

Searching for light dark particles with η and η' mesons

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14th International Workshop on e^+e^- collisions from Phi to Psi

Pisa, 8-11 June, 2026

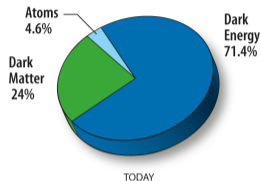


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Beyond the Standard Model

- The SM fails to explain:

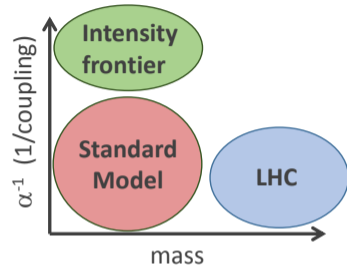
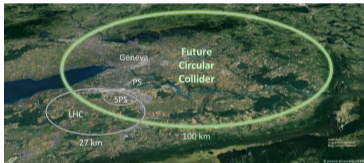
- **Dark matter:** what is the most prevalent kind of matter in our Universe?
- **Dark Energy:** what drives the accelerated expansion of the Universe?
- **Neutrino** masses and oscillations: why do neutrinos have mass? what makes neutrinos disappear and then re-appear in a different form?
- **Baryon asymmetry** of the Universe: what mechanism created the tiny matter-antimatter imbalance in the early Universe?
- Several **anomalies in data:** $(g - 2)_\mu$, B -physics anomalies, KOTO anomaly ($K_L \rightarrow \pi^0 \nu \bar{\nu}$), ^8Be excited decay, ...



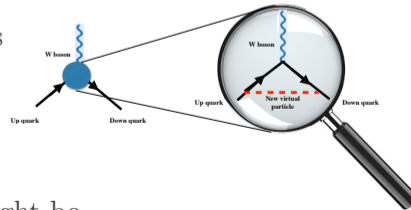
- Where can we look for BSM physics?

Finding New Physics: energy vs precision frontier

- **Energy frontier:** smash protons as hard as you can, and see what comes out (LHC, FCC 9,000M€)
 - create **new** (heavy) particles and/or study their **effects** on rare processes



- **Intensity frontier:** new feebly interacting particles
 - search for tiny indirect effects, with no (or very precisely known) SM background

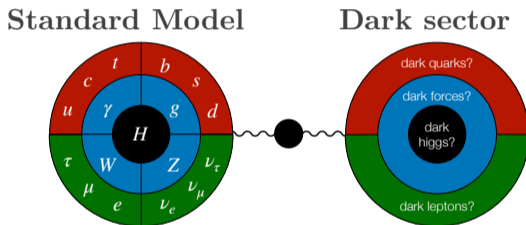


- We don't know in which **direction** BSM physics might be

Dark sector portals to the Standard Model

- How can we **access** (and test) the **dark sector**?

⇒ **Portal** interactions with the SM, only a few are allowed by the SM symmetries



Portal

Vector

Scalar

Neutrino

Axion

Mediators

Dark photon

Dark scalar

Sterile Neutrino

Axion

Portal interactions

$$\epsilon B^{\mu\nu} A'_{\mu\nu}$$

$$\kappa |H|^2 |S|^2$$

$$y H L N$$

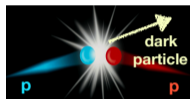
$$\frac{a}{f_a} \tilde{G}_{\mu\nu} G^{\mu\nu}$$

A broad program of searches of dark particles

- Vigorous effort of the community proposing **new** experiments & measurements

Energy frontier

LHC



Novel search strategies are needed!

Flavor-factories

High-luminosity e^+e^- colliders



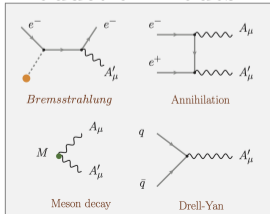
Unique access to dark sectors!

