Inclusive R Measurement at BESIII

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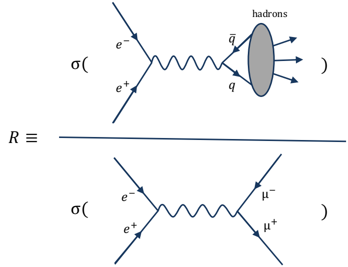
Definition of *R*

The *R* value is defined as the leading-order production cross section ratio of inclusive hadrons and muon pairs in the annihilation of electron-positron:

$$R \equiv \frac{\sigma^0(e^+e^- \to \text{hadrons})}{\sigma^0(e^+e^- \to \mu^+\mu^-)} \equiv \frac{\sigma^0_{\text{had}}}{\sigma^0_{\mu\mu}}$$

That is,



A direct result from the QED:

$$\sigma^0_{\mu\mu}(s) = \frac{4\pi\alpha^2}{3s}\,\frac{\beta_\mu(3-\beta_\mu^2)}{2}, \text{ with } \beta_\mu = \sqrt{1-4m_\mu^2/s}$$

Running of QED coupling constant: $\Delta\alpha(s)$

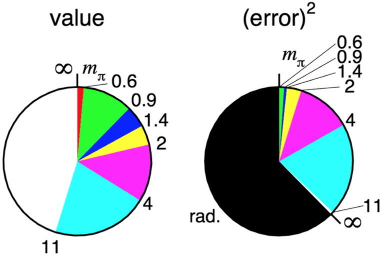
$\Delta\alpha(s)$ receives contributions from three parts:

$$\Delta\alpha(s) = 1 - \alpha(0)/\alpha(s) = \Delta\alpha_{\text{lepton}}(s) + \Delta\alpha^{(5)}_{\text{had}}(s) + \Delta\alpha_{\text{top}}(s)$$

- $\Delta\alpha_{\text{lepton}}(s)$ is calculated with perturbative approach and $\Delta\alpha_{\text{top}}(s)$ is usually small
- $\Delta\alpha^{(5)}_{\text{had}}(s)$ should be calculated by using R value at low energy:

$$\Delta\alpha^{(5)}_{\text{had}}(s) = -\frac{\alpha s}{3\pi}\,\text{Re}\int_{E^2_{\text{th}}}^{\infty} ds' \frac{R(s')}{s'(s'-s-i\varepsilon)}$$

Fractional contribution to $\Delta\alpha^{(5)}_{\text{had}}(M^2_Z)$:



Eur. Phys. J. C 80, 241 (2020)

Source	Contribution($\times 10^4$)
$\Delta\alpha_{\text{lepton}}(M^2_Z)$	314.979 ± 0.002
$\Delta\alpha^{(5)}_{\text{had}}(M^2_Z)$	276.0 ± 1.0
$\Delta\alpha_{\text{top}}(M^2_Z)$	-0.7180 ± 0.0054

To compute $\Delta\alpha^{(5)}_{\text{had}}(s)$, R values over a very wide energy range are needed!

Muon anomaly: $a_\mu^{\text{LO-HVP}}$

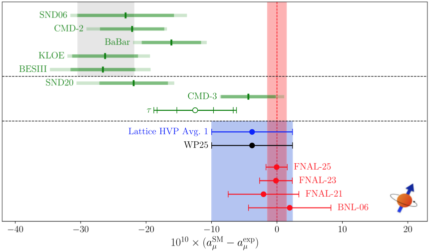
Based on the optical theorem, the leading-order hadronic VP contribution to muon anomaly, i.e., $a_\mu^{\text{LO-HVP}}$, is evaluated with R:

$$a_\mu^{\text{LO-HVP}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{E_{\text{th}}^2}^{\infty} \mathrm{d}s \, \frac{R(s)K(s)}{s^2}$$

- $R(s)$ in low energy region ($\sqrt{s} < 1$ GeV) contributes to $a_\mu^{\text{LO-HVP}}$ significantly.
- Above sufficient high energy, e.g., 12 GeV, $a_\mu^{\text{LO-HVP}}$ is calculated according to pQCD.

