Measurement of the relative phase between strong and electromagnetic amplitudes in the charmonia decays at BESIII

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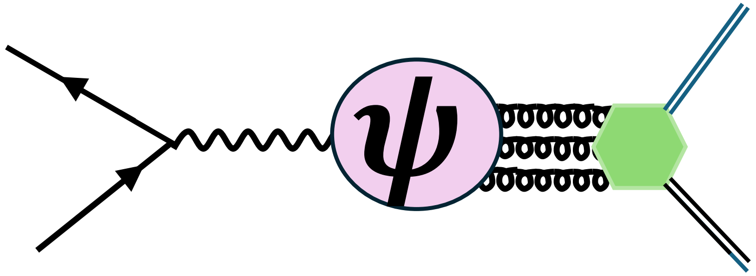
Outline

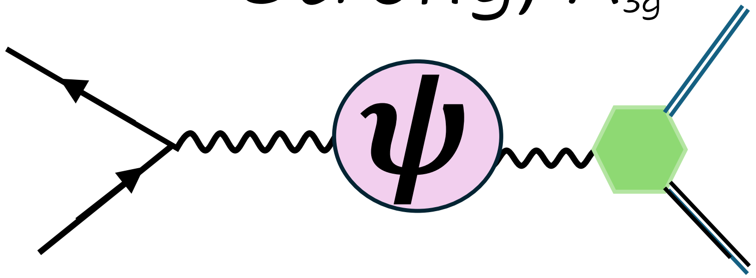
- Introduction
- Model dependent evidences
- BESIII model independent approach
- BESIII measurements
- Summary and prospects

Hadronic production in e^+e^- annihilation charmonia decays

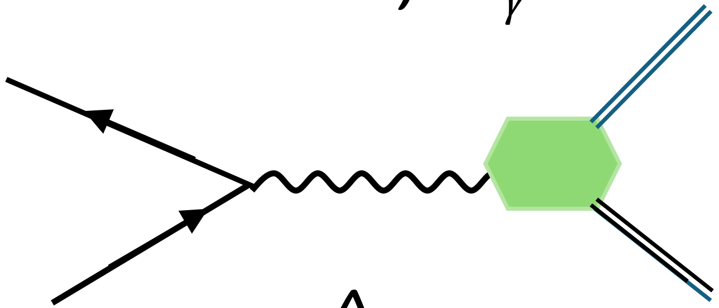
In e^+e^- annihilation in the vicinity of a charmonium resonance



Strong, A_{3g}



EM, A_γ



A_{cont}

Interplay between three possible diagrams (depending on final state) at least,

With both resonant and non-resonant contributions

Continuum process is produced directly in the annihilation.

Relative phases between the amplitudes should be taken into account.

Based on pQCD:

A_{3g} and A_γ amplitudes are real, $\phi_{\gamma,3g}$ between strong and electromagnetic amplitudes must be 0^o or 180^o

[V.L. Chernyak and I.R. Zhinitsky, Nuclear Physics B 246, 52 (1998)]

Model dependent evidences (exclusive J/ψ decays)

Based on SU(3) and SU(3) breaking amplitudes models, for $\phi_{\gamma,3g}$

- ♛ $PP(0^-0^-)(\pi^+\pi^-, K^+K^-, K_SK_L)$ $(89 \pm 10)^\circ$ /$(73\pm5)^\circ$ [1]
- ♛ $VP(1^-0^-)(\rho\pi, \omega\pi^0, \phi\pi^0, \rho\eta, \omega\eta, \phi\eta, \rho\eta', \omega\eta', \phi\eta', \bar{K}^*K) \to (76^{+9}_{-10})^\circ$ [2]
- ♛ $VP(1^+0^-)\left(K_1^\pm(1400)K^\mp, K_1^\pm(1270)K^\mp\right)$~tends to 90° [3]
- ♛ $VV\ (1^-1^-)(\rho^+\rho^-, K^{*+}K^{*-}, K^{*0}\bar{K}^{*0})$ tends to 90° [4]
- ♛ $B\bar{B}(p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}, \Sigma^0\bar{\Sigma}^0, \Sigma^+\Sigma^-, \Xi^0\bar{\Xi}^0, \Xi^+\Xi^-, \Sigma^0\bar{\Lambda} + \bar{\Sigma}^0\Lambda) \to (89^\circ \pm 9)^\circ$[5]$(73 \pm 8)^\circ$ [5*]

$(-89.29\pm0.9)^\circ$, or $(+90.71\pm0.93)^\circ$ [6]

[1] M. Suzuki, Phys. Rev. D **60**, 051501(R) (1999); Metreveli et al.Phys. Rev. D **85**, 092007 (2012),
[2] M.Suzuki, Phys. Rev. D **58**, 111504 (1998). J. L. Rosner, Phys. Rev. D **60**, 074029 (1999).
[3] M. Suzuki, Phys. Rev. D63, 054021 (2001)
[4] L. Kopke and N. Wermes, Phys. Rep.174, 67 (1989).
[5] BESIII, Phys. Rev. D 86, 032014 (2012) [$p\bar{p}, n\bar{n}$ $only$]
[5*] R. Baldini Ferroli et al. Phys. Lett. B 799 135041 (2019)
[6] X.H. Mo, J.Y. Zhang, Physics Letters B, Volume 826, 136927 (2022)

Model dependent evidences (exclusive $\psi(2S)$ decays)

Phenomenological SU(3) models

- $PP(0^-0^-)(\pi^+\pi^-, K^+K^-, K_S K_L)$ $(95 \pm 15)^\circ$, $(110^{+16}_{-9})^\circ$[1]
- $VP(1^-0^-) \sim -90^\circ$[2]
- $B\bar{B}$ $(-98 \pm 25)^\circ$ or $(+134 \pm 25)^\circ$[3][4], (87 ± 15)[4]

More recent phenomenological works seem to rule out big differences in phase with J/ψ, found previously

$\psi(3770)$ Ozi suppressed decays $\sim -90^\circ$[5]

[1] Metreveli et al.,Phys. Rev. D **74**, 011105, Metreveli et al. Phys. Rev. D **85**, 092007(2012

[2] P.Wang et al, Phys. Rev. D 69,057502 (2004).

[3] K. Zhu, X. H. Mo, C. Z. Yuan, Inter. J. Mod. Phys. A, 30, 1550148 (2015)

[4] R. Baldini et al., PhysRevD.103.016005 (2021)

[5] P. Wang hep-ph/0410028 (2004)

The long standing questions are:
is this relative phase universal for all $q\bar{q}$?
or dependent on final states?
Which is its sign?

Experimentally , the interference can affect the cross section and the measured branching fraction of the decays.

$$\sigma_B = |A_{3g}e^{i\phi_{g,\gamma}} + A_\gamma e^{i\phi_{\gamma,cont}} + A_{cont}|^2$$

How much?

An evaluation of the contribution of interference between continuum and resonant amplitudes has been done accounting for a few percent level, depending on the final states for narrow resonances and more for broad ones.[C.Z.Yuan and Y. Guo, Phys. Rev. D 105 (2022) 114001]

$$\delta B = 2\sqrt{\frac{\sigma_0}{\sigma_\psi}} A_{3g} sin\phi_{g,\gamma}$$

Non-resonant cross section (σ_0)

$$\sigma_\psi = (12\pi/m^2)B(\psi \to e^+e^-)$$

BaBar, PhysRevD.92.072008 (2015)

In the high statistics era of BESIII, high precision of branching fraction measurements is reached (few %) in many final states. It’s crucial to know the interference contribution (at the same level).

The approach

THE IDEA: an approach independent on SU(3) amplitudes models, fitting the cross section lineshape around the resonance

$$\sigma_B = |A_{3g}e^{i\phi_{g,\gamma}} + A_\gamma e^{i\phi_{\gamma,cont}} + A_{cont}|^2$$

Needed data samples both on resonance and off resonance to constrain the continuum (and EM) amplitudes

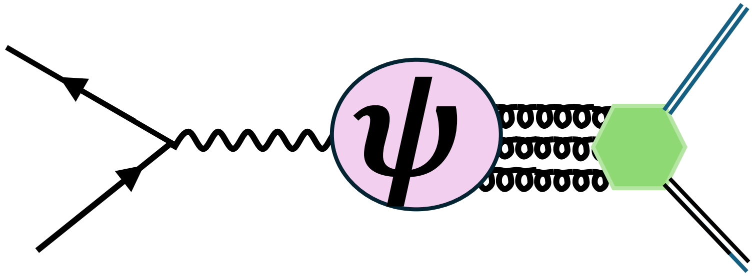
Data samples

Direct scans around the ψ, ψ', ψ'' resonances were collected (dedicated data takings +others). This allows the fit of the lineshape for various exclusive channels.

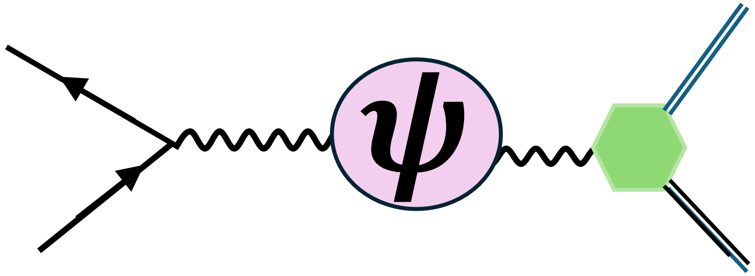
- ★ J/ψ → 2012 scan around 100 pb^{-1} (16 cme) +additional CME points (2015-2018-2019)
- ★ $\psi(2S)$ →2018 scan around 499 pb^{-1} (9 cme)+2018 τ mass sample (6 cme) 138 pb^{-1} +fast scan on peak 75 pb^{-1}
- ★ $\psi(3770)$ →26.5 fb^{-1} from 3.51 GeV to 4.95 GeV

Cross-section lineshape fit-method

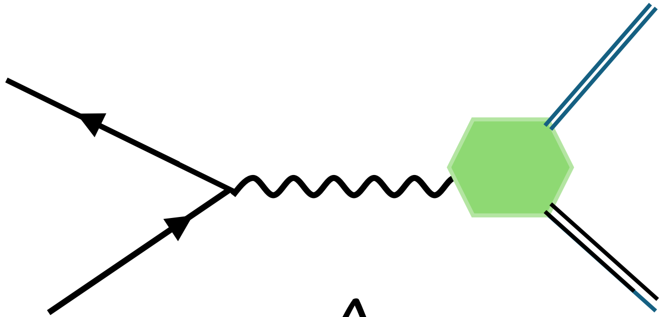
In e^+e^- annihilation:



A_{3g}



A_γ



A_{cont}

$$\sigma_B = |A_{3g}e^{i\phi_{g,\gamma}} + A_\gamma e^{i\phi_{\gamma,cont}} + A_{cont}|^2$$

If we assume $\phi_{\gamma,cont} = 0$ ⟶ $\sigma_B = |A_{3g}e^{i\phi_{g,\gamma}} + A_\gamma + A_{cont}|^2$

FF

$\frac{|A_{3g}|}{|A_\gamma|}$

e.g. $$\sigma_B(W) = (\frac{A}{W^n})^2 \frac{4\pi\alpha^2}{W^2} \left| 1 + \frac{3W^2\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}(1 + Ce^{i\phi_{g,\gamma}})}{\alpha M(W^2 - M^2 + iM\Gamma)} \right|$$

W dependence depending on final state

- To take into account the effects of energy spread and initial state radiation, a two-fold numerical integration of the Born cross section σ_B is done:

$$\sigma'(W) = \int_0^{1-\left(\frac{W_{\min}}{W}\right)^2} dx F(x,W)\sigma_B(W\sqrt{1-x}),$$

$F(x,W)$ -radiator function By Kuraev and Fadin

Yad. Fiz. 41, 733 (1985)
[Sov. J. Nucl. Phys. 41, 466(1985)];

$$\sigma''(W) = \int_{W-nS_E}^{W+nS_E} \frac{1}{\sqrt{2\pi}S_E}\exp\left(\frac{-(W-W')^2}{2S_E^2}\right)\sigma'(W')dW'.$$

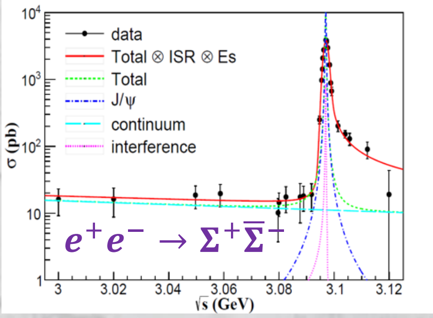
Energy spread of the colliding beams included by convolution with a Gaussian function with width S_E assumed to be constant in limitated energy range

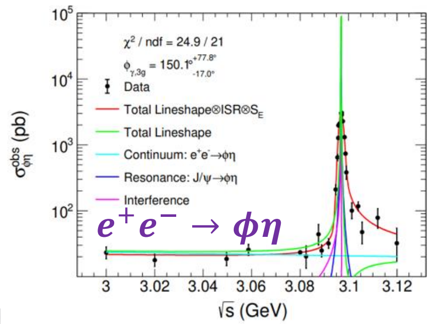
Used to fit the experimental observed cross section (in most of the cases)

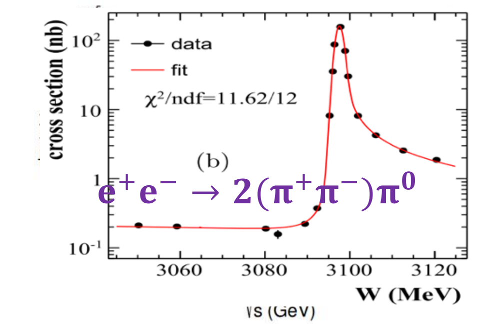
$$\sigma_{obs} = \frac{N_{sig}}{\mathcal{L}\epsilon(\mathcal{B})}$$