Other ongoing/future experiments

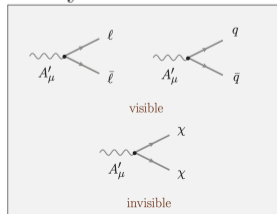


- Plenty of dark particles can be produced from **meson decays!!**

Production modes



Decay modes



High-intensity programs in meson factories

- η/η' -factories



$\sim 3 \times 10^7$



KLOE-2 (Frascati)

$\sim 10^9$



$\sim 4 \times 10^7$



$\sim 4.5 \times 10^8$



CB@MAMI

$\sim 3 \times 10^8$



$\eta \sim 10^{12}$

Experiment	Technique	Total mesons
<i>GlueX@JLAB (running)</i>	$\gamma_{12 \text{ GeV}} \text{ p} \rightarrow \eta \text{ X} \rightarrow \text{neutrals}$	$5.5 \times 10^7 / \text{yr}$
<i>JEF@JLAB (approved)</i>	$\gamma_{12 \text{ GeV}} \text{ p} \rightarrow \eta \text{ X} \rightarrow \text{neutrals}$	$1.5 \times 10^8 / \text{yr}$
<i>HIAF (approved)</i>		$\sim 10^{13} / \text{yr}$
REDTOP (proposing)	$p_{1.8 \text{ GeV}} \text{ Li} \rightarrow \eta \text{ X}$	$3.4 \times 10^{13} / \text{yr}$

- π -factories (PIENU, PIONEER), K -factories (E949, E391, NA62, KOTO), B -factories (LHCb, Belle-(II))

η/η' laboratory for dark sectors

- The η is a pNGB, with $m_\eta \simeq 548$ MeV and $\Gamma_\eta = 1.31$ keV
- The η' : not a pNGB due to $U(1)_A$ anomaly, $m_{\eta'} \simeq 958$ MeV, $\Gamma_{\eta'} = 196$ keV
- Eigenstates of the C, P, CP and G operators: $I^G J^{PC} = 0^+ 0^{-+}$
- Flavor **conserving** decays \Rightarrow laboratory for symmetry tests
- All their EM and strong decays are **suppressed** at LO $\sim \mathcal{O}(\alpha_{\text{em}}^2)$ or $\mathcal{O}((m_u - m_d)^2)$
- **Search** strategies (visible final states):
 - Resonance searches (bump hunting)
 - Displaced vertices (long-lived decays)
 - Rare decays: new physics process mimics highly-suppressed SM channels
- Other possibilities: Invisible (or partially-invisible) decays
- Perfect **laboratory** to stress-test the SM in search for BSM physics

Rich physics program at η, η' factories

Standard Model highlights

- Theory input for light-by-light scattering for $(g-2)_\mu$
- Extraction of light quark masses
- QCD scalar dynamics

Fundamental symmetry tests

- P,CP violation
- C,CP violation

[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]

Dark sectors (MeV—GeV)

- Vector bosons (dark photon, B boson, X boson)
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

Channel	Expt. branching ratio	Discussion
$\eta \rightarrow 2\gamma$	39.41(20)%	chiral anomaly, η - η' mixing
$\eta \rightarrow 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \rightarrow \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χ PT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \rightarrow \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χ PT, axion-like particles (ALPs)
$\eta \rightarrow 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [52]
$\eta \rightarrow \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \rightarrow \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \rightarrow \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χ PT, ALPs
$\eta \rightarrow e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \rightarrow \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \rightarrow e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \rightarrow \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \rightarrow \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-\ell^+\ell^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \rightarrow e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \rightarrow \pi^+e^-\bar{\nu}_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \rightarrow \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [53]	P/CP violation
$\eta \rightarrow 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation Gan, Kubis, Passemar, ST (2020)
$\eta \rightarrow 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation (2020)