Phys. Rep. 1143, 1 (2025)

Source	Value ($\times 10^{11}$)
QED	116584718.8(2)
EW	154.4(4)
HVP LO (e^+e^-)	6931(40)
HVP LO (lattice)	7132(61)
HVP NLO (e^+e^-)	−99.6(1.3)
HVP NNLO (e^+e^-)	12.4(1)
HLbL	115.5(9.9)
a_μ^{SM}	116592033(62)
a_μ^{exp}	116592071.5(14.5)
Δa_μ	38(63)

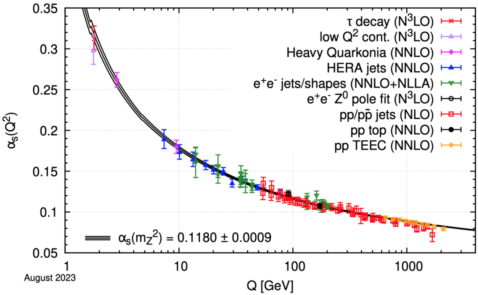


QCD coupling constant: $\alpha_s(s)$

According to pQCD, R is predicted with the coupling constant $\alpha_s(s)$:

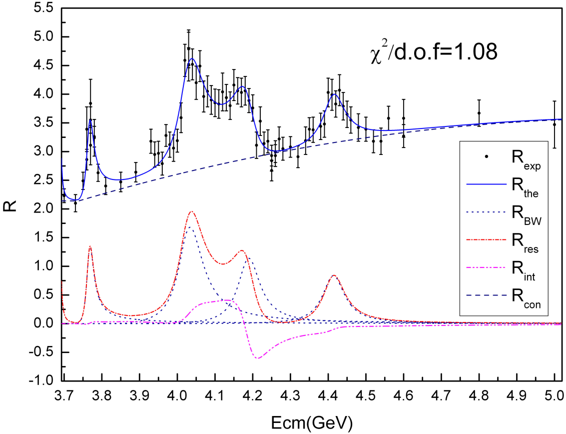
$$R_{\text{QCD}}(s) = R_{\text{EW}}(s)\left[1 + \delta_{\text{QCD}}(s)\right]$$

where $R_{\text{EW}} = N_c \sum_f Q_f^2$ and $\delta_{\text{QCD}}(s) = \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_s(s)}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{s^2}\right)$, $N_c = 3$ and Q_f are number of colors and charge carried by each of N_f activated quarks.

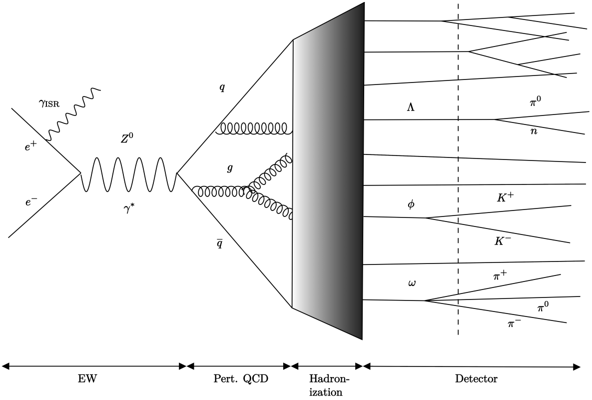


More physics potential of *R*

- Search for higher excitation states or exotic states with $J^{PC} = 1^{--}$
- Constraint charm quark mass with precise R value in open-charm region



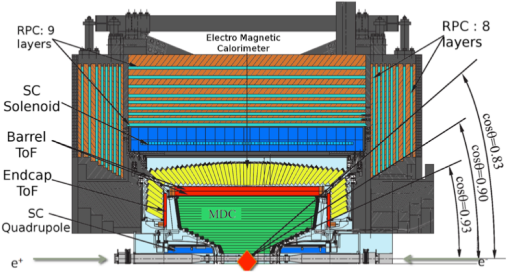
Production of a hadronic event



- R value measurement is based-on experimental data taken at e^+e^- collider
- Initial state radiation reduces the c.m. energy thereby requires a correction
- Hadronization of partons are simulated by phenomenological models

