Light Meson decays at BESIII

Kailu Li

(on behalf of the BESIII Collaboration)

Lanzhou University, Gansu, China

International Workshop on e^+e^- collisions from Phi to Psi

Outline

- Light meson physics
- Light meson factory
- Recent η/η' decays at BESIII
- Summary

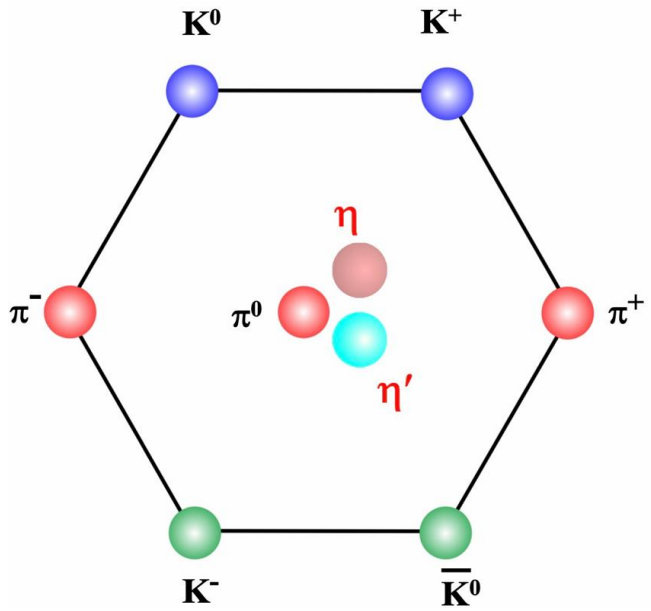
Light Meson Physics

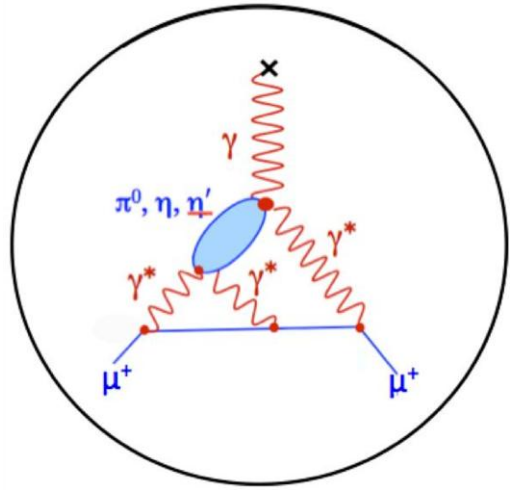
Light meson

- **Important roles in particle physics**
  - Strong interactions, Quark Model, CP violation …

Light meson decays

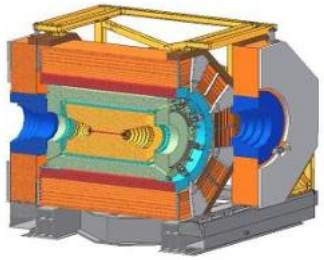
- **Test theoretical model**
  - Chiral symmetry and anomalies, NREFT, VMD…
- **Transition Form factors**
  - Important input for HLbL contributions $(g-2)_\mu$
- **Test fundamental symmetries and search for new physics beyond the Standard Model**





Source of η/η' events

- η/η': Light masses, narrow widths and simple decay final-state topologies.
- A hot topic of theoretical discussions and an important research subject for many experimental collaborations.



BESIII

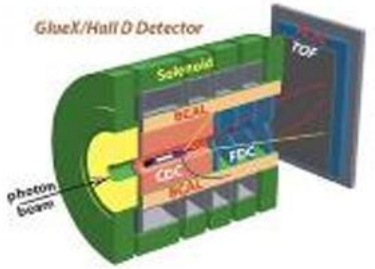
Crystal Ball

WASA-at-COSY

KLOE

KLOE-2

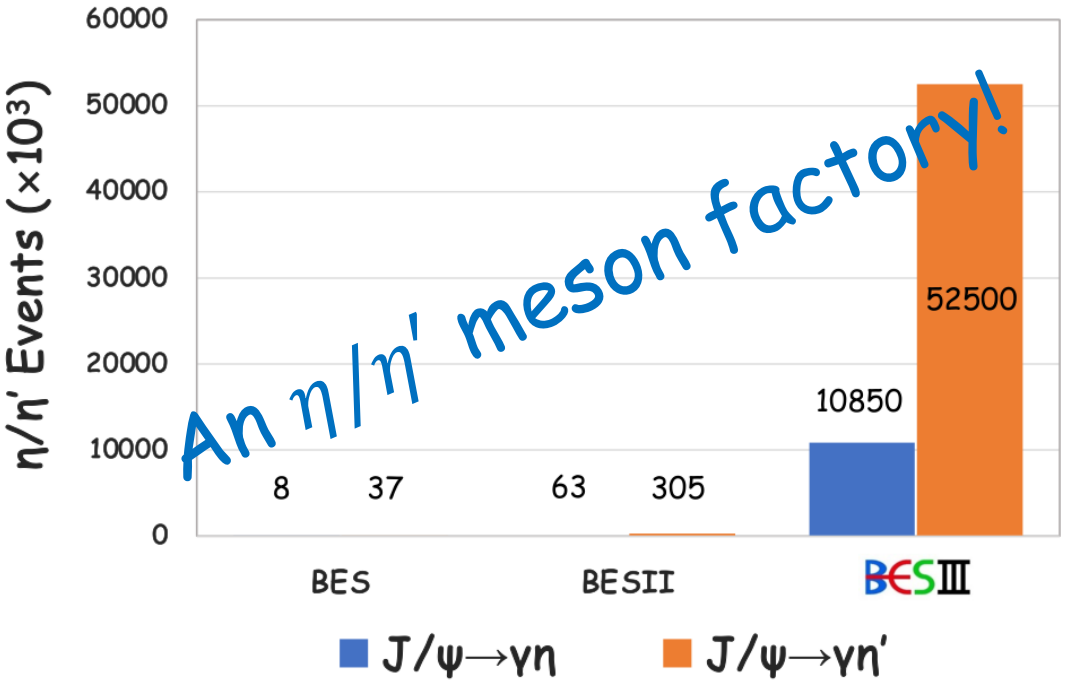
CLAS(12)

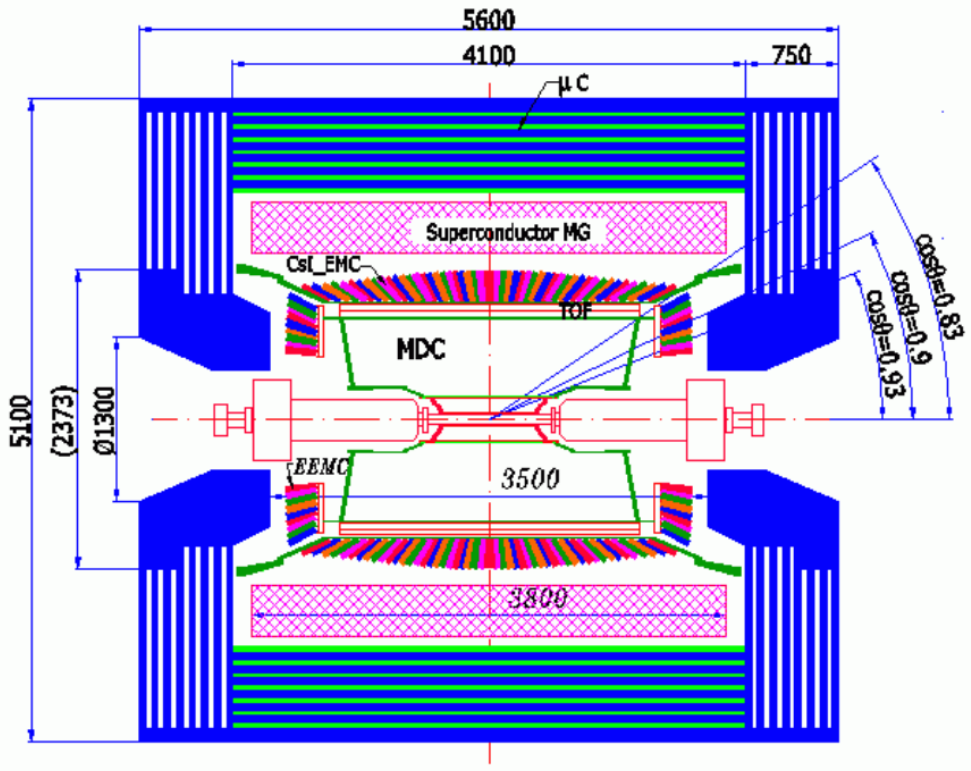


GlueX

BESIII Detector

- The BESIII detector records symmetric e^+e^- collisions provided by the BEPCII storage ring.
- The facility is used for studies of τ-charm physics.
- Collected 10 billion J/ψ Events!



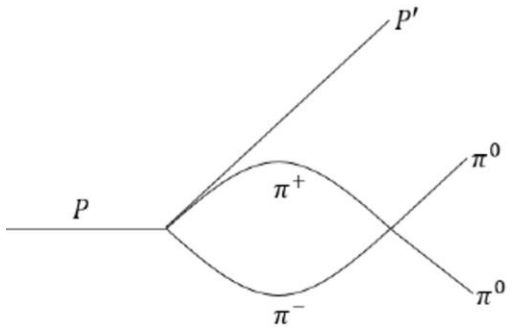


Evidence of Cusp Effect in $\eta' \to \pi^0\pi^0\eta$

PRL 130, 081901 (2023)

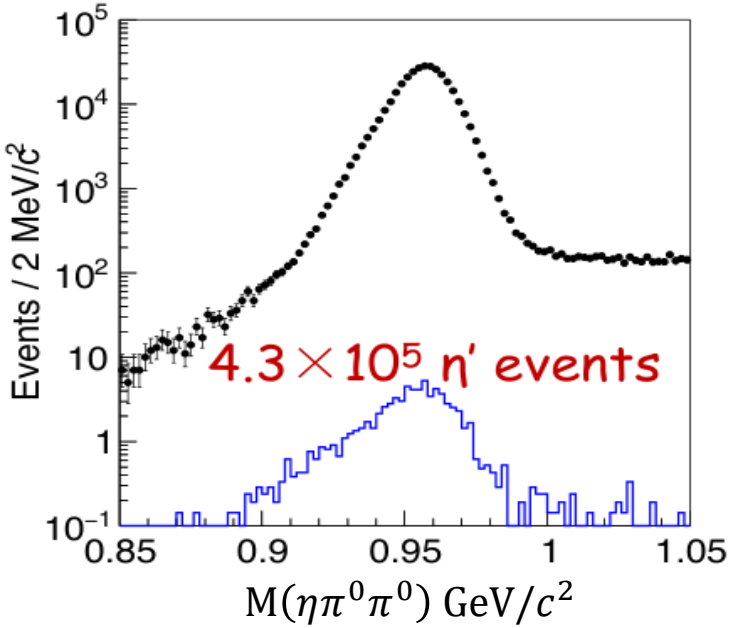
- In $\pi\pi$ interaction, one of the prominent features is the loop contribution to the ππ scattering: The S-wave charge exchange rescattering $\pi^+\pi^- \to \pi^0\pi^0$ causes a prominent cusp at the center of mass energy corresponding to the summed mass of two charged pions.

