

# Recent developments in Phokhara

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On behalf of the **Phokhara** team



# Motivation - The new $(g-2)_\mu$ landscape

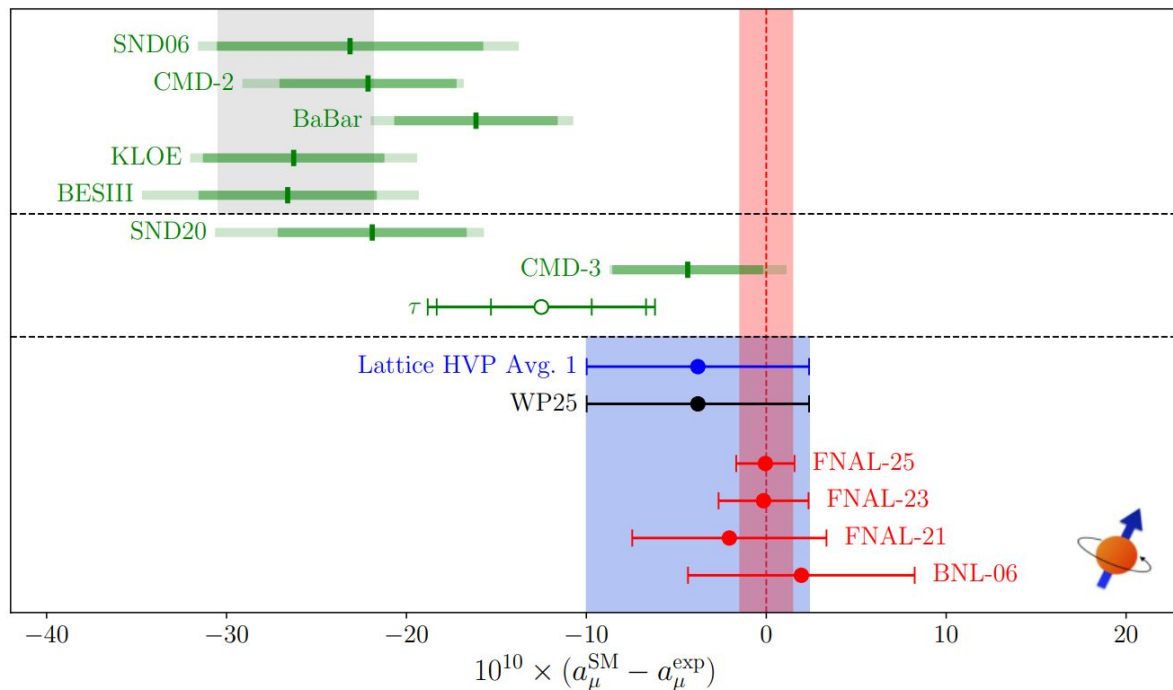


Figure from WP2025 [2505.21476]

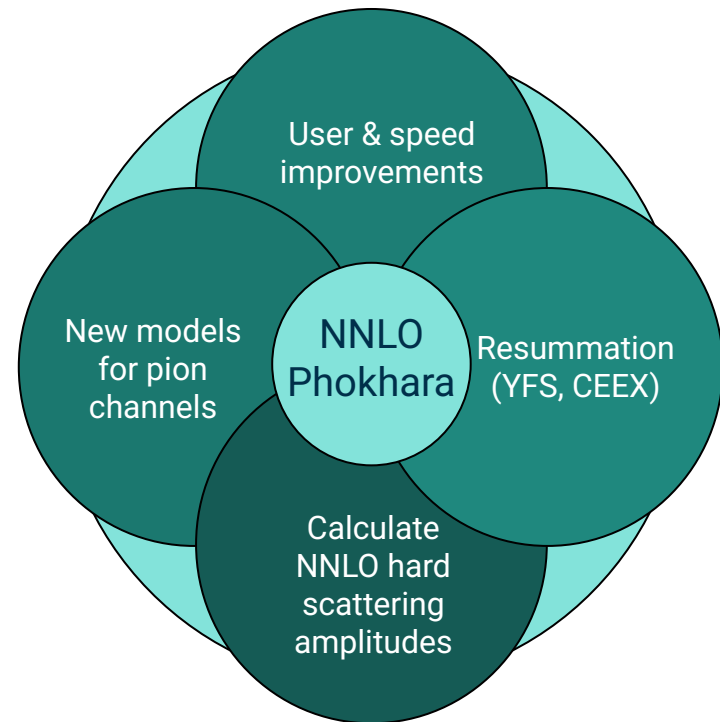
In WP2025...