Detection of a hadronic event at BESIII

In annihilation of e^+e^-, a hadronic event is detected by a composite detector:



- Multilayer drift chamber: $\sigma_{r\phi} \sim 130\ \mu\text{m}$ (single wire), $\sigma_{p_t}/p_t \sim 0.5\%$@ 1 GeV/$c$
- Time-of-Flight system: $\sigma_t \sim 68$ ps (barrel), $\sigma_t \sim 110 \rightarrow 65$ ps (end-cap)
- Electromagnetic calorimeter: $\sigma_E/E < 2.5\%$ (barrel), $\sigma_E/E < 5.0\%$ (end-cap) at 1 GeV
- Resistive plate chamber Muon counter: $\Delta\Omega/4\pi = 93\%$

- Unstable initial state hadrons are produced at interaction point and decay in detector
- Not all the relatively stable hadrons are detected due to limited acceptance
- Various background processes are also stored in data sample, e.g., QED events
- Not all the collision events are recorded by detector due to imperfect trigger system

Determination of R value

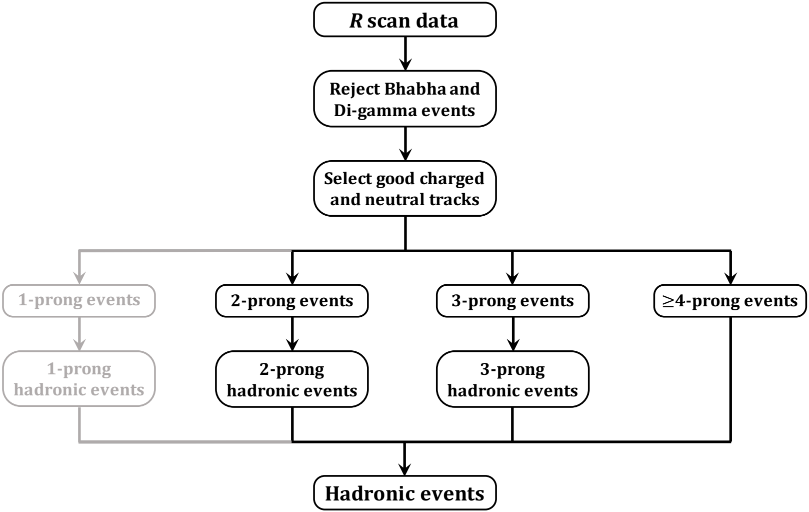
Experimentally, R value is determined by

$$R = \frac{N_{\text{had}}^{\text{obs}} - N_{\text{bkg}}}{\sigma_{\mu\mu}^{0} \mathcal{L}_{\text{int.}} \varepsilon_{\text{trig}} \varepsilon_{\text{had}} (1+\delta)}$$

- $N_{\text{had}}^{\text{obs}}$: Numbers of observed hadronic events.
- N_{bkg}: Number of the residual background events.
- $\sigma_{\mu\mu}^{0}(s) = 86.85\ \text{nb}/s$: Leading order QED cross section for $e^+e^- \to \mu^+\mu^-$.
- $\mathcal{L}_{\text{int.}}$: Integrated luminosity is measured by analyzing Bhabha events.
- $\varepsilon_{\text{trig}}$: Trigger efficiency $\sim 100\%$.
- ε_{had}: Detection efficiency of the hadronic events.
- $(1+\delta)$: Initial state radiation (ISR) correction factor.