Dark particles in η/η' decays

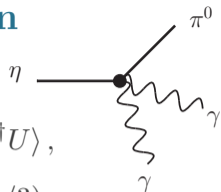
BSM particle	Decay mode	Signal channel	Search strategy
Dark photon (A')	$\eta/\eta' \rightarrow \gamma^{(*)} A'$	$A' \rightarrow \ell^+ \ell^-$	Bump-hunt in $d\Gamma/dm_{\ell\ell}$
		$A' \rightarrow \pi^+ \pi^-$	Bump-hunt in $d\Gamma/dm_{\pi\pi}$
Leptophobic boson (B)	$\eta \rightarrow \gamma B$	$B \rightarrow \gamma \pi^0$	Enhancement in $m_{\pi^0 \gamma}$
		$B \rightarrow \pi^+ \pi^-$	Isospin suppressed
	$\eta' \rightarrow \gamma B$	$B \rightarrow \gamma \pi^0, \pi^+ \pi^-, \pi^+ \pi^- \pi^0, \gamma \eta$	Enhancement in $m_{\pi^0 \gamma}$
ALPs (a)	$\eta \rightarrow \pi \pi a$	$a \rightarrow \gamma \gamma, \ell^+ \ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma}$
	$\eta' \rightarrow \pi \pi a$	$a \rightarrow \gamma \gamma, \ell^+ \ell^-, \pi^+ \pi^- \gamma, 3\pi$	Bump-hunt in $d\Gamma/dm_{\gamma\gamma}$
	$\eta^{(\prime)} \rightarrow \ell^+ \ell^-$		$\eta^{(\prime)}$ - a mixing
Scalar boson (S)	$\eta/\eta' \rightarrow \pi^0 S$	$S \rightarrow \gamma \gamma, \ell^+ \ell^-, \pi \pi$	Bump-hunt in $d\Gamma/dm_{\gamma\gamma}$
	$\eta' \rightarrow \eta S$	$S \rightarrow \gamma \gamma, \ell^+ \ell^-, \pi \pi$	Bump-hunt in $d\Gamma/dm_{\gamma\gamma}$

$\eta \rightarrow \pi^0 \gamma \gamma$ decays: Theoretical motivation

- $\mathcal{O}(p^2)$ tree-level χ PT:

$$\mathcal{L}_2 = \frac{F_\pi^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle + \frac{F_\pi^2}{4} \langle U^\dagger \chi + \chi^\dagger U \rangle,$$

where $D_\mu = \partial_\mu + ieA_\mu[Q, U]$, $Q = \text{diag}(2/3, -1/3, -1/3)$.



$\mathcal{O}(p^2), \mathcal{O}(p^4)$

Ametller et.al. Phys.Lett.B 276 (1992)

$\rightarrow [Q, U] = 0$ for neutral pseudoscalars $\Rightarrow \boxed{\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)|_{p^2}^{\text{tree}} = 0}$

- $\mathcal{O}(p^4)$ tree-level χ PT:

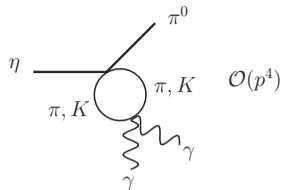
$$\begin{aligned} \mathcal{L}_4 = & L_1 \frac{F_\pi^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle^2 + \dots + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\ & - ieL_9 F_{\mu\nu} \langle Q D^\mu U D^\nu U^\dagger + Q D^\mu U^\dagger D^\nu U \rangle + e^2 L_{10} F_{\mu\nu} F^{\mu\nu} \langle Q U Q U^\dagger \rangle, \end{aligned}$$

L_9 : vanishes because $F_{\mu\nu}$ (antisymmetric) contracted to a symmetric tensor

L_{10} : $F_{\mu\nu} F^{\mu\nu} \langle Q U Q U^\dagger \rangle = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} \{ \langle [Q, U][Q, U^\dagger] \rangle - 2 \langle Q^2 \rangle \}$

$$\Rightarrow \boxed{\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)|_{p^4}^{\text{tree}} = 0}$$

$\eta \rightarrow \pi^0 \gamma \gamma$ decays: Theoretical motivation



Ametller et.al. Phys.Lett.B 276 (1992)