- **Disagreement** between data-driven and Lattice QCD.
- Friction between data of different  $e^+e^-$  experiments.
- Need **improved MC** tools.
- **Beyond NLO** in  $e^+e^- \rightarrow \mu^+\mu^-\gamma$ ,  $e^+e^- \rightarrow \pi^+\pi^-\gamma$ .

# Phokhara – Past, Present & Future

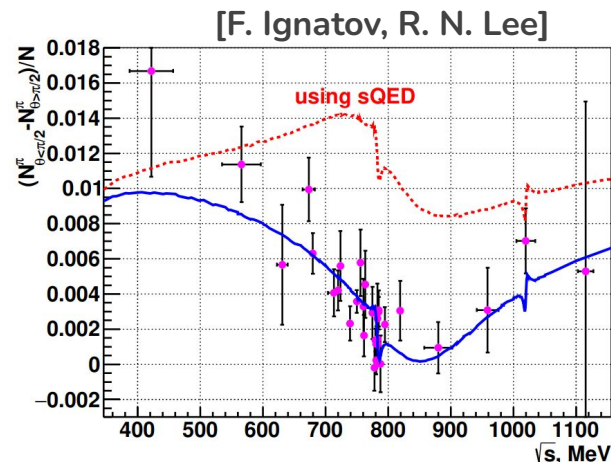
- **Phokhara**, MC generator for low energy  $e^+e^-$  colliders, with +20 years of development. [**PhysRevD.100.076004**]
- Mostly used in **2 to 3** processes with massive leptons and hadrons, widely used by the community.
- Recently updated with quality of life improvements, and **compared** against other MC in [**SciPost 10.21468**].

$e^+e^- \rightarrow$	Order	VP	VFF	Extras
$\mu^+\mu^-$	LO	alphaQED,		Narrow resonances
$\mu^+\mu^-\gamma$	NLO with full mass dependence	from [320, 321] or NSK	-	of $J/\psi$ and $\psi(2S)$
$\pi^+\pi^-$	LO	alphaQED,	F×sQED	Narrow resonances
$\pi^+\pi^-\gamma$	NLO with full mass dependence	from [320, 321] or NSK	choice of 3 VFF	of $J/\psi$ and $\psi(2S)$ Radiative $\phi$ decays
$X$	$X \in 2\pi^0\pi^+\pi^-, 2\pi^+2\pi^-, p\bar{p}, n\bar{n}, K^+K^-, K^0\bar{K}^0, \pi^+\pi^-\pi^0, \Lambda(\rightarrow\pi^-p)\bar{\Lambda}(\rightarrow\pi^+\bar{p}), \eta\pi^+\pi^-, \pi^0\gamma, \eta\gamma, \eta'\gamma, \chi_{c1} \rightarrow J/\psi(\rightarrow\mu^+\mu^-)\gamma, \chi_{c2} \rightarrow J/\psi(\rightarrow\mu^+\mu^-)\gamma$			