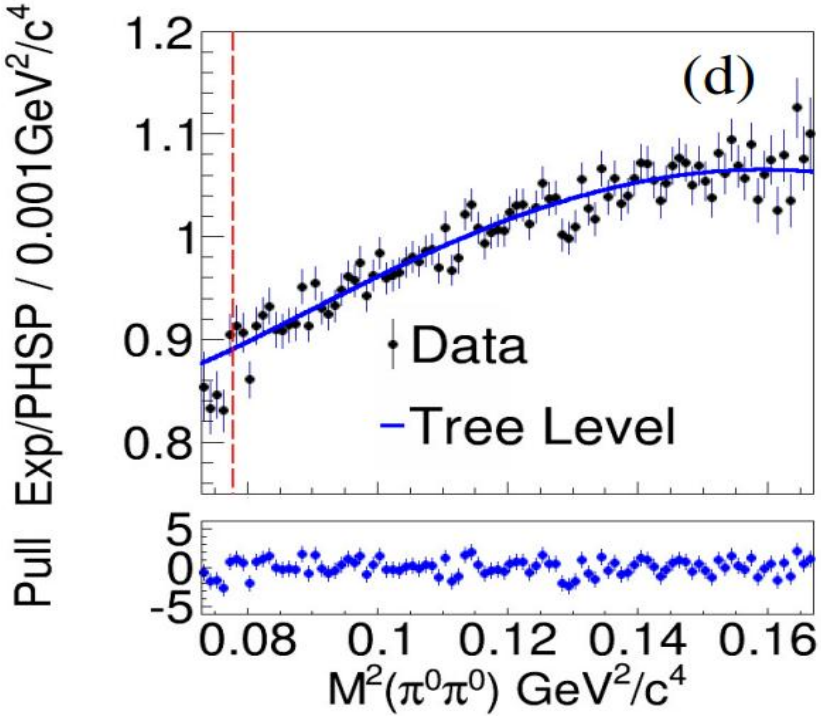
- By determining the strength of the S-wave $\pi\pi$ interaction to study the fundamental properties of QCD at low energies.



high term of $\pi\pi$ rescattering

EPJC 62, 511 (2009)



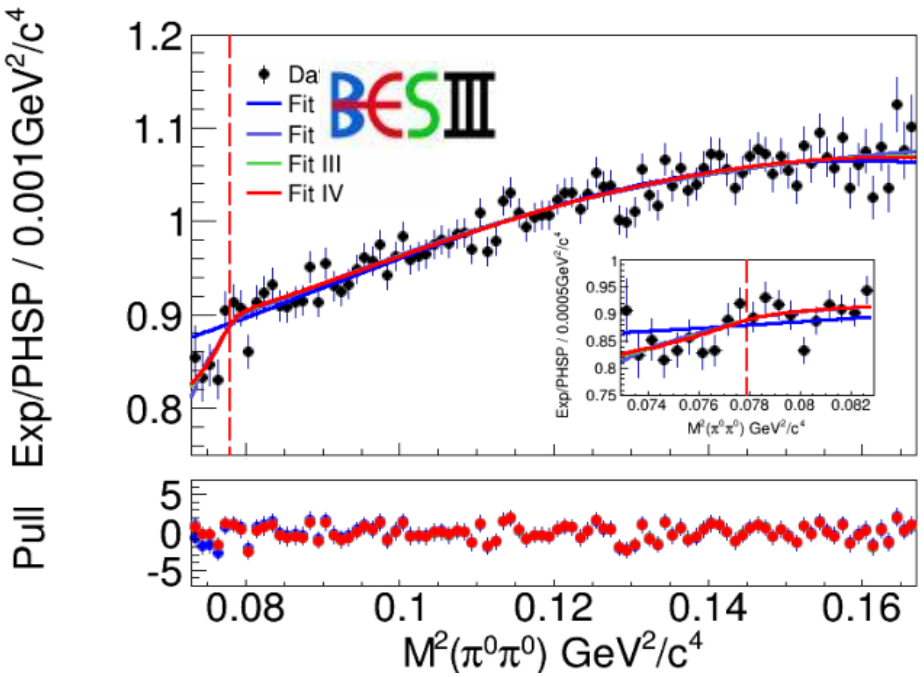


Fit I: Only the tree level contribution

The cusp effect is sizeable in this decay.

Evidence of Cusp Effect in $\eta' \to \pi^0\pi^0\eta$

PRL 130, 081901 (2023)



$$M = M_{tree} + M_{one-loop} + M_{two-loop}$$

Fit	Notes
I	Only the tree level contribution
II	Consider the loop level contribution
III	Fix some parameters based on Fit II to reduce the correlations
IV	Ignore the non-cusp terms from the loop contributions

- ✧ Non-relativistic effective field theory
- ✧ Evidence of the cusp effect around 3.5σ.

$\pi - \pi$ scattering pars

Parameters	Fit I	Fit II	Fit III	Fit IV
a	$-0.075 \pm 0.003 \pm 0.001$	-0.207 ± 0.013	-0.143 ± 0.010	$-0.077 \pm 0.003 \pm 0.001$
b	$-0.073 \pm 0.005 \pm 0.001$	-0.051 ± 0.014	-0.038 ± 0.006	$-0.066 \pm 0.006 \pm 0.001$
d	$-0.066 \pm 0.003 \pm 0.001$	-0.068 ± 0.004	-0.067 ± 0.003	$-0.068 \pm 0.004 \pm 0.001$
$a_0 - a_2$	-	0.174 ± 0.066	0.225 ± 0.062	$0.226 \pm 0.060 \pm 0.012$
a_0	-	0.497 ± 0.094	With cusp effect	
a_2	-	0.322 ± 0.129		
Statistical Significance	-	3.4σ	3.7σ	3.6σ

Dalitz Plot Analysis of $\eta \to \pi^0\pi^0\pi^0$

Phys. Rev. D 107, 092007 (2023)

- The density distribution of the Dalitz plot has threefold symmetry due to the three identical π^0s in the final state.

$$Z = X^2 + Y^2 = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_\eta} - 1\right)^2$$

$$|A(X,Y)|^2 \propto 1 + 2\alpha Z$$

- KLOE: ~420 pb^{-1} ($\simeq 1.4 \cdot 10^9$ ϕ) data

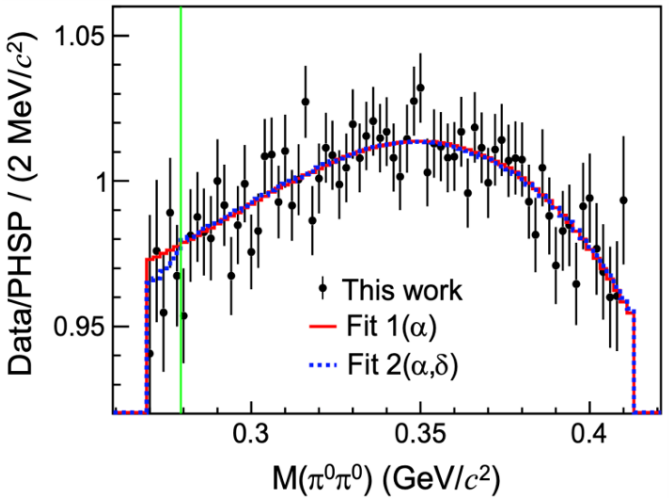
$$\alpha = -0.0301 \pm 0.0035_{stat}\ ^{+0.0022}_{-0.0035}\ _{syst}$$

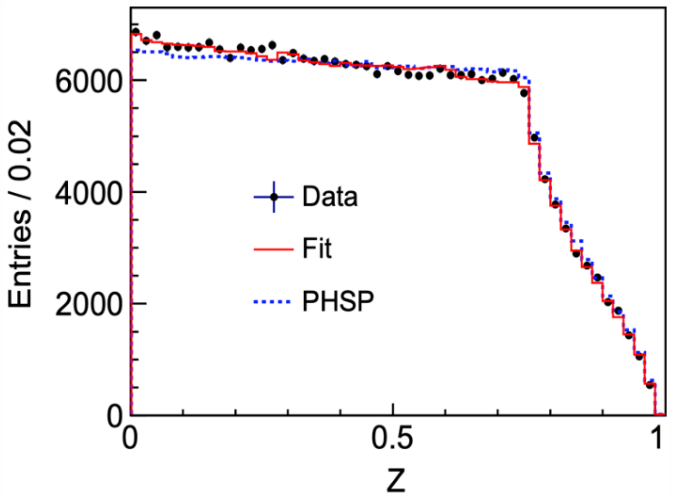
- BESIII: 10 billion J/ψ radiative decays

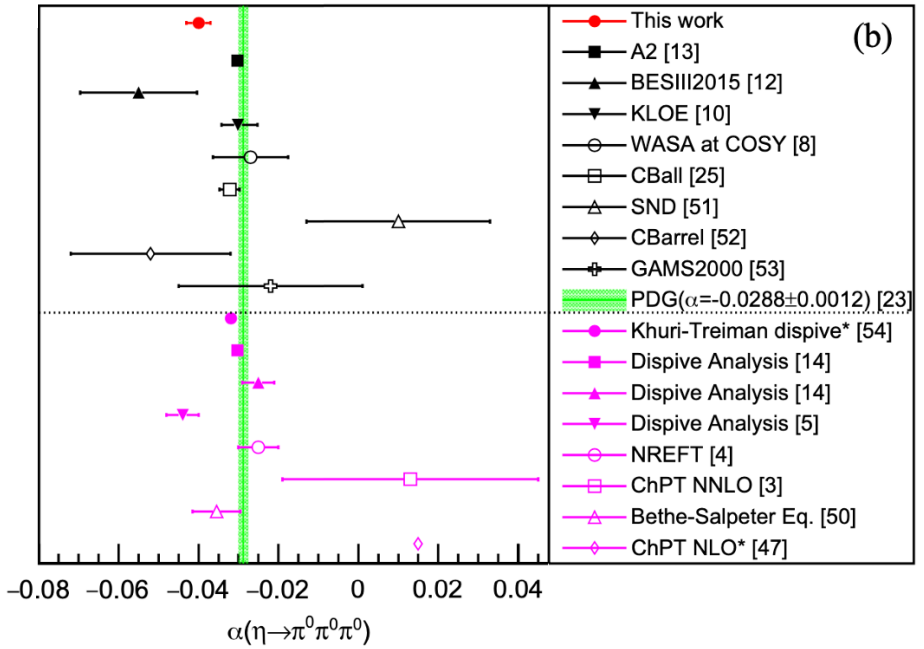
$$\alpha = -0.0406 \pm 0.0035 \pm 0.0008$$

- α is consistent with A2 and KLOE measurement within 2.8σ

- The cusp effect is investigated, but no obvious contribution is found.







Dalitz Plot Analysis of $\eta \to \pi^+\pi^-\pi^0$

Phys. Rev. D 107, 092007 (2023)

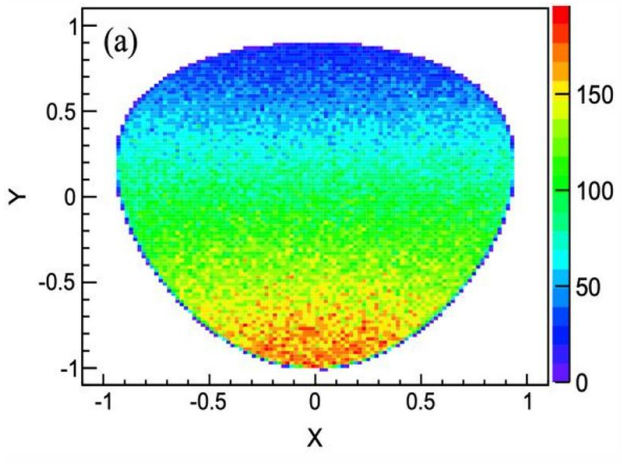
- The Dalitz plot is generally described as:

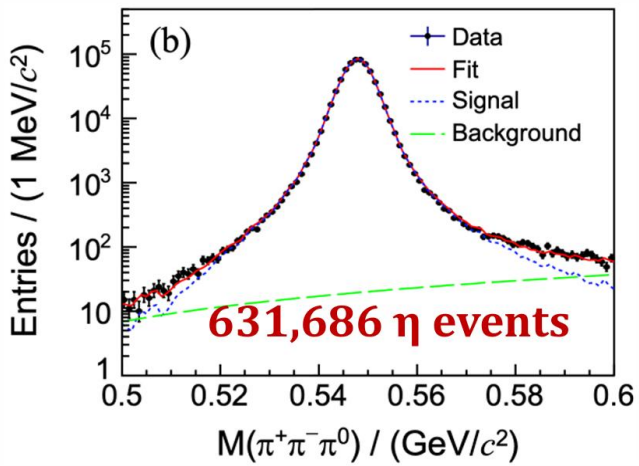
$$X = \frac{\sqrt{3}}{Q_\eta}(T_{\pi^+} - T_{\pi^-}), Y = \frac{3T_{\pi^0}}{Q_\eta} - 1$$

- The squared amplitude can be expanded as:

$$|A(X,Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + gX^2Y + ..$$

  - a, b, c, d, e, f, g are the Dalitz plot matrix elements.
  - c and e are related to charge conjugation violation.



