The strong coupling from hadronic tau decays including $\tau \to \pi^- \pi^0 \nu_\tau$ from Belle

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with Diogo Boito, Aaron Eiben, Maarten Golterman, Kim Maltman, and Santiago Peris

Based on

2502.08147 Boito, Eiben, Golterman, Maltman, Mansur, and Peris, *Phys. Rev. D 111 (2025) 7, 074010*
2012.10440 Boito, Golterman, Maltman, Peris, Rodrigues, and Schaaf, *Phys. Rev. D 103 (2021) 3, 034028*

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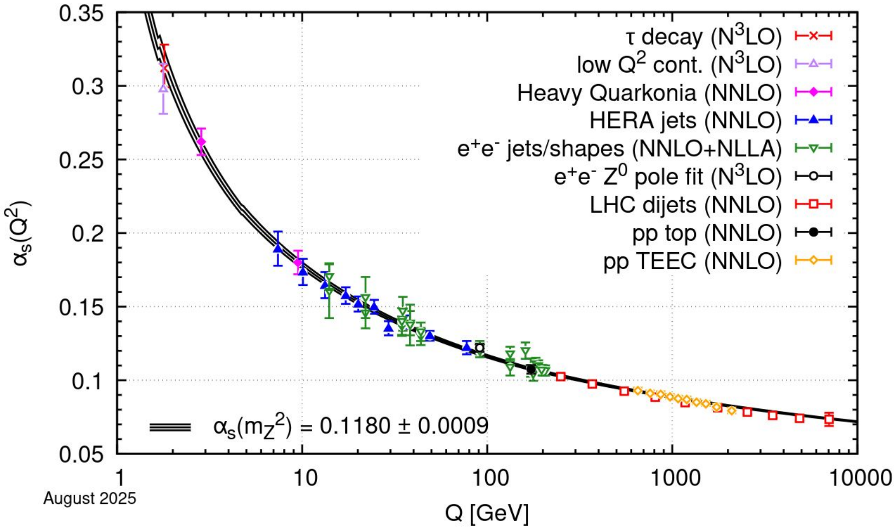
Strong coupling from hadronic tau decays



The **strong coupling**, α_s, is

1. One of the least precisely known parameters in the SM;
2. Key for hadronic physics: enters as input for **all** theoretical hadronic cross-sections and strong decays;
3. Determined through independent methods.

[PDG, 2024. *Phy. Rev. D* 110]

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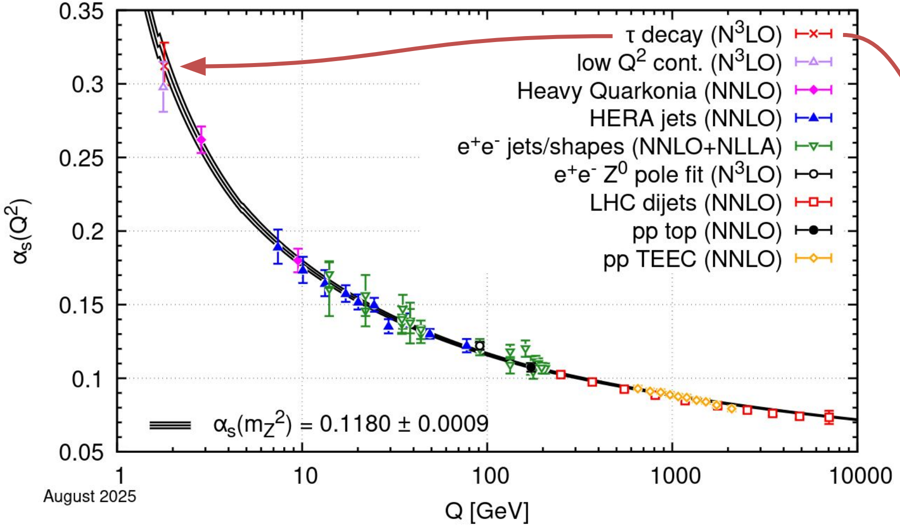
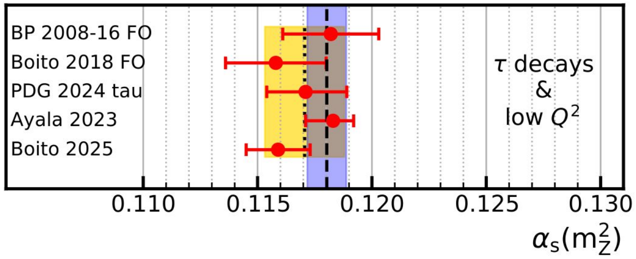
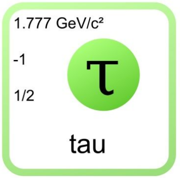


[Boito, et al. (2025) 2502.08147]





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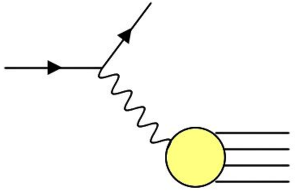
The extraction of the **strong coupling** using **hadronic tau decays** is

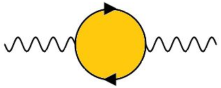
1. Important test of **asymptotic freedom** (low energy regime);
2. Larger coupling at lower energies: **more sensitivity to QCD corrections**, less precision required from experimental data;
3. Larger **non-perturbative contributions** (OPE, Duality Violations).

Excellent experimental and theoretical laboratory for **Quantum Chromodynamics!**

Theory overview

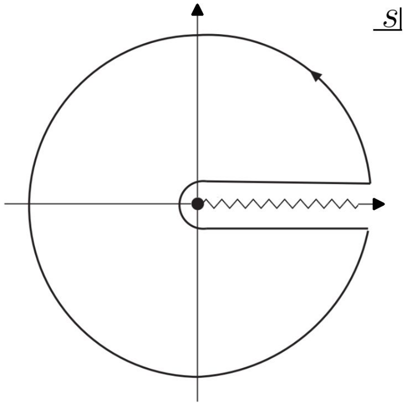


Hadronic tau decays are described by the **2-point *V(A)* correlation function**



$$\Pi_{\mu\nu}(q^2) = i \int d^4x\, e^{iqx}\, \langle 0|T\{J_\mu(x)J_\nu(0)^\dagger\}|0\rangle \qquad J_\mu = \bar{u}\gamma_\mu(\gamma^5)d\ \ldots$$

Can be studied using **Finite Energy Sum Rules (FESRs)**: [e.g., Braaten, Narison, Pich. (1992)]



From the Cauchy's Theorem in the complex plane

General analytic (polynomial, in practice) function in *s*

$$\frac{1}{s_0}\int_0^{s_0} ds\; w_i(s)\rho^{(1+0)}(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} \frac{ds}{s_0}\; w_i(s)\Pi^{(1+0)}(s)$$

EXPERIMENTAL DATA **THEORY**

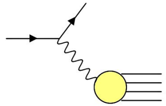
- FESRs connects the **experimental information** with the **theoretical description** of correlators.
- The tau width is obtained from $w_\tau = (1-x)^2(1+2x)$
- Here we focus on the **light-quark current**. Tiny mass corrections can then be neglected.
- **No** net strangeness in final states.
- We extract the coupling from the **vector-isovector channel**.

Theory overview

On the **theoretical side**, FESRs in hadronic tau decays receive four contributions

$$\int_0^{s_0} \frac{\mathrm{d}s}{s_0}\, w_i(s/s_0)\, \rho^{(1+0)}_{V/A;ud}(s) = \frac{N_c}{24\pi^2}\left[\delta^{(\mathrm{tree})}_{w_i}(s_0) + \delta^{(0)}_{w_i}(s_0) + \sum_{D\geq 4} \delta^{(\mathrm{OPE})}_{w_i} + \delta^{(\mathrm{DV})}_{w_i}\right]$$

1) Partonic contribution: $\delta^{(\mathrm{tree})}_{w_i} = \frac{2}{s_0}\int_0^{s_0} \mathrm{d}s\; w_i(s/s_0).$



Tree level computation

2) Perturbative contribution:

$$\delta^{(0)}_{\mathrm{FO},w_i} = \sum_{n=1}^{\infty} b_n^{(w_i)}\, a^n(s_0)$$

Parameters: $\alpha_S(s_0)$

[Gorishnii, Kataev, Larin. (1991)] [Surguladze, Samuel. (1991)] [Baikov, Chetyrkin, Kühn. (2008)]

$$\begin{array}{ccccc} & \alpha_s^1 & \alpha_s^2 & \alpha_s^3 & \alpha_s^4 \end{array}$$

$$\delta^{(0)}_{w_\tau} = 0.1001 + 0.0521 + 0.0264 + 0.0127 = 0.1914$$

(fixed order perturbation theory for R_τ, $\alpha_s(m_\tau^2) = 0.3144$)

- Perturbative series in the **strong coupling** .
- Corresponds to a correction of ~**20%** to the FESR.
- In this work, we will employ **Fixed Order Perturbation Theory** (**FOPT**) only! CIPT is **incompatible** with the standard form of OPE condensates.

[Hoang, Regner. (2020)(2021)] [Golterman, Maltman, Peris. (2023)]

- 5th order estimate to supplement known 4th order result: [Beneke, Jamin. (2008)] [Boito, Masjuan, Oliani. (2018)]

[Caprini. (2019)]

$$c_{5,1} = 283 \pm 140$$

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3) OPE condensates contribution: $\delta^{(\mathrm{OPE},D)}_{w_i} = \frac{6\pi i}{(-s_0)^{D/2}} \oint dx \frac{w_i(x)}{x^{D/2}} C_D(x s_0)$ ⇨ **Parameters**: C_D *(D-dimension OPE Condensates).*

$$\delta^{D\geq 4}_{\mathrm{OPE}} = c^{(4)}_w \frac{C_4}{Q^4} + c^{(6)}_w \frac{C_6}{Q^6} + c^{(8)}_w \frac{C_8}{Q^8} + \cdots$$

4) Duality Violations (DV): $\delta^{(\mathrm{DV})}_{w_i} = -\frac{24\pi^2}{N_c s_0} \int_{s_0}^{\infty} ds\, w_i(s) \rho^{DV}_{V/A}(s)$

We rely on an asymptotic ansatz for DV contribution to the spectral function:

$$\rho^{DV}_{V/A}(s) = \frac{1}{\pi} \mathrm{Im}\Delta_{V/A}(s) = e^{-\delta_{V/A} - \gamma_{V/A} s} \sin(\alpha_{V/A} + \beta_{V/A} s)$$

oscillation and exponential decay

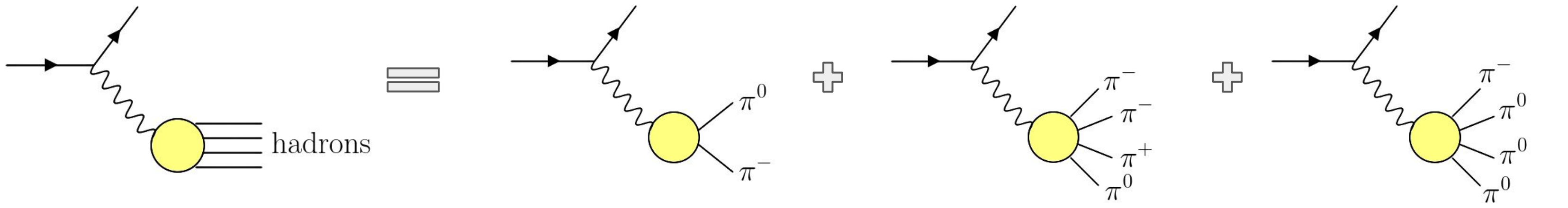
Parameters: *DV's (α, β, γ, δ).*

- Ansatz based on widely accepted assumptions about QCD: **asymptotic Regge behavior** of the spectrum and **large-Nc**.
- **Main expected large-s corrections**: logarithmic and powers of 1/s.

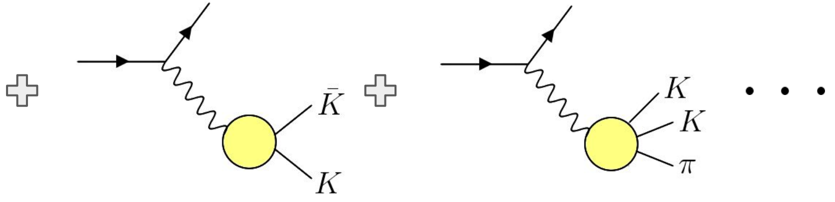
[Boito, Caprini, Golterman, Maltman, Peris. (2018)]

Experimental data sets

On the **experimental side**, the tau lepton decays into **several** hadronic modes



Dominant Modes: **2π channel** ($\pi^- \pi^0$) and **4π channel** ($\pi^- 3\pi^0 + 2\pi^- \pi^+ \pi^0$).



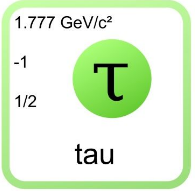
Residual / Sub-dominant Modes:

KK_S, $KK\pi$, $KK\pi\pi$,
6π, $\eta\pi^-\pi^0$, $\eta 4\pi$,
$\pi^- \omega(\rightarrow \text{non } 3\pi)$,
$(3\pi)^- \omega(\rightarrow \text{non } 3\pi)$,
$\pi^- \eta\omega(\rightarrow \text{non } 3\pi)$,
…

What do we need for the **strong coupling?**

A **fully inclusive** vector-isovector spectral function data set!

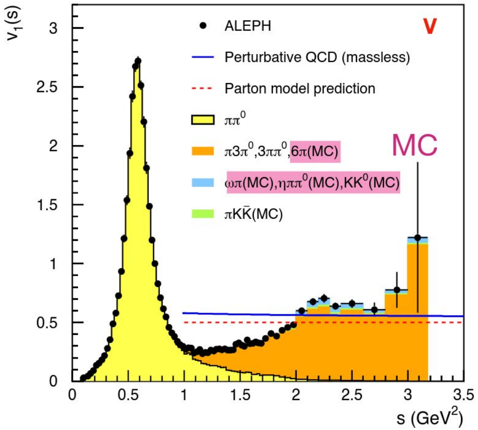
Experimental data sets



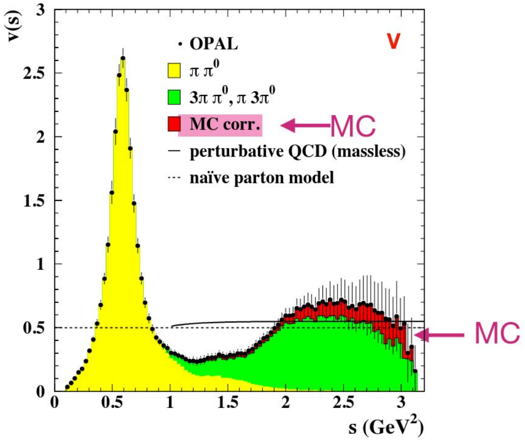
Available experimental data sets for *vector-isovector* channels:

- 2π Channel : **ALEPH**, **OPAL**, **Belle** and **CLEO** experiments
- 4π Channels: **ALEPH** and **OPAL** experiments
- Residual Channels: Babar $\tau \to K K_S \nu_\tau$ and **electroproduction data**

[ALEPH Collaboration. (2014)]

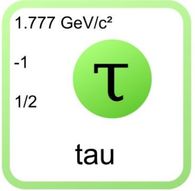


[OPAL Collaboration. (1998)]



- **ALEPH** and **OPAL** collaborations used **Monte Carlo data** as input for **several subdominant** (residual) modes.
- High-statistics data set from **Belle** for the 2π channel.
- **No access** to **CLEO** systematic correlation information.