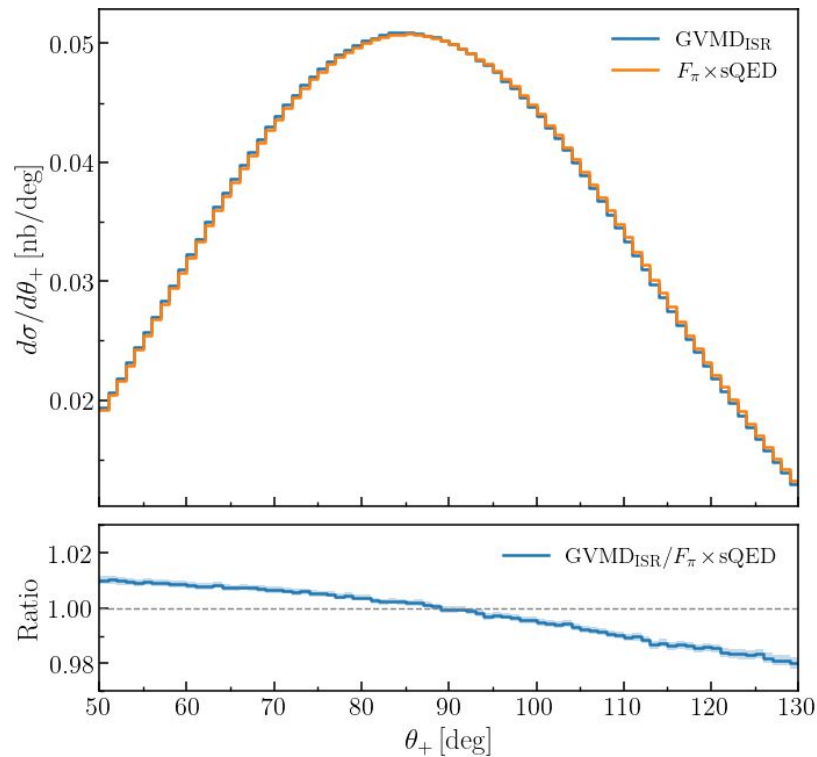
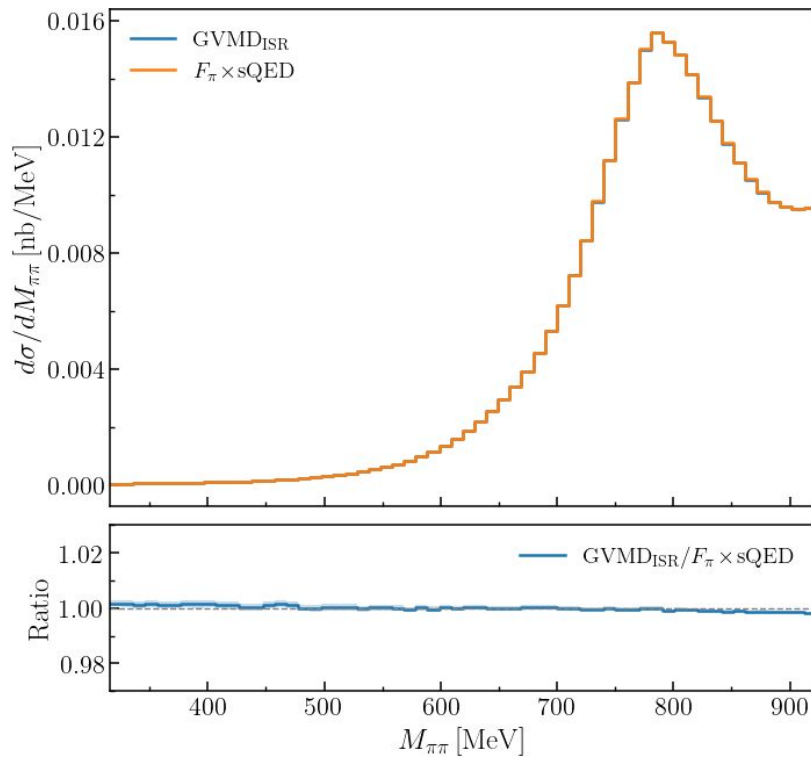
# Quick recap on GVMD

- The full pion Compton tensor is difficult to embed in loop calculations.
- Approximate (but accurate!) methods are needed for MC generators.
- In scan mode, **FxsQED** approach fails to capture the pion behaviour in TPE diagrams.
- What if we introduce the **FF in the Feynman rules?**
- How does this extend to radiative return?