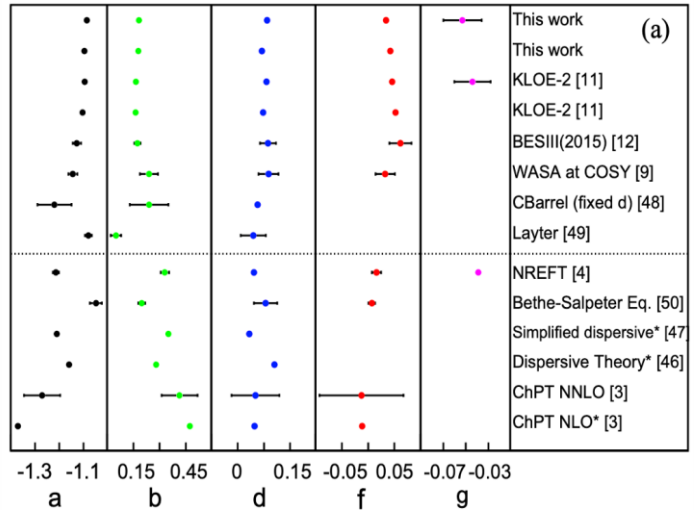


- Ignoring the high-order term

$$a = -1.097 \pm 0.005 \pm 0.001$$
$$b = 0.158 \pm 0.006 \pm 0.003$$
$$d = 0.070 \pm 0.006 \pm 0.001$$
$$f = 0.134 \pm 0.010 \pm 0.003$$

- Included the cubic term

$$a = -1.086 \pm 0.006 \pm 0.001$$
$$b = 0.162 \pm 0.006 \pm 0.003$$
$$d = 0.083 \pm 0.007 \pm 0.001$$
$$f = 0.118 \pm 0.011 \pm 0.003$$
$$g = -0.053 \pm 0.017 \pm 0.003$$



➡ All parameters related with C symmetry breaking are consistent with zero

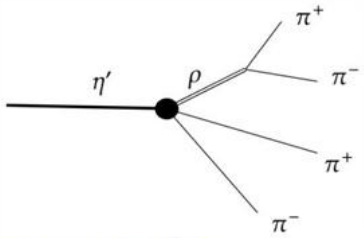
Amplitude analysis for $\eta' \to \pi^+\pi^-\pi^+\pi^-$

PRD 109, 032006 (2024)

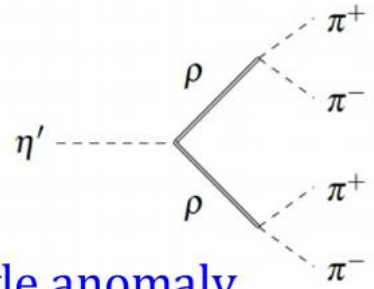
✧ Decay amplitude is constructed with the combination of the Chiral Perturbation Theory (ChPT) and VMD model.

PRD 85, 014014 (2012)

$$F(s_{12}, s_{34}) = \left[\frac{s_{12}}{D_\rho(s_{12})} + \frac{s_{34}}{D_\rho(s_{34})} - \frac{s_{14}}{D_\rho(s_{14})} - \frac{s_{23}}{D_\rho(s_{23})}\right] + \frac{c_3}{c_1 - c_2}\left[\frac{m_\rho^2(s_{12} - s_{34})}{D_\rho(s_{12})D_\rho(s_{34})} - \frac{m_\rho^2(s_{14} - s_{23})}{D_\rho(s_{14})D_\rho(s_{23})}\right]$$



Box anomaly

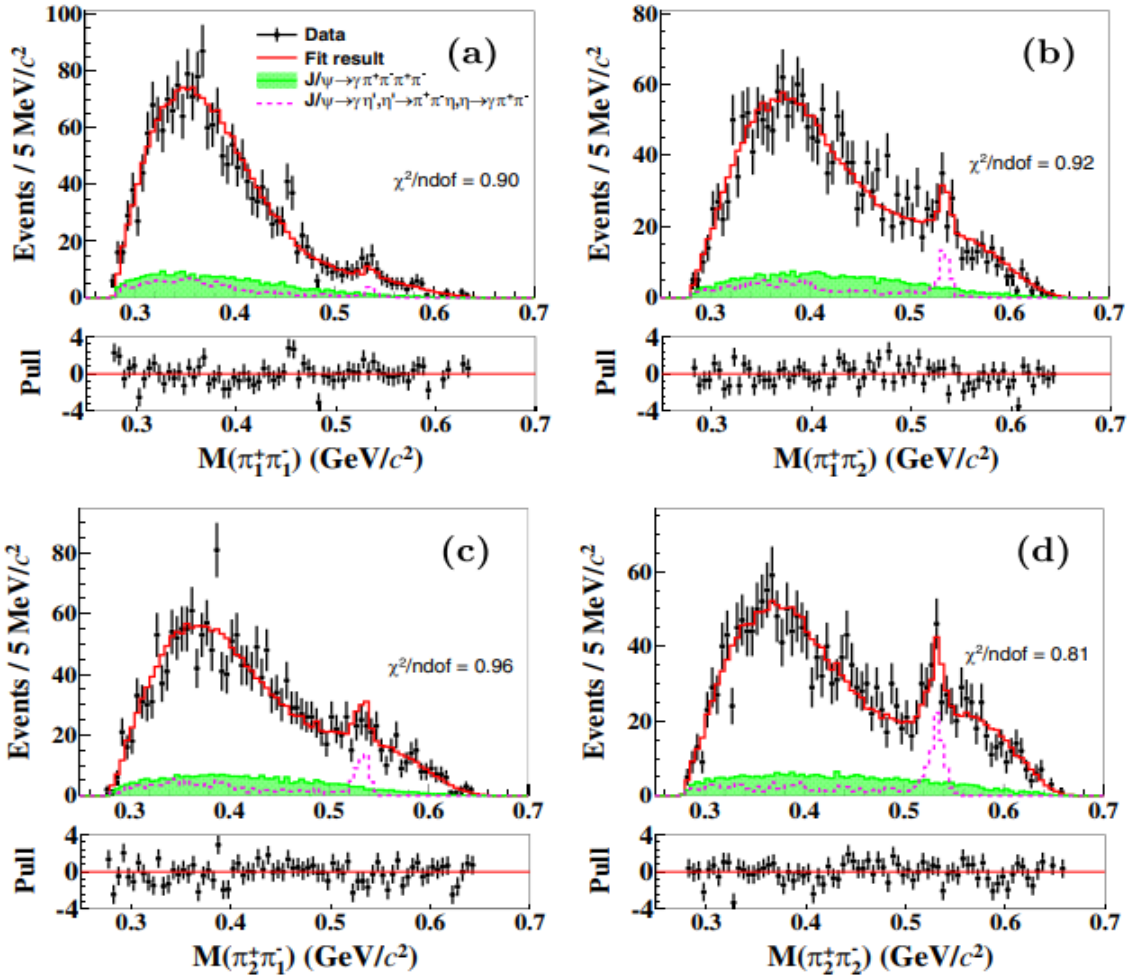


Triangle anomaly

✧ Measurement of the doubly virtual isovector form factor :

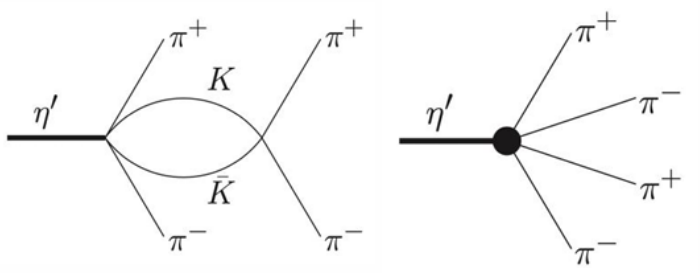
$$\frac{c_3}{c_1 - c_2} = 1.22 \pm 0.29 \pm 0.04$$

✧ If $\frac{c_3}{c_1 - c_2} \approx 1$, triangle anomaly would be dominated.



Study of $\eta' \to \pi^{+(0)}\pi^{-(0)}\pi^{+(0)}\pi^{-(0)}$

PRD 109, 032006 (2024)

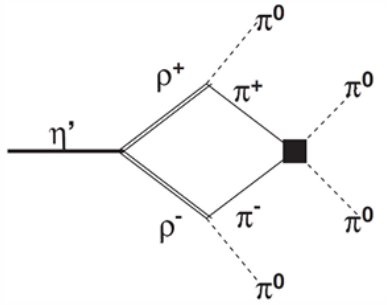


Loop and counter term at $O(p^6)\Rightarrow$

$Br(\eta' \to 2(\pi^+\pi^-)) = (1.0 \pm 0.3) \times 10^{-4}$

$Br(\eta' \to \pi^+\pi^- 2\pi^0) = (2.4 \pm 0.7) \times 10^{-4}$

F. K. Guo, B. Kubis, A. Wirzba, PRD 85,014014 (2012)



D-wave pion loop

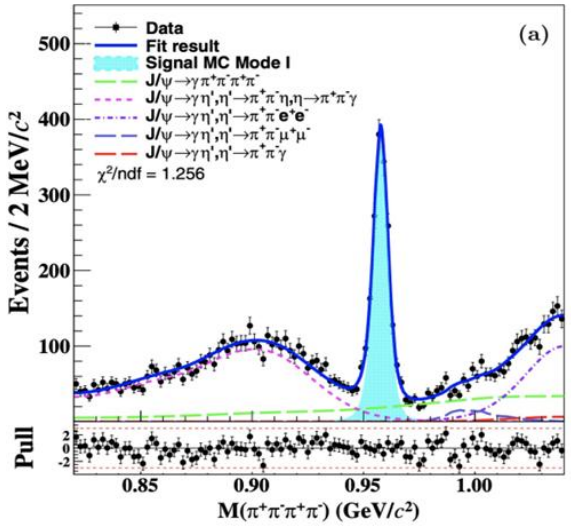
$\Rightarrow$ Br~10^{-8}

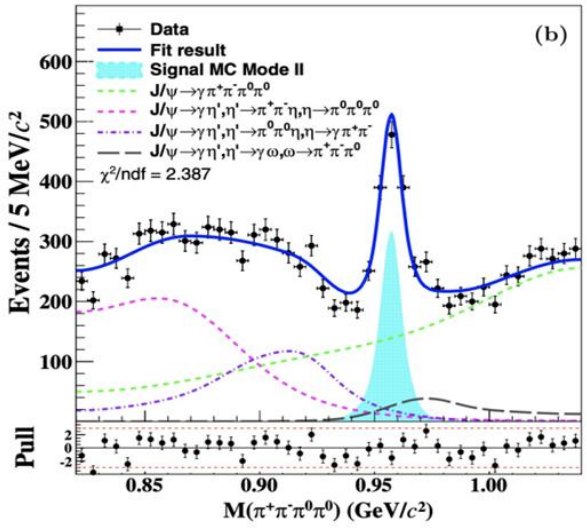
✧ The branching fraction:

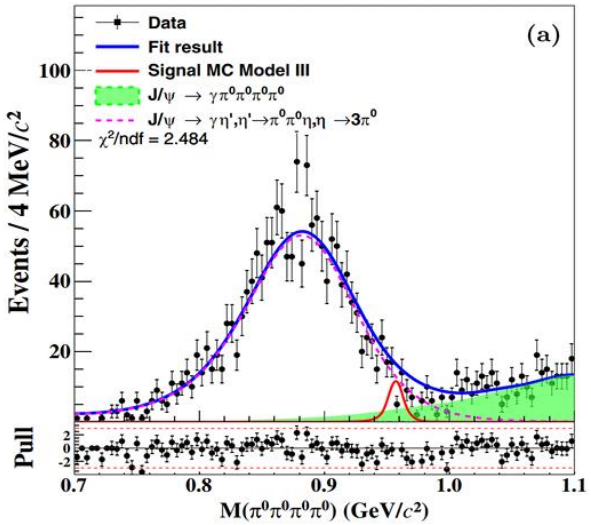
$$B(\eta' \to \pi^+\pi^-\pi^+\pi^-) = (8.56 \pm 0.25 \pm 0.23) \times 10^{-5}$$

$$B(\eta' \to \pi^+\pi^-\pi^0\pi^0) = (2.12 \pm 012 \pm 0.10) \times 10^{-4}$$

✧ No event left for $\pi^0\pi^0\pi^0\pi^0$ hypothesis in η' signal region and the UL is set as 1.24×10^{-5} at the 90% C.L.