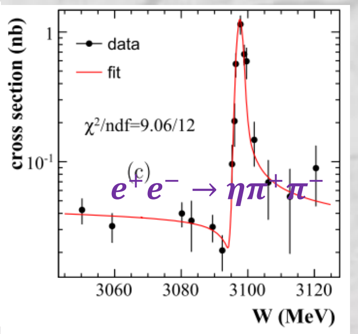
Typical fitted parameters are relative phase and ratio between the strong and EM amplitudes, continuum parameters and energy spread.
Multiple solutions are mathematically expected.

scan measurements menu



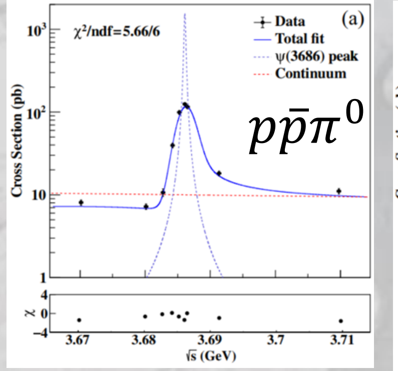


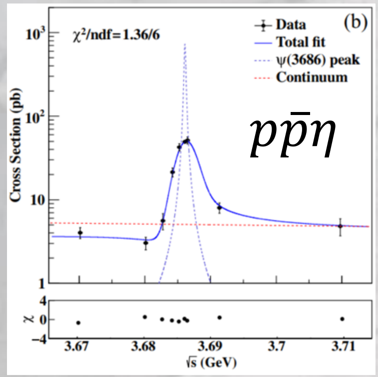


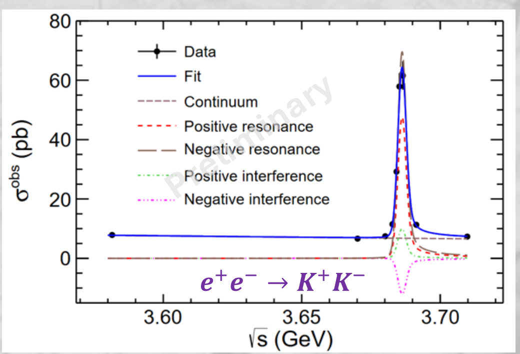


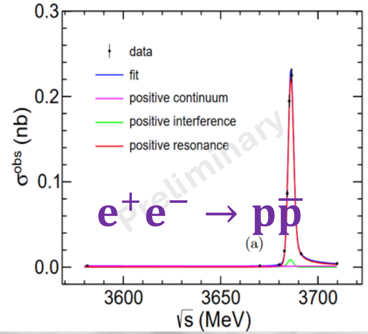
Around J/ψ

Many exclusive channels (2 and 3 body) on-going



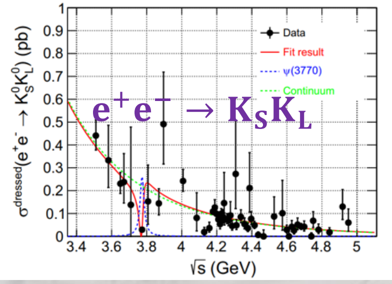


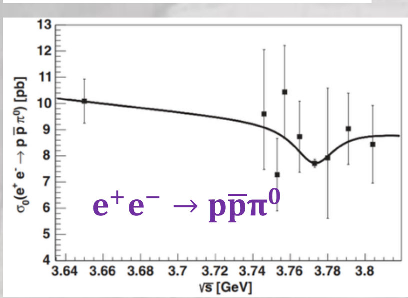


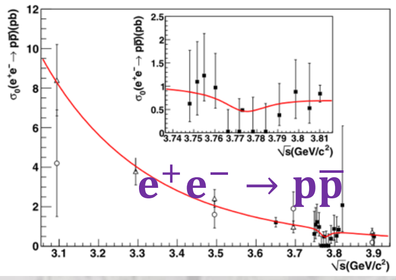


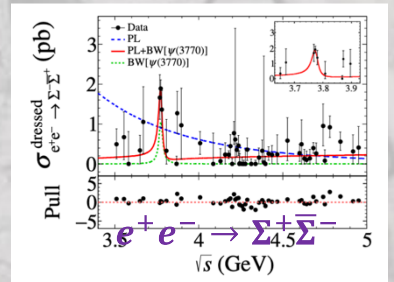
Around $\psi(3686)$

Many exclusive channels (2 and 3 body) on-going





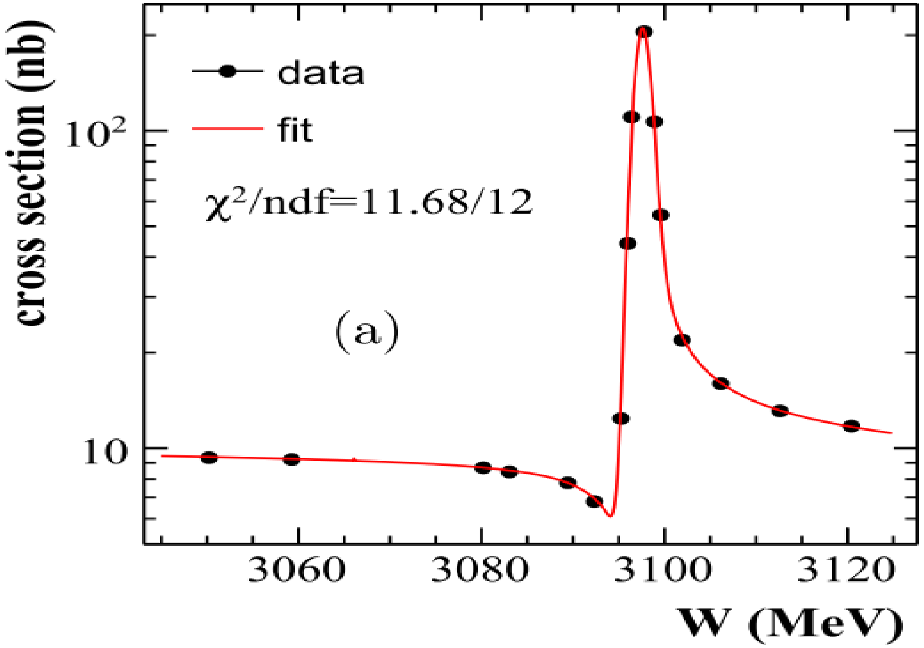


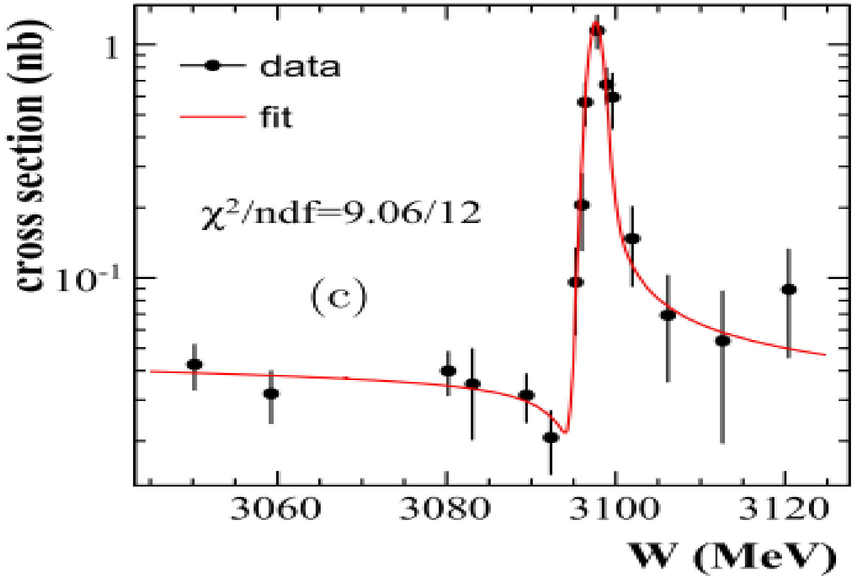


Around $\psi(3770)$

BESIII Collaboration, Phys. Lett. B 791, 375 (2019)

$e^+e^- \to \eta\pi^+\pi^-$





Pure electromagnetic process.
The relative phase btw A_γ and A_{cont} can be measured

$$\sigma^0(W) = \frac{4\pi\alpha^2}{W^2}\left|1 + \frac{3W^2\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}e^{i\Phi_{\gamma,\mathrm{cont}}}}{\alpha M(W^2 - M^2 + iM\Gamma)}\right|^2$$

- $\Phi_{\gamma,cont.} = (3.0 \pm 10.0)^\circ$
- $S_E = (0.90 \pm 0.03)$ MeV

$$\sigma^0(W) = \left(\frac{\mathcal{A}}{W^2}\right)^2 \frac{4\pi\alpha^2}{W^2}\left|1 + \frac{3W^2\sqrt{\Gamma_{ee}\Gamma_{\mu\mu}}C_1 e^{i\Phi_{\gamma,\mathrm{cont}}}(1 + C_2 e^{i\Phi})}{\alpha M(W^2 - M^2 + iM\Gamma)}\right|^2$$

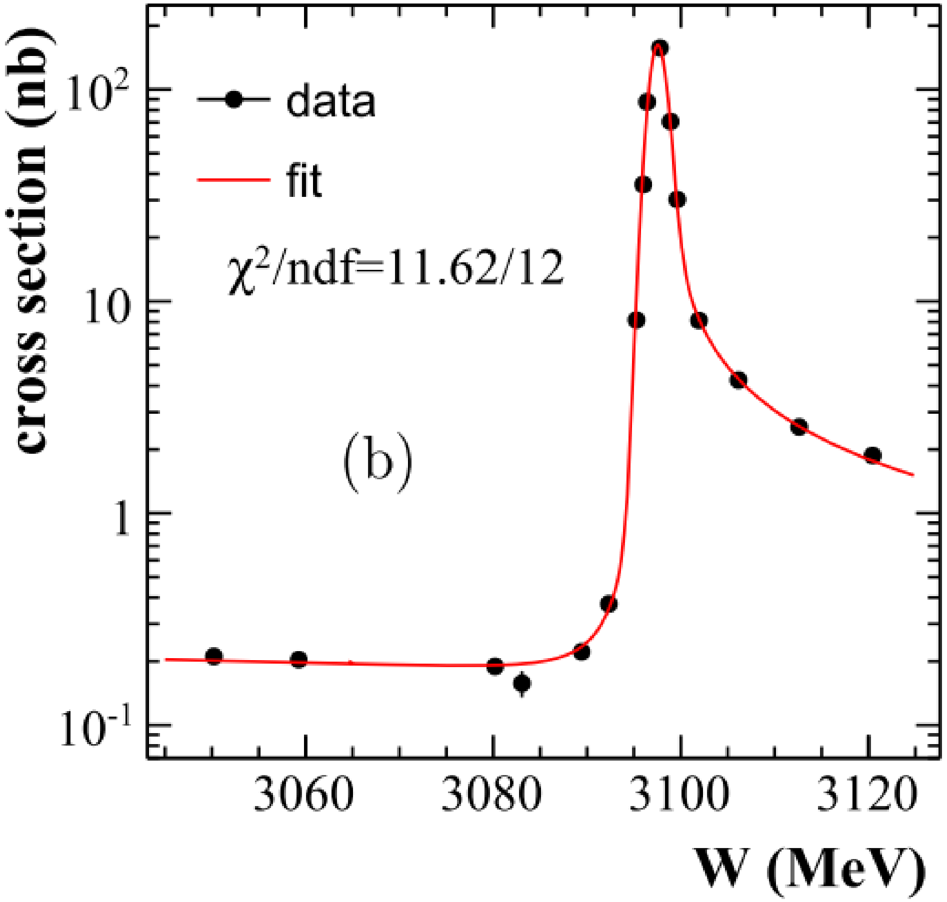
Φ represents the interference between $J/\psi \to \eta\rho^0$ and $J/\psi \to \eta\omega$

- $\Phi_{\gamma,cont.} = (-2 \pm 36)^\circ$ or $(-22 \pm 36)^\circ$
- $Br(J/\psi \to \eta\pi^+\pi^-) = (3.78 \pm 0.66) \times 10^{-4}$
- $Br_{PDG}(J/\psi \to \eta\pi^+\pi^-) = (4.0 \pm 1.7) \times 10^{-4}$

the phase between A_γ and $A_{cont.}$ is compatible with zero, as expected.

$\phi_{\gamma,cont} = 0$ Assumption is confirmed!

$e^+e^- \to 2(\pi^+\pi^-)\pi^0$



- Detection efficiency is simulated with MCGPJ generator for the ISR effect around J/ψ narrow peak
- Intermediate resonances are considered in simulation without interference

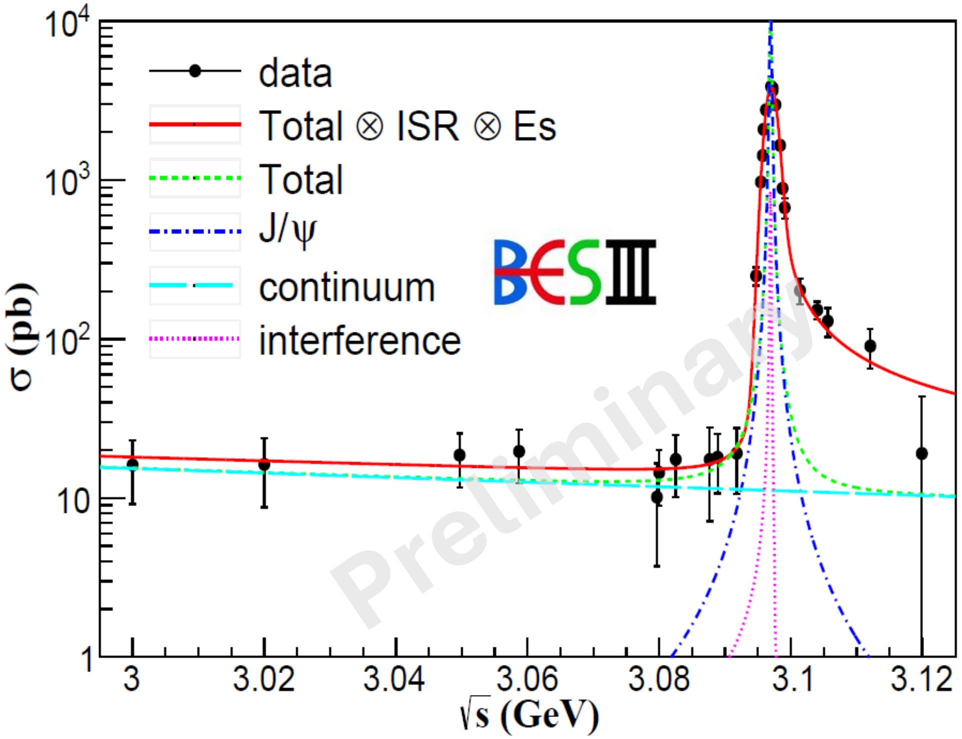
	$\Phi_{g,EM}$	$\mathcal{B}_{5\pi}$ (%)
Solution I	$(84.9 \pm 3.6)^\circ$	4.73 ± 0.44
Solution II	$(-84.7 \pm 3.1)^\circ$	4.85 ± 0.45

$$\mathcal{B}_{PDG} = (4.1 \pm 0.5)\%$$

The phase between A_γ and A_{3g} is found being consistent with 90°.