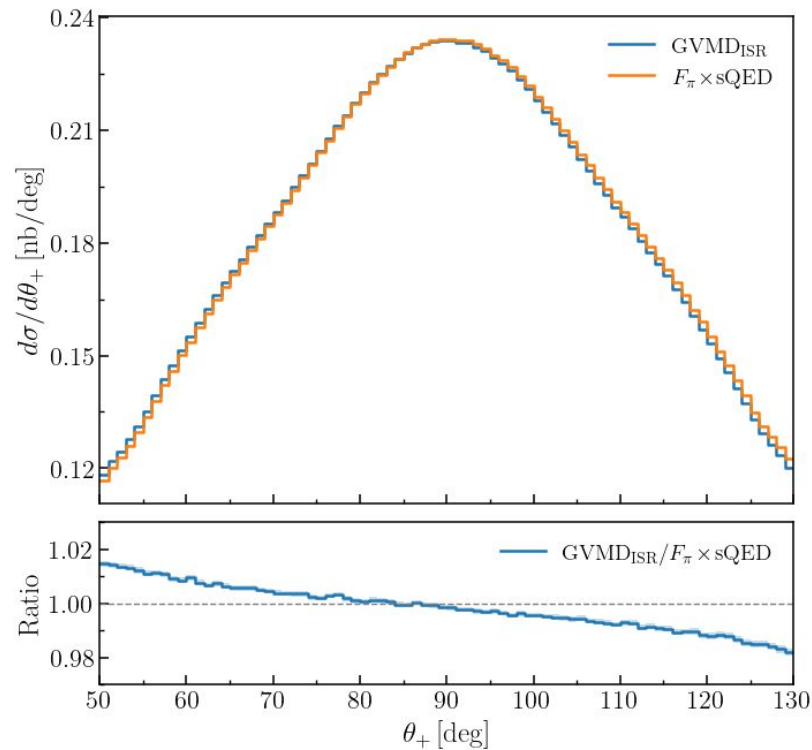
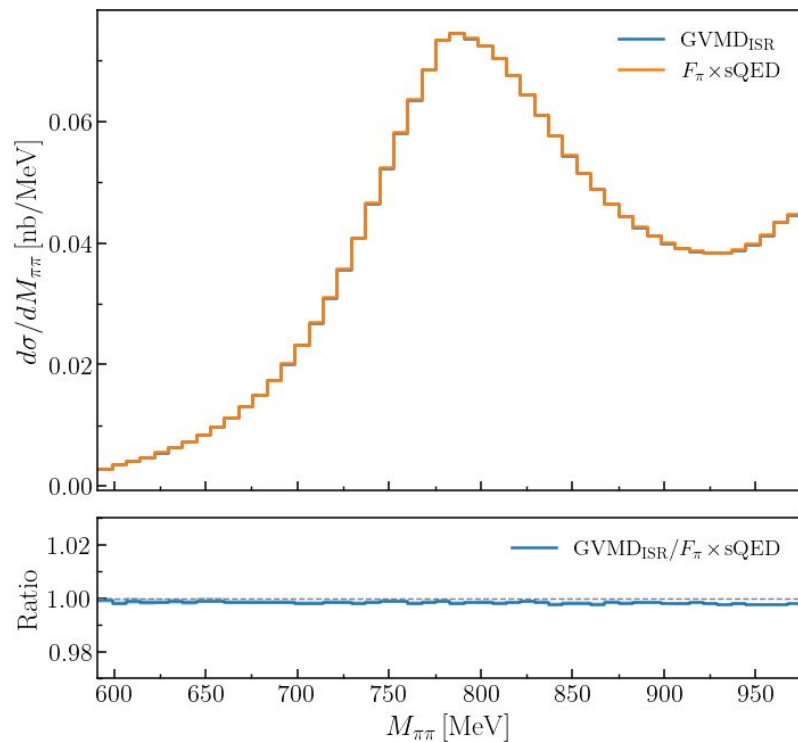