Experimental data sets



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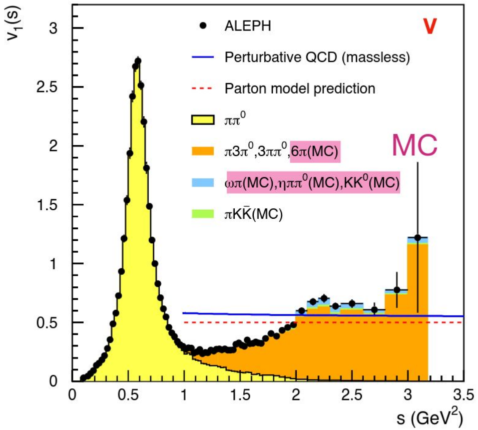

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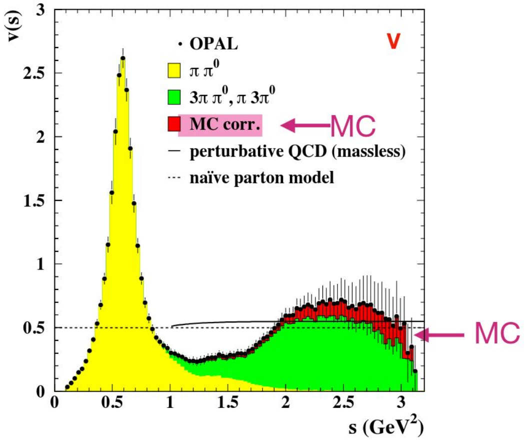

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GOAL

Construct an **improved** non-strange vector-isovector spectral function based **solely** on experimental data.

[Boito, et al. (2025) 2502.08147]

New algorithm to perform a channel-by-channel data combination for the dominant modes:

- 2π channel combination of **ALEPH**, **OPAL**, and the high-statistics **Belle** data set. [KNT19. (2018)]
- 4π channels combination of **ALEPH** and **OPAL** data sets, with a statistical treatment for non-χ^2 fits, and correction for d'Agostini bias.

[Bruno, Sommer. (2023)] [NNPDF Collaboration. (2009)]

See **Extras - Backup** for data combination procedure.

Residual channels obtained using Babar $\tau \to K K_S \nu_\tau$ and **electroproduction data**, related by CVC (results were already employed in previous work).

Experimental data sets

1.777 GeV/c²
-1
1/2
τ
tau

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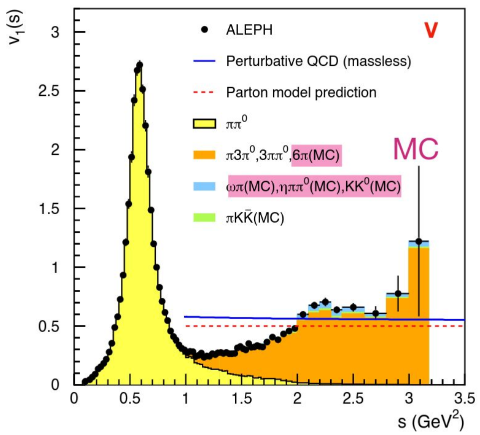

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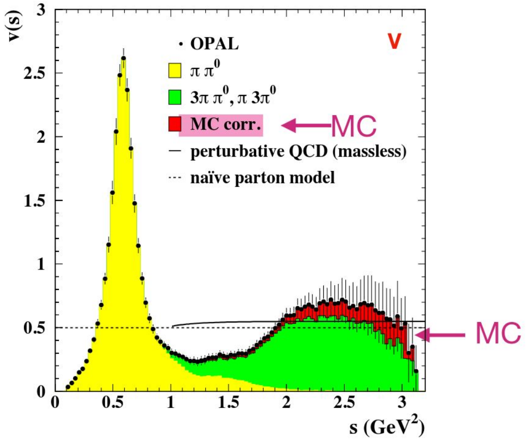

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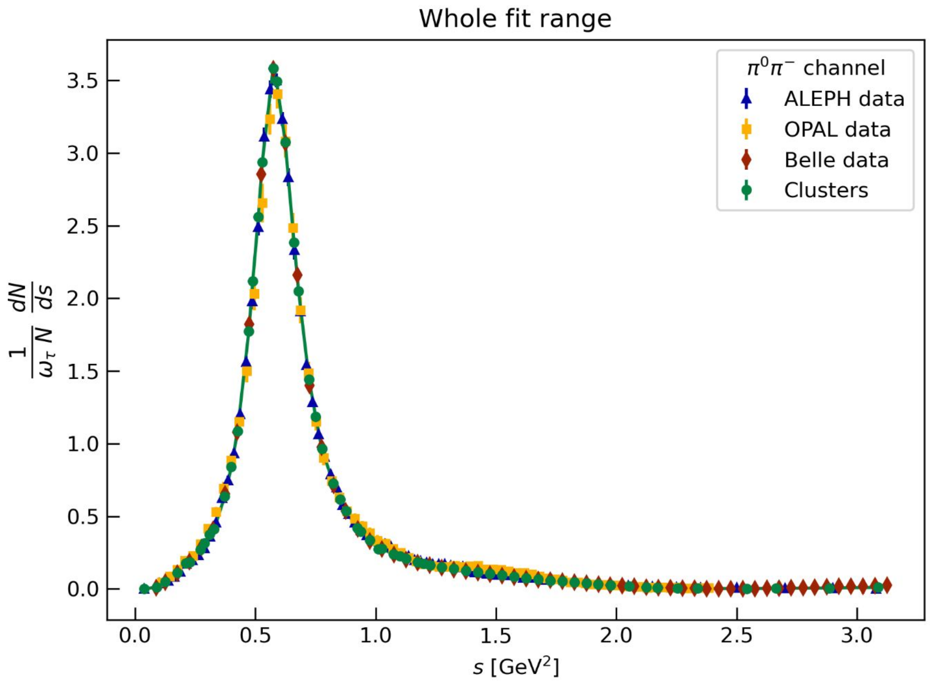
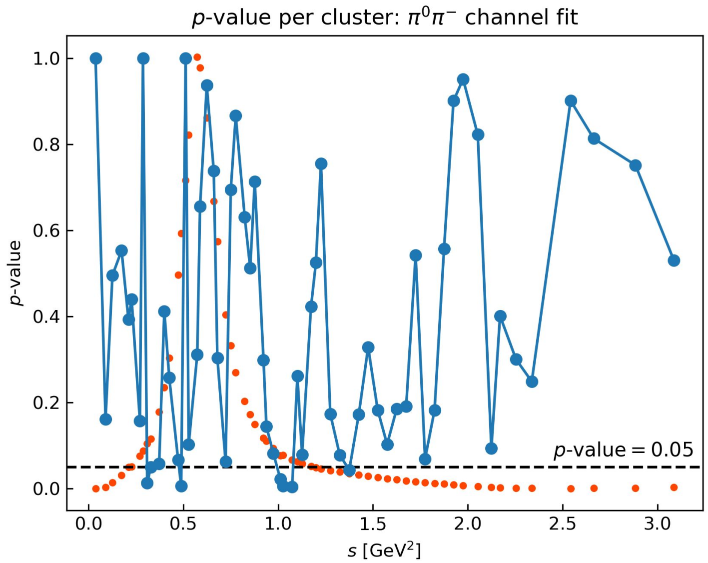
Impact of **CLEO** data set studied in detail separately (**Extra - Backup**).

2π channel combination

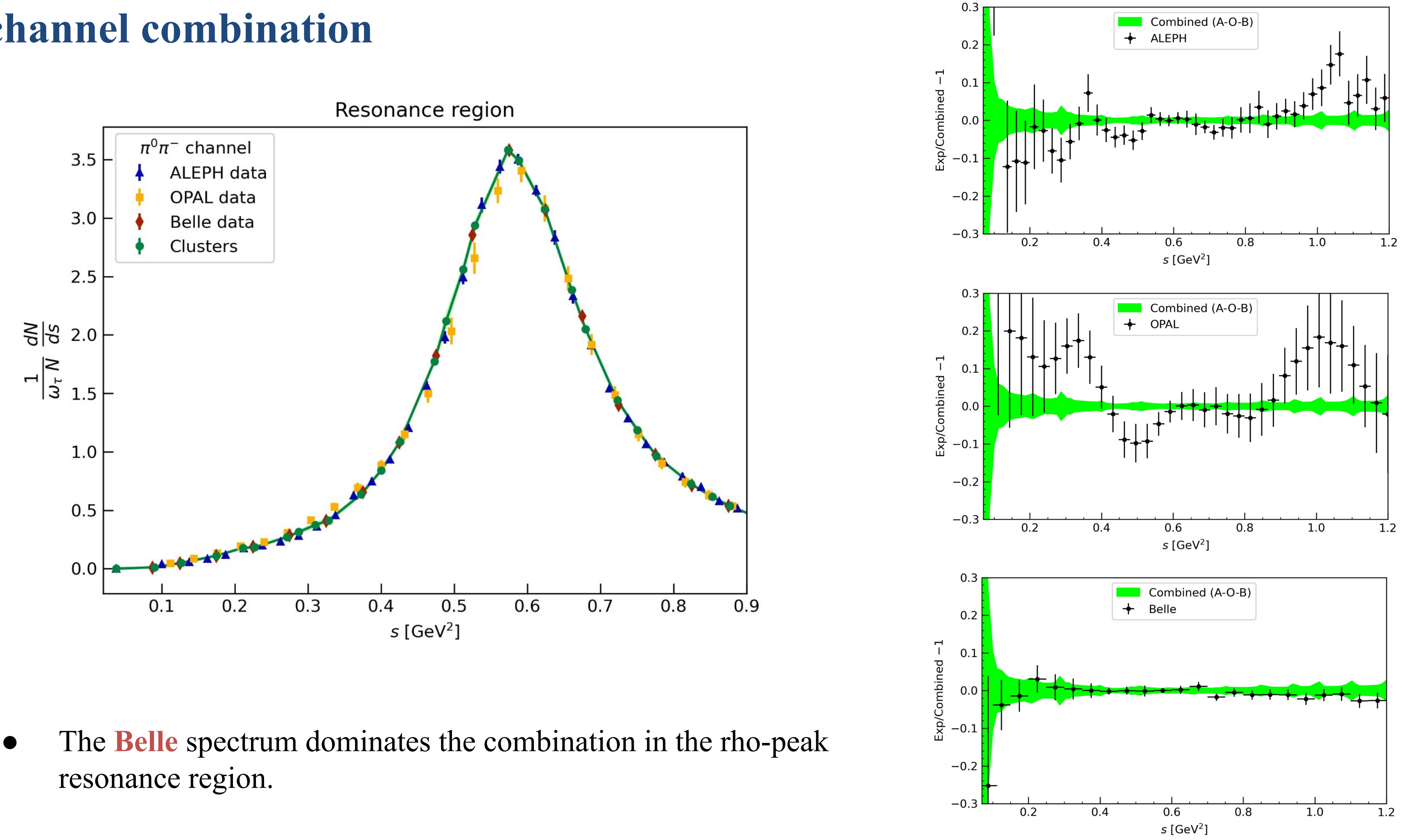




- For this channel: standard χ^2 fit combination with full covariance matrix input.
- Our data combination in this channel produced **good local *p*-values** (only 3 clusters with *p*-value < 0.01).
- Impact of error inflation is **minimal.**

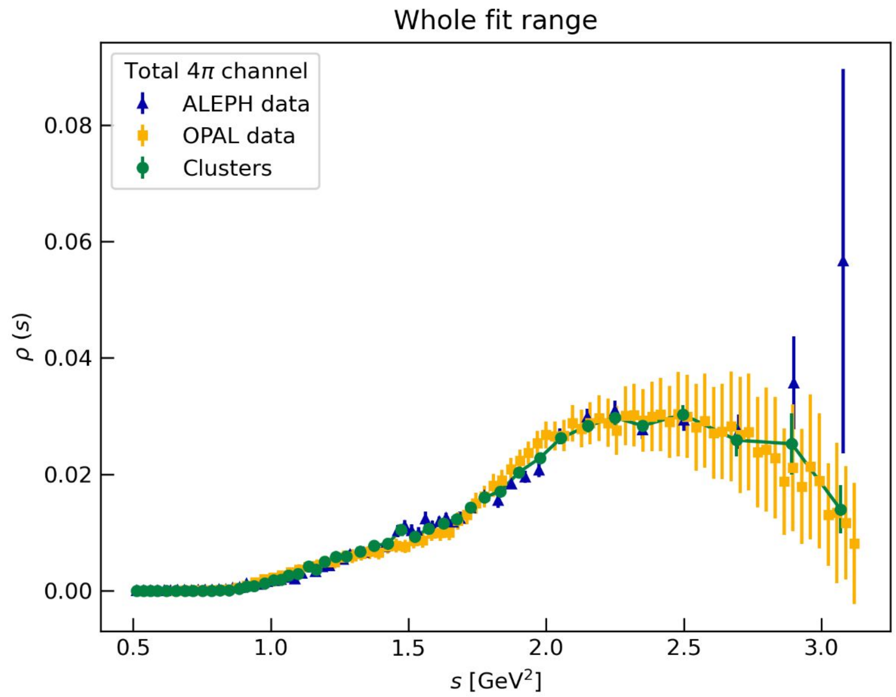
The **combined (green)** unit-normalized distribution from the **ALEPH**, **OPAL**, and **Belle** experiments.

2π channel combination



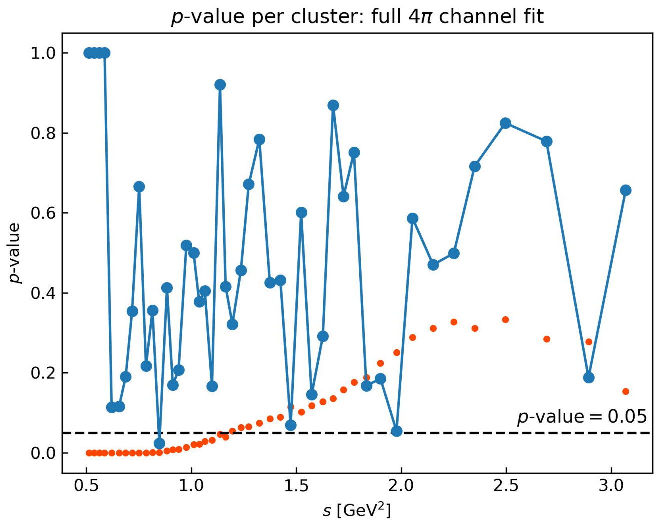
- The **Belle** spectrum dominates the combination in the rho-peak resonance region.

4π channel combination



The **combined (green)** spectral function from **ALEPH** and **OPAL**, using the updated values of branching ratios from HFLAV 2024.