Transition Form Factor of $\eta/\eta' \to \gamma e^+ e^-$

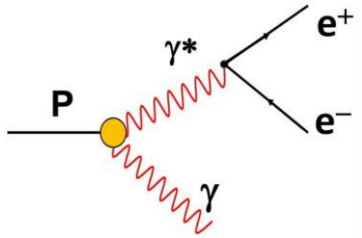
PRD 109, 072001 (2024)

- Performed an unbinned fit based on the amplitude formula.

$$\frac{\mathrm{d}\Gamma(P \to \gamma l^+ l^-)}{dq^2 \Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_l^2}{q^2}}\left(1+\frac{2m_l^2}{q^2}\right)\left(1-\frac{q^2}{M_P^2}\right)|F(q^2)|^2$$

- Single-pole: $F(q^2) = \frac{1}{1-q^2/\Lambda^2}$

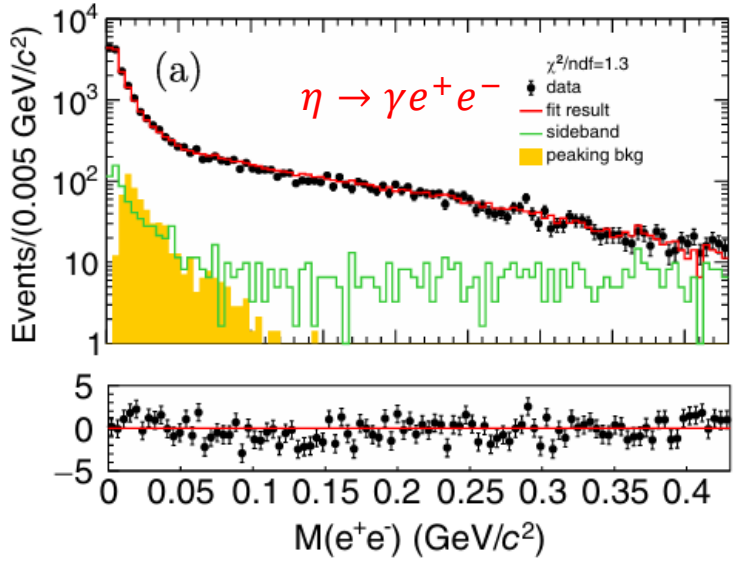
$$\Lambda_\eta = (0.749 \pm 0.026 \pm 0.008)\ \mathrm{GeV}/c^2$$

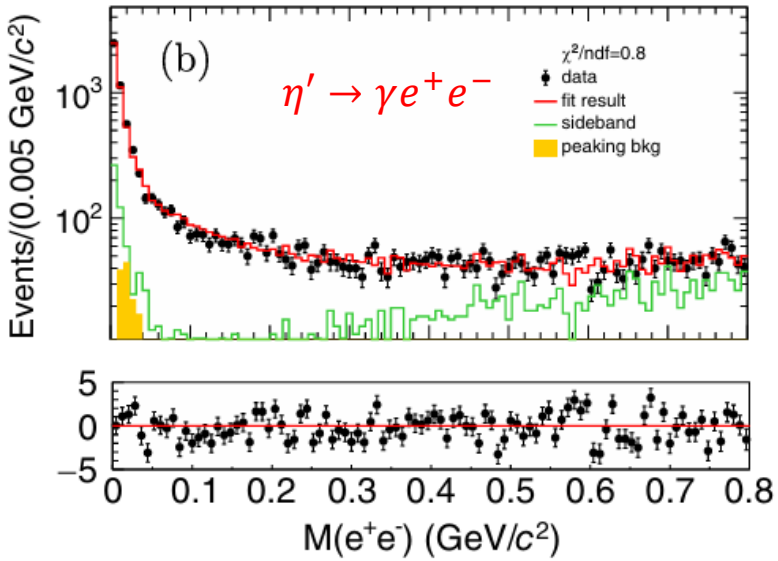


- Multi-pole: $|F(q^2)|^2 = \frac{\Lambda^2(\Lambda^2+\gamma^2)}{(\Lambda^2-q^2)^2+\Lambda^2\gamma^2}$

$$\Lambda_{\eta'} = (0.802 \pm 0.007 \pm 0.008)\ \mathrm{GeV}/c^2$$

$$\gamma_{\eta'} = (0.113 \pm 0.009 \pm 0.002)\ \mathrm{GeV}/c^2$$





Transition Form Factor of $\eta' \to \pi^+\pi^-\eta, \eta \to \gamma l^+ l^-$

PRD 112, 052007 (2025)

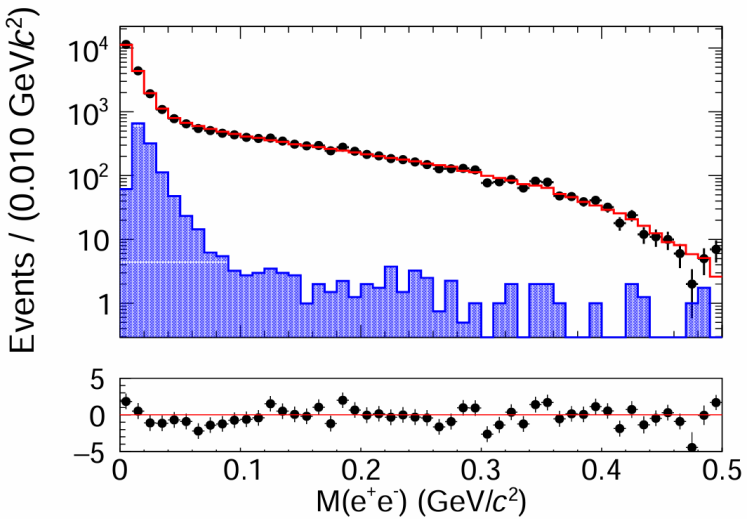
- Using the transition form factor predicted by the VMD mode
  - Single-pole: $F(q^2) = \frac{1}{1-q^2/\Lambda^2}$
  - Slope parameter: $b_p = \left.\frac{d|F(q^2)|}{dq^2}\right|_{q^2=0} = \Lambda^{-2}$

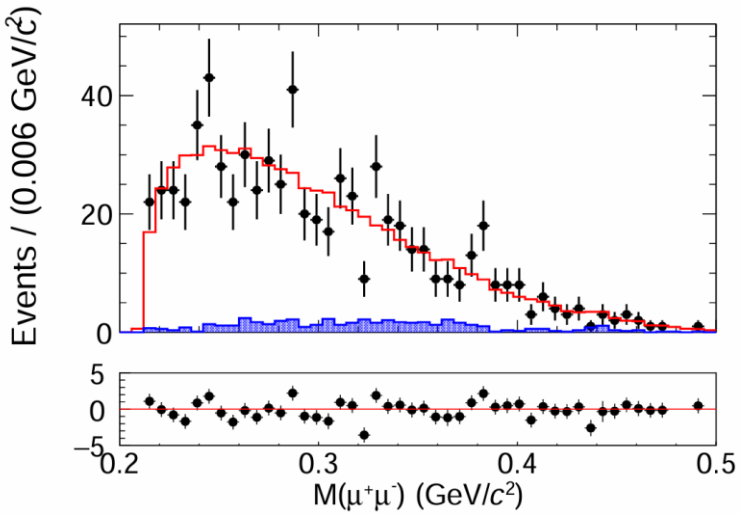
- For $\eta \to \gamma e^+ e^-$: $\Lambda^{-2} = 1.668 \pm 0.093 \pm 0.024 (\mathrm{GeV}/c^2)^{-2}$

- For $\eta \to \gamma \mu^+ \mu^-$: $\Lambda^{-2} = 1.645 \pm 0.343 \pm 0.017 (\mathrm{GeV}/c^2)^{-2}$

- By combining with the results based on the $J/\psi \to \gamma\eta, \eta \to \gamma e^+ e^-$ events from the previous BESIII measurement:

$$\Lambda^{-2} = 1.707 \pm 0.076 \pm 0.029 (\mathrm{GeV}/c^2)^{-2}$$





Transition Form Factor of $\eta' \to \pi^+\pi^- l^+ l^-$ JHEP07, 135(2024)

✧ Decay amplitude:

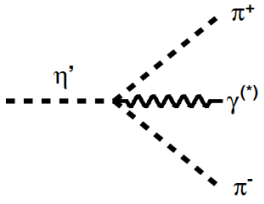
T. Petri, *Anomalous decays of pseudoscalar mesons*, Master's thesis, [1010.2378].

$$\overline{\left|\mathcal{A}_{\eta'\to\pi^+\pi^- l^+ l^-}\right|}^2(s_{\pi\pi}, s_{ll}, \theta_\pi, \theta_1, \phi) = \frac{e^2}{8k^2}|M(s_{\pi\pi}, s_{ll})|^2 \times \lambda\left(m_{\eta'}^2, s_{\pi\pi}, s_{ll}\right) \times \left[1 - \beta_1^2 \sin^2\theta_1 \sin^2\phi\right] s_{\pi\pi}\beta_\pi^2 \sin^2\theta_\pi$$

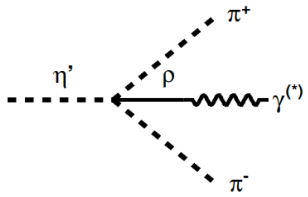
✧ $M(s_{\pi\pi}, s_{ll}) = \mathcal{M}_{mix} \times VMD(s_{\pi\pi}, s_{ll})$ contains the information of the decaying particle and the form factor.