# Results: KLOE LA



# Results: KLOE SA



# The YFS framework

YFS FF (contains IR divergences)

Regularised amplitudes

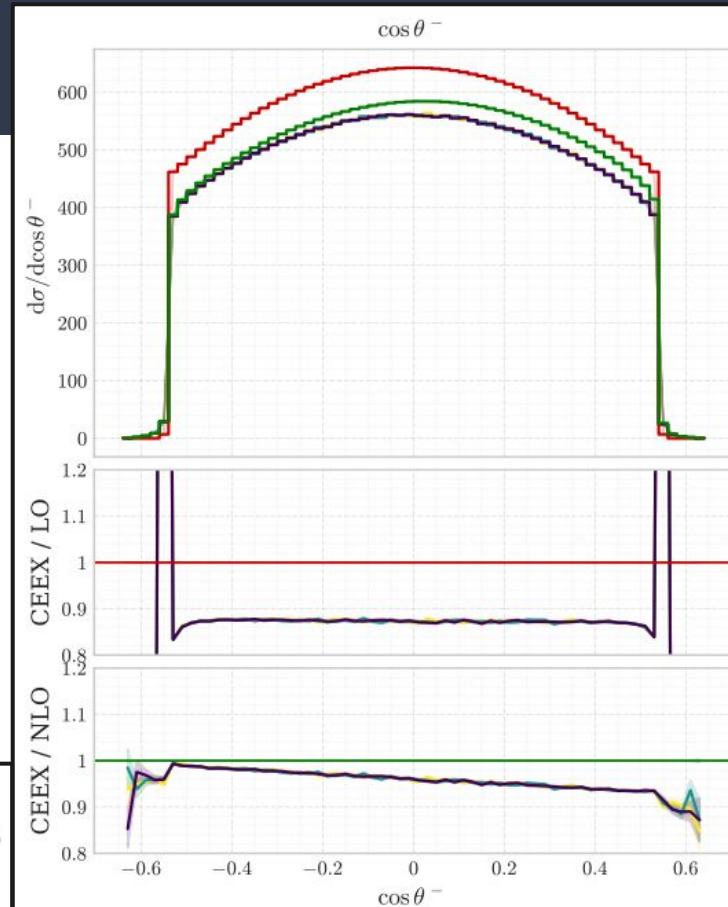
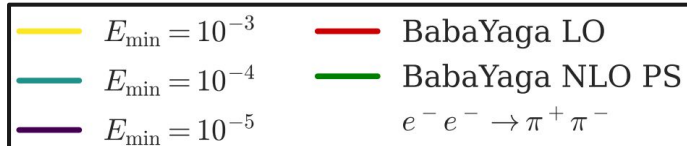
$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{\exp Y(\Omega)}{n_\gamma!} d\Phi_Q \frac{1}{4} \sum_h \left| \sum_\phi \left[ \prod_{i=1}^{n_\gamma} d\phi_i^\gamma S(k_i) \Theta(k_i, \Omega) \right] \left( \beta_0 + \sum_{j=1}^{n_\gamma} \frac{\beta_1(k_j)}{S(k_j)} + \sum_{1=j < k}^{n_\gamma} \frac{\beta_2(k_j, k_k)}{S(k_j)S(k_k)} + \dots \right) \right|^2$$

Partitions (ISR/FSR)
Eikonal factors
Resolved domain

- Resummation of all soft photon contributions to all orders.
- Ability to truncate and recover fixed order contributions
- No approximations!