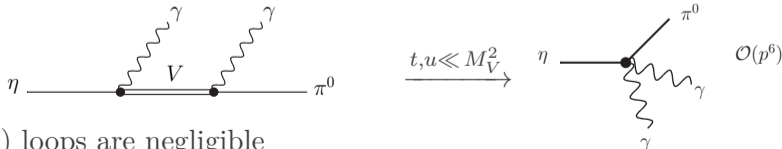
- $\mathcal{O}(p^4)$ χ PT π, K loops: $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)|_{p^4}^{\text{loops}} = 1.87 \times 10^{-3} \text{ eV}$

Reference	$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)$ [eV]
$\mathcal{O}(p^2), \mathcal{O}(p^4)$ tree-level χ PT	0
$\mathcal{O}(p^4)$ $\pi + K$ loops	1.87×10^{-3}
Experimental value (A2@MAMI'14)	0.33(3)
Experimental value (KLOE'26)	0.13(2)

- $\mathcal{O}(p^6)$ tree-level χ PT: first sizable contribution $\mathcal{O}(\Gamma_{\text{exp}})$

$$\mathcal{L}_6 = d_1 F_{\mu\nu} F^{\mu\nu} \langle Q^2 \partial^\lambda U \partial_\lambda U^\dagger \rangle + d_2 F_{\mu\alpha} F^{\mu\beta} \langle Q^2 \partial^\alpha U^\dagger \partial_\beta U \rangle,$$

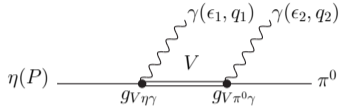
$d_{1,2}$: saturated with meson resonances $d_{1,2} = d_{1,2}^V + d_{1,2}^S + d_{1,2}^T$



- $\mathcal{O}(p^6)$ loops are negligible

$\eta \rightarrow \pi^0 \gamma \gamma$: Vector meson exchange contributions

- Six **diagrams** corresponding to the exchange of $V = \rho^0, \omega, \phi$



$$A_{\eta \rightarrow \pi^0 \gamma \gamma}^{\text{VMD}} = \sum_{V=\rho^0, \omega, \phi} g_{V\eta\gamma} g_{V\pi^0\gamma} \left[\frac{(P \cdot q_2 - m_\eta^2) \{a\} - \{b\}}{D_V(t)} + \left\{ \begin{array}{l} q_2 \leftrightarrow q_1 \\ t \leftrightarrow u \end{array} \right\} \right],$$

Decay	Branching ratio (pdg)	$ g_{VP\gamma} \text{ GeV}^{-1}$
$\rho^0 \rightarrow \pi^0 \gamma$	$(4.7 \pm 0.6) \times 10^{-4}$	0.22(1)
$\rho^0 \rightarrow \eta \gamma$	$(3.00 \pm 0.21) \times 10^{-4}$	0.48(2)
$\eta' \rightarrow \rho^0 \gamma$	$(28.9 \pm 0.5)\%$	0.40(1)
$\omega \rightarrow \pi^0 \gamma$	$(8.40 \pm 0.22)\%$	0.70(1)
$\omega \rightarrow \eta \gamma$	$(4.5 \pm 0.4) \times 10^{-4}$	0.135(6)
$\eta' \rightarrow \omega \gamma$	$(2.62 \pm 0.13)\%$	0.127(4)
$\phi \rightarrow \pi^0 \gamma$	$(1.30 \pm 0.05) \times 10^{-3}$	0.041(1)
$\phi \rightarrow \eta \gamma$	$(1.303 \pm 0.025)\%$	0.2093(20)
$\phi \rightarrow \eta' \gamma$	$(6.22 \pm 0.21) \times 10^{-5}$	0.216(4)

$$t, u = (P - q_{2,1})^2 = m_\eta^2 - 2P \cdot q_{2,1},$$

$$\{a\} = (\epsilon_1 \cdot \epsilon_2)(q_1 \cdot q_2) - (\epsilon_1 \cdot q_2)(\epsilon_2 \cdot q_1),$$