✧ Within the VMD model, TFF can be parameterized into three separate parts

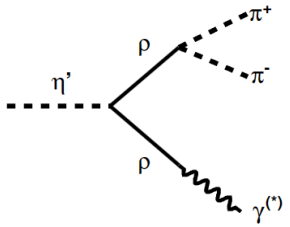
$$VMD(s_{\pi\pi}, s_{ll}) = 1 - \frac{3}{4}(c_1 - c_2 + c_3) + \frac{3}{4}(c_1 - c_2 - c_3)\frac{m_V^2}{m_V^2 - s_{ll} - im_V\Gamma(s_{ll})} + \frac{3}{2}c_3\frac{m_V^2}{m_V^2 - s_{ll} - im_V\Gamma(s_{ll})}\frac{m_{V,\pi}^2}{m_{V,\pi}^2 - s_{\pi\pi} - im_{V,\pi}\Gamma(s_{\pi\pi})}$$



Axial anomaly



VMD contribution

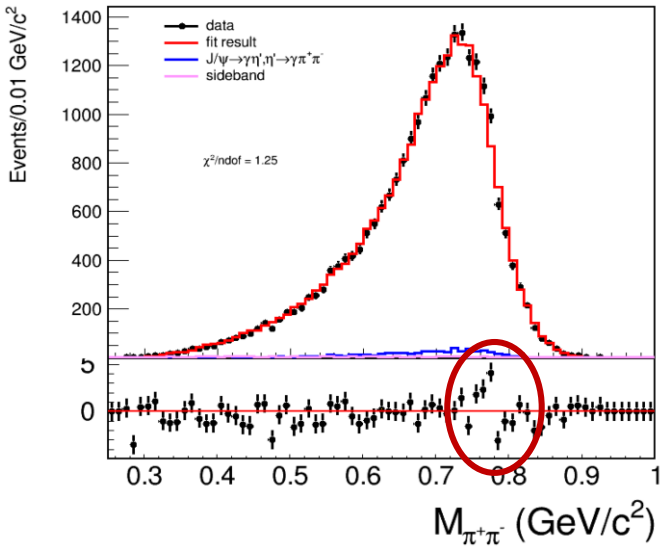


VMD contribution

✧ For $\eta' \to \pi^+\pi^- e^+ e^-$ decay

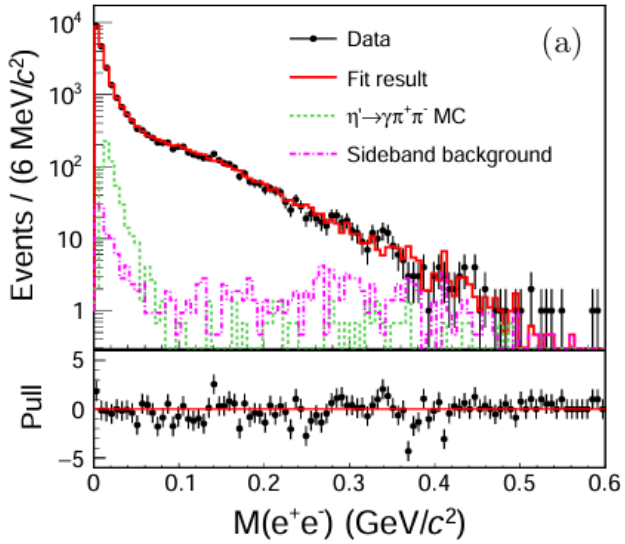
✓ ρ^0 only can not describe data well.

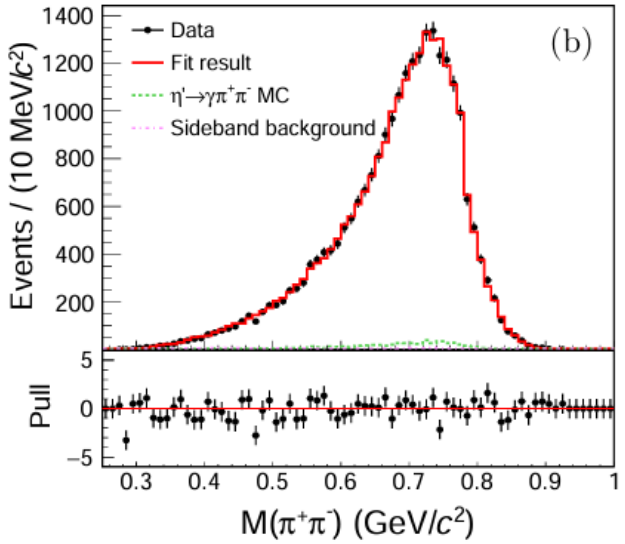
✓ $\omega \to \pi^+\pi^-$ decay is necessary!

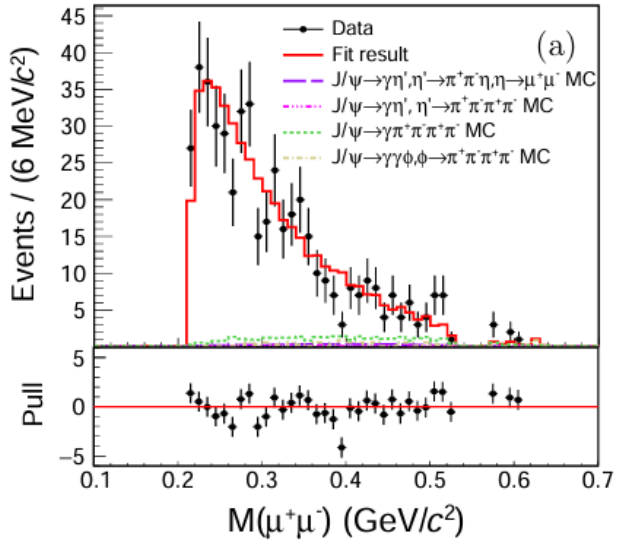


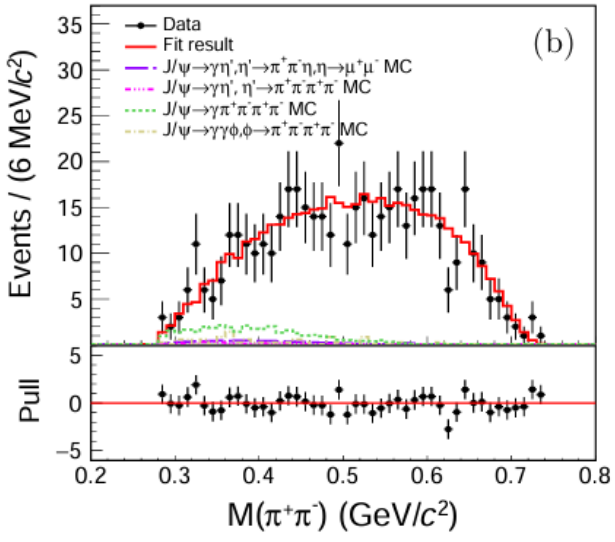
Transition Form Factor of $\eta' \to \pi^+\pi^- l^+ l^-$

JHEP07, 135(2024)



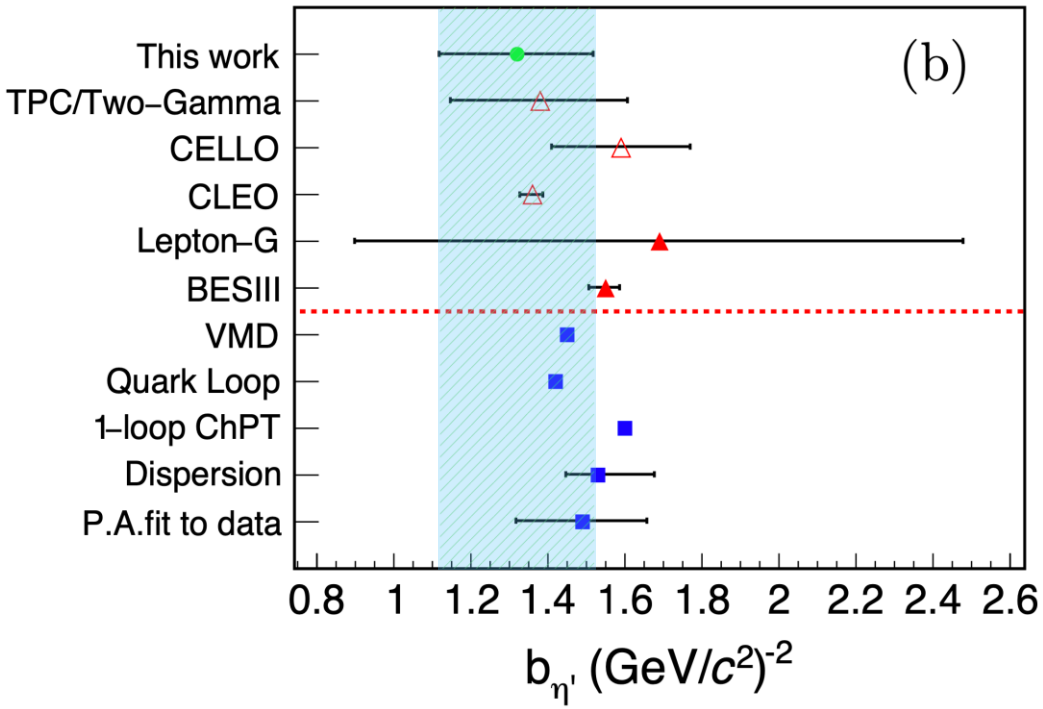






- First time to study form factors with $\eta' \to \pi^+\pi^- l^+ l^-$:

$$b_{\eta'} = 1.30 \pm 0.19\ \left(\mathrm{GeV}/c^2\right)^{-2}$$



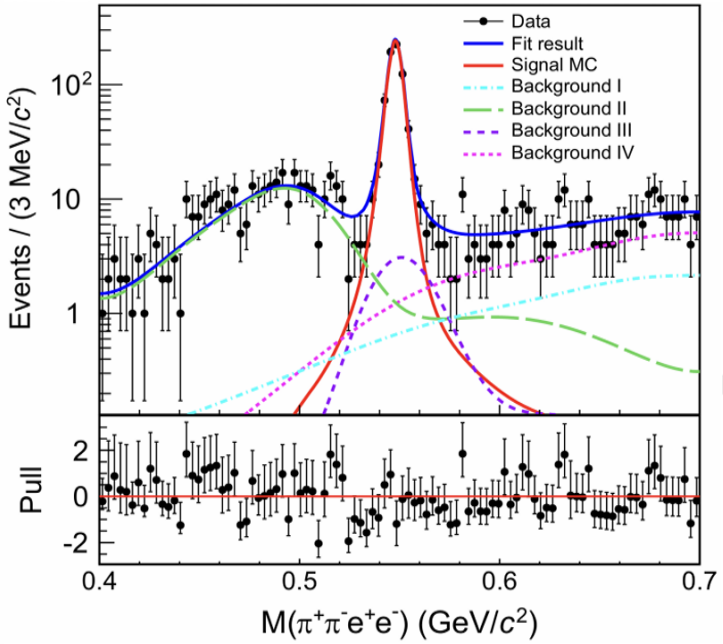
Study of $\eta \to \pi^+\pi^- l^+ l^-$

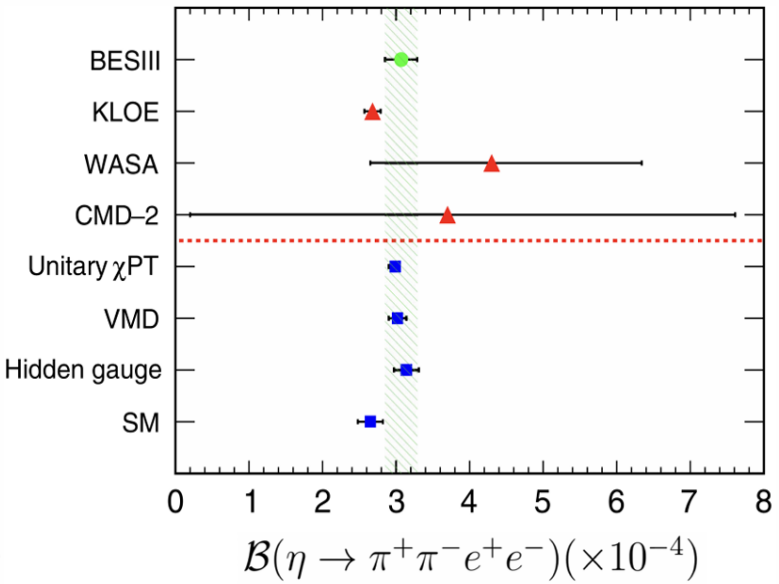
Phys. Rev. D 111, 072008 (2025)

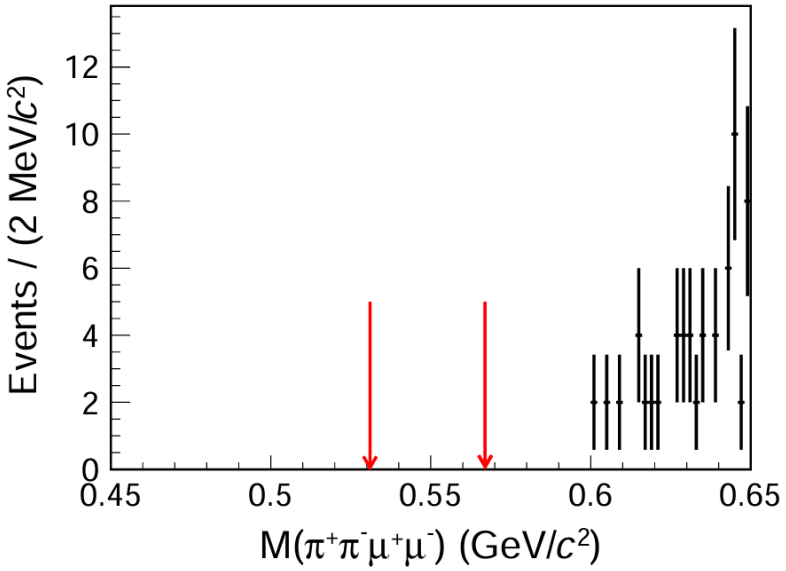
✧ The branching fraction:

$$B(\eta \to \pi^+\pi^- e^+ e^-) = (3.07 \pm 0.12 \pm 0.19) \times 10^{-4}$$

✧ No event left for $\pi^+\pi^-\mu^+\mu^-$ hypothesis in η signal region and the UL is set as 4.0×10^{-7} at the 90% C.L.



