distance).
We notice that Schioppa's example and the previous result imply that there exists no Lipschitz transport in dimension d>1.