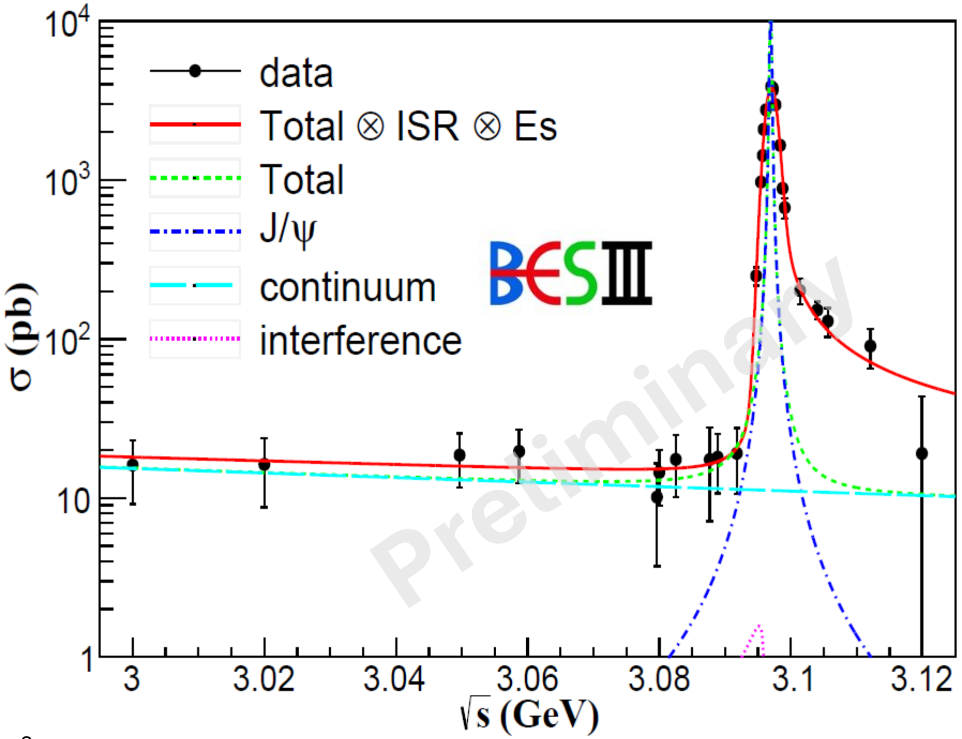
BESIII Collaboration, Phys. Lett. B 791, 375 (2019)

$e^+e^- \to \Sigma^+\bar{\Sigma}^-$ Around J/ψ



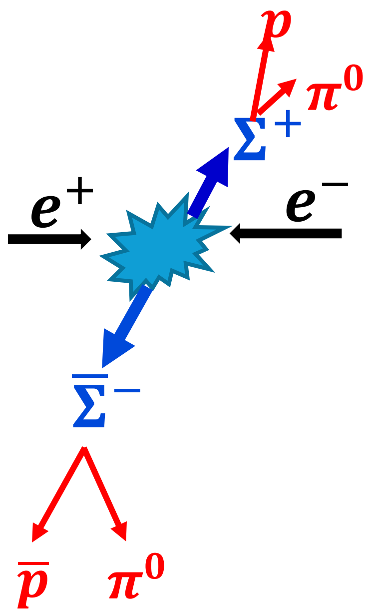
Positive phase $\Phi_{3g,\gamma} = (107.9 \pm 24.9)^\circ$

$$BF = (1.14 \pm 0.04) \times 10^{-3}$$



Negative phase : $\Phi_{3g,\gamma} = (-107.6 \pm 24.3\,)^\circ$

$$BF = (1.19 \pm 0.04) \times 10^{-3}$$



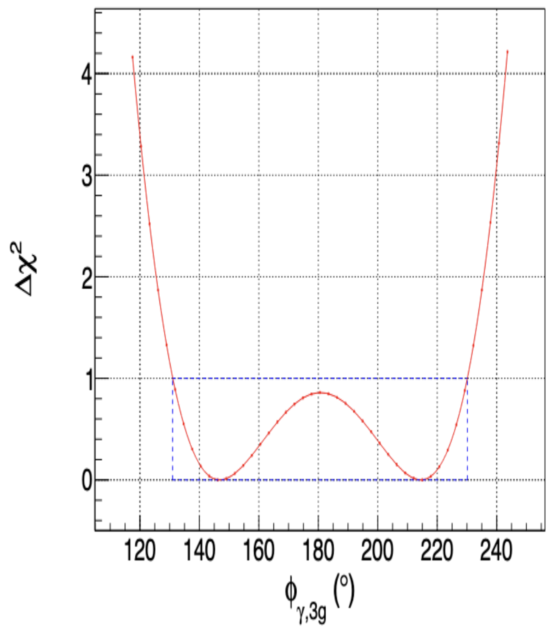
$\mathbf{1.07 \pm 0.04}$	OUR AVERAGE	$(\times 10^{-3})$				
$1.061 \pm 0.004 \pm 0.036$		87k	ABLIKIM	2021AT	BES3	$J/\psi \to p\pi^0\bar{p}\pi^0$
$1.50 \pm 0.10 \pm 0.22$		399	ABLIKIM	2008O	BES2	$e^+e^- \to J/\psi$

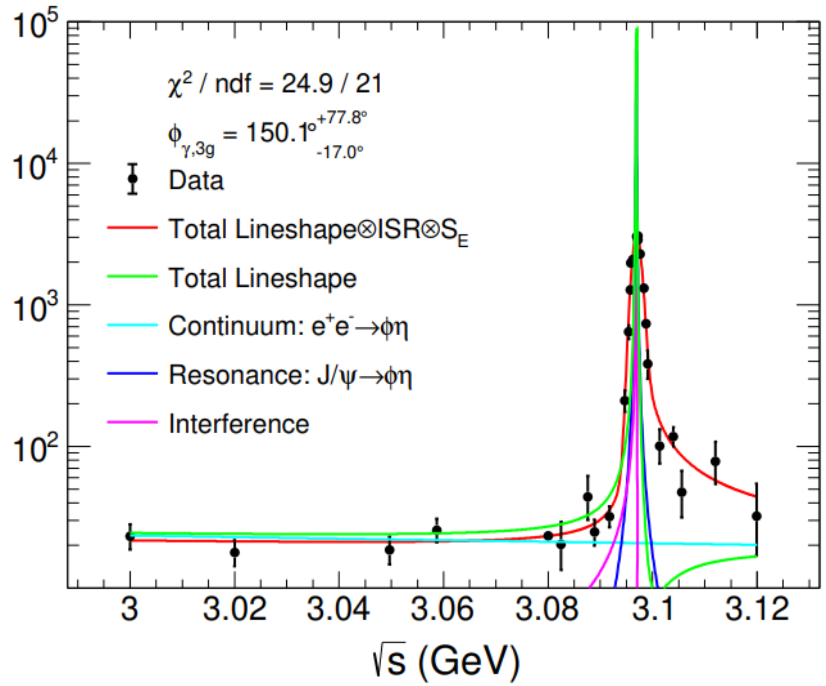
$e^+e^- \to \phi\eta$

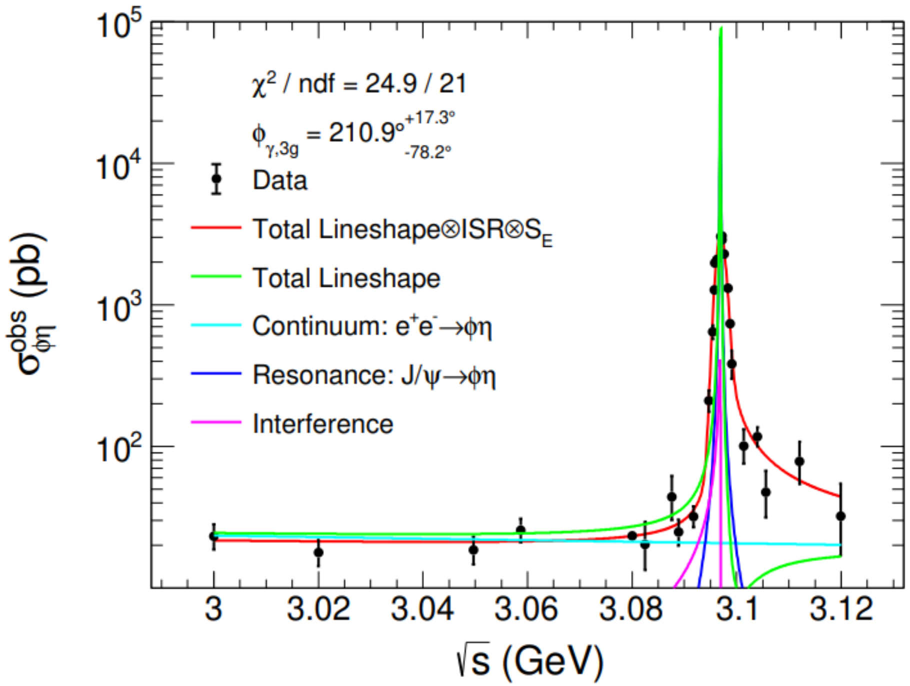
2012 scan+2015/2018/2019 additional data points at 26 CM Energies, total 452 pb^{-1}

JHEP11(2025)077

first direct measurement in VP decay channel







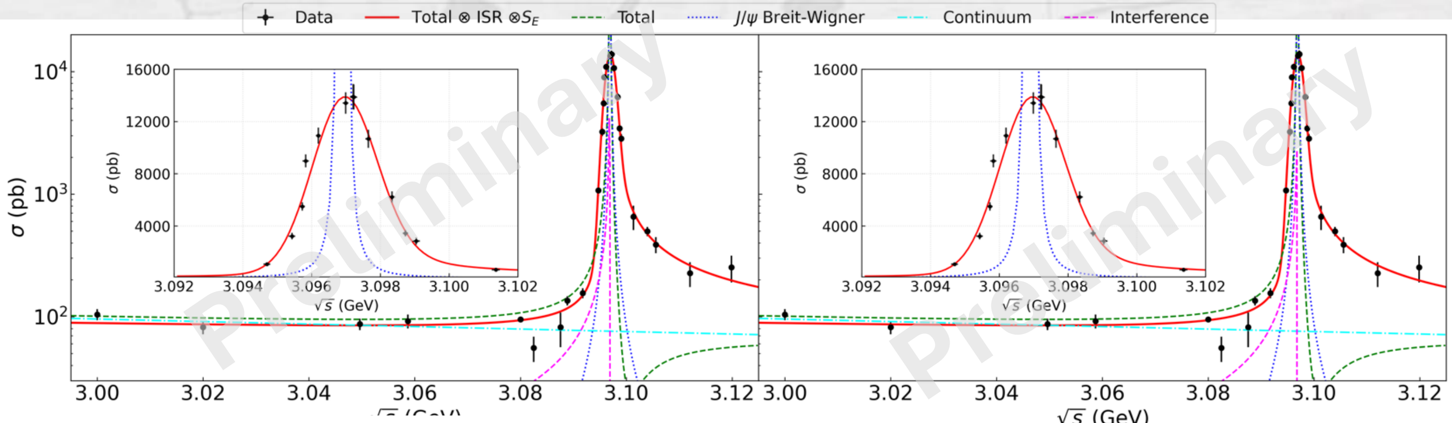
	Positive phase	Negative phase
χ^2/ndf	24.9/21	24.9/21
$\phi_{\gamma,3g}$ (°)	150^{+78}_{-17}	211^{+17}_{-78}
$\mathcal{F}$	0.11±0.01	
C	3.3±0.4	
S_E (MeV)	0.88±0.03	
f	0.99±0.04	