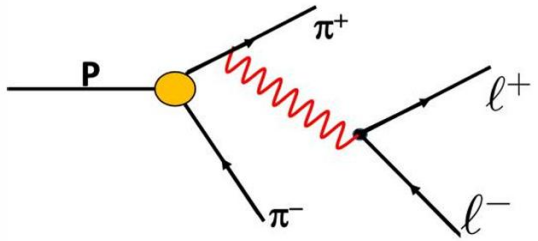


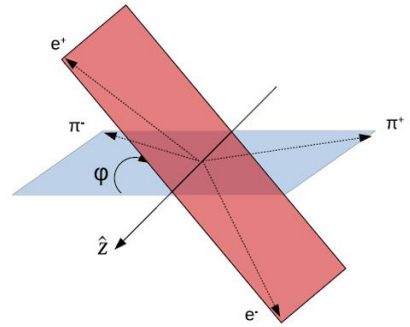


Asymmetry in $\eta/\eta' \to \pi^+\pi^-l^+l^-$

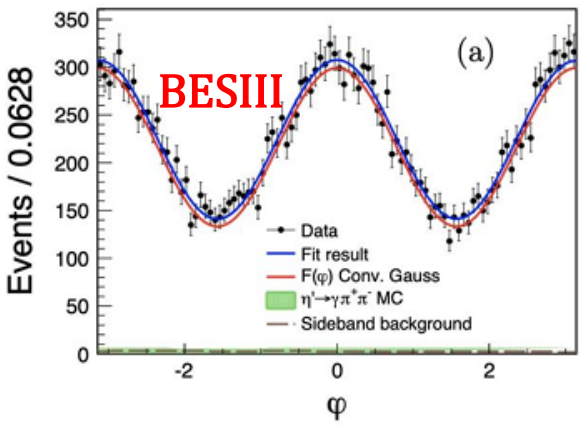
JHEP07, 135 (2024)
Phys. Rev. D 111, 072008 (2025)
PLB 675 (2009) 283

- Test of non-CKM CP Violation
- Arises from interference between CP conserving magnetic and CP violating electric transition



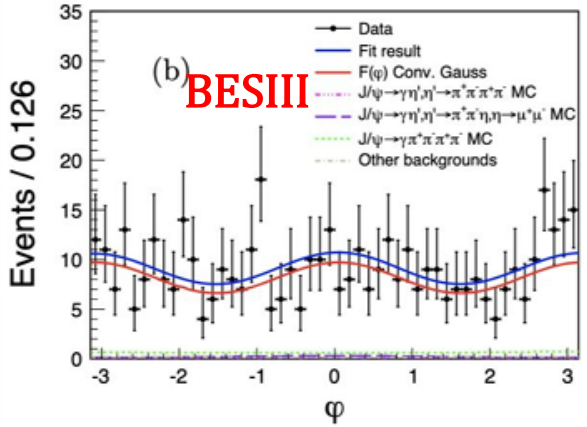


$\eta' \to \pi^+\pi^-e^+e^-$



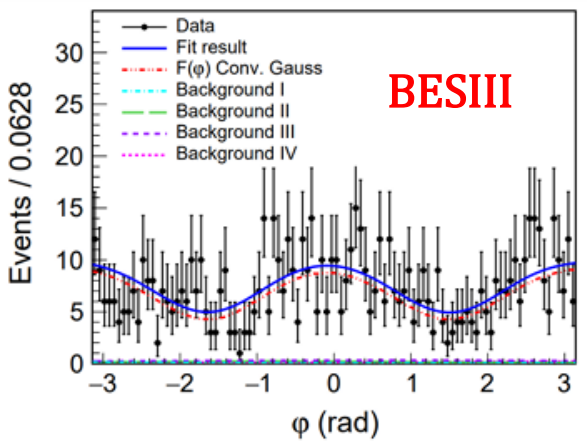
$A_{CP} = (-0.21 \pm 0.73 \pm 0.01)\%$

$\eta' \to \pi^+\pi^-\mu^+\mu^-$



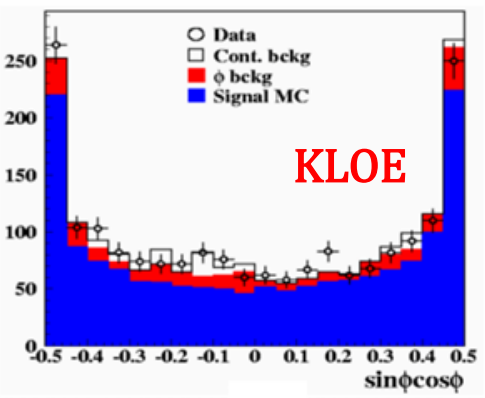
$A_{CP} = (0.62 \pm 4.71 \pm 0.08)\%$

$\eta \to \pi^+\pi^-e^+e^-$



$A_{CP} = (-4.04 \pm 4.69 \pm 0.14)\%$

$\eta \to \pi^+\pi^-e^+e^-$



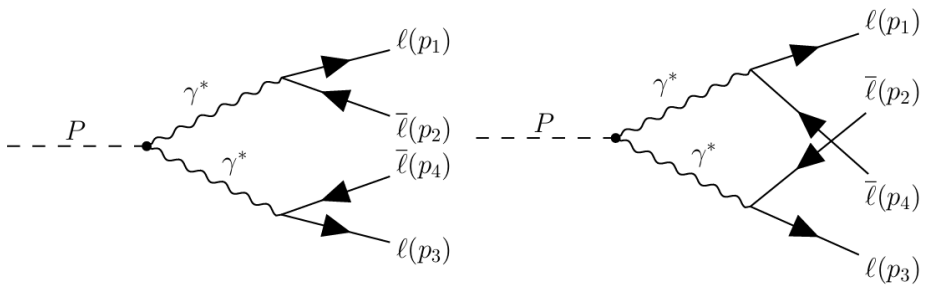
$A_{CP} = (-0.6 \pm 2.5 \pm 1.8)\%$

➡ All asymmetries consistent with zero at 10^{-3} level

Study of $\eta/\eta' \to l^+l^-l^+l^-$

Phys. Rev. D 105,112010 (2022)
Phys. Rev. D 111, 052002 (2025)
arXiv: 2605.04898 (Submit to PRD)

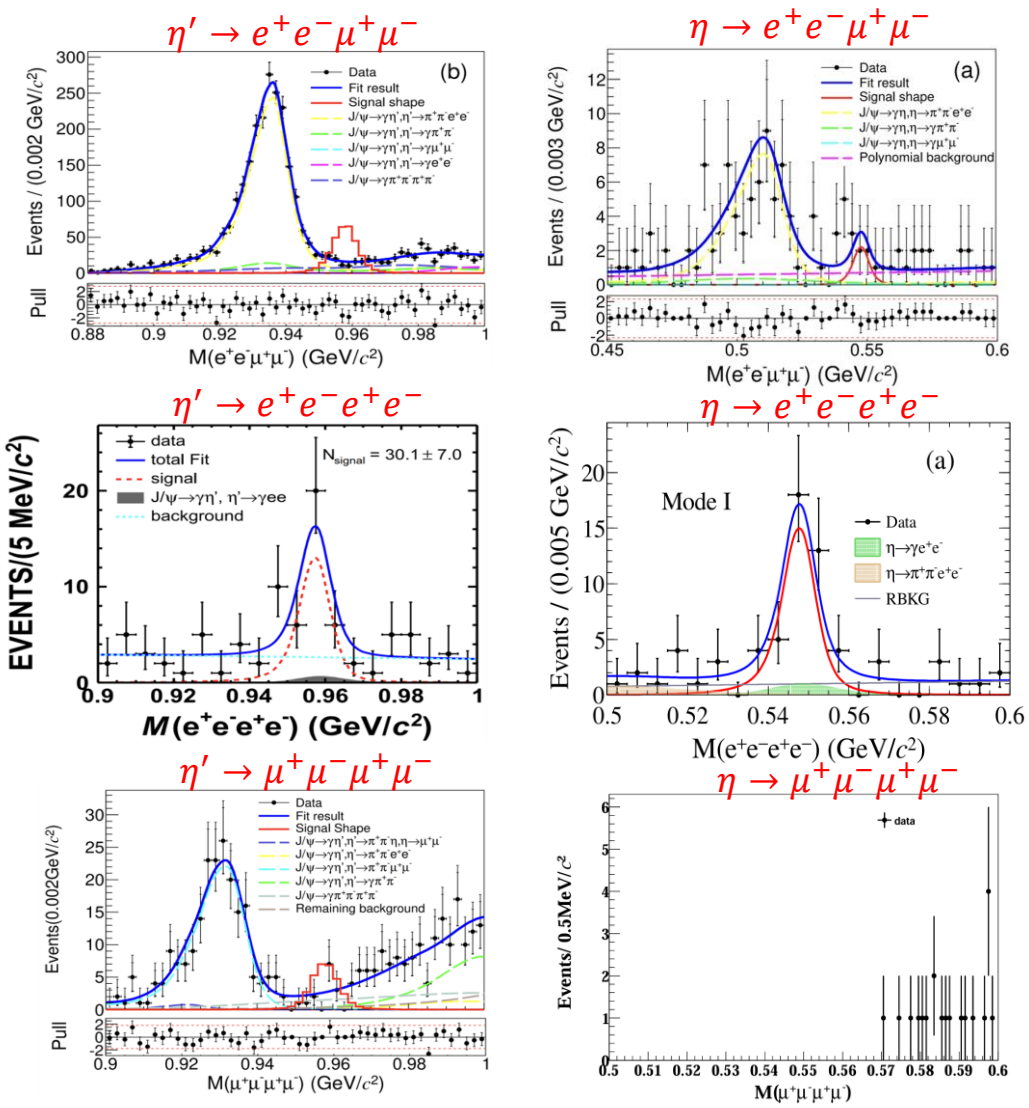
✧ The double Dalitz decays through two virtual photons intermediate with internal photon conversion to l^+l^- pairs.