[Banerjee, et al. HFLAV group. (2024)]

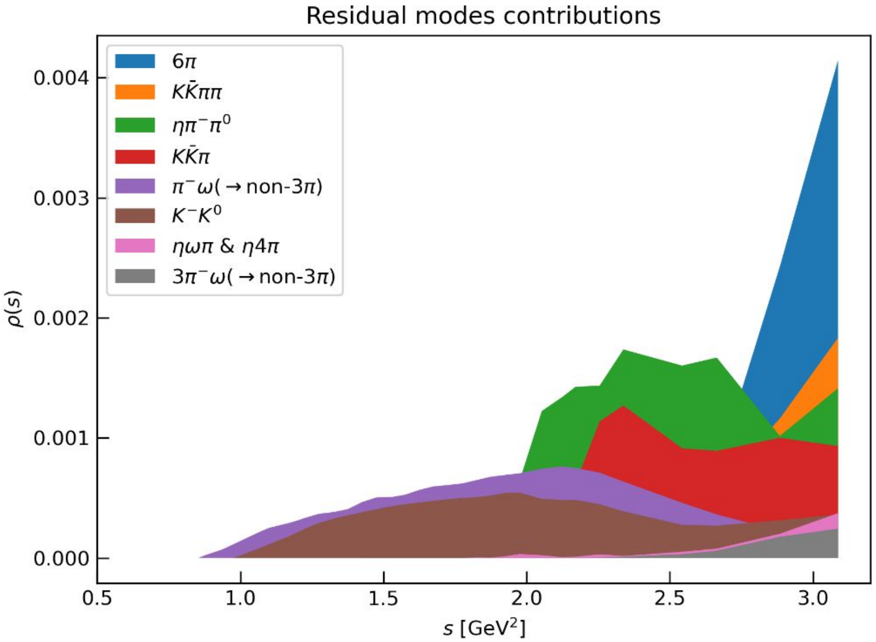


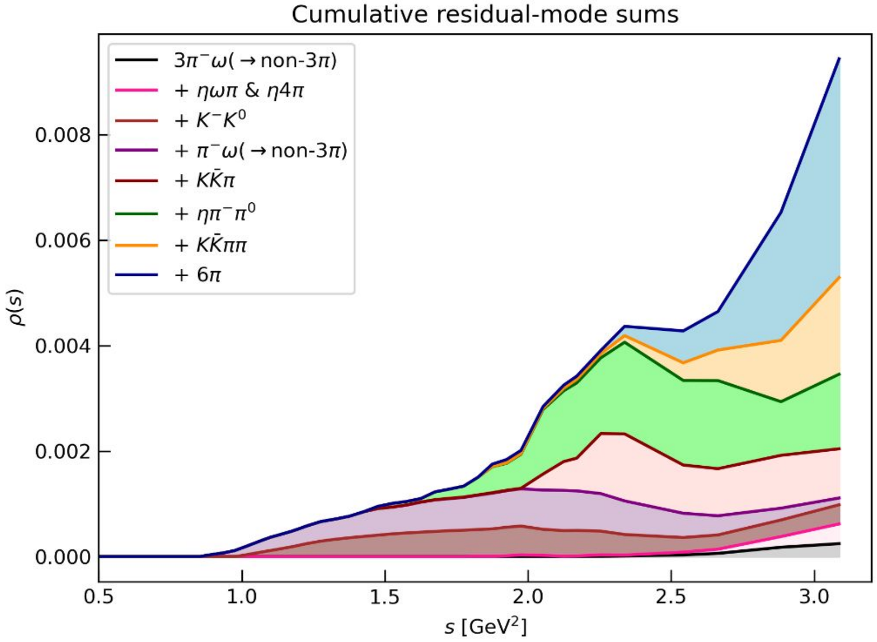
- Due to poorly behaved covariance matrices, only possible to combine **total 4*π* mode**.
- Problems with strong correlations in **OPAL** $\pi^- 3\pi^0$ channel => not included in the minimization, but still used in the error propagation.
- Our data combination in this channel produced **good local *p*-values** (only one cluster with *p*-value < 0.05). See **Extras - Backup** for details.

Residual channels [Boito, et al. (2020) 2012.10440]

- 7 residual modes extracted from e^+e^- data (+CVC) + Babar data for $\tau \to K K_S \nu_\tau$
- Dramatic improvement in errors for higher multiplicity modes — **higher *s*-values**
- Isospin breaking corrections on already small residual modes (1 - 2%) would not affect final results

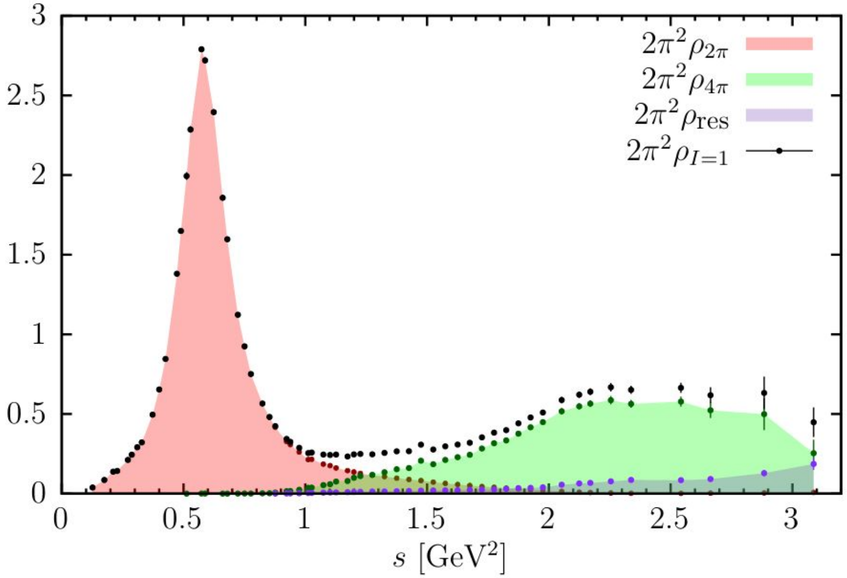
NO MONTE CARLO INPUTS

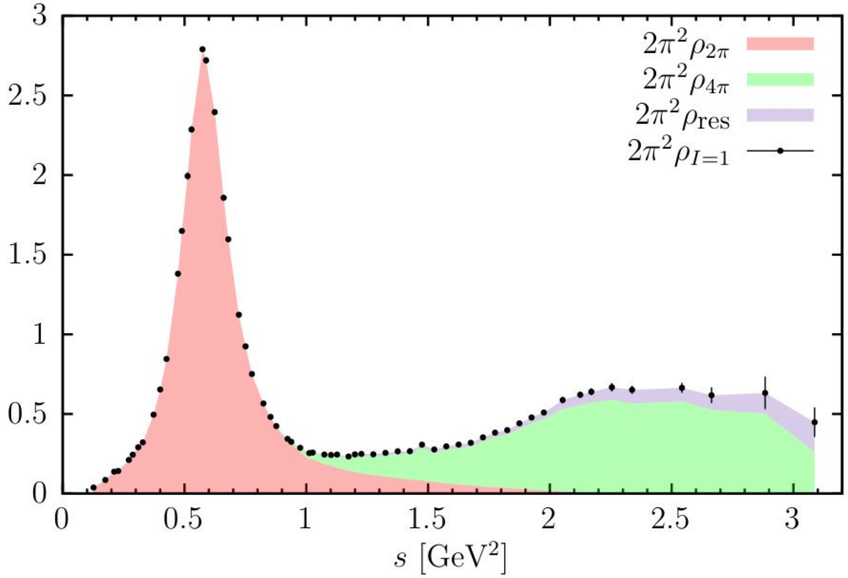




Final spectral function

- Final spectral function (**2π + 4π + Residual Modes**) constructed with **all** the relevant correlations between branching ratios taken into account. [Boito, et al. (2025) 2502.08147]





- The errors for high s-values in the spectrum are basically coming only from **4π modes.**

Fitting strategy — extraction of strong coupling and other parameters

$$\int_0^{s_0} \frac{ds}{s_0}\, w_i(s/s_0)\, \rho^{(1+0)}_{V/A;ud}(s) = \frac{N_c}{24\pi^2}\left[\delta^{(\mathrm{tree})}_{w_i}(s_0) + \delta^{(0)}_{w_i}(s_0) + \sum_{D\geq 4} \delta^{(\mathrm{OPE})}_{w_i} + \delta^{(\mathrm{DV})}_{w_i}\right]$$

Parameters: $\alpha_S(s_0)$, C_D *(D-dimension OPE Condensates), DV's (α, β, γ, δ).*

Any analytical weight function can be used in the FESRs (normally polynomials are used)

Desired properties from the choice of weight functions:

1. Good perturbative behavior
2. Small condensate contributions
3. Suppression of DVs

Choice of weights in this work:

$w_0(y) = 1$	Tiny condensate contributions, sensitive to DVs
$w_2(y) = 1 - y^2$	$D = 6$
$w_3(y) = (1-y)^2(1+2y)$	$D = 6$ and 8: Tau kinematical Moment (R_τ)
$w_4(y) = (1-y^2)^2$	$D = 6$ and 10

Extensively studied in the past:

[Boito, Cata, Golterman, Jamin, Maltman. (2011)]
[Beneke, Boito, Jamin. (2012)]
[Boito, Golterman, Maltman, Peris. (2016)]
[Boito, Oliani. (2020)]
[Boito, Golterman, Maltman, Peris. (2024)]

OPE condensate, DV contributions sub-leading cf. D=0 at s_0 of the analysis; model-dependent DV terms suppressed for doubly pinched w_3, w_4 (y).

Other strategies also exists in literature (Truncated OPE - tOPE). [Pich, Rodriguez-Sanchez. (2016)] [Davier, Hoecker, Malaescu, Yuan, Zhang. (2013)]

Truncated OPE contains problems recently unraveled in **Boito et al. 2402.05588 (PRD):** extracts strong coupling from perturbation theory **only,** with problems in the OPE coefficients obtained from tOPE fits .

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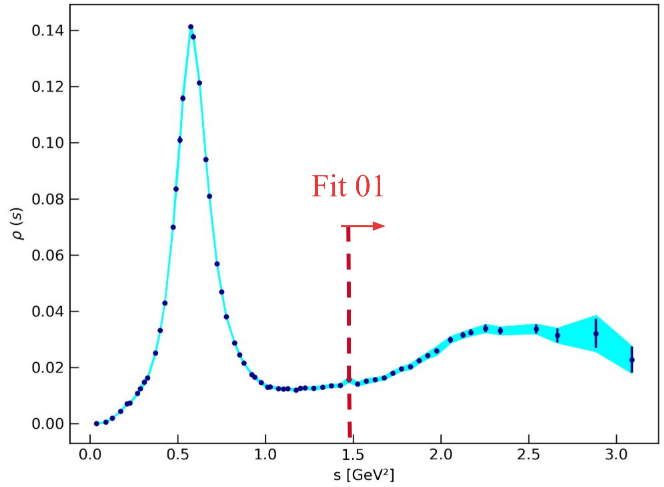

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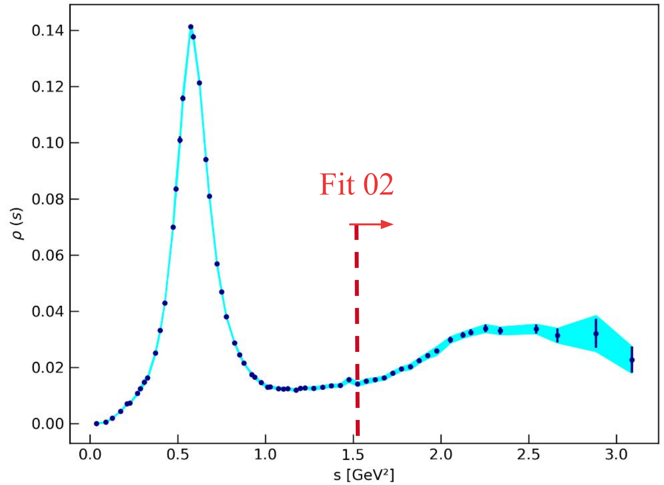

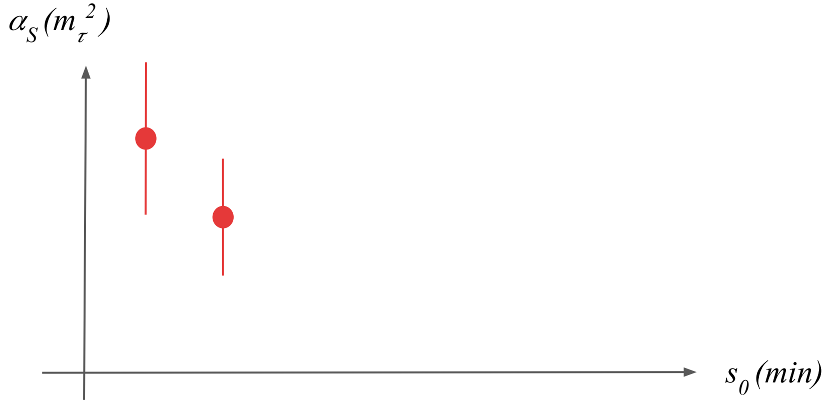

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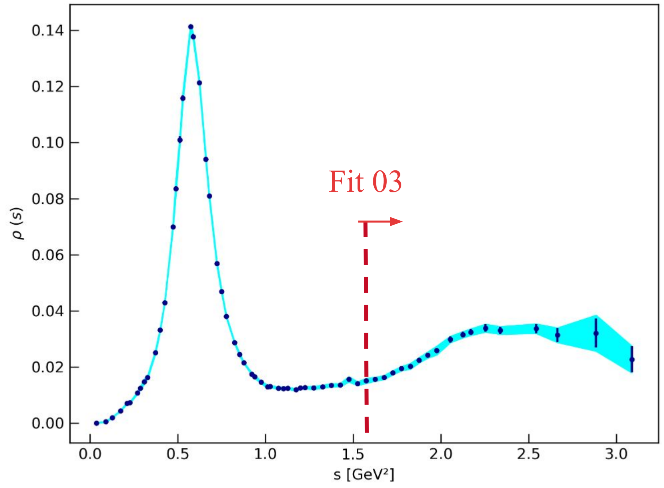

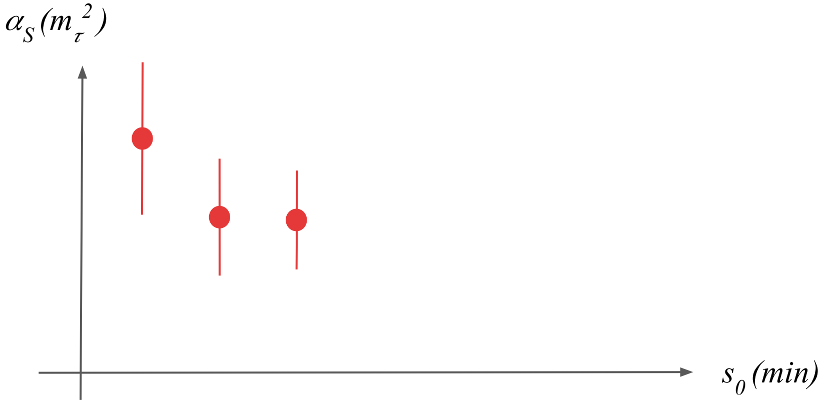

Fitting strategy — extraction of strong coupling and other parameters

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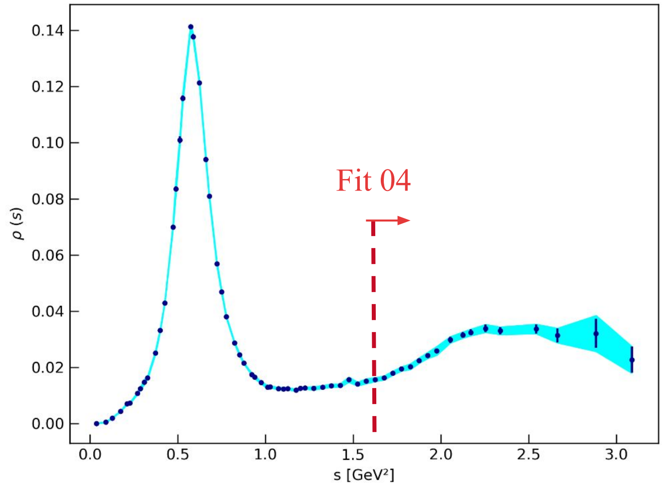