$a_0 = 1.5$

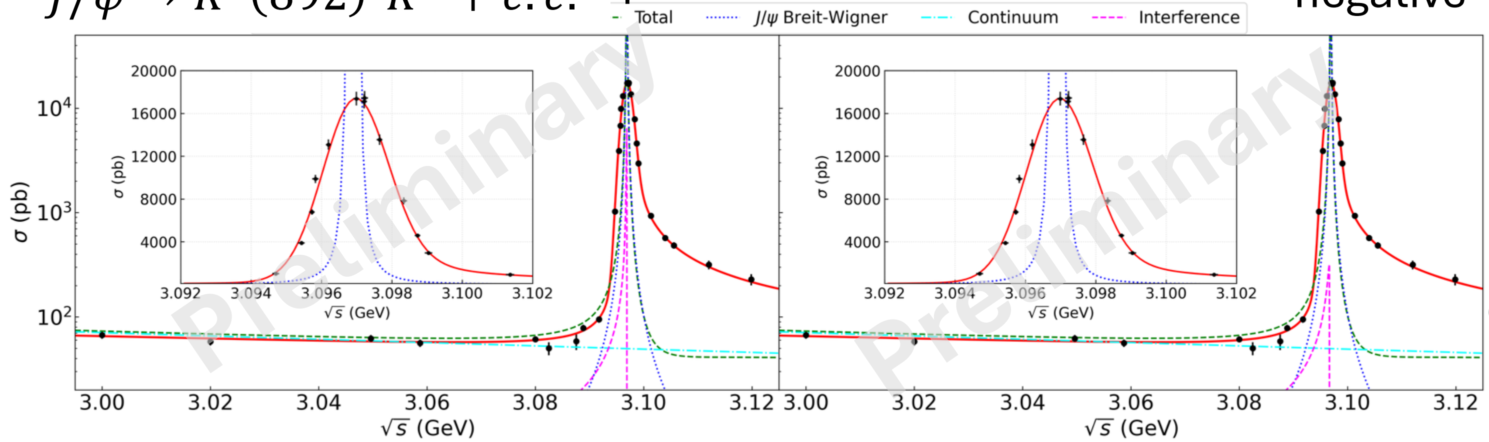
$$\sigma(s) = \mathcal{P}_{\phi\eta}(s)\cdot\left(\frac{\mathcal{F}}{s^{a_0}}\right)^2\cdot\frac{4\pi\alpha^2}{3s}\cdot\left|1+\frac{3}{\alpha}\frac{s}{M}\frac{\Gamma_{ee}\cdot(1+C\cdot e^{i\phi_{\gamma,3g}})}{(s-M^2)+iM\Gamma}\right|^2.$$

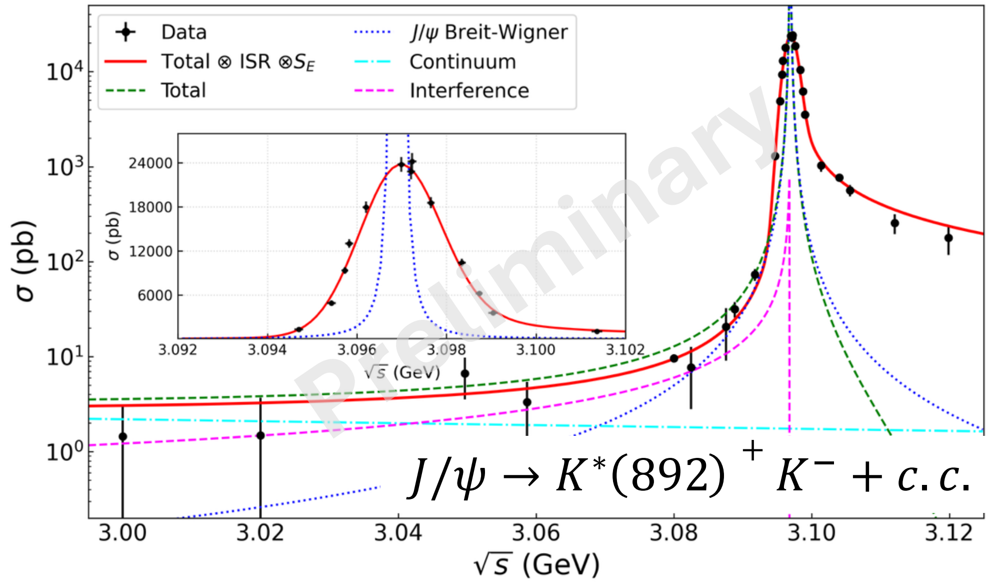
Two solutions indistinguishable within 1σ confidence, $\Phi_{3g,\gamma}\in[\mathbf{133.1°},\mathbf{229.2°}]$

$e^+e^- \to K_S^0 K^+ \pi^-$



$J/\psi \to K^*(892)^0\overline{K}^0 + c.c.$ positive negative





$J/\psi \to K^*(892)^+ K^- + c.c.$

$$\sigma^f_{\rm Born} = \frac{4\pi\alpha^2}{3s}\left|1 + (1 + \mathcal{C}e^{i\phi_{\gamma,3g}})\frac{s}{M}\frac{3\sqrt{\Gamma^0_{ee}\Gamma^0_{\mu\mu}}/\alpha}{s - M^2 + iM\Gamma}e^{i\phi_{\gamma,\rm cont}}\right|^2 \cdot \mathcal{P}(s)\cdot\frac{\mathcal{F}^2}{s^n},$$

$J/\psi \to K_S^0 K^+\pi^- + c.c.$

PWA analysis
Isobar model

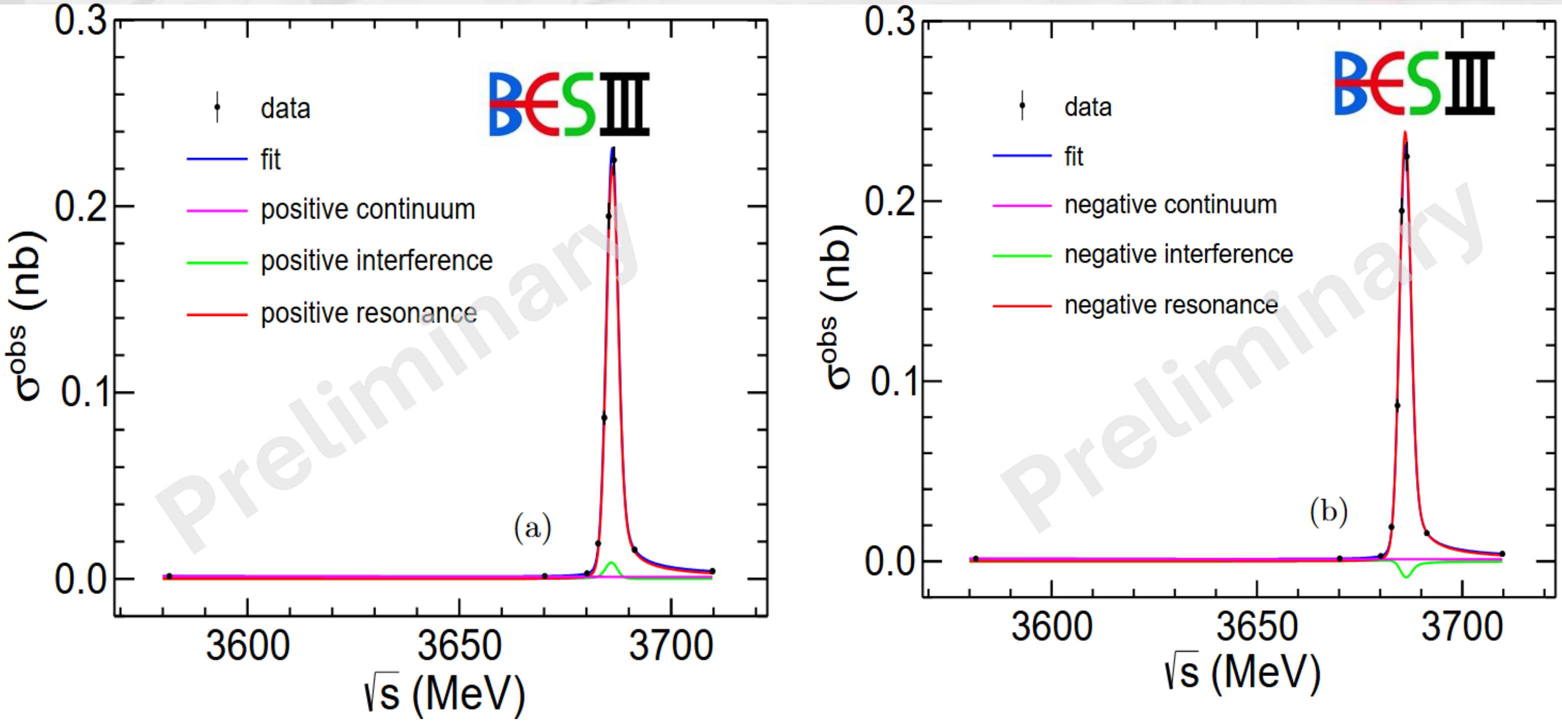
Channels	$\phi_{\gamma,3g}$ sign	$\mathcal{C}$	$\mathcal{B}$ $(\times 10^{-3})$	$\phi_{\gamma,3g}$ (°)	S_E (MeV)	χ^2/ndf
$J/\psi \to K_S^0 K^+\pi^-$	Positive	4.31 ± 0.22	5.17 ± 0.20	123.7 ± 5.3	0.90 ± 0.02	16.0/22
	Negative	4.38 ± 0.22	5.36 ± 0.20	-123.1 ± 5.2	0.90 ± 0.02	16.0/22
$J/\psi \to \bar{K}^0 K^*(892)^0$	Positive	3.67 ± 0.27	4.18 ± 0.18	155.2 ± 15.5	0.92 ± 0.03	30.1/22
	Negative	3.71 ± 0.25	4.31 ± 0.19	-154.1 ± 15.5	0.92 ± 0.03	30.1/22
$J/\psi \to K^+ K^*(892)^-$	...	25.06 ± 2.51	7.09 ± 0.28	180.1 ± 31.8	0.88 ± 0.02	21.2/22

Preliminary

BESIII

$e^+e^- \to p\bar{p}$

Around $\psi(3686)$



$$\sigma(s) = \frac{4\pi\alpha^2\beta C}{3s}\left(\frac{G_0}{s^2}\right)^2\left(1+\frac{1}{2\tau}\right)\left|1+\left(1+\mathcal{C}e^{i\Phi_{g,EM}}\right)\frac{s}{M}\frac{3\Gamma_{ee}/\alpha}{s-M_\psi^2+i\Gamma_\psi M_\psi}\right|^2$$

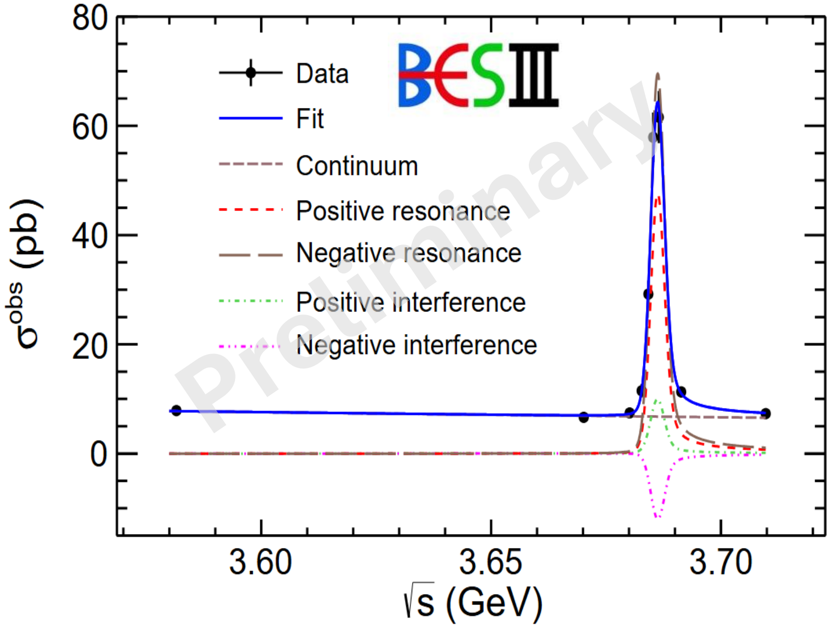
Parameter	Positive	Negative
$\Phi_{g,EM}(^\circ)$	105.9 ± 9.4	-105.3 ± 9.1
M_ψ (MeV/c^2)	3685.99 ± 0.08	3685.99 ± 0.08
$\mathcal{C}$	16.02 ± 1.25	16.63 ± 1.23
G_0 (GeV^4)	2.38 ± 0.15	2.38 ± 0.15
$\mathcal{B}$ (10^{-4})	3.21 ± 0.13	3.47 ± 0.19

Preliminary

2.94 ± 0.09	OUR FIT Error includes scale factor of 1.3.				
3.02 ± 0.08	OUR AVERAGE				
$3.05 \pm 0.02 \pm 0.12$	19k	ABLIKIM	2018T	BES3	$e^+e^- \to \psi(2S) \to p\bar{p}$
$3.08 \pm 0.05 \pm 0.18$	4.5k	[1] DOBBS	2014		$e^+e^- \to \psi(2S) \to p\bar{p}$
$3.36 \pm 0.09 \pm 0.25$	1.6k	ABLIKIM	2007C	BES	$e^+e^- \to \psi(2S) \to p\bar{p}$
$2.87 \pm 0.12 \pm 0.15$	557	PEDLAR	2005	CLEO	$e^+e^- \to \psi(2S) \to p\bar{p}$
1.4 ± 0.8	4	BRANDELIK	1979C	DASP	$e^+e^- \to \psi(2S) \to p\bar{p}$
2.3 ± 0.7		FELDMAN	1977	MRK1	$e^+e^- \to \psi(2S) \to p\bar{p}$

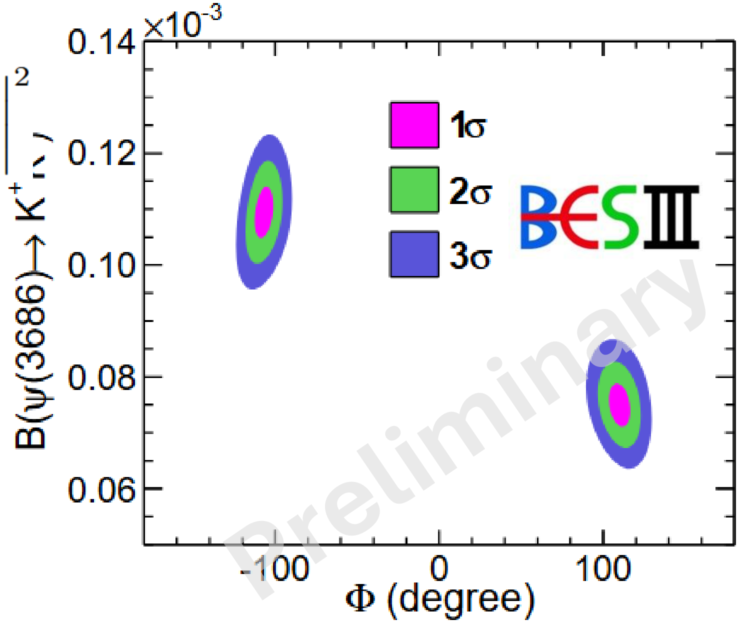
Around $\psi(3686)$

$e^+e^- \rightarrow K^+K^-$



$$\sigma^0(\sqrt{s}) = \beta_K^3 \left(\frac{\mathcal{F}}{s}\right)^2 \frac{4\pi\alpha^2}{3s} \frac{1}{|1-\Pi_0(s)|^2} \cdot \left|1 + (1+\mathcal{C}e^{i\Phi}) \frac{s}{M} \frac{3\Gamma_{ee}^0/\alpha}{s-M^2+iM\Gamma}\right|^2$$

$$\mathcal{B} = \beta_K^3 \left(\mathcal{F}/M^2\right)^2 \left|1+\mathcal{C}e^{i\Phi}\right|^2 \Gamma_{ee}/\Gamma$$