$$\{b\} = (\epsilon_1 \cdot q_2)(\epsilon_2 \cdot P)(P \cdot q_1)$$

$$+ (\epsilon_2 \cdot q_1)(\epsilon_1 \cdot P)(P \cdot q_2)$$

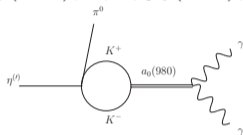
$$- (\epsilon_1 \cdot \epsilon_2)(P \cdot q_1)(P \cdot q_2)$$

$$- (\epsilon_1 \cdot P)(\epsilon_2 \cdot P)(q_1 \cdot q_2)$$

- The decays $\eta' \rightarrow \{\pi^0, \eta\} \gamma \gamma$ are formally identical: $g_{V\eta\gamma} g_{V\pi^0\gamma} \rightarrow g_{V\eta'\gamma} g_{V\{\pi^0, \eta\}\gamma}$

$L\sigma M$ for the scalar resonance contributions

- χ PT loops complemented by the exchange of scalar resonances, e.g. $a_0(980)$, κ , σ , $f_0(980)$,



$$\mathcal{A}_{\eta^{(\prime)} \rightarrow \pi^0 \gamma \gamma}^{\text{L}\sigma\text{M}} = \frac{2\alpha}{\pi} \frac{1}{m_{K^+}^2} L(s_K) \{a\} \times \mathcal{A}_{K^+ K^- \rightarrow \pi^0 \eta^{(\prime)}}^{\text{L}\sigma\text{M}},$$

- Scalar amplitudes:

$$\mathcal{A}_{K^+ K^- \rightarrow \pi^0 \eta^{(\prime)}}^{\text{L}\sigma\text{M}} = \frac{1}{2f_\pi f_K} \left\{ (s - m_{\eta^{(\prime)}}^2) \frac{m_K^2 - m_{a_0}^2}{D_{a_0}(s)} \cos \varphi_P + \frac{1}{6} \left[(5m_{\eta^{(\prime)}}^2 + m_\pi^2 - 3s) \cos \varphi_P - \sqrt{2}(m_{\eta^{(\prime)}}^2 + 4m_K^2 + m_\pi^2 - 3s) \sin \varphi_P \right] \right\},$$

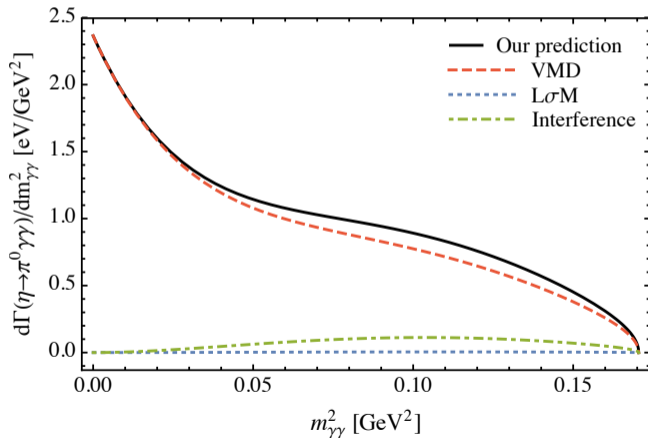
- Complete one-loop propagator for the scalar resonances:

$$D_R(s) = s - m_R^2 + \text{Re}\Pi(s) - \text{Re}\Pi(m_R^2) + i\text{Im}\Pi(s).$$

$\eta \rightarrow \pi^0 \gamma \gamma$ predictions

- Our theoretical prediction: $\text{BR} = 1.36(18) \times 10^{-4}$
(updated from [Escribano, SGS, Jora, Royo, Phys.Rev.D 102, 034026 \(2020\)](#))