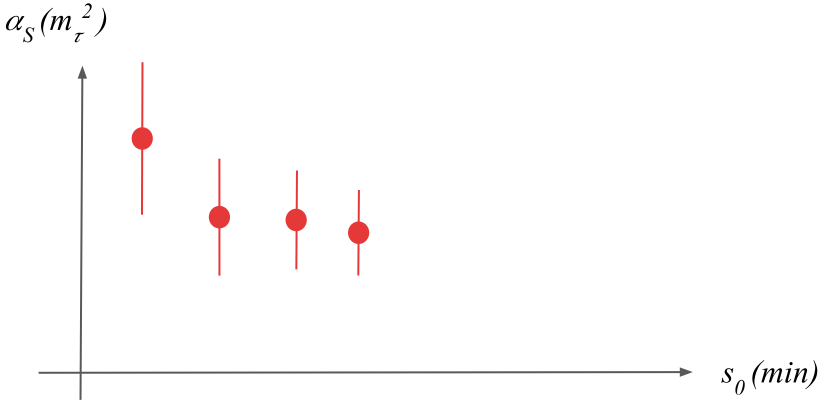

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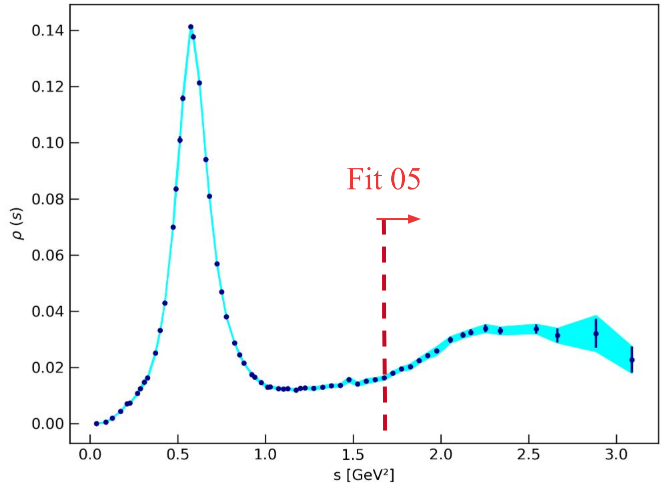

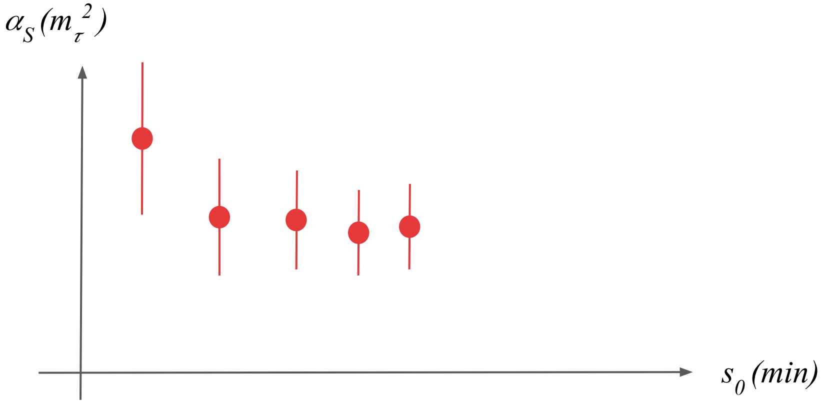

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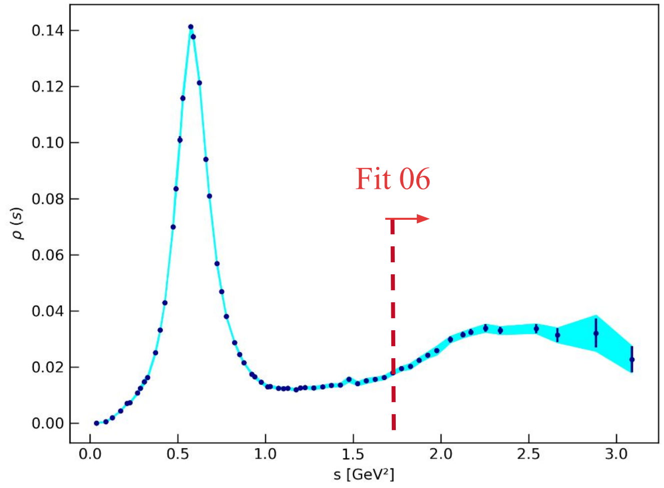

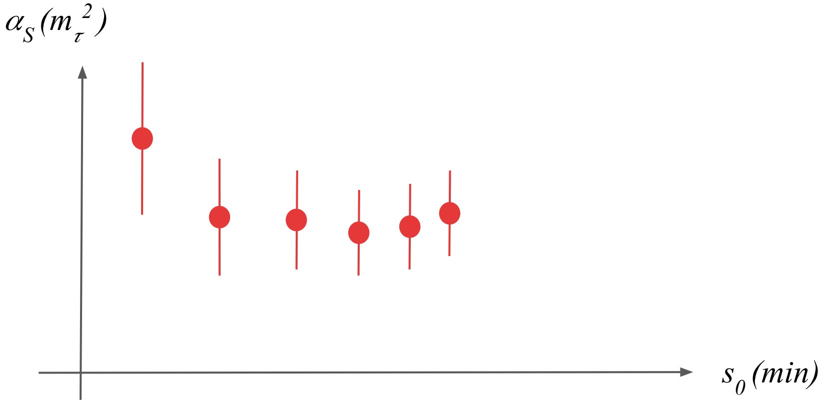

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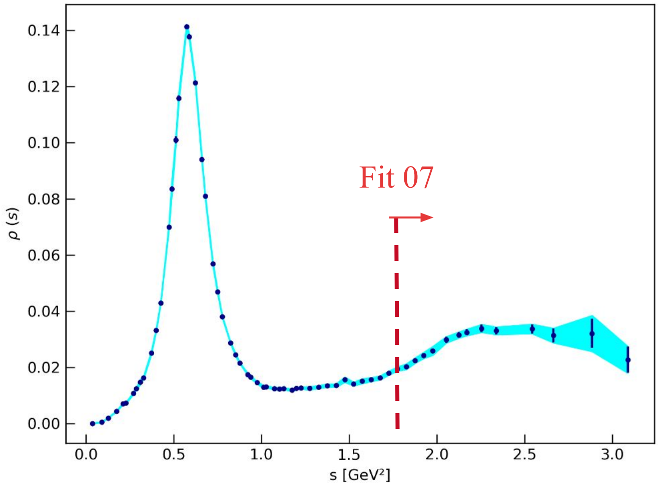

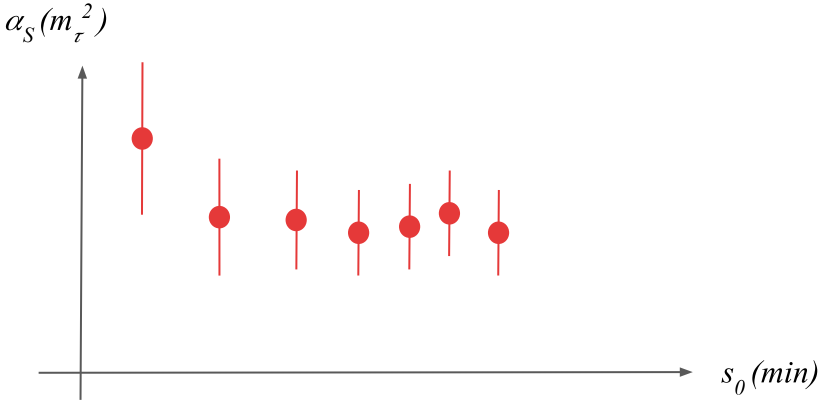

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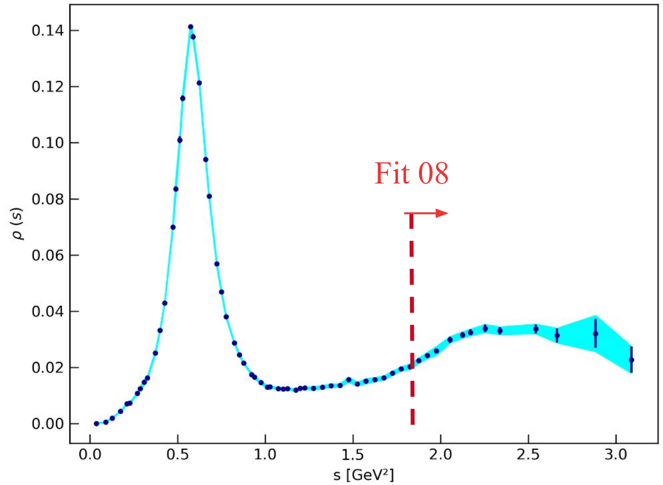

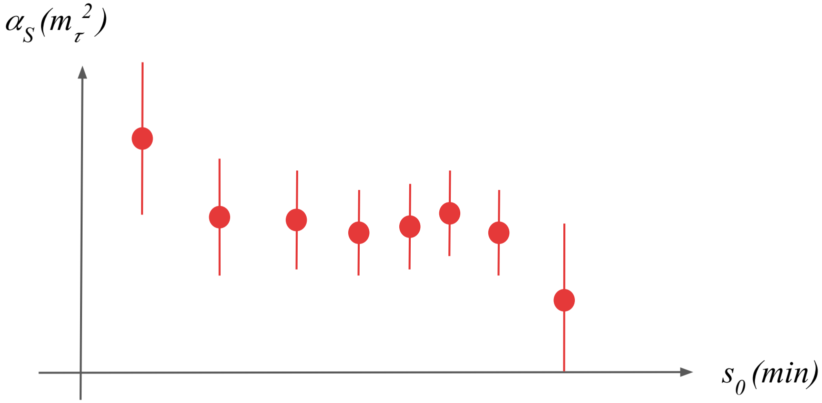

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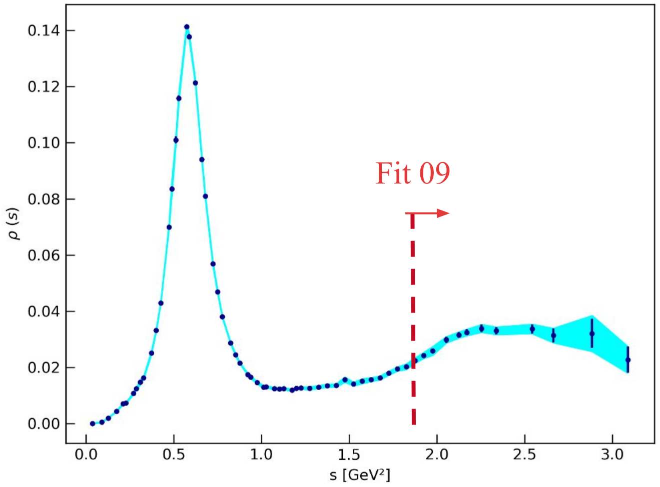

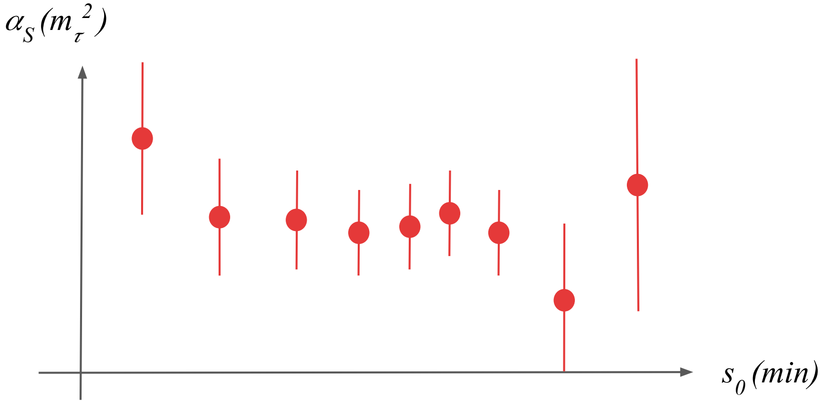

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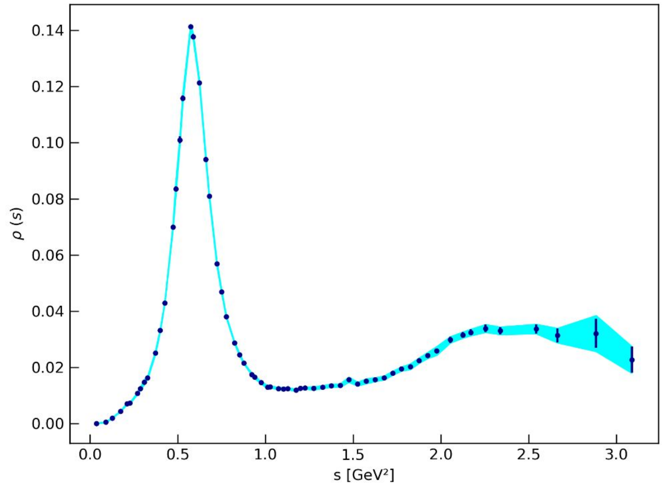
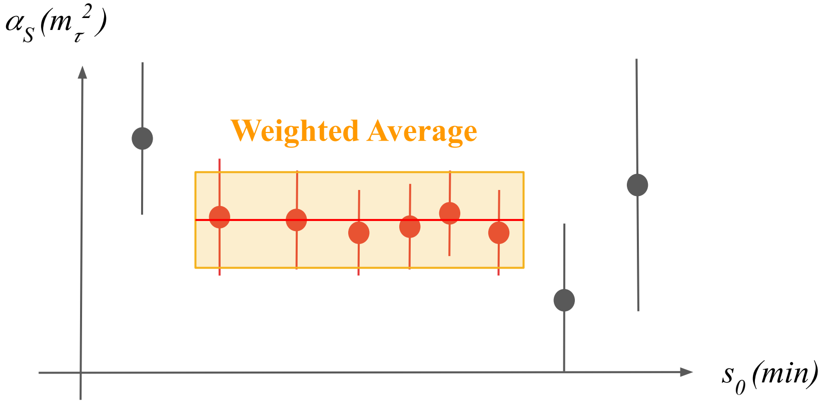
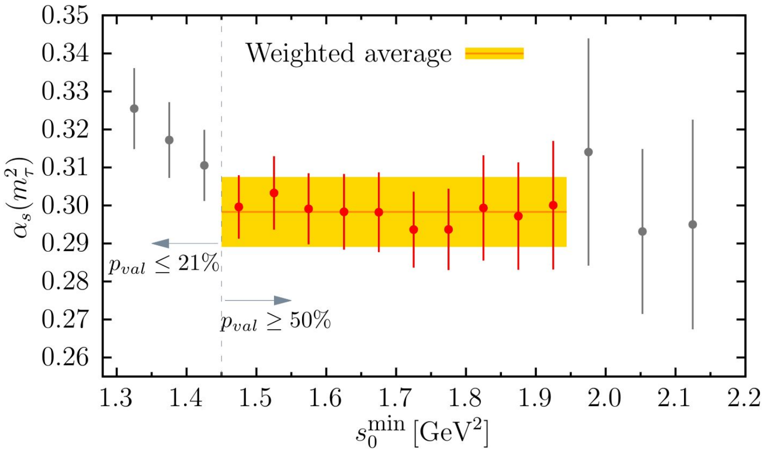




- <u>Final result</u> from a **weighted average**, with correlations taken into account in error propagation.
- Fitted region chosen based on **good** *p*-value of the fits, and **stability** of the parameters.
- **DV's parameters** determined using the <u>same</u> fit window.