# Scan mode at LO+YFS

- Implemented the YFS framework.
- Scan mode at LO+YFS and NLO available.
- Final stages of NLO+YFS implementation.
- Extension to radiative return channel also in the final stages.



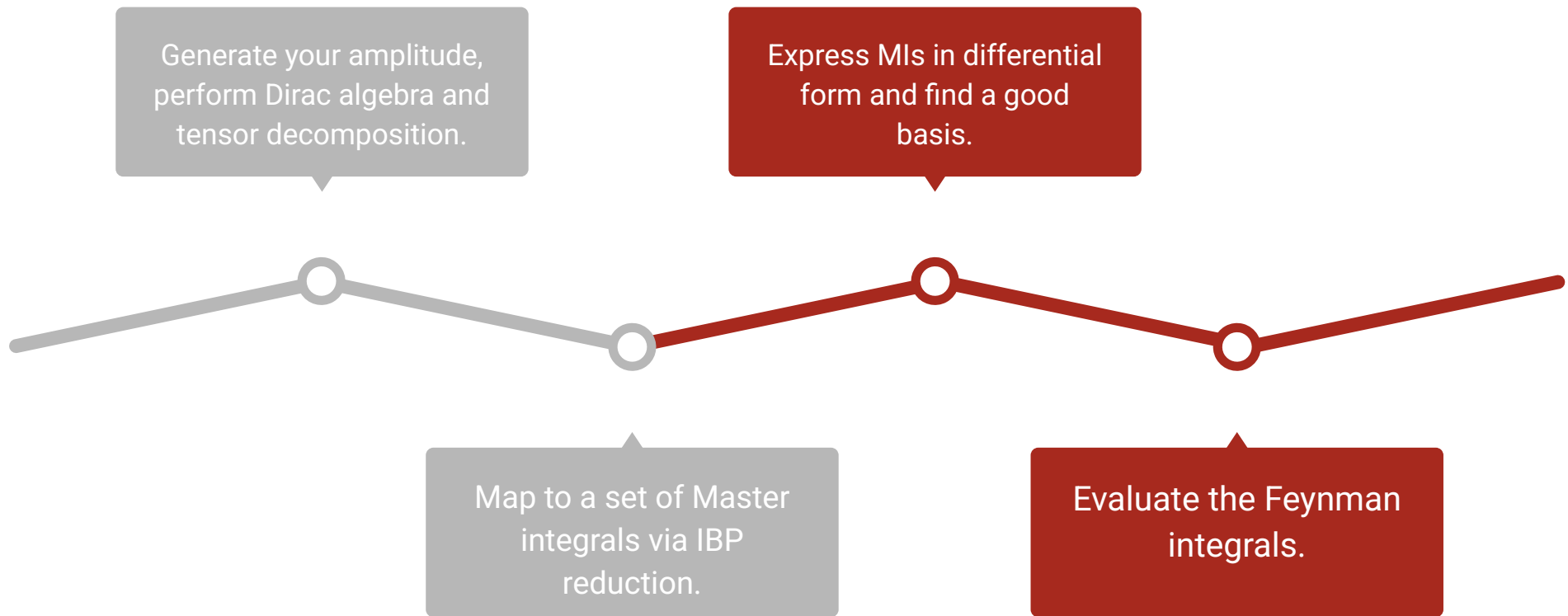
# The pipeline of NNLO

Generate your amplitude,  
perform Dirac algebra and  
tensor decomposition.

Express MIs in differential  
form and find a good  
basis.

Map to a set of Master  
integrals via IBP  
reduction.

Evaluate the Feynman  
integrals.