✧ The branching fraction:

Decay mode	Theoretical predictions		Experiment
	VMD	Data driven approach	BESIII
$\eta \to e^+e^-e^+e^-$	$2.67(13) \times 10^{-5}$	$2.71(2) \times 10^{-5}$	$(2.6 \pm 0.3 \pm 0.2) \times 10^{-5}$
$\eta' \to e^+e^-e^+e^-$	$2.32(4) \times 10^{-6}$	$2.10(45) \times 10^{-6}$	$(4.5 \pm 1.0 \pm 0.5) \times 10^{-5}$
$\eta \to e^+e^-\mu^+\mu^-$	$2.15(22) \times 10^{-6}$	$2.39(7) \times 10^{-6}$	$< 6.88 \times 10^{-6}$
$\eta' \to e^+e^-\mu^+\mu^-$	$7.97(31) \times 10^{-7}$	$6.39(9) \times 10^{-7}$	$< 1.75 \times 10^{-6}$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$3.80(26) \times 10^{-9}$	$3.98(15) \times 10^{-9}$	$< 3.59 \times 10^{-7}$
$\eta' \to \mu^+\mu^-\mu^+\mu^-$	$2.19(10) \times 10^{-8}$	$1.69(36) \times 10^{-8}$	$< 5.28 \times 10^{-7}$

➡ The ULs are quite close to the SM predictions.

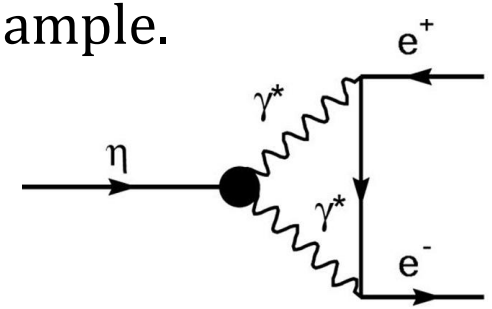


Study of $\eta \to l^+l^-$

PRD 113, 072002 (2026)

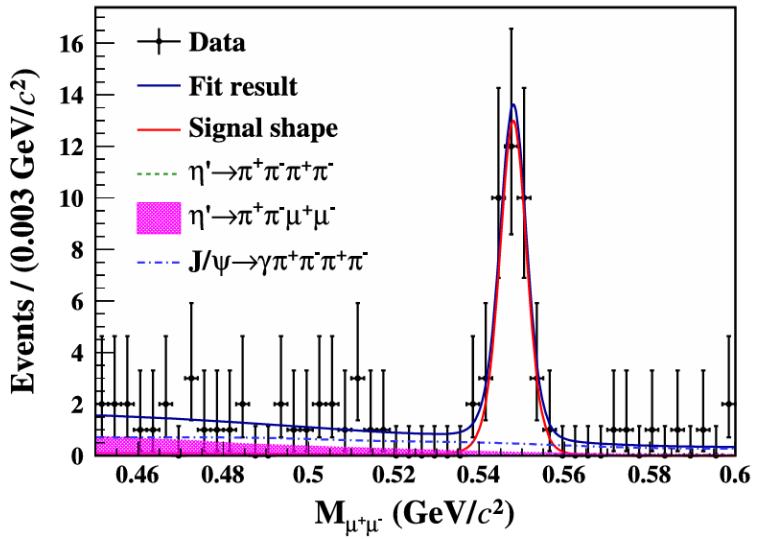
✧ Decays of pseudoscalar mesons to lepton pairs $\eta \to l^+l^-$ are rare processes that proceed primarily through a two-photon intermediate state. These decays are strongly suppressed.

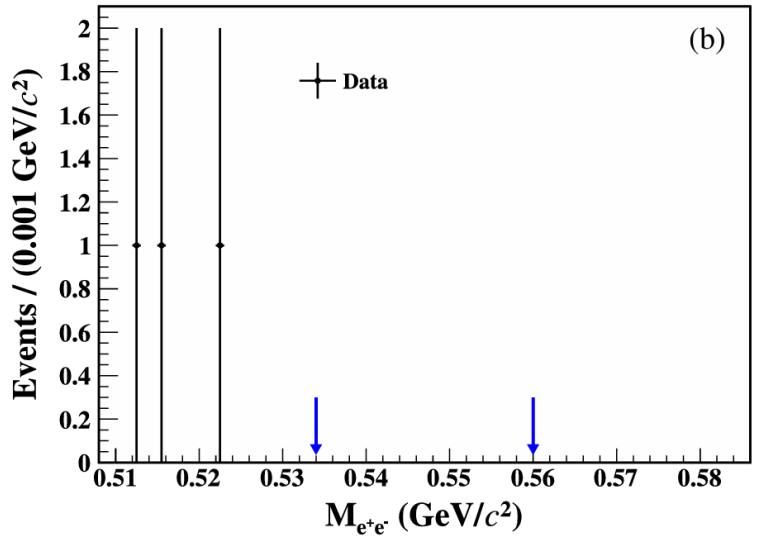
✧ $J/\psi \to \gamma\eta', \eta' \to \pi^+\pi^-\eta, \eta \to l^+l^-$ decay channel is provide abundant η sample.



✧ The branching fraction: $B(\eta \to \mu^+\mu^-) = (4.54 \pm 0.04 \pm 0.02) \times 10^{-6}$

$B(\eta \to e^+e^-) < 2.2 \times 10^{-7}$





Summary

- **10 Billion J/ψ events at BESIII:**
  - A worldwide unique laboratory for studying light mesons
  - Allow to study light meson decays with high precision

- **Significant progresses achieved on η/η' decays:**
  - Decay dynamics, rare decays, transition form factors, discrete symmetries …

- **More results are expected to come soon!**

THANKS

Backup

Decay list of light meson in BESIII

Rich η/η' Physics

Standard Model Tests:
- Chiral symmetry and anomalies
- Extract $\eta - \eta'$ mixing angle and quark mass ratio
- Theory inputs to HLbL for $(g-2)_\mu$
- QCD scalar dynamics

Fundamental Symmetry Tests:
- C, CP violations
- P, CP violations
- Lepton flavor violations

BSM Physics in Dark Sector:
- Vector bosons (B boson, dark photon and X boson)
- Dark scalars
- Pseudoscalars (ALPs)
- BSM weak decays

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	Chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$<1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$<2.8 \times 10^{-4}$	$<10^{-11}$[55]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	Chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$<2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	Theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$<7 \times 10^{-7}$	Theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	Theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$<3.6 \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	Theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$<1.6 \times 10^{-4}$	Theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$<3.6 \times 10^{-4}$	Theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$<5 \times 10^{-4}$	Direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$<1.7 \times 10^{-4}$	Second-class current
$\eta \to \pi^+\pi^-$	$<4.4 \times 10^{-6}$ [56]	P/CP violation
$\eta \to 2\pi^0$	$<3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$<6.9 \times 10^{-7}$	P/CP violation

Phys. Rept. 945 (2022) 1-105

Decay list of light meson in BESIII

Rich η/η' Physics

Standard Model Tests:
- Chiral symmetry and anomalies
- Extract $\eta - \eta'$ mixing angle and quark mass ratio
- Theory inputs to HLbL for $(g-2)_\mu$
- QCD scalar dynamics

Fundamental Symmetry Tests:
- C, CP violations
- P, CP violations
- Lepton flavor violations

BSM Physics in Dark Sector:
- Vector bosons (B boson, dark photon and X boson)
- Dark scalars
- Pseudoscalars (ALPs)
- BSM weak decays

Channel	Expt. branching ratio	Discussion
$\eta' \to \eta\pi^+\pi^-$	42.6(7)%	Large-N_c χPT, light Higgs scalars
$\eta' \to \pi^+\pi^-\gamma$	28.9(5)%	Chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta' \to \eta\pi^0\pi^0$	22.8(8)%	Large-N_c χPT
$\eta' \to \omega\gamma$	2.489(76)% [58]	Theory input for singly-virtual TFF and $(g-2)_\mu$
$\eta' \to \omega e^+e^-$	$2.0(4) \times 10^{-4}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \to 2\gamma$	2.331(37)% [58]	Chiral anomaly, η–η' mixing
$\eta' \to 3\pi^0$	2.54(18)% (*)	$m_u - m_d$
$\eta' \to \mu^+\mu^-\gamma$	$1.09(27) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta' \to e^+e^-\gamma$	$4.73(30) \times 10^{-4}$	Theory input for $(g-2)_\mu$, dark photon
$\eta' \to \pi^+\pi^-\mu^+\mu^-$	$<2.9 \times 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \to \pi^+\pi^-e^+e^-$	$2.4\left(^{+1.3}_{-1.0}\right) \times 10^{-3}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, dark photon, ALPs
$\eta' \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta' \to \pi^+\pi^-\pi^0$	$3.61(17) \times 10^{-3}$	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta' \to 2(\pi^+\pi^-)$	$8.4(9) \times 10^{-5}$	Theory input for doubly-virtual TFF and $(g-2)_\mu$
$\eta' \to \pi^+\pi^-2\pi^0$	$1.8(4) \times 10^{-4}$	
$\eta' \to 2(\pi^+\pi^-)\pi^0$	$<1.8 \times 10^{-3}$	ALPs
$\eta' \to K^\pm\pi^\mp$	$<4 \times 10^{-5}$	Weak interactions
$\eta' \to \pi^\pm e^\mp \nu_e$	$<2.1 \times 10^{-4}$	Second-class current
$\eta' \to \pi^0\gamma\gamma$	$3.20(24) \times 10^{-3}$	Vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \to \eta\gamma\gamma$	$8.3(3.5) \times 10^{-5}$ [59]	Vector and scalar dynamics, B boson, light Higgs scalars
$\eta' \to 4\pi^0$	$<4.94 \times 10^{-5}$ [60]	(S-wave) P/CP violation
$\eta' \to e^+e^-$	$<5.6 \times 10^{-9}$	Theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \to \mu^+\mu^-$		Theory input for $(g-2)_\mu$, BSM weak decays
$\eta' \to \ell^+\ell^-\ell^+\ell^-$		Theory input for $(g-2)_\mu$
$\eta' \to \pi^+\pi^-\pi^0\gamma$		B boson
$\eta' \to \pi^+\pi^-$	$<1.8 \times 10^{-5}$	P/CP violation
$\eta' \to 2\pi^0$	$<4 \times 10^{-4}$	P/CP violation

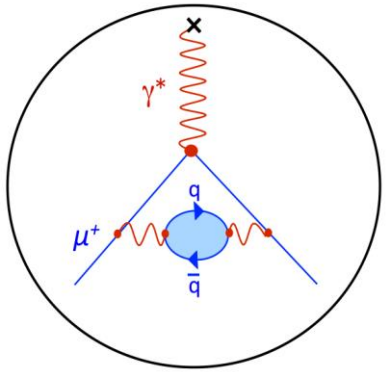
Phys. Rept. 945 (2022) 1-105

Form Factor Physics

- ✓ Describe the complex internal structure or intermediate processes
- ✓ It determines the size of hadronic quantum corrections in the calculation of the $(g-2)_\mu$

$$a_\mu = \frac{1}{2}(g-2)_\mu$$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadron}, a_\mu^{hadron} = a_\mu^{HVP} + a_\mu^{HLbL}$$