Strong coupling from new spectral function

- Several fits, single moments or in combination
- Many fit windows: $[s_{\min}, m_\tau^2]$
- Consistency between different fits (α_S, condensates, DV parameters).



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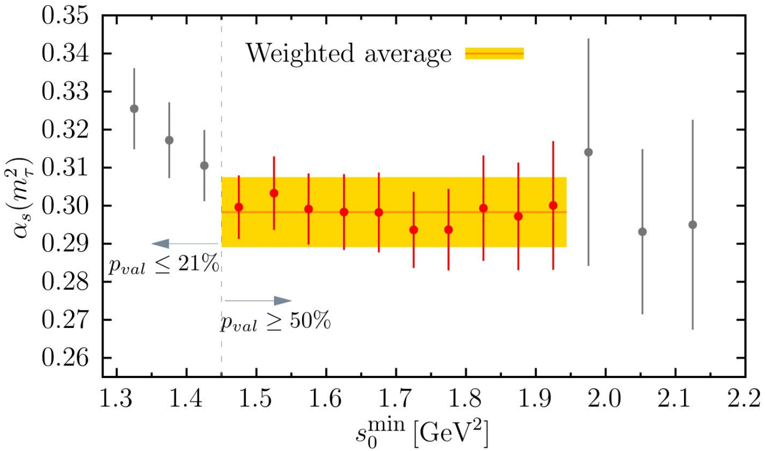
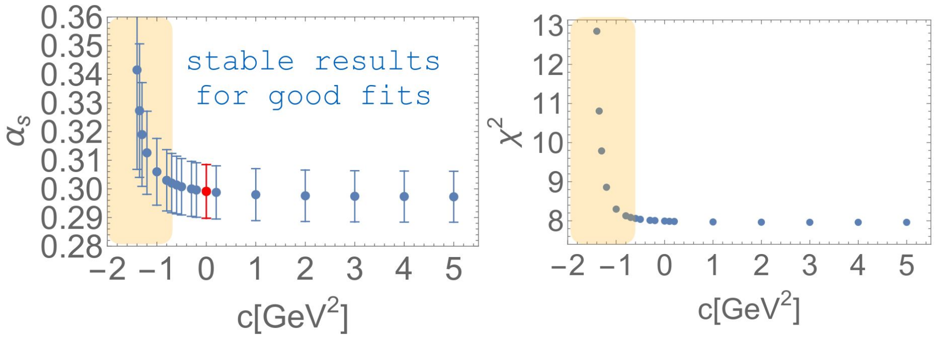
[Boito, Caprini, Golterman, Maltman, Peris. (2018)]

[Pich, Rodriguez-Sanchez. (2022)]

$$\rho^{\rm DV}(s) = \left(1 + \frac{c}{s}\right) e^{-\delta-\gamma s} \sin(\alpha + \beta s)$$





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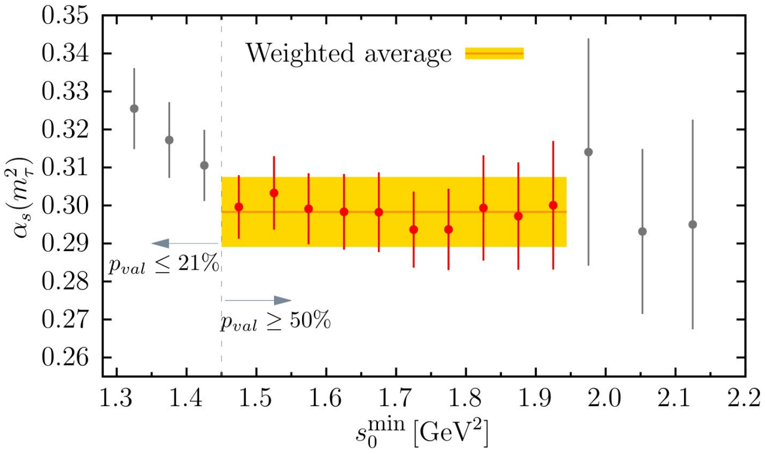

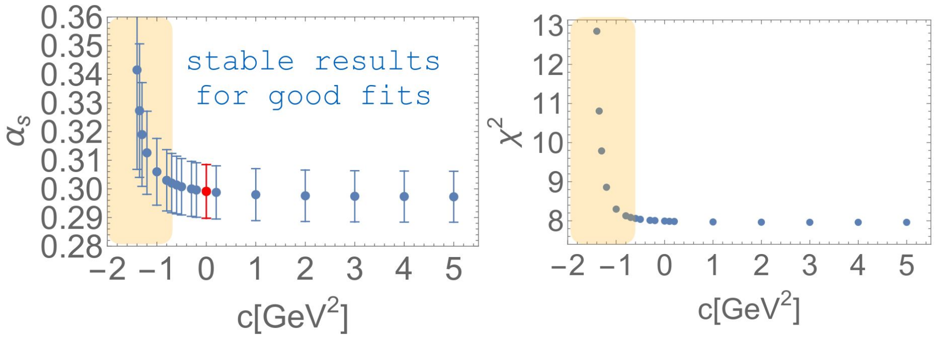

$$\alpha_s(m_\tau^2) = 0.2983 \pm 0.0092_{\rm stat} \pm 0.0026_{\rm fit} \pm 0.0022_{\rm pert} \pm 0.0025_{\rm DVs}$$
$$= 0.2983 \pm 0.0101 \qquad (n_f = 3)$$

At the Z-boson mass scale:

$$\alpha_s(m_Z^2) = 0.1159 \pm 0.0014 \qquad (n_f = 5)$$

[Boito, et al. (2025) 2502.08147]

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New value lower than our previous result (~1.2σ):

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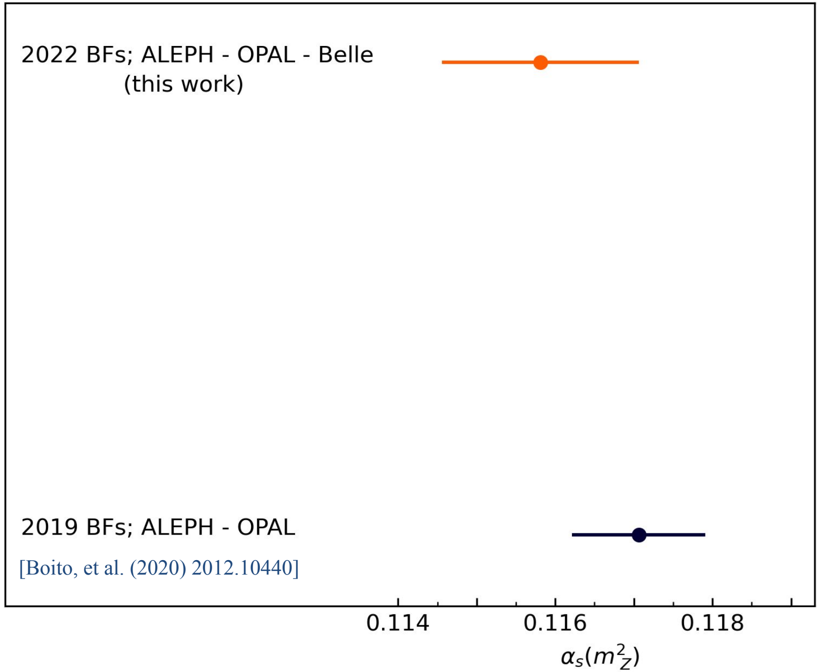
*(Statistical errors only)



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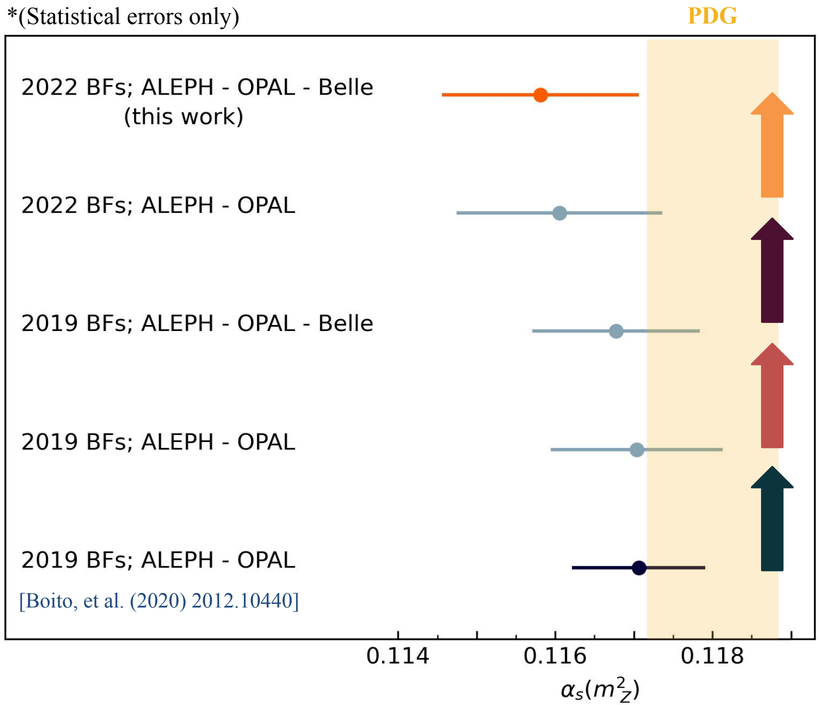


Final result: including **Belle**, new **branching fractions** and **new algorithm**

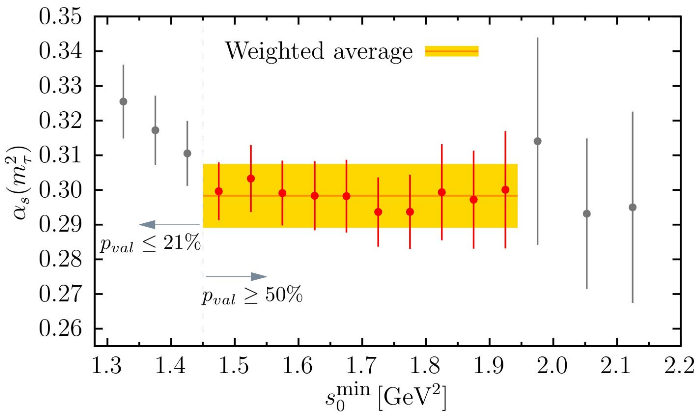
Largest shift due to new **4π branching fraction inputs (mostly from $\pi^- 3\pi^0$)**!

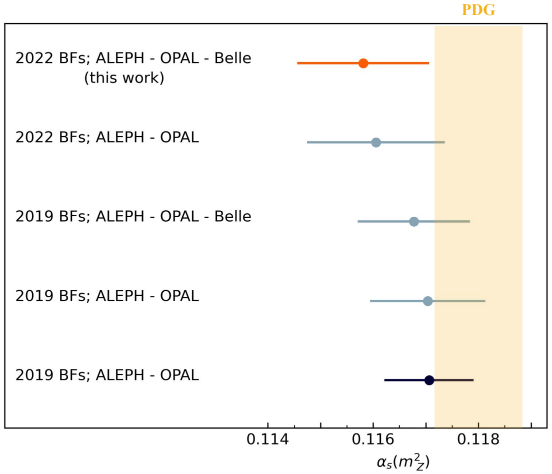
Inclusion of the **Belle** spectrum leads to a small shift towards smaller values.

Changes in our **data combination** algorithm lead to slightly larger errors but no shift

Conclusions

- Vector channel in hadronic tau decays is **special**: e^+e^- data + CVC allows for **improvement** near kin. end point.
- **New vector-isovector spectral function** purely based on exp. data, **no MC input needed**.
- Inclusion of high-statistics **Belle** 2π spectrum in an inclusive hadronic tau decay analysis.
- Our analysis can **immediately incorporate** any **new spectrum** for 2π or 4π tau decay channels.
- Improvements of this type not possible for the axial channel.
- Final value for the strong coupling lower than from 2021 analysis **mainly due to changes in input experimental 4π BFs**.
- **NEW**: applications of spectral function to **HVP calculations** and comparison with lQCD. **See Diogo Boito's talk tomorrow!**





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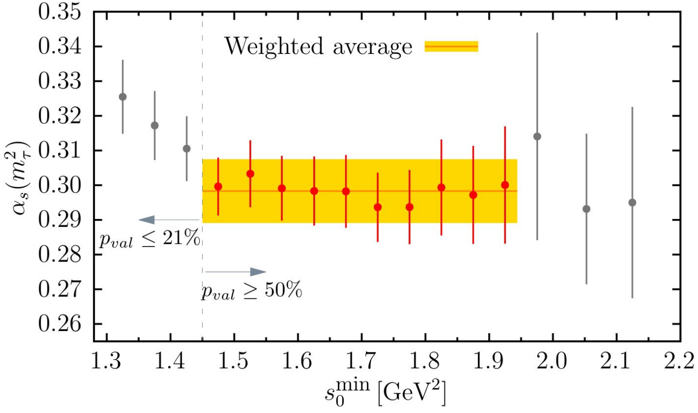

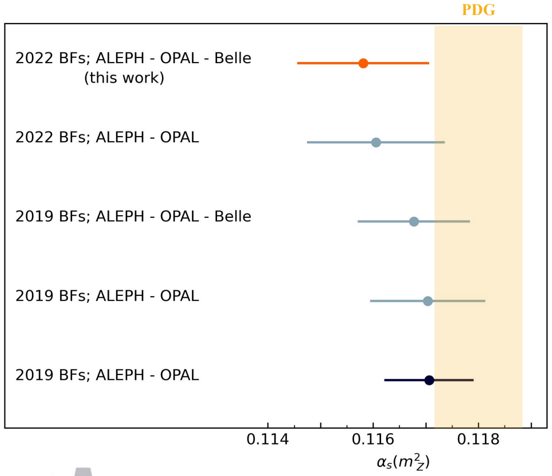

FAPESP SÃO PAULO RESEARCH FOUNDATION DFG Deutsche Forschungsgemeinschaft Hadrons CRC 1660 Nuclei JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Extras - Backup