Parameter	This work Positive	This work Negative	BES	CLEO	Seth *et al.*
Φ (°)	110.1 ± 6.7	-106.8 ± 5.7	$(89 \pm 35)^\circ$	$(93 \pm 20)^\circ$	$(66.6 \pm 11.5)^\circ$
$\mathcal{C}$	3.18 ± 0.16	3.77 ± 0.15	$2.6^{+0.9}_{-1.4}$	$2.8^{+1.2}_{-2.8}$	–
$\mathcal{F}$ (GeV^2)	0.467 ± 0.009	0.467 ± 0.009	–	–	
$\mathcal{B}$ (10^{-5})	7.49 ± 0.41	10.94 ± 0.49	6.1 ± 2.1	6.3 ± 0.7	7.48 ± 0.45

w/o interference correction

https://doi.org/10.1103/PhysRevD.85.092007

Interference effect accounts for ~30% difference in the branching fraction, negative solution is in agreement with PDG value

... analyses with lineshape fitting w/o disantengling EM and strong contributions both for $\psi(3686)$ and $\psi(3770)$ can give other pieces of information..

$$\sigma_{\rm Born} = |A_{\rm con} + A_{\rm res} \times e^{i\phi}|^2$$

$$A_{\rm con}(s) = a/s^n$$

$$A_{\rm res}(s) = \frac{\sqrt{12\pi\Gamma_{ee}\Gamma_{\rm tot}\mathcal{B}_f}}{s - M^2 + iM\Gamma_{\rm tot}}$$

Experimentally measured parameters are σ_{cont}, Γ_f and ϕ

They can be rewritten in terms of

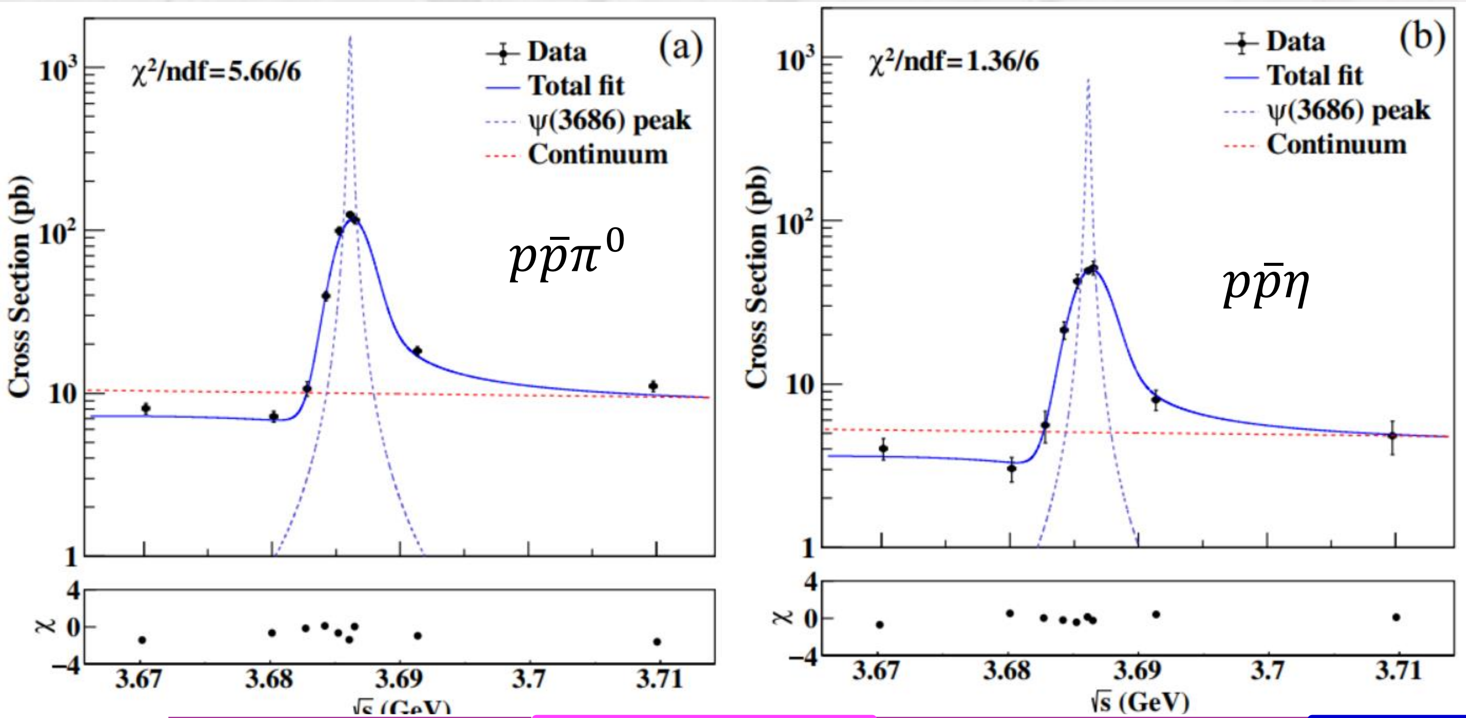
$$\Gamma_f = \beta^{2l+1}\left(\frac{\mathcal{F}}{s^n}\right)^2 \Gamma_{ee}|1 + Ce^{i\phi_{\gamma,g}}|^2 \qquad \mathcal{B}_f = \Gamma_f/\Gamma_{tot}$$

$$\phi = arg(1 + Ce^{i\phi_{\gamma,3g}}) \qquad \sigma_{cont} = \beta^{2l+1}\left(\frac{\mathcal{F}}{s^n}\right)^2 \frac{4\pi\alpha^2}{3s}$$

$e^+e^- \to p\bar{p}\,\eta/\pi^0$

Around $\psi(3686)$

BESIII Collaboration, Phys. Rev. D 111, 032011 (2025)



PWA performed

$$A_{\rm con}(s) = a/s^n$$

$$A_{\rm res}(s) = \frac{\sqrt{12\pi\Gamma_{ee}\Gamma_{\rm tot}\mathcal{B}_f}}{s - M^2 + iM\Gamma_{\rm tot}}$$

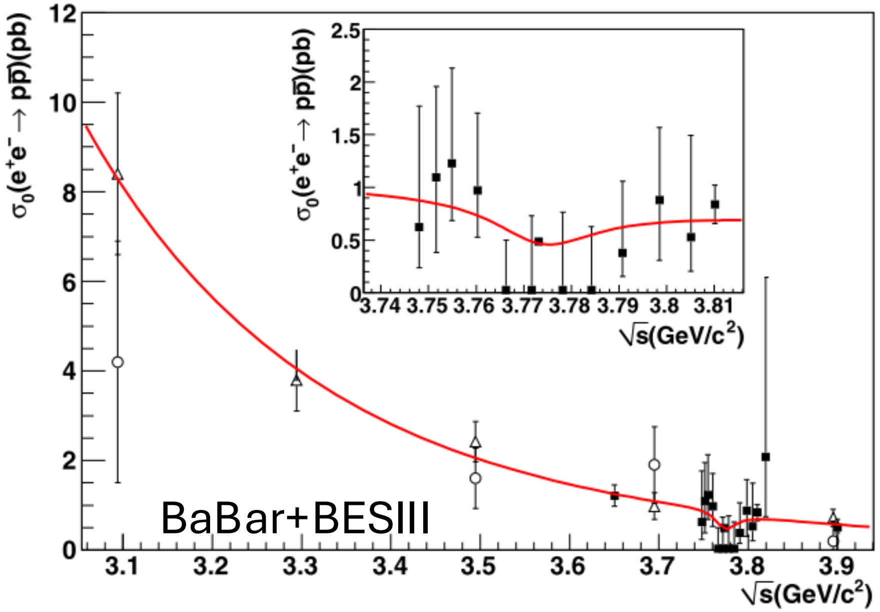
channel	ϕ (°)	Br ($\times 10^{-6}$)	Φ (°)	Br_{PDG}($\times 10^{-6}$)
$\psi(2S) \to p\bar{p}\pi^0$	65.0 ± 6.7	$133.9 \pm 11.2 \pm 2.3$	82.5 ± 8.9	153 ± 7
	-68.9 ± 5.7	$183.7 \pm 13.7 \pm 3.2$	-83.8 ± 7.1	
$\psi(2S) \to p\bar{p}\eta$	58.9 ± 14.1	$61.5 \pm 6.5 \pm 1.1$	77.3 ± 22.1	60 ± 4
	-63.8 ± 12.1	$84.4 \pm 6.9 \pm 1.4$	-79.5 ± 17.5	

Courtesy dott. F. Rosini

- The interference changes branching fraction results.
- Relative phase between strong and EM amplitudes is consistent with $\pm 90°$ after conversion

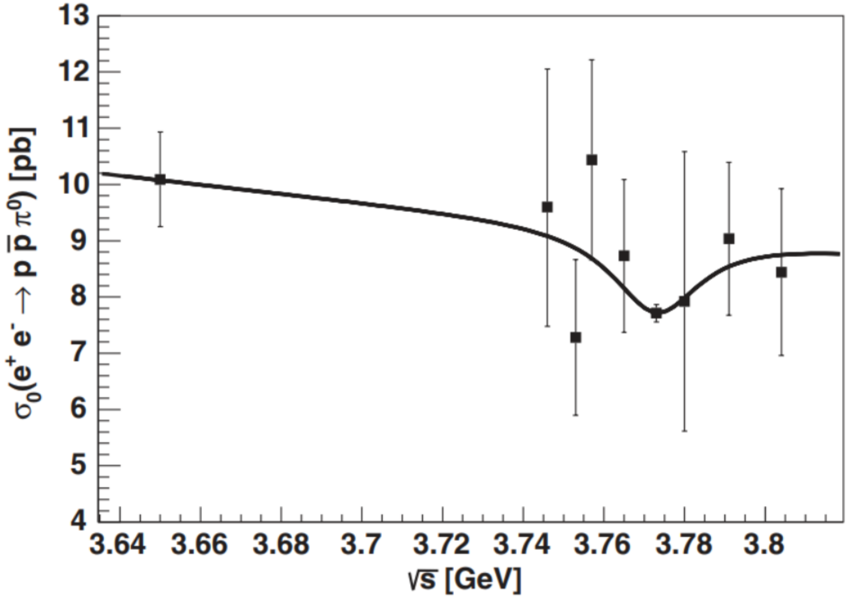
$e^+e^- \to p\bar{p}$ | $e^+e^- \to p\bar{p}\pi^0$

BESIII Collaboration, Phys. Lett. B 735, 101 (2014)



Around $\psi(3770)$

BESIII Collaboration, Phys. Rev. D 90, 032007 (2014)



Resonant cross section and continuum cross section are floating parameters as well	$Br(\times 10^{-4})$	$\phi(^\circ)$
	$7.1^{+8.6}_{-2.9}$	$255.8^{+39.0}_{-26.6} \pm 4.8$
	3.1 ± 0.3	$266.9^{+6.1}_{-6.3} \pm 0.9$

- The phase $\Phi_{3g,\gamma}$ is still close to -90° assuming A_{3g} is much larger than A_γ

Solution	ϕ_{Fit} [$^\circ$]
1	$269.8^{+52.4}_{-48.0} \pm 11.0$
2	$269.7 \pm 2.3 \pm 0.3$

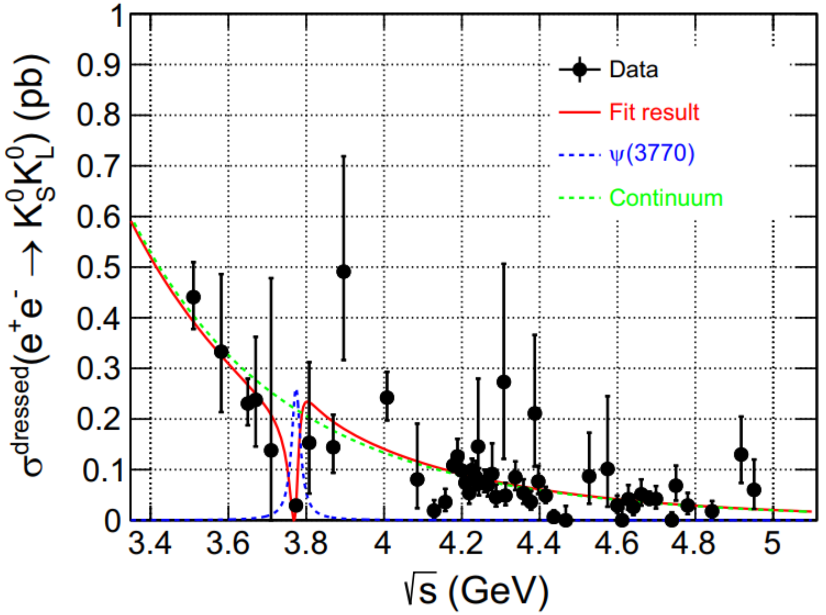
The phase $\Phi_{3g,\gamma}$ is still close to -90° with large uncertainties assuming A_{3g} is much larger than A_γ