# The pipeline of NNLO

Generate your amplitude,  
perform Dirac algebra and  
tensor

Express MIs in differential  
form and find a good

	PB <sub>A</sub>	PB <sub>B</sub>	PB <sub>C</sub>	all MIs	special func. (all)	special func. (non-polylog.)
$\langle T \rangle$	43 s	77 s	66 s	309 s	297 s	16 s
$\sigma$	7 s	17 s	14 s	27 s	65 s	3 s

**Table 2:** Average time per segment and standard deviation for the solution of the DEs for the MIs of each 2-loop family, cumulatively for all 2-loop MIs, for all special functions, and for the non-polylogarithmic special functions alone. The time for all MIs keeps into account that PB<sub>A</sub> and PB<sub>B</sub> are needed in two permutations of the external legs, and that the time for the one-loop family (P<sub>A</sub>) is  $\approx 2s$ .

From [2412.13876]. Time per segment for the MIs of  $gg \rightarrow ttg$ .

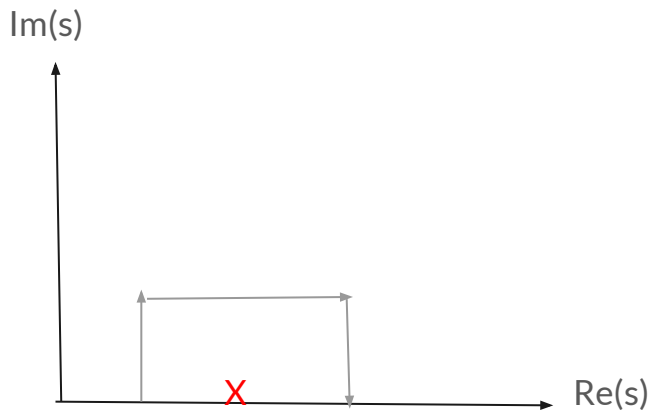
integrals via IBP  
reduction.

integrals.

man

# The differential equation method

- Feynman integrals can be cast into **differential form**.
- Highly non trivial to obtain a nice form, but
  - once obtained, nice **analytic structure**.
- Series expansion is then used with packages DiffExp, SeaSyde, Line.
  - Slow because Mathematica and method.
  - And, for  $2 \rightarrow 3$ , we **can't** have **interpolation grids**.



$$d\vec{J} = \sum_{k=0}^{\infty} \epsilon^k \left[ \sum_i \mathcal{A}_{k,i} d \log(\alpha_i(\vec{x})) + \sum_j \mathcal{B}_{k,j} \Omega_j(\vec{x}) \right] \vec{J}$$

# Two-Loop $pp \rightarrow tt + \text{jet}$

- Test integrator with two-loop one mass process. **88** and **121 MIs** respectively, expanded in **four orders** in epsilon.
- Differential equation built in [2404.12325]. No canonical form for PBb, but polynomial in dim. regulator.
- Fast in double precision, might not be enough.
- However, quad precision still **faster** than DiffExp and easy to parallelize generation of grids.