Event selection strategy

For an inclusive measurement, the signal event is not selected specifically:



***R* scan data**

Reject Bhabha and Di-gamma events

Select good charged and neutral tracks

1-prong events

2-prong events

3-prong events

≥4-prong events

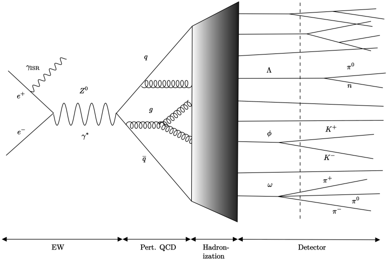
1-prong hadronic events

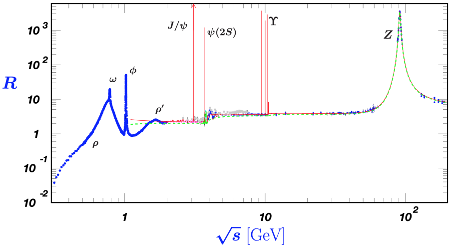
2-prong hadronic events

3-prong hadronic events

Hadronic events

Signal simulation: A key challenge



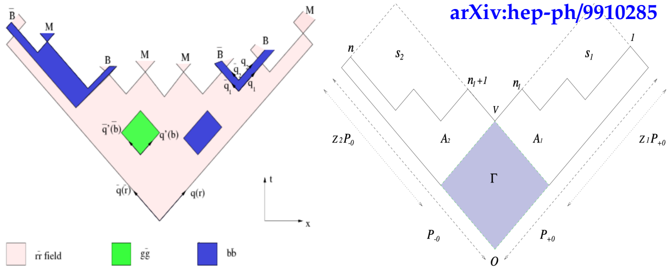


Reliable simulation of inclusive hadronic event at c.m. energy $\sqrt{s}$ requires:

- precise probability of emitting an ISR photon with specific energy and angle.
- accurate knowledge of all the allowed 1^{--} resonance states below $\sqrt{s}$.
- an effective phenomenological model to realize hadronization of partons.
- reliable production fractions and kinematic variables of some few-body channels, e.g., $e^+e^- \to \pi^+\pi^-$, 3π, 4π, K^+K^-, and almost all the open-charm channels.
- comparable multiplicities and momenta distributions of $p(\bar{p})$, $K^\pm$, and $\pi^\pm$ to data

Inclusive simulation model: LUARLW

An inclusive simulation model of hadronic events at low c.m. energy ($\sqrt{s} < 5$ GeV):



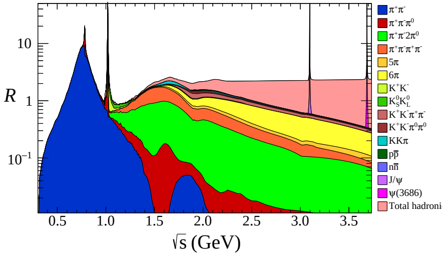
arXiv:hep-ph/9910285

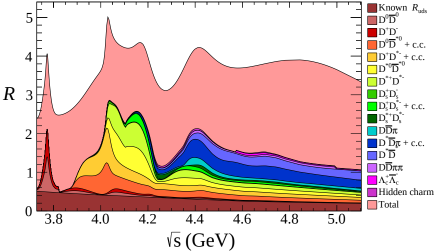
Main features of the LUARLW model:

- ✓ A self-consistent inclusive generator developed based on JETSET.
- ✓ Initial-state radiation (ISR) process is implemented from $2m_\pi$ to given $\sqrt{s}$.
- ✓ Kinematic quantities of initial hadrons are sampled by the Lund area law.
- ✓ Phenomenological parameters are tuned based on comparisons with data.

Exclusive point of view: HYBRID

An alternative model is proposed by replacing the phenomenological hadronization scheme with published experimental data of exclusive measurements:





The main features of the HYBRID model:

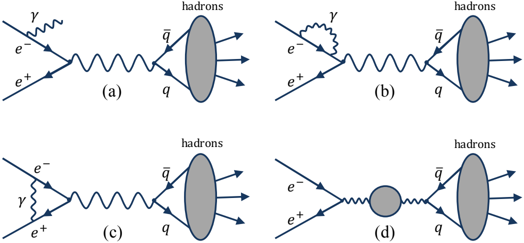
- ✓ Combination of THREE different well-established simulation models.
- ✓ As much as currently known experimental knowledge is implemented, especially almost all the known open-charm processes have been included.
- ✓ Different ISR and VP schemes are used for a conservative cross check.