- Significance for $\psi(3770)$ resonance ~ $\mathbf{1.5\,\sigma}$

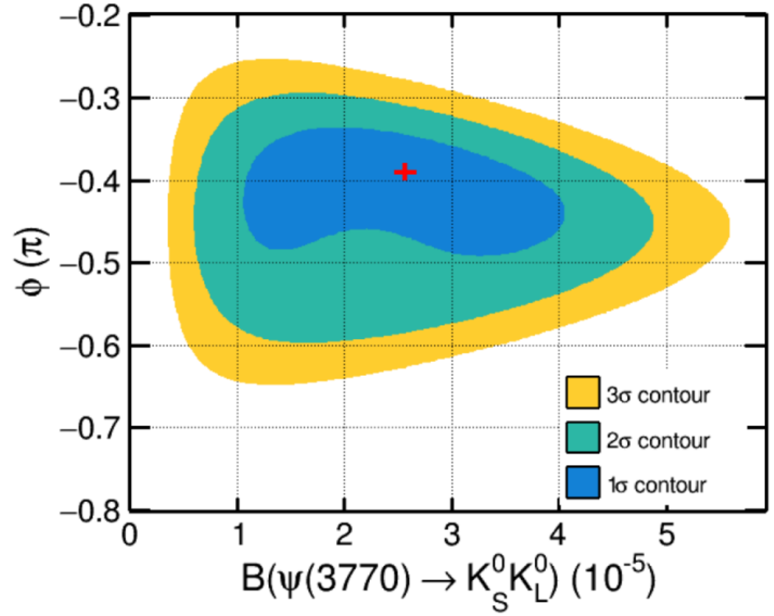
$e^+e^- \to K_SK_L$

Around $\psi(3770)$

BESIII Collaboration, Phys. Rev. Lett. 132, 131901 (2024)



51 energy points; total integrated luminosity of 26.5 fb^{-1}



$$\sigma^{\mathrm{dressed}} = \left| BW \cdot e^{i\phi} + \frac{a}{(\sqrt{s})^n} \cdot \sqrt{\Phi(\sqrt{s})} \right|^2$$

$$BW = \frac{\sqrt{12\pi\Gamma_{ee}\Gamma B}}{s - M^2 + iM\Gamma}\sqrt{\frac{\Phi(s)}{\Phi(M)}},\ \Phi(s) = \frac{q^3}{s}$$

Here, the relative phase is between resonant and not resonant amplitudes

- $\boldsymbol{\mathcal{B}} = (\mathbf{2.63}^{+1.40}_{-1.59}) \times \mathbf{10^{-5}}$ and $\phi = (-0.39^{+0.05}_{-0.10})\pi$ within 1σ likelihood contour.
- Significance of $\psi(3770)$ resonance contribution determined to be **10σ**.
- First observation the charmless decay $\psi(3770) \to K_SK_L$.

Assuming negligible contribution of EM mechanism in resonant amplitude, phase between STRONG and EM amplitudes is found consistent with $-90°$

Summary and prospects

- The relative phase between strong and EM amplitudes $\phi_{\gamma,3g}$ can be measured with scan method in BESIII This allows a model independent approach and relies on all data collected by the BESIII experiment. From the fit procedure two solutions are mathematically found.

- Experimental results of direct scan of J/ψ, $\psi(3686)$ and $\psi(3770)$ decays were shown.

- More analyses are on-going on 2 and 3-body exclusive channels and the results are close to come!!!

- Most of our experimental results point to an orthogonal relative phase $\phi_{\gamma,3g}$, except the two VP decays of J/ψ .
- In some cases the issue of the sign of the phase can be solved by precise BF measurements with channels in which the interference with the continuum is negligible...e.g. using the J/ψ produced in the $\psi(2S)$ decay to $\pi\pi J/\psi$

- Large statistical samples of charmonia in BESIII require the precise knowledge of the interference to get accurate results!

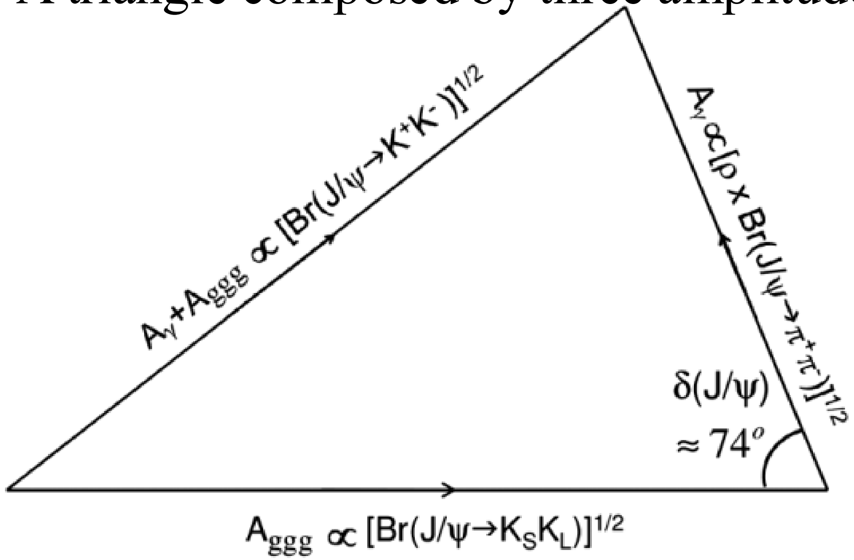
In memory of Rinaldo Baldini
who proposed the idea, the dedicated data takings
and strongly supported and inspired
these analyses

Thank you for your attention!!!

spares

Phase in $J/\psi \to P\bar{P}$

A triangle composed by three amplitudes



$$\delta(\psi)_{\rm PP} = \cos^{-1}\left(\frac{\mathcal{B}(K_SK_L) + \rho\mathcal{B}(\pi^+\pi^-) - \mathcal{B}(K^+K^-)}{|2\sqrt{\mathcal{B}(K_SK_L) \times \rho \times \mathcal{B}(\pi^+\pi^-)}|}\right)$$

	Mark III [10]	BES [16]	CLEO 2012
$\mathcal{B}(J/\psi \to \pi^+\pi^-) \times 10^4$	1.58 ± 0.25	...	**1.47 ± 0.18**
$\mathcal{B}(J/\psi \to K^+K^-) \times 10^4$	2.39 ± 0.33	...	**2.86 ± 0.21**
$\mathcal{B}(J/\psi \to K_SK_L) \times 10^4$	1.01 ± 0.18	1.82 ± 0.13	**2.62 ± 0.21**
$\delta(J/\psi)$	$(88 \pm 11)^\circ$	...	**$(73.6 \pm 5.6)^\circ$**

Z. Metreveli, PRD 85, 092007 (2012)

With only two $K\bar{K}$ amplitudes

BABAR, PRD 92, 072008 (2015)

$$\mathcal{B}(\psi \to K^+K^-) = |A_\gamma^{K^+K^-} + A_s e^{i\varphi}|^2,$$
$$\mathcal{B}(\psi \to K_SK_L) = |\kappa A_\gamma^{K^+K^-} + A_s e^{i\varphi}|^2,$$

$J/\psi \to K_SK_L$	φ	$\varphi(\kappa = 0)$
BES [35]	$(97 \pm 5)^\circ$ $-(97 \pm 5)^\circ$	$(98 \pm 4)^\circ$ $-(96 \pm 4)^\circ$
Seth *et al.* [18]	$(111 \pm 5)^\circ$ $-(109 \pm 5)^\circ$	$(108 \pm 4)^\circ$ $-(107 \pm 4)^\circ$

$\mathcal{B}(\psi \to K^+K^-) \times 10^4$		J/ψ
Measured value		$3.36 \pm 0.20 \pm 0.12$
Corrected with $\sin\varphi > 0$		$3.22 \pm 0.20 \pm 0.12$
Corrected with $\sin\varphi < 0$		$3.50 \pm 0.20 \pm 0.12$
$e^+e^- \to K^+K^-$ average		2.43 ± 0.26 [3,31,32]
$\psi(2S) \to J/\psi\pi^+\pi^-$, $J/\psi \to K^+K^-$	CLEO 2012	2.86 ± 0.21
$\boldsymbol{\psi(2S) \to J/\psi\pi^+\pi^-, J/\psi \to K^+K^-}$		$(3.072 \pm 0.023 \pm 0.050)$

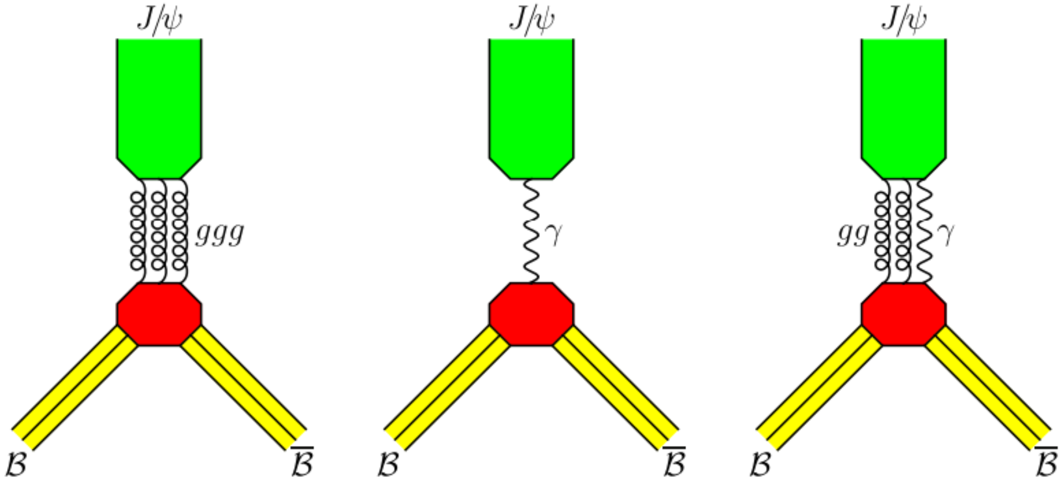
BESIII recent result

PRD 110, 032006 (2024)

Model dependent experimental evidences

from J/ψ decays

R. Baldini, A. Mangoni, S. Pacetti, K. Zhu;
Phy. Lett. B 799, 135041 (2019)



$\mathcal{B}\overline{\mathcal{B}}$	$\mathrm{BR}^{\mathrm{exp}}_{\mathcal{B}\overline{\mathcal{B}}} \times 10^3$	$\mathrm{BR}_{\mathcal{B}\overline{\mathcal{B}}} \times 10^3$
$\Sigma^0\overline{\Sigma}^0$	1.164 ± 0.004	1.160 ± 0.041
$\Lambda\overline{\Lambda}$	1.943 ± 0.003	1.940 ± 0.055
$\Lambda\overline{\Sigma}^0 +$ c.c.	0.0283 ± 0.0023	0.0280 ± 0.0024
$p\overline{p}$	2.121 ± 0.029	2.10 ± 0.16
$n\overline{n}$	2.09 ± 0.16	2.10 ± 0.12
$\Sigma^+\overline{\Sigma}^-$	1.50 ± 0.24	1.110 ± 0.086
$\Sigma^-\overline{\Sigma}^+$	/	0.857 ± 0.051
$\Xi^0\overline{\Xi}^0$	1.17 ± 0.04	1.180 ± 0.072
$\Xi^-\overline{\Xi}^+$	0.97 ± 0.08	0.979 ± 0.065

- Consider the small contribution from $A_{gg\gamma}$
- Assume $A_{gg\gamma}$ has the same phase as A_{3g} to A_γ
- Perform SU(3) analysis based on experimental branching ratios of J/ψ decaying to baryons

$$\Phi = (73 \pm 8)^\circ$$

Br result from SU(3) very close to PDG

The so called "$\rho\pi$ puzzle"

Analyses on-going. Stay tuned!

pQCD expectation of the ratio between **J/ψ** and **ψ'** decays:

$$R \equiv \frac{BR(\psi' \to hadrons)}{BR(J/\psi \to hadrons)} \simeq \frac{BR(\psi' \to e^+e^-)}{BR(J/\psi \to e^+e^-)} \simeq 12\%,$$

"$\rho\pi$ puzzle" 12% rule is severly violeted

in $\rho\pi$, $K^*\bar{K}$ and more VP decay modes as well as VT decay modes

J/ ψ – enhancement :

- J/ ψ -glueball mixing:

Freund and Nambu, Hou and Soni, Brodsky, Lepage and Tuan

- Final state interaction:Li, Bugg and Zou
- Intrinsic charmonium component within light vectors:Brodsky and Karliner, Feldman and Kroll

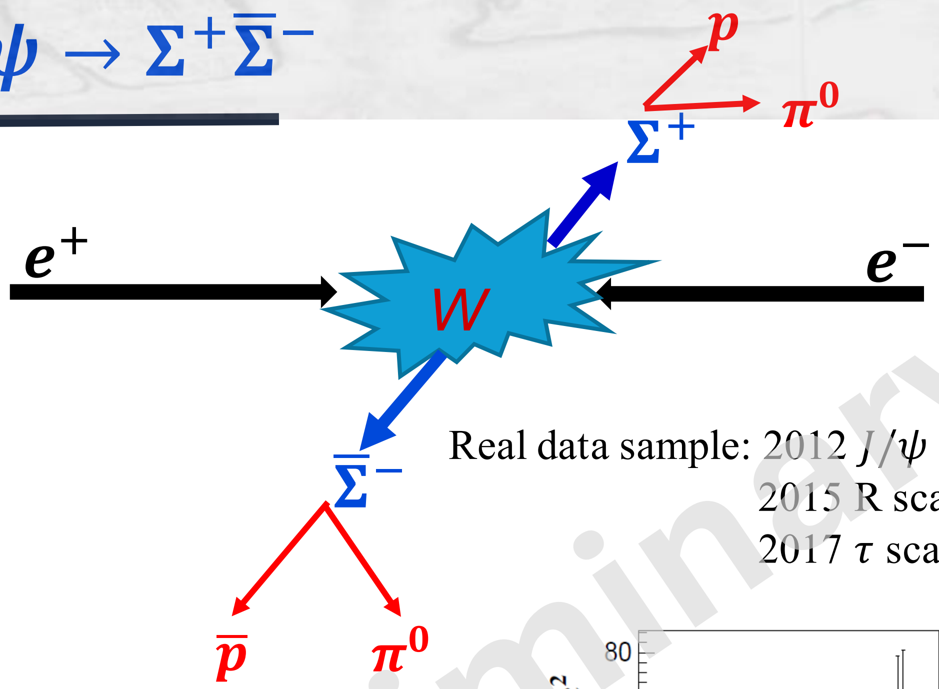
ψ' – suppression:

Chen and Braaten: color octet Fock state dominance in J/

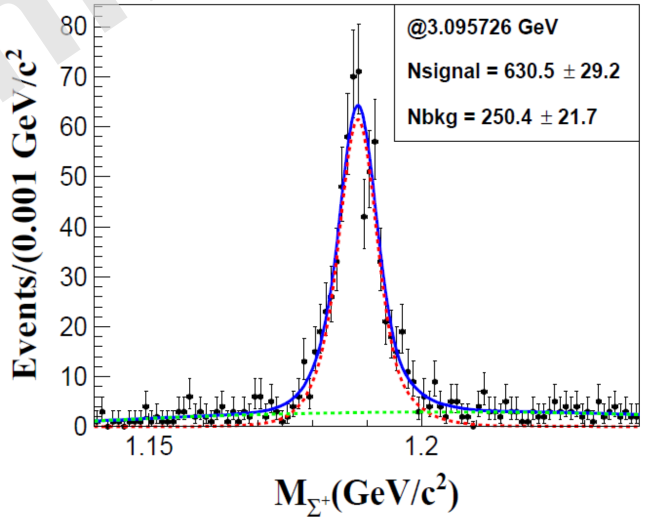
- Rosner **J/ψ** and **ψ'** mixing

Event selection

$J/\psi \rightarrow \Sigma^+\bar{\Sigma}^-$



Real data sample: 2012 J/ψ scan data
2015 R scan data
2017 τ scan data



☐ Final state : $p\bar{p}\pi^0\pi^0$

- **Good Charged Tracks:**
  - $|cos\theta| < 0.93$
  - $V_r < 2\text{cm}, |V_z| < 10\text{cm}$
  - $N_{charged} \geq 2$
- **PID(dE/dX + TOF):**
  - $prob(p) > prob(\pi)\ \&\ prob(p) > prob(k)$
  - $prob(\bar{p}) > prob(\pi)\ \&\ prob(\bar{p}) > prob(k)$
  - $N_p = N_{\bar{p}} = 1$
- **Good Neutral Tracks:**
  - Barrel: $|cos\theta| < 0.8, E_\gamma \geq 25\text{MeV}$
  - Endcap: $0.86 < |cos\theta| < 0.92, E_\gamma \geq 50\text{MeV}$
  - EMC TDC: $0 \leq t \leq 700\text{ns}$
  - $N_{shower} \geq 2$

$\chi_a^2 = (M_{p\pi^0} - M_{\Sigma_{PDG}})^2 + (M_{\bar{p}\pi^0_{miss}} - M_{\Sigma_{PDG}})^2$

$\chi_b^2 = (M_{p\pi^0_{miss}} - M_{\Sigma_{PDG}})^2 + (M_{\bar{p}\pi^0} - M_{\Sigma_{PDG}})^2$

- **1C fit for π^0:**
  - $M_{\gamma\gamma} \in (M_{\pi^0} - 0.06, M_{\pi^0} + 0.04)\ \text{GeV/c}^2;\ \chi^2_{1C} < 25;\ N_{\pi^0} \geq 1$
  - **2C Kinematic Fit:** Loop all π^0 candidate and miss the other π^0. Do kinematic fit on $p\bar{p}\pi^0\pi^0$ hypothesis →Constraint on π^0 (1C) invariant mass
  - Add π^0 (miss) invariant mass
  - The smallest fitting χ^2 is applied to select the best π^0.
  - After kinematic fit, select the Σ candidate from best combination with minimum χ_a^2/χ_b^2.

☐ Unbinned maximum likelihood is performed to Σ^+ invariant mass.

☐ **Total Fit (Blue)** = **MC shape** ⊗ **Gaussian** + **Bkg (2nd order Polynomial**

Preliminary

Phase Measurement (The Born cross section)

$$\sigma(W) = \left| N_0 \frac{Se^{i\Phi}+E}{M_{J/\psi}-W-i\Gamma_{J/\psi}/2} + C \right|^2$$

- $\mathbf{B_{J/\psi\to\Sigma^+\bar{\Sigma}^-} = |Se^{i\Phi} + E|^2}$

1. Incorporating the radiative correction $F(x, W)$ as

$$\sigma'(W) = \int_0^{1-\left(\frac{W_{min}}{W}\right)^2} dx F(x,W)\sigma(W\sqrt{1-x})$$

2. Energy spread S_E is included by convolving with Gaussian function by set the width of S_E, expected cross section becomes

$$\sigma''(W) = \int_{W-nS_E}^{W+nS_E} \frac{1}{\sqrt{2\pi}S_E} \exp\left(\frac{-(W-W')^2}{2S_E^2}\right) \sigma'(W')dW'$$

- **Minimization Function:**
- The fitting parameters are obtained by means of χ^2 −minimization function defined as $\chi^2 = \Delta X^T M^{-1} \Delta X$
  - ΔX is difference between the measured and observed value.
  - M is the covariance matrix.

$$\sigma_{cont}(W) = \sigma_0 \left(\frac{W_0}{W}\right)^{10} = C^2$$

$$W_0 = 3.0 \text{ GeV}$$

$$C = -\sqrt{\frac{\sigma_0 \cdot (W_0)^{10}}{W^{10}}}$$

$$B_{out}^{EM} = \sqrt{\frac{C^2}{\sigma_{ee\to\mu^+\mu^-}} B_{J/\psi\to\mu^+\mu^-}}$$

$$E = \sqrt{B_{out}^{EM}} = \sqrt{\sigma_0 \left(\frac{W_0}{M_{J/\psi}}\right)^{10} \frac{1}{86.8} \left(\frac{M_{J/\psi}}{1000}\right)^2 B_{in}}$$

$$N_0 = \sqrt{\frac{3\pi\Gamma^2 B_{in}(hc)^2 \cdot 10^{10}}{W^2}}$$

$e^+e^- \rightarrow p\bar{p}$

- **2 good charged tracks**

  -$|R_{xy}| < 1cm, |R_z| < 10cm$

  -|cosθ|<0.8

- **veto bhabha**

  -each track has energy deposit in EMC

  -$E_{EMC}/P_{MDC}<0.5$ (only for proton)

- **Particle Identification for both p and $\bar{p}$ combining with dE/dx and TOF information**

  -$Prob(p)>prob(\mu)\&\&Prob(p)>prob(\pi)$ && $Prob(\bar{p})>prob(\mu^-)\&\&Prob(\bar{p})>prob(\pi^-)$

- **veto cosmic rays or beam related background**

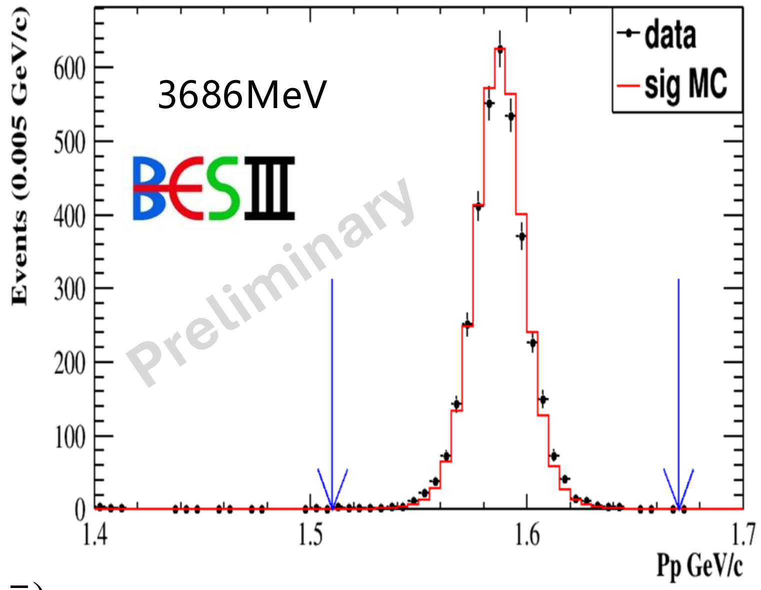
  -each track has TOF information

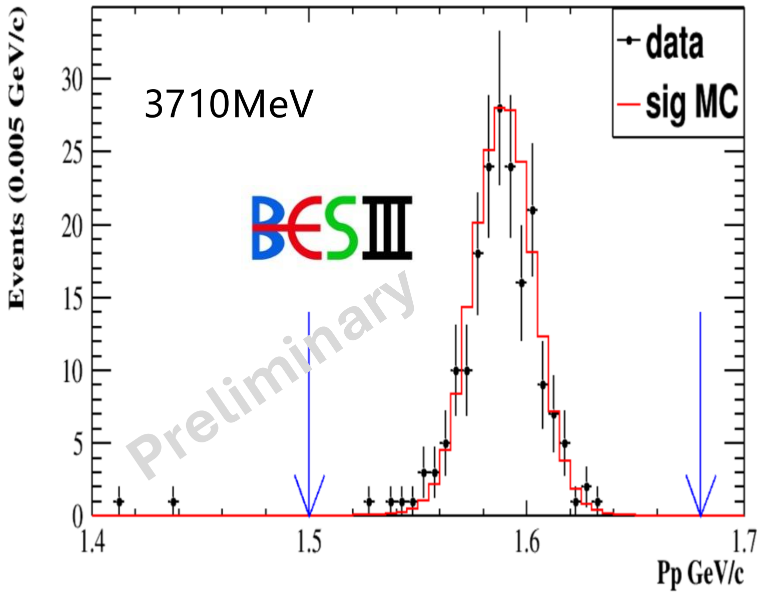
  -$|\Delta T|=|Tof(p)-Tof(\bar{p})|<3$ ns

- **veto multi-tracks**

  -$178° < \theta_{p\bar{p}} < 180°$

  $\theta_{p\bar{p}}$ *is the polar angle between the two tracks*





The background is negligible in the signal region.

$e^+e^- \to \phi\eta$

$e^+e^- \to K^+K^-\gamma\gamma$:

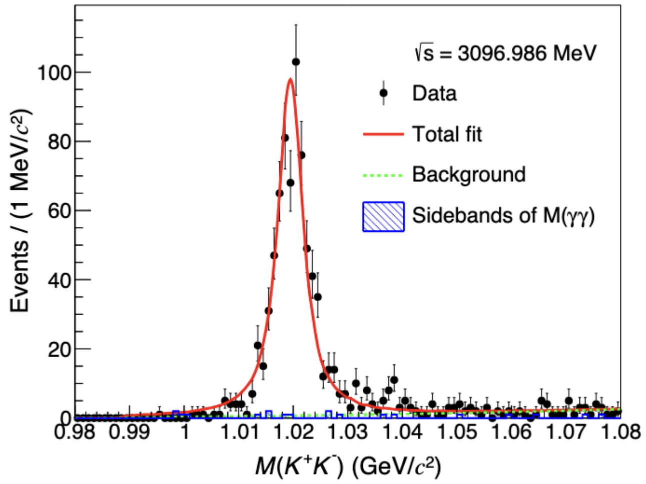
at least two candidate
charged kaons with opposite charge and at least two candidate photons

$|\cos\theta| < 0.93$ For charged tracks

R_{xy}<10 cm ; R_z<1
PID TOF/dE/dx
photon candidates in the barrel region (| cos θ| < 0.80) of the EMC with at least 25 MeV (and 0.86 < | cos θ| < 0.92) with at least 50 MeV of energy deposition.
Opening angle between a candidate shower and the closest charged than 10°

A four-constraint (4C) kinematic fit under the hypothesis e+e− → K+K−γ constraining the measured four-moment, χ<85

$|M(\gamma\gamma) - M_\eta| < 30\ \mathrm{MeV}/c^2$ $0.98 < M(K^+K^-) < 1.08\ \mathrm{GeV}/c^2$