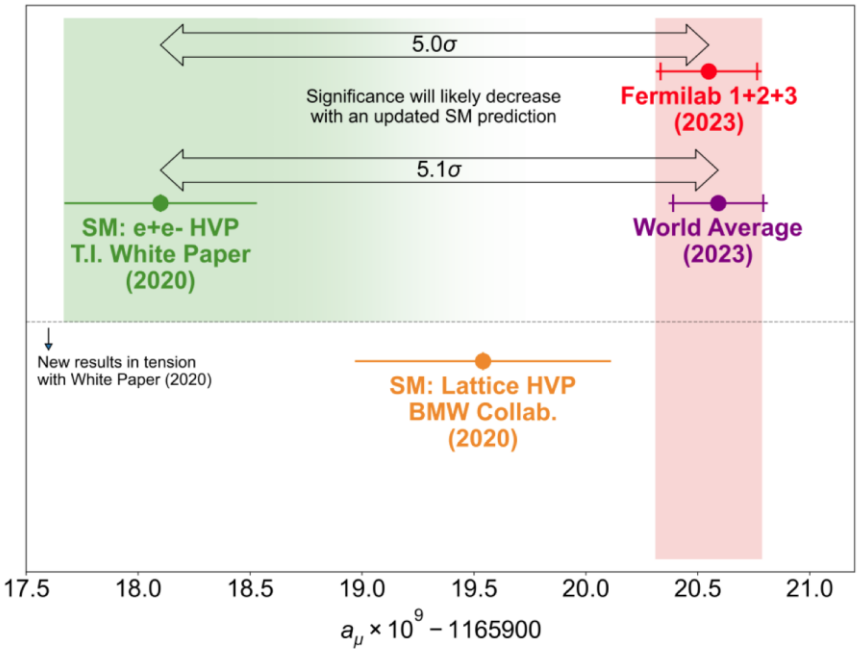


$(40)\times10^{-11}$

Polarization(LO)

$a_\mu^{HVP} = 6845(40) \times 10^{-11}$
Hadronic Vacuum Polarization(LO)

$a_\mu^{HLbL} = 92(18) \times 10^{-11}$
Hadronic Light-by-Light

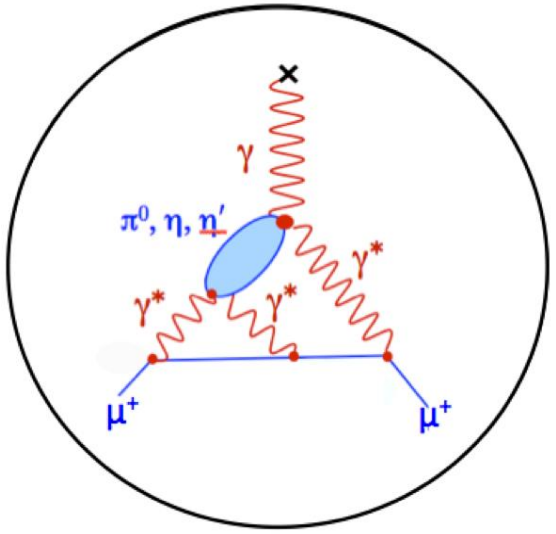


- ✓ Experimental input is needed to improve the precision of predictions!

Form Factor Physics

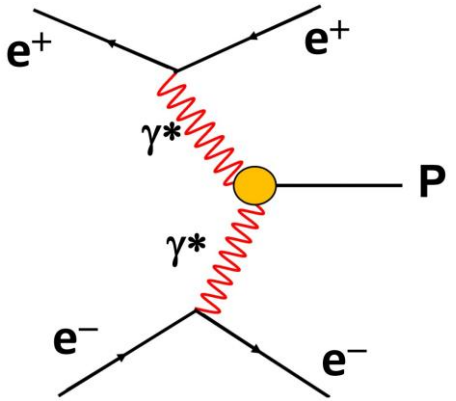
- ✓ The coupling of π^0, η, and η' with photon in HLbL can be described using transition form factor (TFF).

TFFs as experimental input!

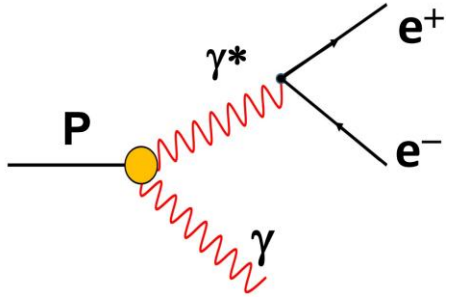


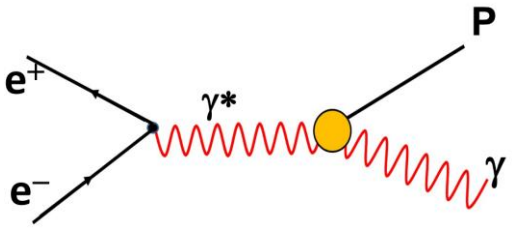
- ✓ TFFs are experimentally accessible in three different processes

Space-like



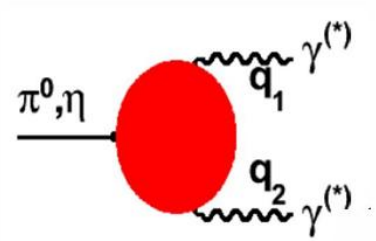
Time-like



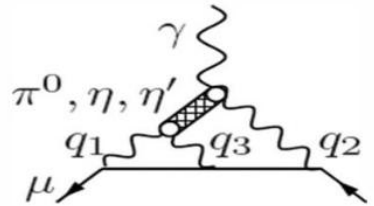


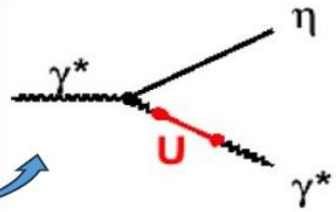
Form Factor Physics

Meson transition form factors

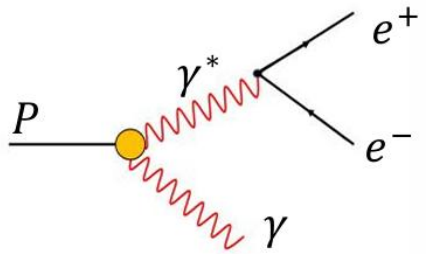


- Low energy QCD:
  - Enters in th. description of QCD processes
  - Evolution with Q^2 predicted by pQCD: models can be tested using data on Q^2 dependance
- Light-by-light contribution to a_μ
- Search for light dark force mediator

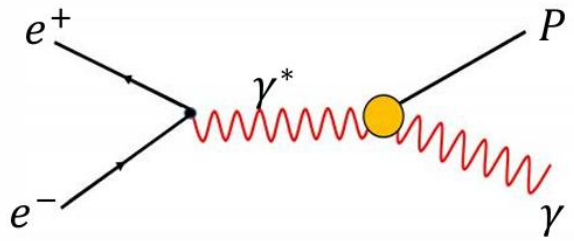




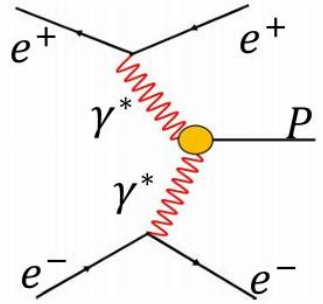
Experimentally



Dalitz decays $0<q^2<M^2$



Annihilation process $q^2>M^2$



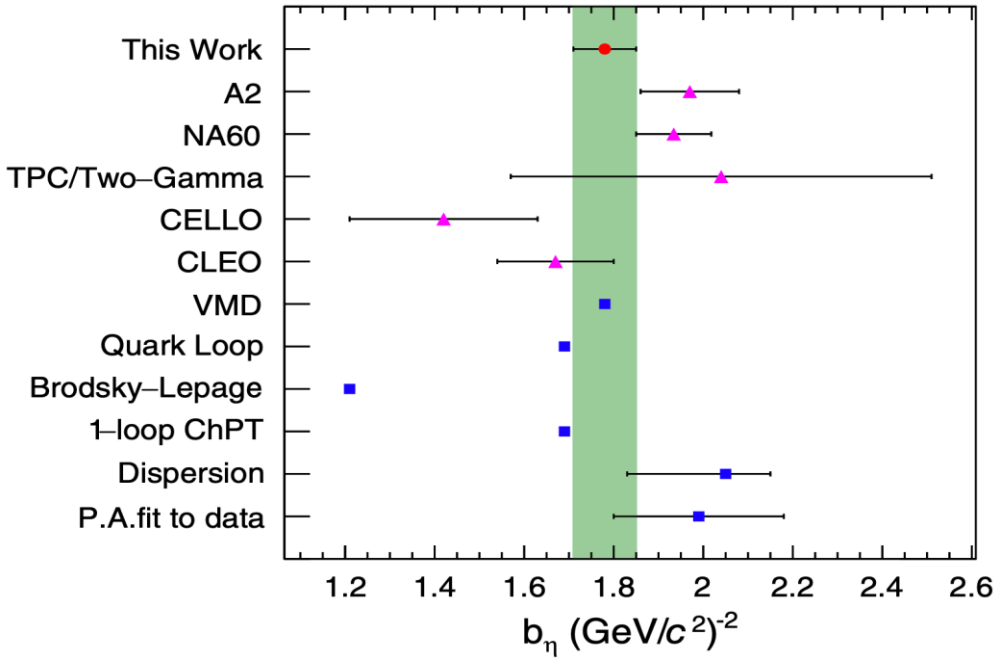
Two photon process $q^2<0$

$\eta/\eta' \to \gamma e^+ e^-$

Time-like: $\eta' \to \gamma l^+ l^-$

Space like: $e^+ e^- \to e^+ e^- \eta'$

Theory



$\eta' \to \pi^+\pi^- l^+ l^-$

TFF Results

$\eta' \to \pi^+\pi^- e^+ e^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V(\mathrm{MeV}/c^2)$	$954.3 \pm 82.5 \pm 36.4$	857.4 ± 74.3	787.5 ± 137.9
$m_{V,\pi}(\mathrm{MeV}/c^2)$	$765.3 \pm 1.1 \pm 20.2$	765.4 ± 1.1	764.8 ± 1.3
$m_\omega(\mathrm{MeV}/c^2)$	$778.7 \pm 1.3 \pm 17.3$	778.7 ± 1.3	778.7 ± 1.4
$\beta(10^{-3})$	$8.5 \pm 1.4 \pm 0.7$	8.5 ± 1.4	8.1 ± 1.4
θ	$1.4 \pm 0.3 \pm 0.1$	1.4 ± 0.3	1.4 ± 0.4
$c_1 - c_2$	1	1/3	-0.03 ± 0.87
c_3	1	1	1.03 ± 0.02
$\chi^2/ndof(e^+e^-, \pi^+\pi^-)$	65.3/82.0, 44.5/65.0	66.1/82.0, 44.3/65.0	66.8/82.0, 42.2/65.0
$b_{\eta'}(\mathrm{GeV}/c^2)^{-2}$	$1.10 \pm 0.19 \pm 0.07$	1.36 ± 0.24	1.61 ± 0.56

$\eta' \to \pi^+\pi^- \mu^+ \mu^-$	Model I	Model II	Model III
	$c_1 - c_2 = c_3 = 1$	$c_1 - c_2 = 1/3, c_3 = 1$	$c_1 - c_2 \neq c_3$
$m_V(\mathrm{MeV}/c^2)$	$649.4 \pm 52.3 \pm 35.6$	601.6 ± 24.0	589.6 ± 24.2
$m_{V,\pi}(\mathrm{MeV}/c^2)$	$757.3 \pm 22.6 \pm 18.0$	765.4 ± 17.6	774.4 ± 40.7
$c_1 - c_2$	1	1/3	0.01 ± 0.42
c_3	1	1	0.98 ± 0.38
$\chi^2/ndof(\mu^+\mu^-, \pi^+\pi^-)$	36.1/34.0, 30.4/46.0	36.1/34.0, 30.4/46.0	37.4/35.0, 29.9/46.0
$b_{\eta'}(\mathrm{GeV}/c^2)^{-2}$	$2.37 \pm 0.38 \pm 0.27$	2.76 ± 0.22	2.88 ± 0.24

→ **Large statistical uncertainty of m_V and $c_1 - c_2$**

- A test with $c_1 - c_2 = c_3$ gives

$$c_1 - c_2 = c_3 = 1.03 \pm 0.02$$

- Provide a weighted average of the slope parameter for $\eta' \to \pi^+\pi^- e^+ e^-$ and $\eta' \to \pi^+\pi^- \mu^+ \mu^-$ based on Model I.

$$b_{\eta'} = 1.30 \pm 0.19 \left(\mathrm{GeV}/c^2\right)^{-2}$$