ISR effects in LUARLW: Feynman Diagram

Definition of initial-state radiation (ISR) factors:

$$(1+\delta)(s) \equiv \sigma^{\text{tot}}_{\text{had}}(s)/\sigma^{0}_{\text{had}}(s)$$

In LUARLW, the Feynman Diagram (FD) scheme is used to simulate ISR correction and calculate $(1+\delta)$. Following ISR procedures are considered:



The total hadronic cross section measured by experiment is the total effect of all these diagrams:

$$\sigma^{\text{tot}}_{\text{had}}(s) = \frac{\delta_{\text{vert}}\,\sigma^{0}_{\text{had}}(s)}{|1-\Pi(s)|^2} + \int_0^{x_m} \frac{F_{\text{FD}}(x,s)\,\sigma^{0}_{\text{had}}(s')}{|1-\Pi(s')|^2}\text{d}x, \text{ and } F_{\text{FD}}(x,s) \equiv \beta\frac{x^{\beta}}{x}\left(1-x+\frac{x^2}{2}\right)$$

ISR effects in HYBRID: Structure Function

The Structure Function (SF) scheme of ISR correction is implemented in HYBRID model:

$$\sigma_{\mathrm{had}}^{\mathrm{tot}}(s) = \int_0^{x_m} F_{\mathrm{SF}}(x,s)\, \frac{\sigma_{\mathrm{had}}^{0}(s')}{|1-\Pi(s')|^2}\mathrm{d}x.$$

Taking the parameterization scheme given in **Nucl. Phys. B318, 1 (1989)** as an example:

$$F_{\mathrm{SF}}(x,s) = \beta x^{\beta-1}\Delta - \beta(1-\frac{1}{2}x) - \frac{1}{8}\beta^2\left[4(2-x)\ln x + \frac{1+3(1-x)^2}{x}\ln(1-x) + 6 - x\right]$$

where

$$\Delta = 1 + \frac{\alpha}{\pi}\left(\frac{3}{2}L + \frac{\pi^2}{3} - 2\right) + \left(\frac{\alpha}{\pi}\right)^2\left\{[\frac{9}{8} - 2\zeta(2)]L^2 + [-\frac{45}{16} + \frac{11}{2}\zeta(2) + 3\zeta(3)]L \right.$$
$$\left. - \frac{6}{5}[\zeta(2)]^2 - \frac{9}{2}\zeta(3) - 6\zeta(2)\ln 2 + \frac{3}{8}\zeta(2) + \frac{57}{12}\right\}.$$

According to the experimental definition of R, its uncertainty is roughly expressed as

$$\left(\frac{\Delta R}{R}\right)^2_{\text{sys}} = \left(\frac{\Delta \tilde{N}}{\tilde{N}}\right)^2 + \left(\frac{\Delta \sigma^0_{\mu\mu}}{\sigma^0_{\mu\mu}}\right)^2 + \left(\frac{\Delta \mathcal{L}_{\text{int.}}}{\mathcal{L}_{\text{int.}}}\right)^2 + \left(\frac{\Delta \varepsilon_{\text{trig}}}{\varepsilon_{\text{trig}}}\right)^2 + \left(\frac{\Delta \varepsilon_{\text{had}}}{\varepsilon_{\text{had}}}\right)^2 + \left[\frac{\Delta(1+\delta)}{(1+\delta)}\right]^2,$$

where

$$\tilde{N} = \frac{N^{\text{net}}_{\text{had}}}{\varepsilon_{\text{had}}} = \frac{N^{\text{obs}}_{\text{had}} - N_{\text{bkg}}}{\varepsilon_{\text{had}}}$$

In practice, the uncertainties are addressed in following different aspects:

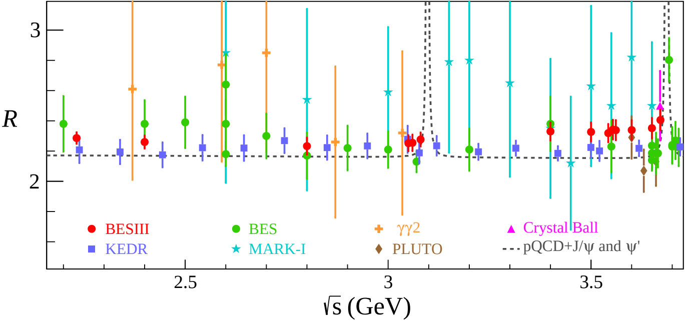
- Event selection: all implemented selection criteria are slightly varied, $0.40 \sim 0.80\%$.
- Background estimation: use different methods and simulation models, $0.30 \sim 0.40\%$.
- $\sigma^0_{\mu\mu}$: uncertainty is negligible due to the high precision of QED.
- $\mathcal{L}_{\text{int.}}$: uncertainty is directly cited from published results, $0.80 \rightarrow 0.50\%$.
- $\varepsilon_{\text{trig}}$: approaches to 100% with an uncertainty less than 0.10%.
- Signal simulation: differences of R resulted by LUARLW and HYBRID is taken, $1.00 \sim 2.50\%$.
- ISR factor: considered in calculation precision and uncertainty in $\sigma^0_{\text{had}}(s)$, $0.50 \sim 1.00\%$.

A total uncertainty no larger than 3.0% could be achieved.

Measured *R* values between 2.2 ∼ 3.7 GeV

After successfully constructing the LUARLW and HYBRID models, the first batch of *R* values has been published in 2022:

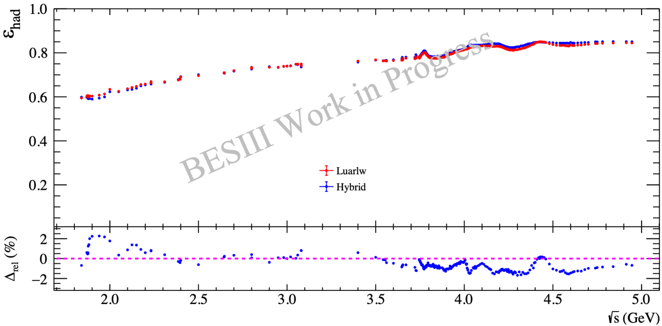
Phys. Rev. Lett. 128, 062004 (2022)



- The accuracy is better than 2.6% below 3.1 GeV and 3.0% above.
- Larger than the pQCD prediction by 2.7σ between 3.4 ∼ 3.6 GeV.
- A plenty of checks have been carried out before and after the publication for *R* above 3.4 GeV, the results are found to be solid.

More results are on the way

- Published result was based on a relatively small data set.
- A lot more data sets (~200 points) has been collected in the meantime.
- Both the LUARLW and HYBRID models are ready.

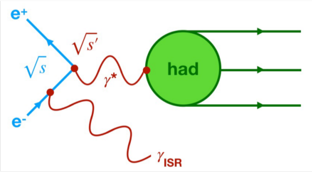


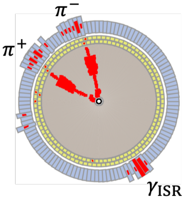
- Reliable and consistent ε_{had} is obtained from $\sqrt{s} = 1.8$ to 5.0 GeV.
- The *R* value could be precisely determined at these data points, where those below 2.0 GeV and above 3.7 GeV are of great interest.

Another approach: measure *R* via ISR

The ISR approach could access the *R* value below $\sqrt{s} = 2.0$ GeV:

- no inclusive measurements have been obtained far below ~2 GeV.
- no inclusive measurements have been obtained via ISR method.





In practice, the signal selection strategy is:

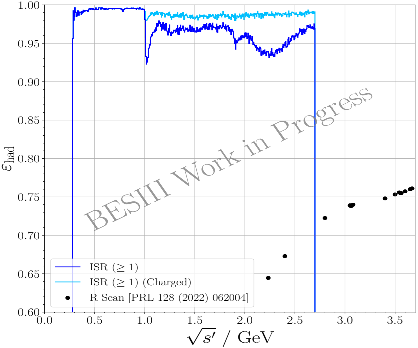
- require an energetic photon in barrel of EMC: $E_\gamma > 1.2$ GeV and $|\cos\theta| < 0.8$.
- there should be at least one charged track in barrel region of the detector.
- suppress Bhabha and Di-gamma background, veto photon produced by mesons.
- reconstruct mass of hadronic final states from recoil of the ISR photon:

$$s' = m^2_{\text{had}} = s - 2E_\gamma\sqrt{s}$$

Inclusive *R* measurement via ISR technique

Advantages:

- ✓ Very high detection efficiency due to the sufficient boost of ISR photon.
- ✓ Less reliant on the simulation of the hadronic events in data sample.
- ✓ Single measurement accesses m_{had} down to threshold of $\pi^+\pi^-$.
- ✓ Fully inclusive for final state radiation and higher order ISR effects.
- ✓ Independent of previous *R* analysis based on energy scan method.

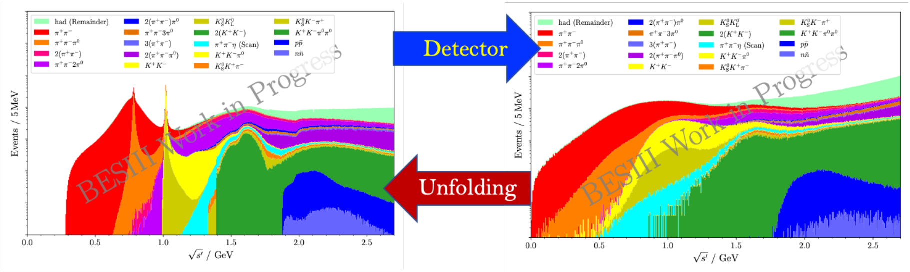


Challenges:

- Significant QED backgrounds due to their higher cross sections: dedicated PID needed.
- Background from non-ISR hadronic events containing π^0/η: dedicated vetoes.
- Limited resolution in m_{had} due to high energy of ISR photon: unfolding of m_{had}.

Inclusive R measurement via ISR technique

Unfolding is a powerful approach to recover the truth m_{had} spectrum from detected one:

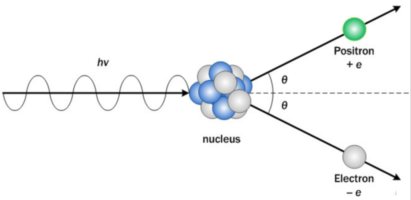


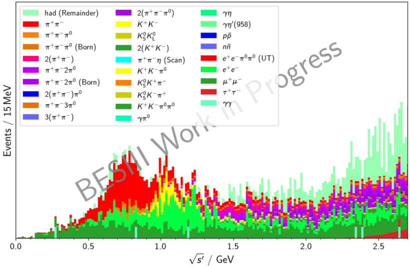
- Large smearing is caused by detector: track lost, photon energy leakage, and so on.
- An un-biased unfolding is crucial to recover the true hadronic mass spectrum.
- Fractions and shapes of $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ channels in the signal MC samples producing response matrix are modified to test the unfolding method.
- Unfolded spectra keep unchanged within the corresponding standard deviation.

Improve m_{had} by photon conversion event

An alternative method to tag the ISR photon:

- Energetic ISR photon could convert to a pair of e^+e^- via interaction with detector material or beam-pipe.
- Tracks of produced e^+e^- pair could be well reconstructed by the tracking sub-detectors.
- Improve the m_{had} resolution by large factors thanks to precisely measured e^+ and e^- momenta.
- As a result, the statistics is significantly reduced due to the low probability of photon conversion.
- High potential for the new high-statistics data sets at BESIII.





Summary

- Precise R value is highly desired in evaluation of $\alpha_{\mathrm{QED}}(s)$ and a_μ^{HVP}.
- BESIII plays an important role in inclusive R measurement.
- New R values from $\sqrt{s} = 1.8$ to 5.0 GeV are around the corner.
- Efforts are made in R measurement via ISR technique:
  - QED and non-ISR hadronic background are understood.
  - Unfolding approach is effective and robust in extracting truth m_{had}.
  - Few percent accuracy is targeted to shed light on current discrepancy in obtained a_μ^{HVP} between data-driven and Lattice QCD.

Thanks for your attention!