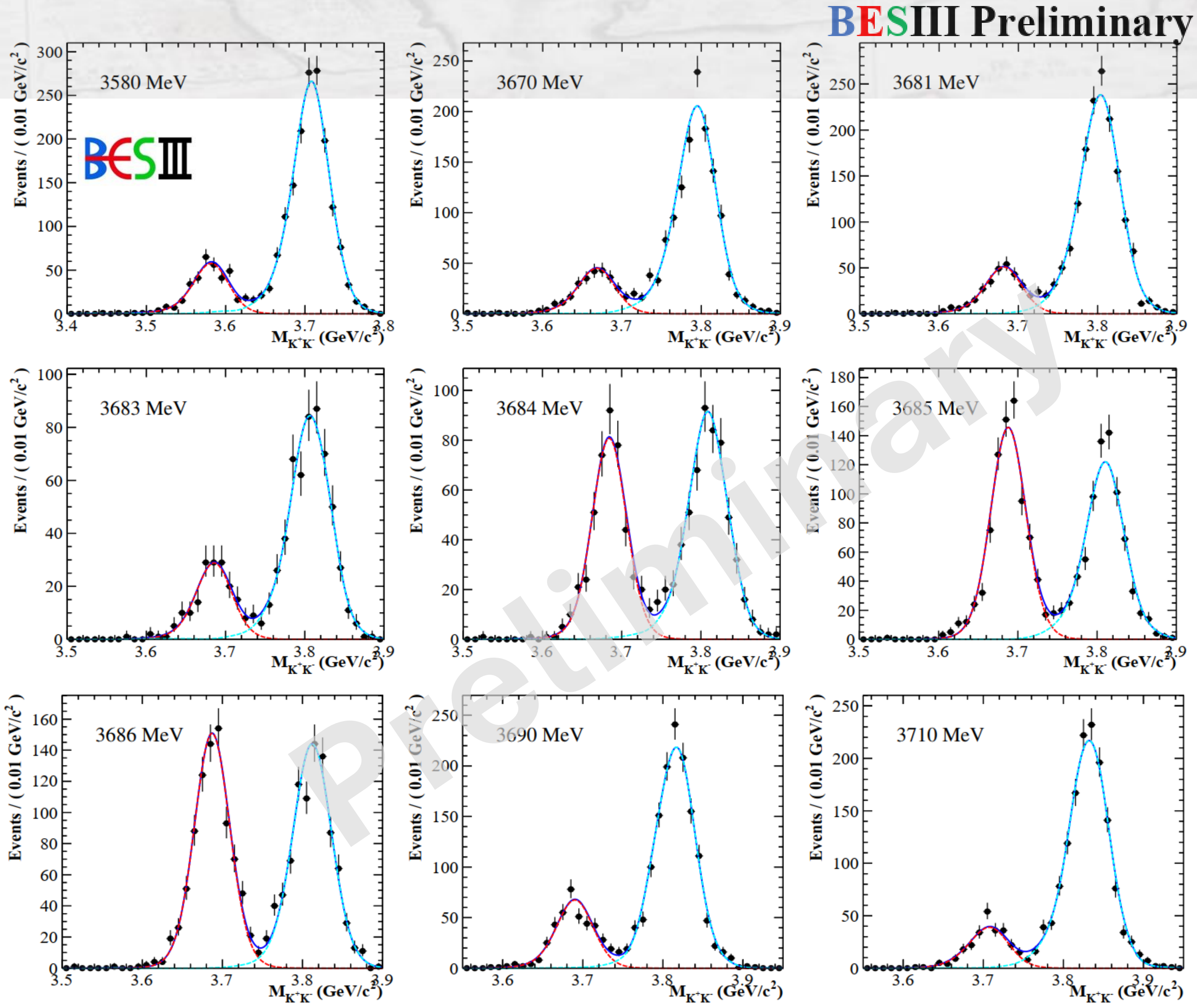


Background negligible

$e^+e^- \to K^+K^-$

Fitting on $M_{K^+K^-}$

- Two good charged tracks:
  - $|R_{xy}| < 1cm, |R_z| < 10cm$
- $E/p < 0.7$ for both tracks;
- PID with de/dx+TOF for both tracks:
  - $P_K > P_\pi, P_K > P_\mu, P_K > 0.001$;
- vertex fit on K^+ and K^- tracks:
  - $\chi^2_{\text{vtx}} < 100$;
- $|\Delta T| = |TOF_{K^+} - TOF_{K^-}| < 3ns$;
- Back-to-back angle:
  - $\theta_{K^+K^-} > 177°$(178°@5th, 6th, 7th points);
- residual momentum:
- Fitting on $M_{K^+K^-}$ spectrums with model: $\mathrm{MC}_{K^+K^-} \otimes \mathrm{GS}_{K^+K^-} + \mathrm{MC}_{\mu^+\mu^-} \otimes \mathrm{GS}_{\mu^+\mu^-}$.
- The MC shapes ($\mathrm{MC}_{K^+K^-}$ and $\mathrm{MC}_{\mu^+\mu^-}$) are extracted from simulation samples.
- The mass resolution functions ($\mathrm{GS}_{K^+K^-}$ and $\mathrm{GS}_{\mu^+\mu^-}$) are set free.



only $\mu^+\mu^-$ channel contributes as backgound

An evaluation of the contribution of interference between continuum and resonant amplitudes has been done accounting for a few percent level, depending on the final states for narrow resonances and more for broad ones.

[C.Z.Yuan and Y. Guo, Phys. Rev. D 105 (2022) 114001]

With the high statistics of BESIII, high precision of branching fraction measurements is reached (few %) in many final states. It's crucial to know the interference contribution (at the same level).

For narrow resonances (extreme case)

Non-resonant cross section

$$\delta B = 2\sqrt{\frac{\sigma_0}{\sigma_\psi}} A_{3g} sin\phi_{g,\gamma}$$

A_{3g} should be determined

$$\sigma_\psi = (12\pi/m^2)B(\psi \to e^+e^-)$$

BaBar, PhysRevD.92.072008 (2015)

$\phi_{\gamma,cont.} = 0$

Found 5% for $J/\psi \to K^+K^-$
And 15% for $\psi(2S) \to K^+K^-$

cos ϕ and A_s from a combined analysis of the $\psi \to K^+K^-$ and $\psi \to K_SK_L$ decays, whose branching fractions depend on the same strong amplitude

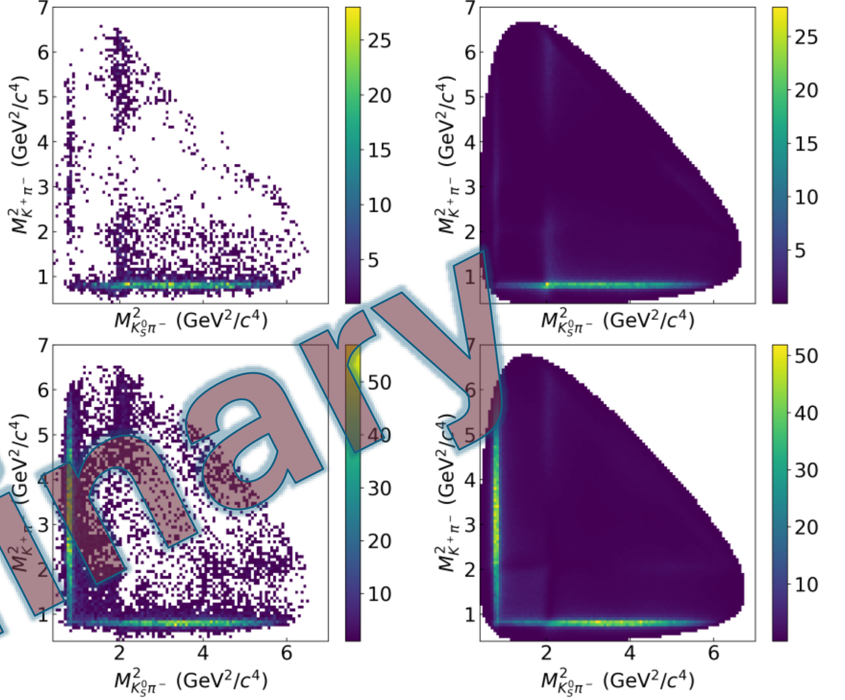
Counting for the total yield of $J/\psi \to K_S^0 K^+ \pi^-$, PWA extract the subdecay yield

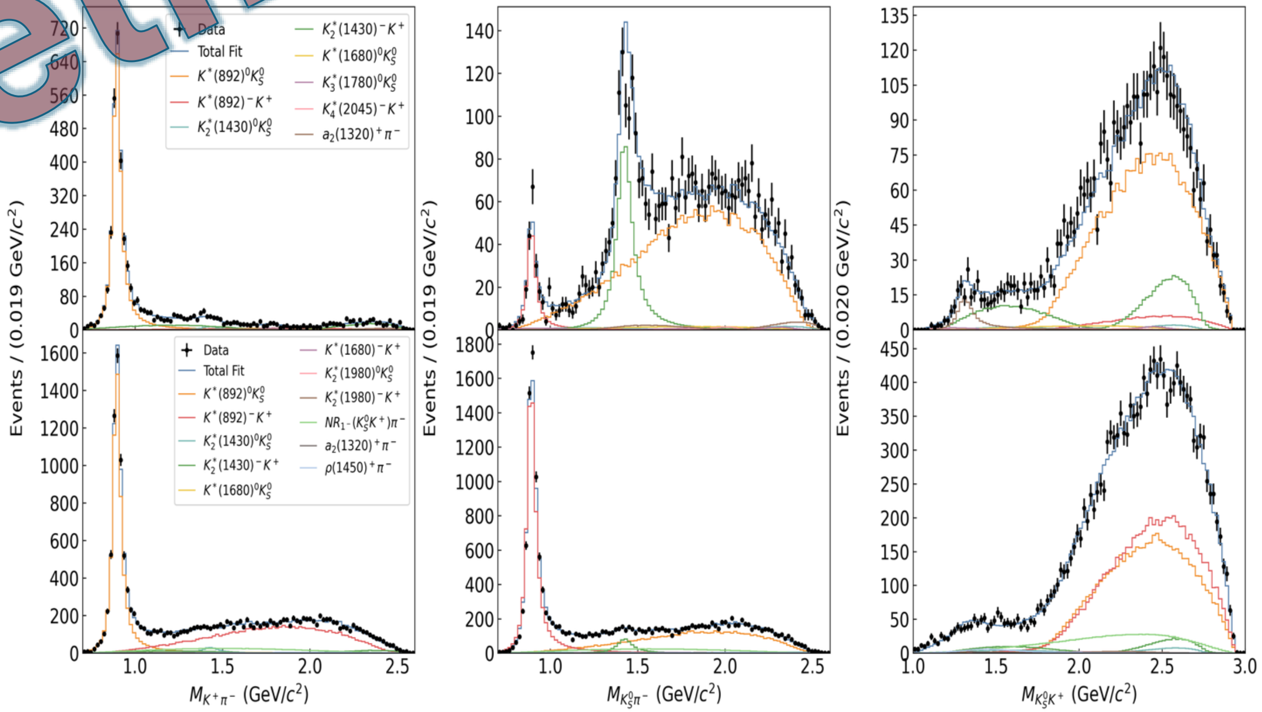
$$|\mathcal{M}|^2 = \sum_{\Lambda,\Lambda'} \rho_{\Lambda,\Lambda'} \sum_{\lambda_{K_S^0}=0} \sum_{\lambda_{K^+}=0} \sum_{\lambda_{\pi^-}=0} \left| \mathcal{M}^{R_{K^+\pi^-}}_{\lambda_{K_S^0},\lambda_{K^+},\lambda_{\pi^-}} + \mathcal{M}^{R_{K_S^0\pi^-}}_{\lambda_{K_S^0},\lambda_{K^+},\lambda_{\pi^-}} + \mathcal{M}^{R_{K_S^0 K^+}}_{\lambda_{K_S^0},\lambda_{K^+},\lambda_{\pi^-}} \right|^2,$$

$$\mathcal{M}^{0\to1+2}_{\lambda_0,\lambda_1,\lambda_2} = \mathcal{H}^{0\to1+2}_{\lambda_1,\lambda_2} D^{J_0*}_{\lambda_0,\lambda_1-\lambda_2}(\phi,\theta,0),$$

$$\mathcal{H}^{0\to1+2}_{\lambda_1,\lambda_2} = \sum_{ls} g_{ls} \sqrt{\frac{2l+1}{2J_0+1}} \langle l0, s\delta \mid J_0, \delta\rangle \langle J_1 J_2, \lambda_1-\lambda_2 \mid s, \delta\rangle \left(\frac{q}{q_0}\right)^l B'_l(q, q_0, d),$$

$\sqrt{s}$ (MeV)	$\sigma_{\rm obs}^{K_S^0 K^+\pi^-}$ (pb)	$\sigma_{\rm obs}^{\bar{K}^0 K^{*0}}$ (pb)	$\sigma_{\rm obs}^{K^+ K^{*-}}$ (pb)
3000.00 ± 0.20	67.0 ± 4.1	104.2 ± 9.5	1.4 ± 1.6
3020.00 ± 0.20	58.0 ± 3.7	82.1 ± 9.4	1.5 ± 2.2
3049.64 ± 0.06	62.0 ± 4.0	87.0 ± 8.6	6.7 ± 3.1
3058.69 ± 0.06	56.3 ± 3.8	92.1 ± 8.8	3.3 ± 2.1
3080.00 ± 0.20	61.0 ± 0.9	95.2 ± 2.1	9.6 ± 0.8
3082.50 ± 0.06	50.1 ± 6.5	55.7 ± 12.1	7.7 ± 4.9
3087.59 ± 0.13	58.4 ± 9.7	82.2 ± 23.5	20.7 ± 11.5
3088.85 ± 0.06	78.3 ± 4.4	135.0 ± 10.4	31.6 ± 5.5
3091.76 ± 0.06	94.7 ± 4.9	155.9 ± 12.9	73.0 ± 9.3
3094.70 ± 0.10	1022.7 ± 40.7	1073.4 ± 98.9	1289.0 ± 101.3
3095.43 ± 0.10	3887.4 ± 86.2	3227.9 ± 162.9	4902.6 ± 247.2
3095.73 ± 0.08	6779.0 ± 91.7	5499.7 ± 172.1	9341.7 ± 281.3
3095.83 ± 0.09	9870.4 ± 126.5	8983.5 ± 281.2	13008.6 ± 340.0
3096.20 ± 0.07	13089.1 ± 97.8	10915.0 ± 196.1	17952.0 ± 293.1
3096.99 ± 0.08	17377.8 ± 143.2	13418.6 ± 258.4	23781.2 ± 453.3
3097.21 ± 0.09	17123.7 ± 169.7	13893.2 ± 304.5	22819.6 ± 481.1
3097.23 ± 0.10	17436.1 ± 193.9	13865.9 ± 359.1	24197.9 ± 556.9
3097.65 ± 0.08	13542.3 ± 103.1	10669.6 ± 201.0	18566.4 ± 321.8
3098.34 ± 0.09	7836.5 ± 111.6	6216.2 ± 204.7	10406.1 ± 317.5
3098.73 ± 0.08	4581.5 ± 54.6	3444.2 ± 98.5	6192.6 ± 179.4
3099.04 ± 0.11	2959.6 ± 117.1	2838.1 ± 233.1	3534.8 ± 290.0
3101.36 ± 0.12	932.4 ± 45.4	655.2 ± 77.5	1026.4 ± 115.9
3104.00 ± 0.08	570.4 ± 19.3	497.3 ± 38.3	766.6 ± 51.4
3105.58 ± 0.10	484.1 ± 28.5	387.4 ± 51.5	563.3 ± 66.9
3112.05 ± 0.11	315.9 ± 25.8	225.9 ± 41.0	254.5 ± 56.3
3119.88 ± 0.13	228.5 ± 26.0	252.9 ± 52.1	178.0 ± 57.3





BESIII Preliminary