1. **2π channel combination:** plots for higher *s*-bins spectral function.
2. **DV Parameters extracted** from the fit.
3. **2π channel combination** including the **CLEO data.**
4. **4pi channel combination** plots for different energy regions.
5. **Correlation matrices** in the 4π data sets from **ALEPH** and **OPAL**.
6. **d'Agostini bias correction** in the 4π channel combination.
7. **Combination Procedure** based on the KNT19.
8. **The problems with the Truncated OPE approach.**

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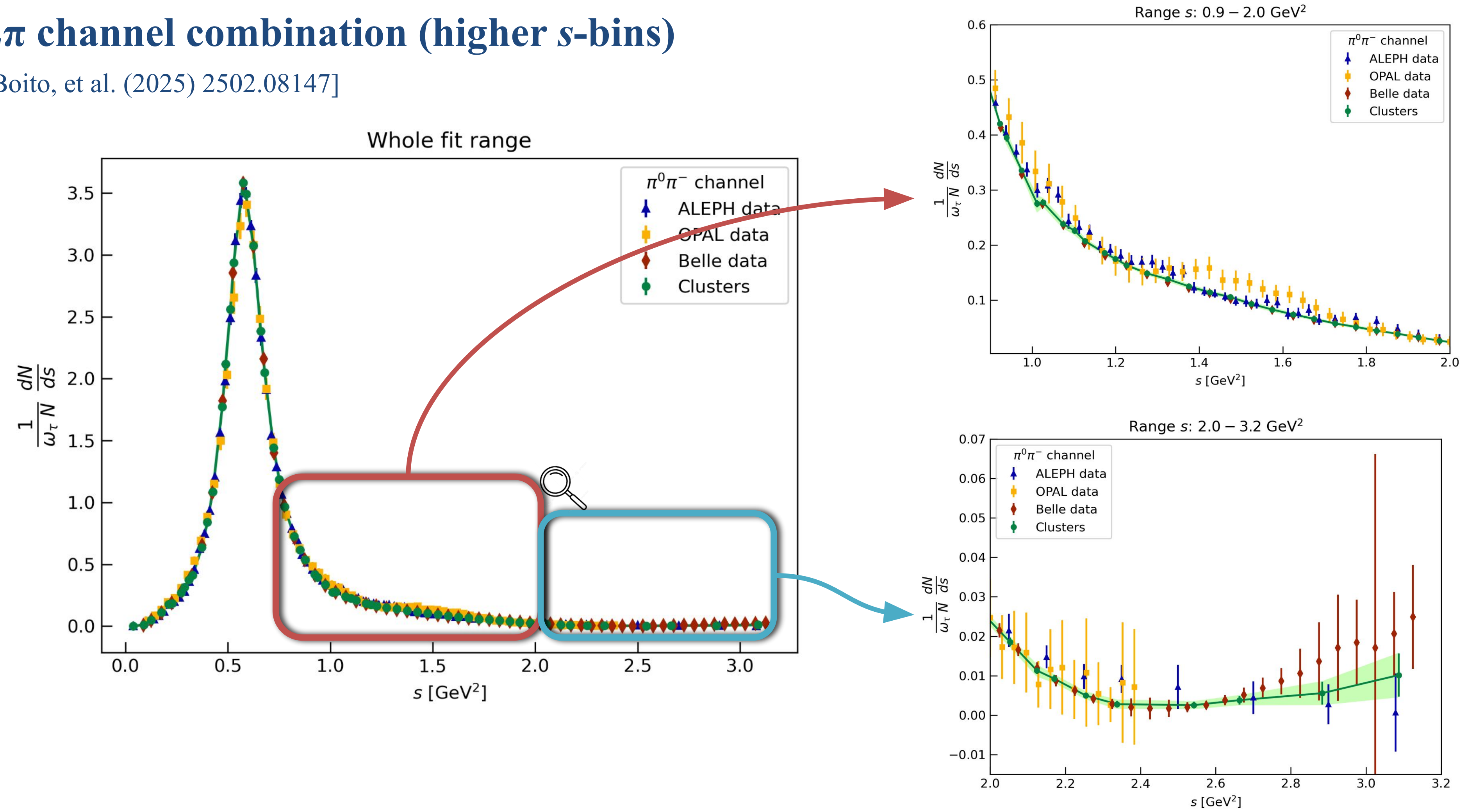
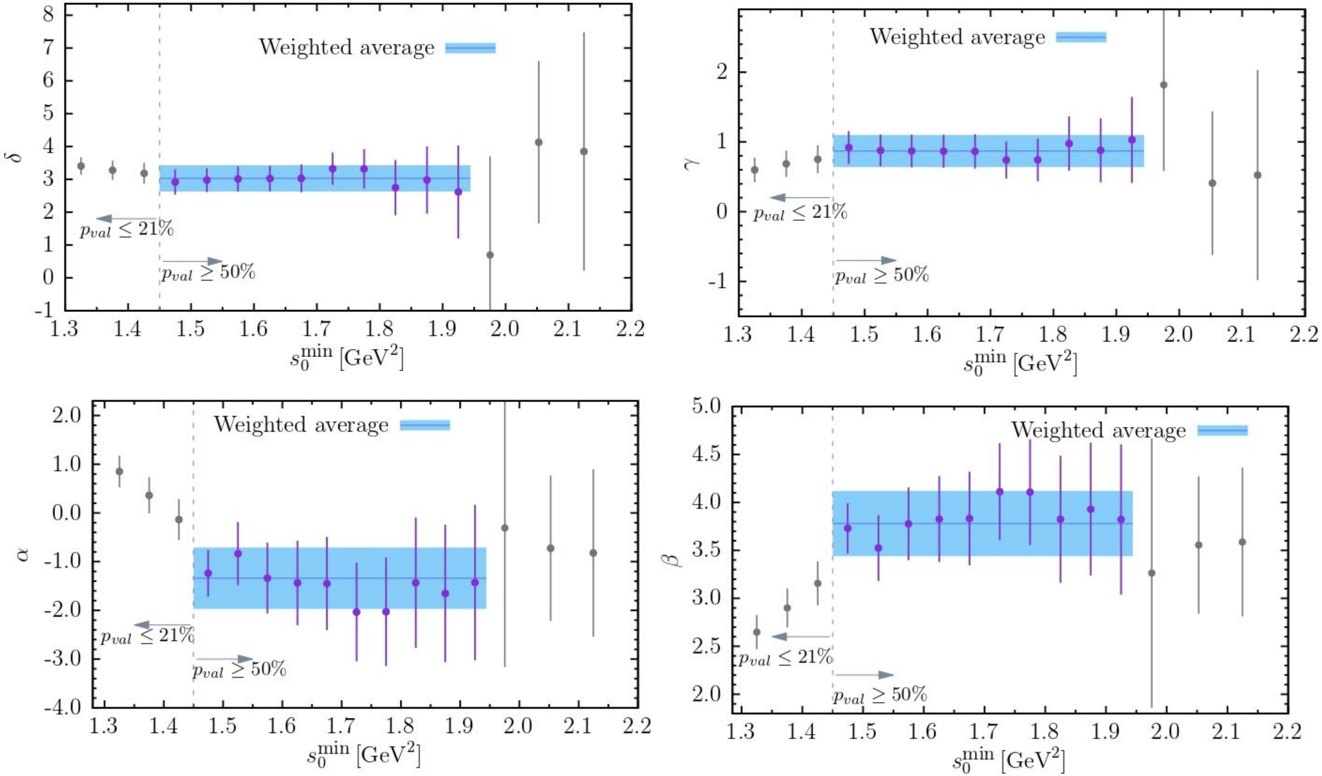
2π channel combination (higher s-bins)

[Boito, et al. (2025) 2502.08147]



DV parameters extracted from the fit [Boito, et al. (2025) 2502.08147]

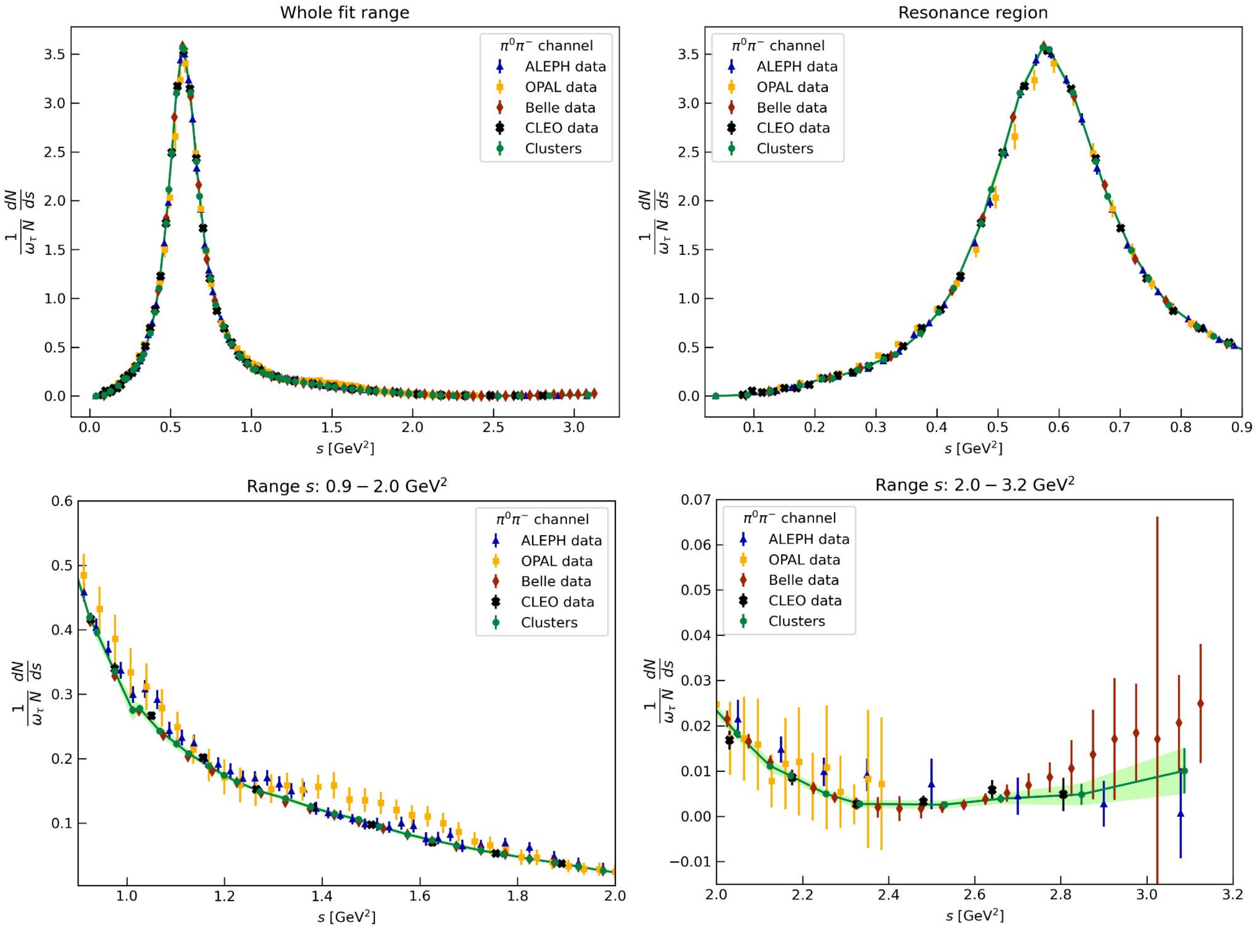


The DV parameters present **good consistency** in the different fits performed!

For *w = 1 :*

$$\delta \;=\; 3.03(40)\ ,$$
$$\gamma \;=\; 0.87(23)\ \text{GeV}^{-2}\ ,$$
$$\alpha \;=\; -1.34(63)\ ,$$
$$\beta \;=\; 3.78(34)\ \text{GeV}^{-2}\ .$$

2π channel combination including CLEO [Boito, et al. (2025) 2502.08147]



The **CLEO** data set was not included in our final result due to a **lack** of **systematic uncertainty** information bin-by-bin.

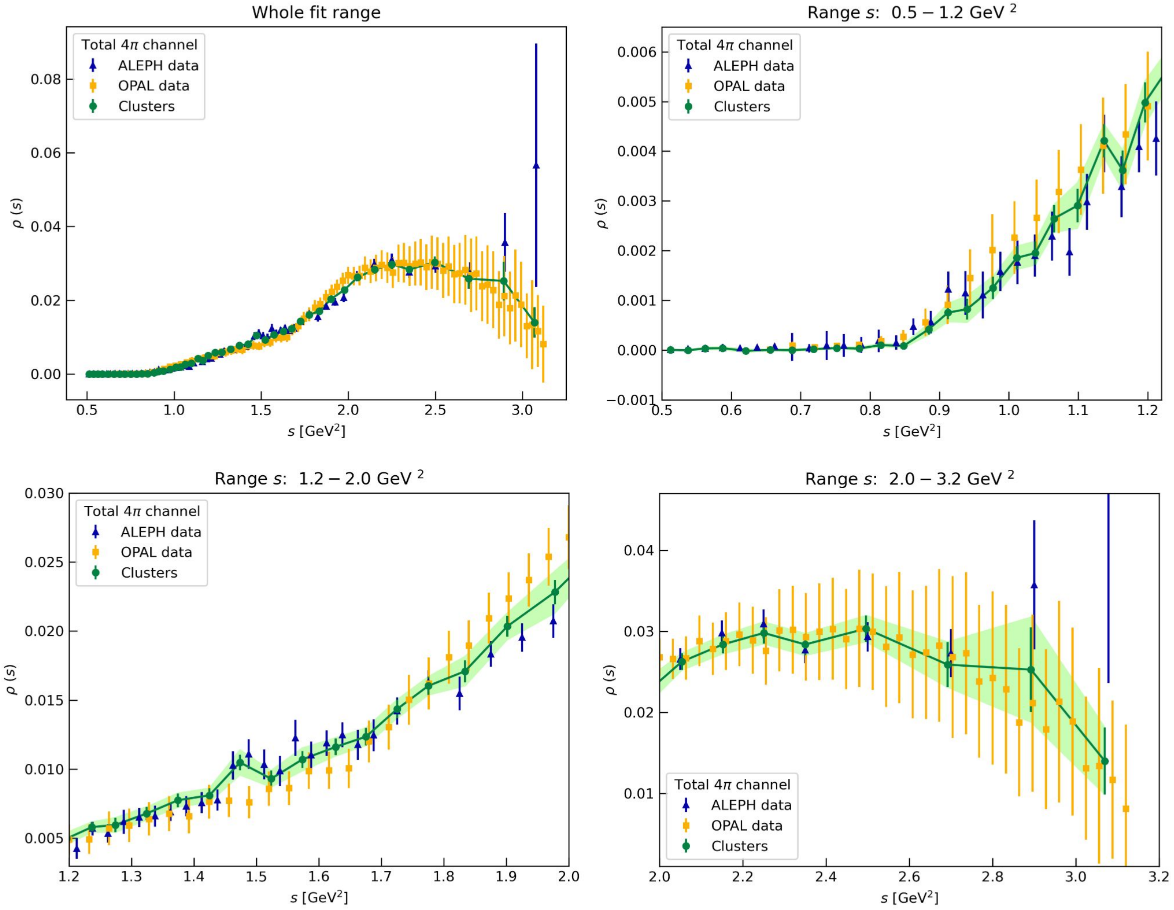
However, we studied the inclusion of this data set in our analysis.