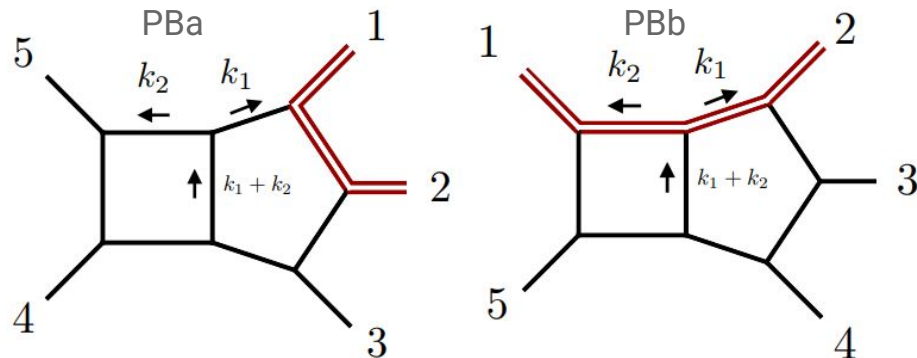
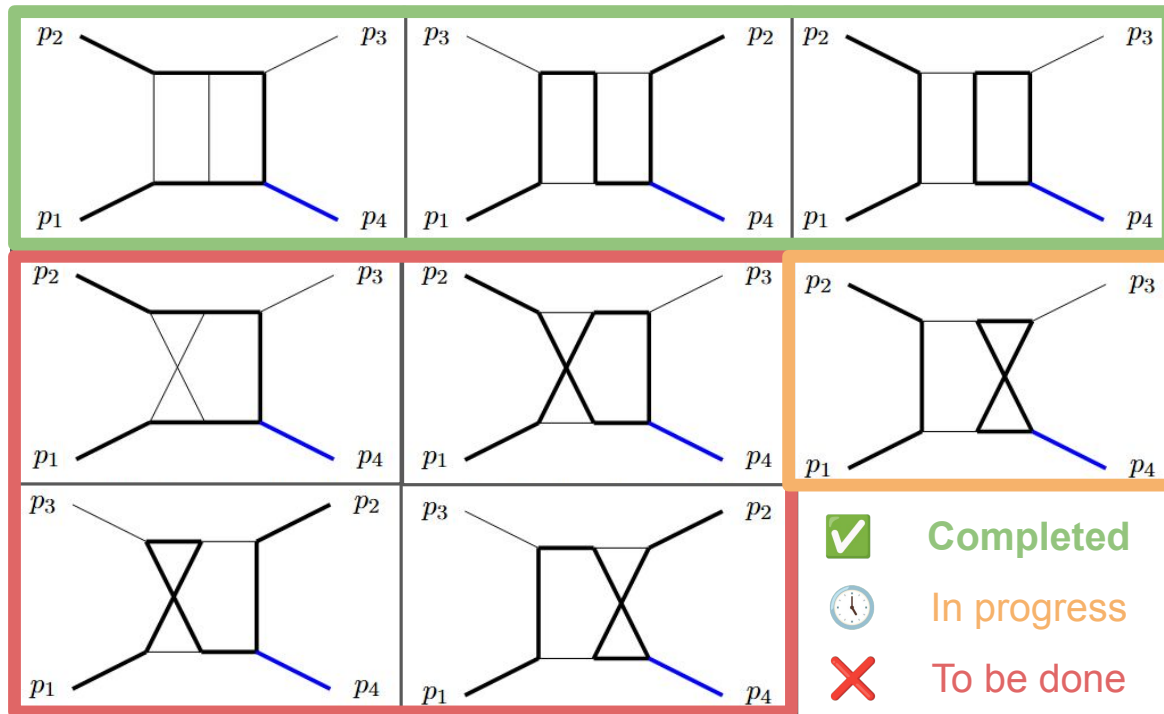


Figure from [2404.12325]

		$\mathcal{T}_A, \mathcal{T}_R$	$\mathcal{R}$	$\langle \tau \rangle$ [s]	DiffExp $\langle \tau \rangle$ [s]
PB <sub>A</sub>	double	$10^{-12}, 10^{-12}$	10	0.0881	580.85
	quad	$10^{-28}, 10^{-28}$	27	51.588	795.516
PB <sub>B</sub>	double	$10^{-12}, 10^{-12}$	10	0.100	555.438
	quad	$10^{-28}, 10^{-28}$	27	89.088	826.219

$$e^+e^- \rightarrow \gamma\gamma^* @ \text{NNLO}$$


Thanks to M. Pozzoli!

# Conclusions

- Calculated the GVMD corrections to radiative return and interfaced it with Phokhara.
  - GVMD captures percent level effects on angular differential xsections.
- YFS implementation in the final stages. Both scan and radiative return modes at NLO+YFS resummation will be available.
- Progress towards understanding the two loop process  $e^+e^- \rightarrow \gamma\gamma^*$ :
  - Developed a method capable of evaluating the needed Feynman integrals,
  - Available solutions for the planar families.

Thanks!