For *w = 1* :

$$\begin{aligned} \alpha_s(m_\tau^2) &= 0.2997(96)\ , \\ \delta &= 3.04(39)\ , \\ \gamma &= 0.85(24)\ \mathrm{GeV}^{-2}\ , \\ \alpha &= -1.30(73)\ , \\ \beta &= 3.76(38)\ \mathrm{GeV}^{-2}\ . \end{aligned}$$

Also, combination has a **good** global p-value in the 2π channel (0.34).

4π channel combination: plots for different *s*-regions.



- Correlation matrices for the 4π modes from **ALEPH** and **OPAL** are ill behaved (negative eigenvalues).
- Adding up the 4π channels softens the problem, but strong correlations in **OPAL** results lead to poor fits.
- The main issue are the strong, but poorly known, correlations in the $\pi^- 3\pi^0$ **OPAL** channel.
- Perform fits where the strong correlations in **OPAL** $\pi^- 3\pi^0$ channel are not included in the minimization, but are still used in the error propagation.

Correlation matrices in the 4π data sets [Boito, et al. (2025) 2502.08147]

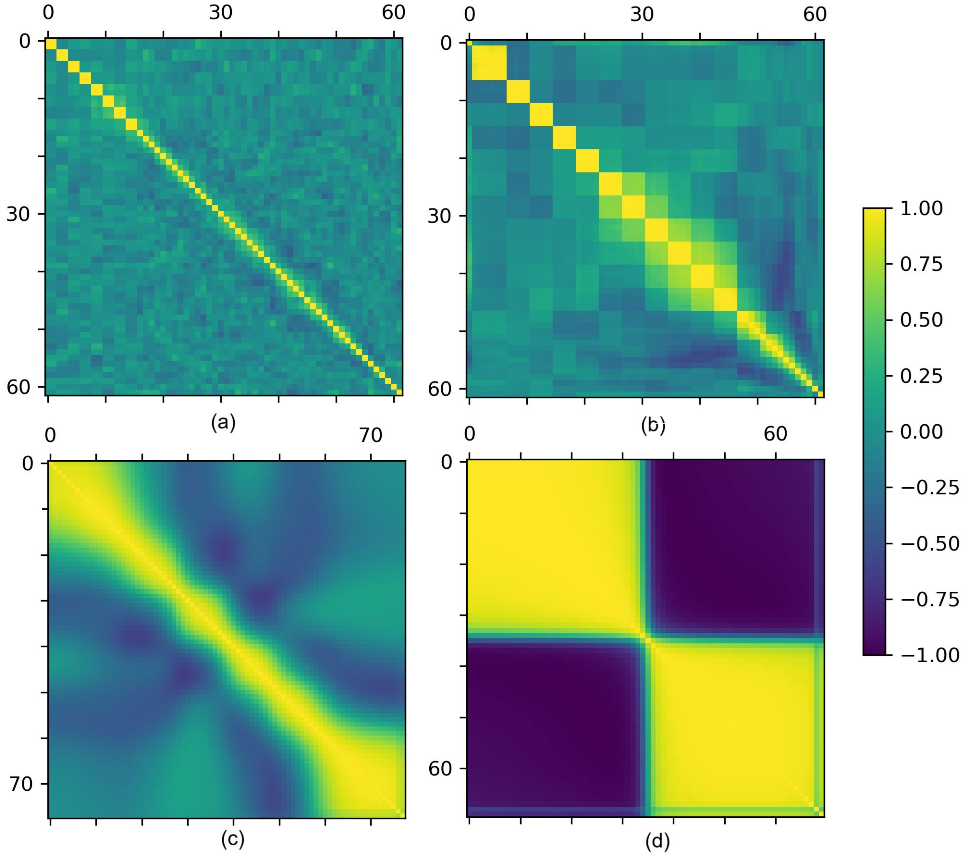
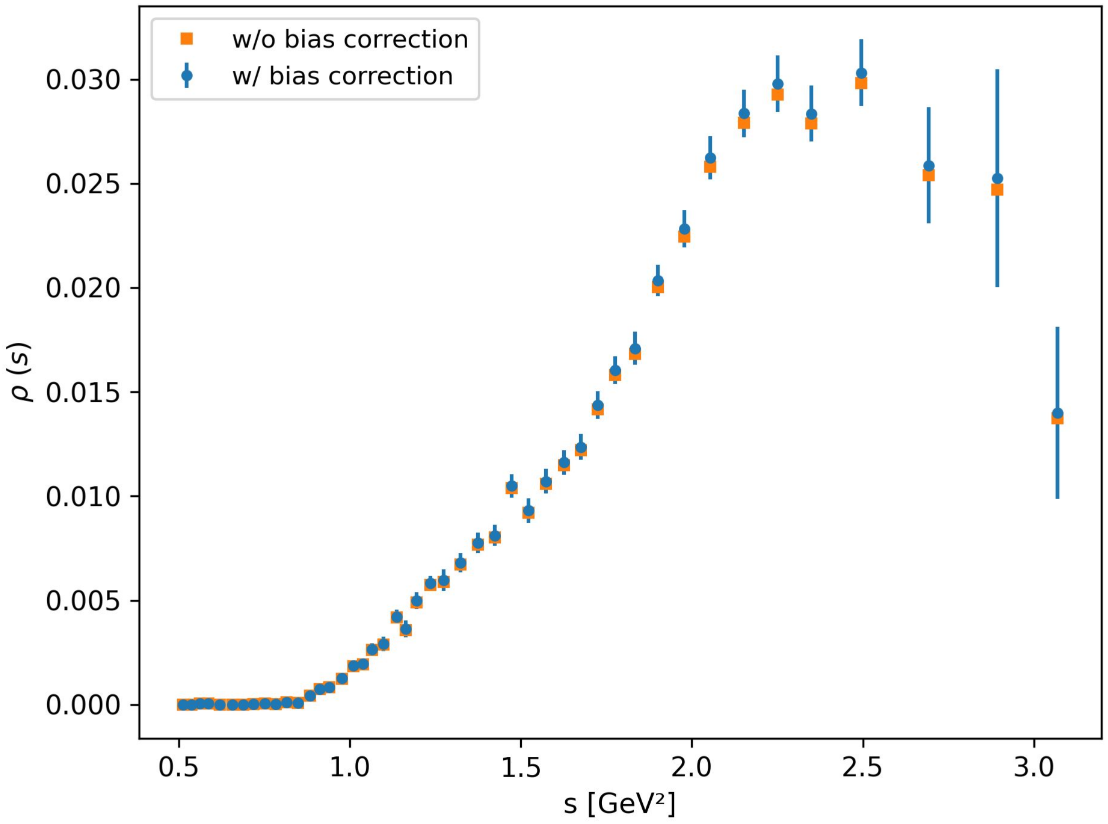


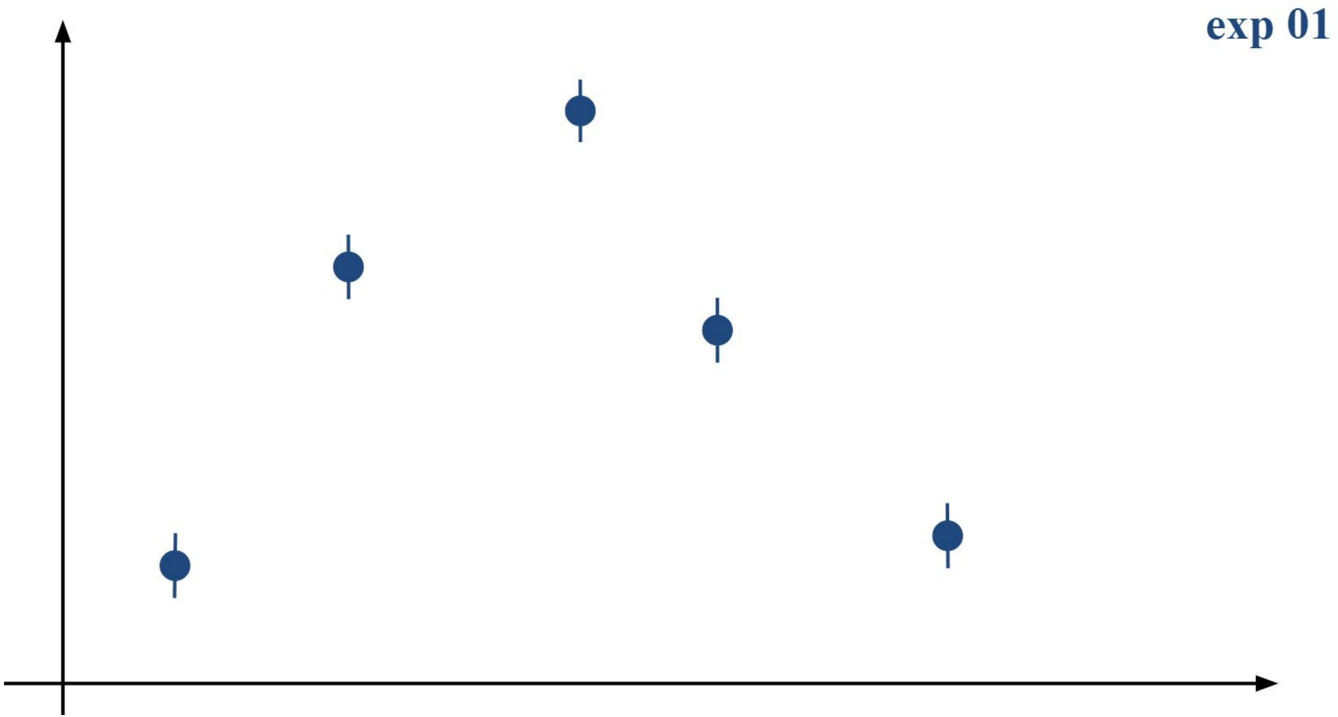
FIG. 4. *Correlations of the unit-normalized, 4π exclusive-mode number distribution data sets. (a) ALEPH $2\pi^-\pi^+\pi^0$, (b) ALEPH $\pi^-3\pi^0$, (c) OPAL $2\pi^-\pi^+\pi^0$ and (d) OPAL $\pi^-3\pi^0$.*

d’Agostini bias correction in the 4π channel [Boito, et al. (2025) 2502.08147]



Combination Procedure

Algorithm based on the KNT19 **[Keshavarzi, Nomura, Teubner. (2018)]**, employed in the context of g-2:



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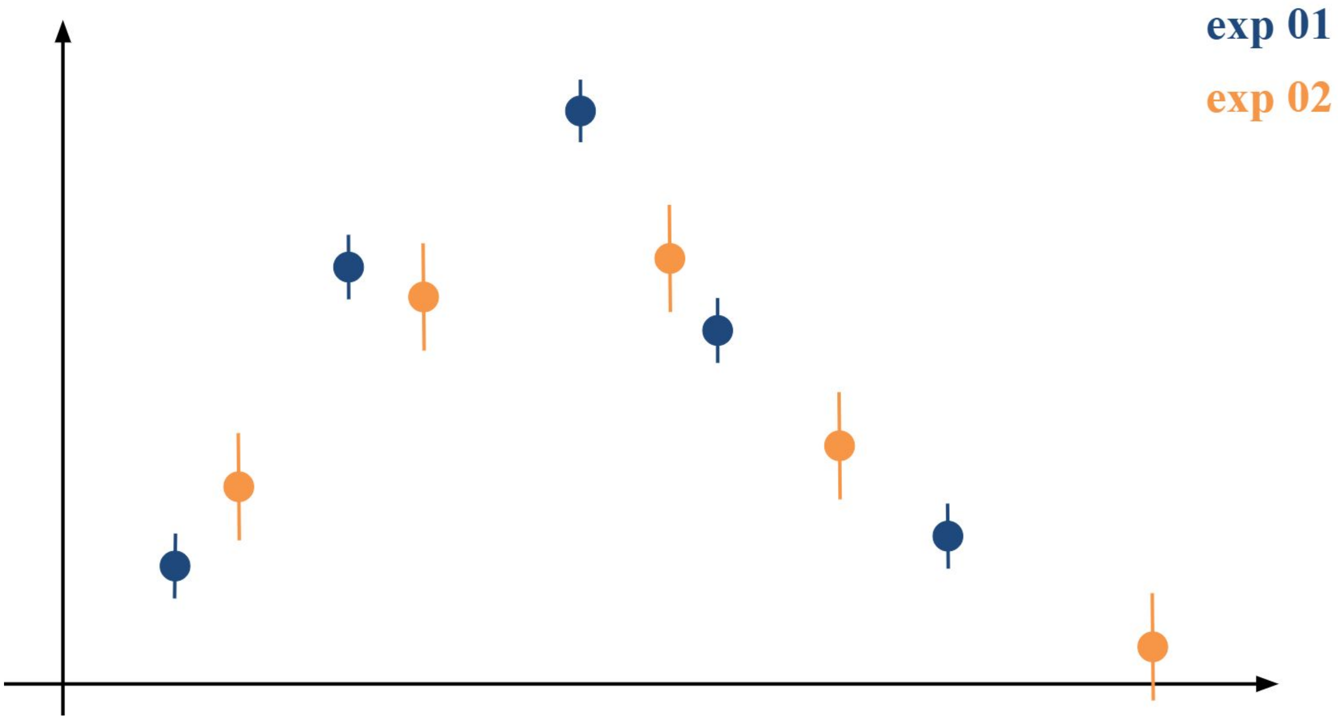

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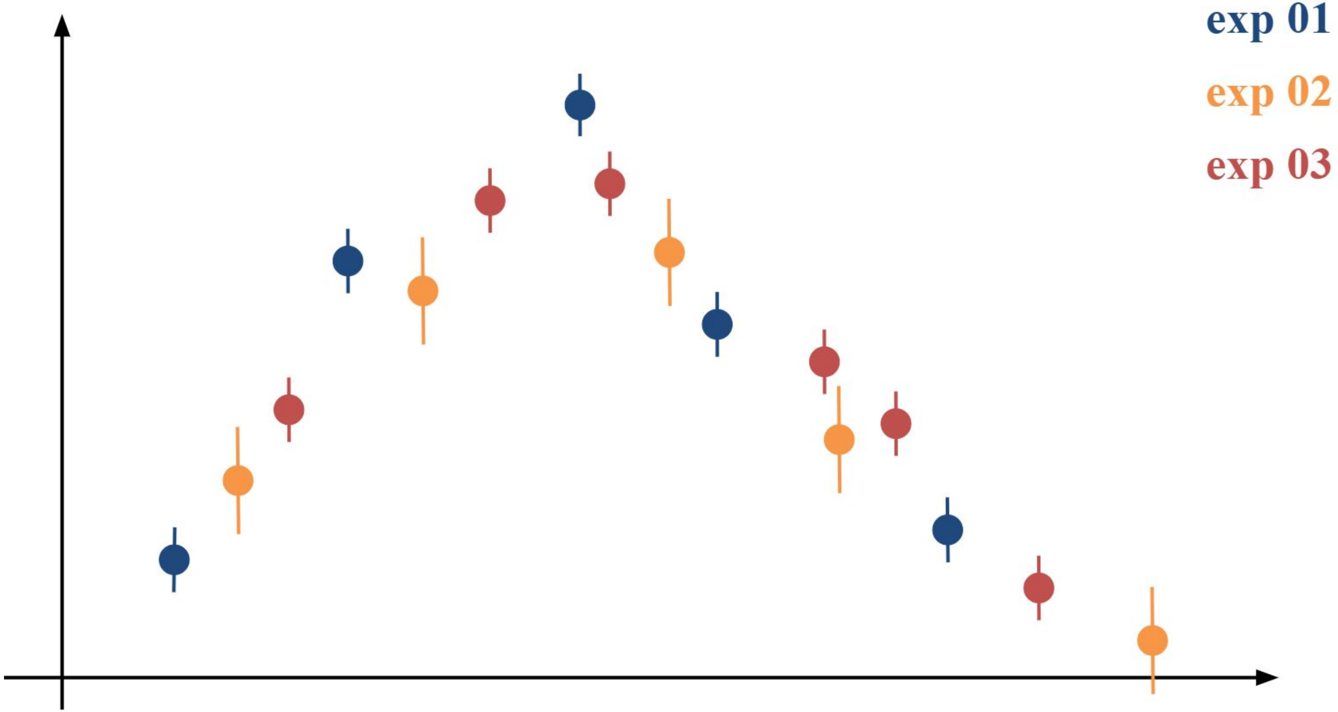

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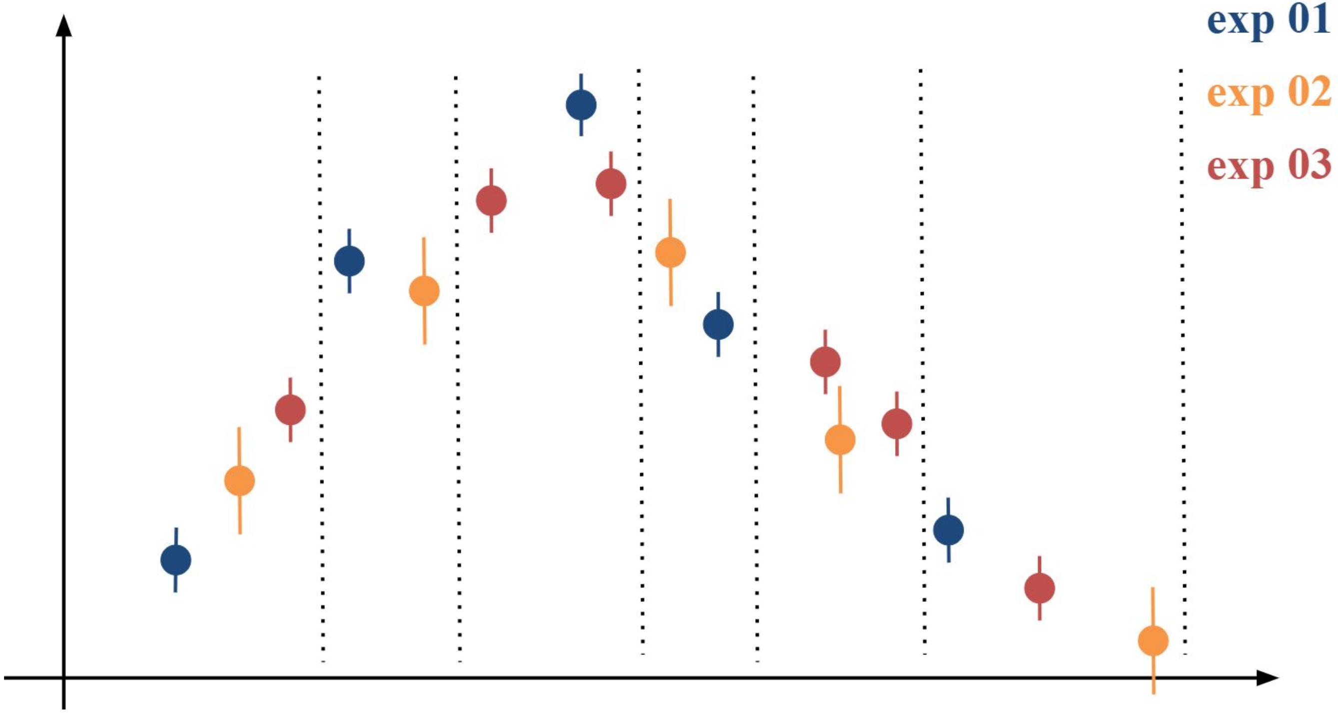

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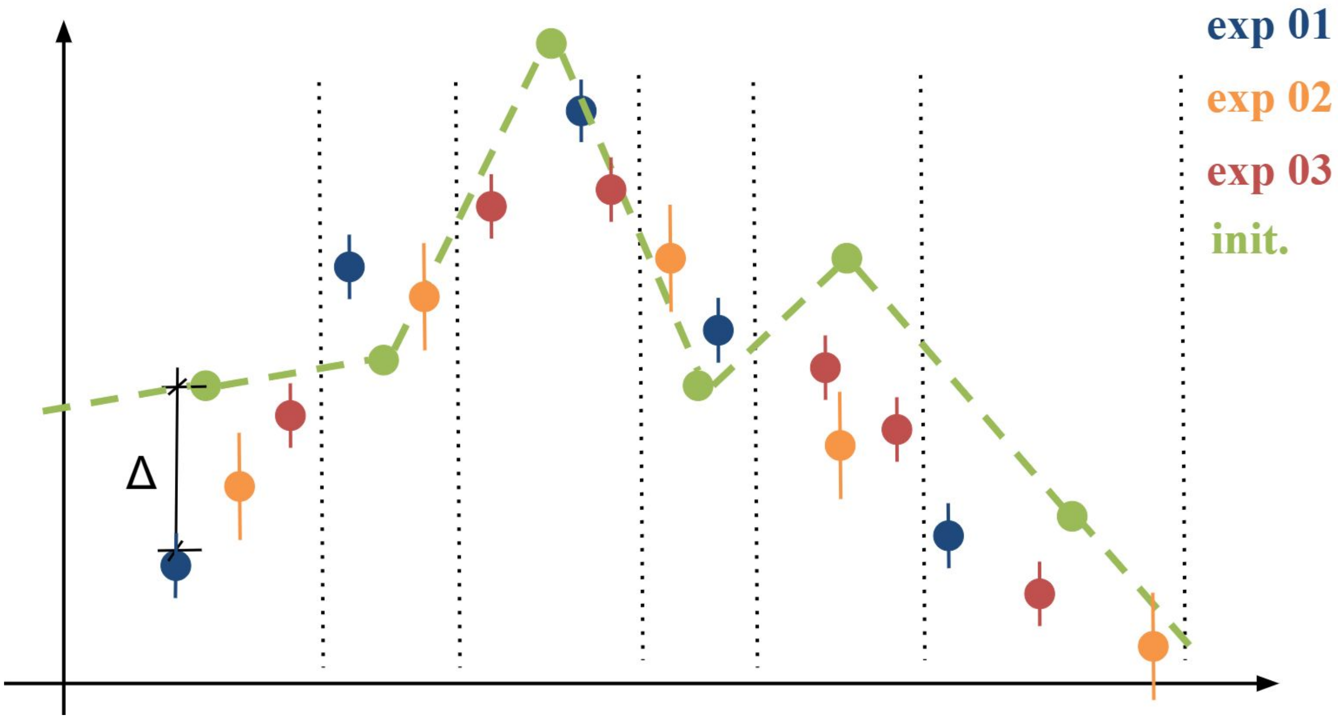

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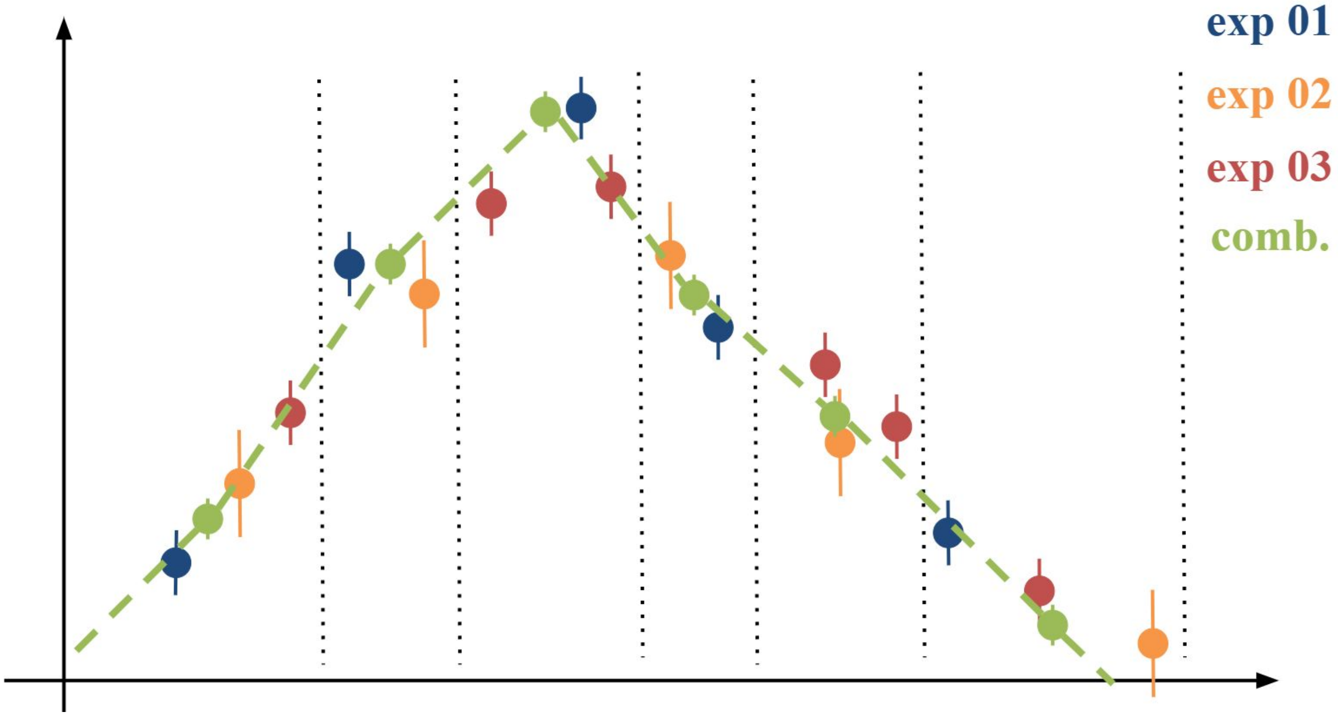

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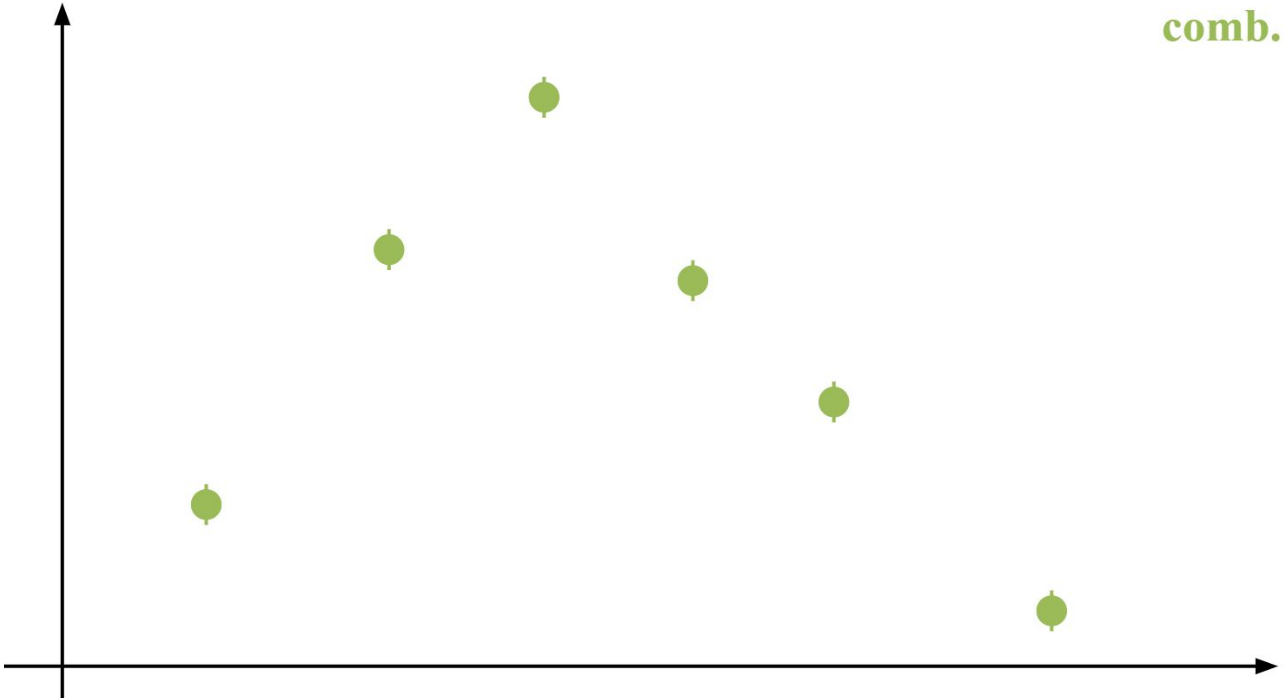
1. Separation of data into clusters and definition of central s-values of the clusters;
2. Minimization of Δ following χ^2 / Q^2 minimization **[Bruno, Sommer. (2023)]**, employing d'Agostini bias correction **[NNPDF. (2009)]**;
3. Standard linear error propagation using the full covariances matrices from experiments.



The problems with the Truncated OPE approach

The truncated OPE strategy consists of using 5 moments to fit 4 parameters.
The OPE must be truncated arbitrarily. Small sensitive to DV's, but large sensitivity to OPE condensates!

[Pich and Rodriguez-Sanchez, 2016]

$$w_{\text{opt}}^{(n)} = (1-x)^2 \sum_{k=0}^{n} (k+1)x^k = 1-(n+2)x^{n+1} + (n+1)x^{n+2}$$ uses moments with n = 1, …, 5

weights	χ^2	$\alpha_s(m_\tau^2)$	$10^3 C_{10}$	$10^3 C_8$	$10^3 C_6$
$w^{(21)}, w^{(22)}, w^{(23)}, w^{(24)}, w^{(25)}$	3.068384	0.31685(0.00253)	0.3464(0.1187)	−0.8720(0.2107)	1.3771(0.2371)

- The strong coupling is determined **using perturbation theory only**, with no reference to non-perturbative contributions. OPE condensates adjust to the data without any constraints from the fit procedures.

weights	χ^2	$\alpha_s(m_\tau^2)$	$10^3 C_{10}$	$10^3 C_8$	$10^3 C_6$
$w^{(24)}, w^{(25)}$	3.068384	0.31685(0.00253)	—	—	—
$w^{(23)}, w^{(24)}, w^{(25)}$	3.068384	0.31685(0.00253)	0.3464(0.1187)	—	—
$w^{(22)}, w^{(23)}, w^{(24)}, w^{(25)}$	3.068384	0.31685(0.00253)	0.3464(0.1187)	−0.8720(0.2107)	—
$w^{(21)}, w^{(22)}, w^{(23)}, w^{(24)}, w^{(25)}$	3.068384	0.31685(0.00253)	0.3464(0.1187)	−0.8720(0.2107)	1.3771(0.2371)

[Boito, Golterman, Maltman, Peris. 2024]