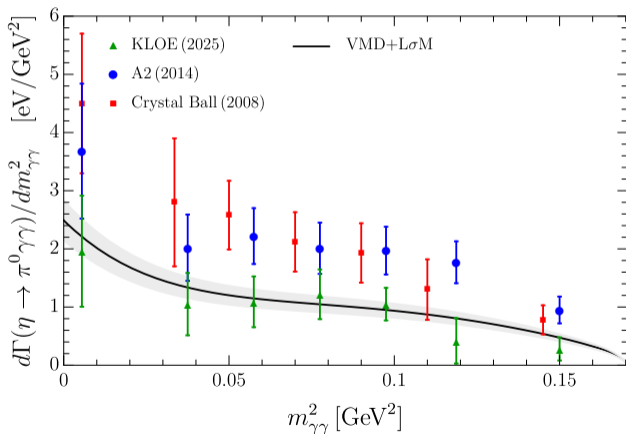
- VMD dominates:
- ρ : 27% of the signal
- ω : 21% of the signal
- ϕ : 0% of the signal
- interference between ρ - ω - ϕ : 52%
- interference between scalar and vector mesons: 7%



$\eta \rightarrow \pi^0 \gamma \gamma$ predictions

- Comparison of our prediction ($\text{BR} = 1.36(18) \times 10^{-4}$) with experimental data (updated from [Escribano, SGS, Jora, Royo, Phys.Rev.D 102, 034026 \(2020\)](#))

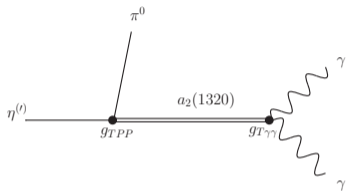
- Shape of the A2
($\text{BR} = 2.54(27) \times 10^{-4}$) and
Crystal Ball
($\text{BR} = 2.21(24)(47) \times 10^{-4}$)
spectra is captured well
(normalization offset)
- Good agreement
with KLOE data,
 $\text{BR} = 0.98(11)(14) \times 10^{-4}$
([JHEP 01 \(2026\) 091](#))



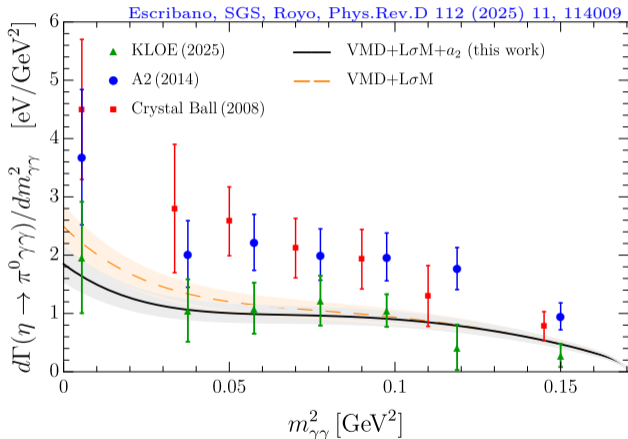
- The experimental situation needs to be **clarified** (A2, JEF, REDTOP)

Inclusion of the $a_2(1320)$ tensor meson contribution

- One **diagram** corresponding to the exchange of $a_2(1320)$ in the s -channel



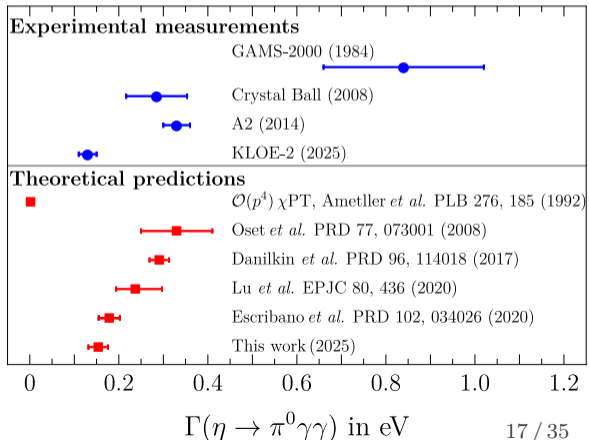
- $|g_{T\pi\pi}|$, $|g_{T\gamma\gamma}|$ fixed from the pdg
BR($a_2 \rightarrow \pi\eta$) and BR($a_2 \rightarrow \gamma\gamma$)



- **Destructive** VMD- a_2 interference $\sim 20\%$ of the signal (could be tested and distinguished from VMD with precise measurements at *e.g.* JEF)

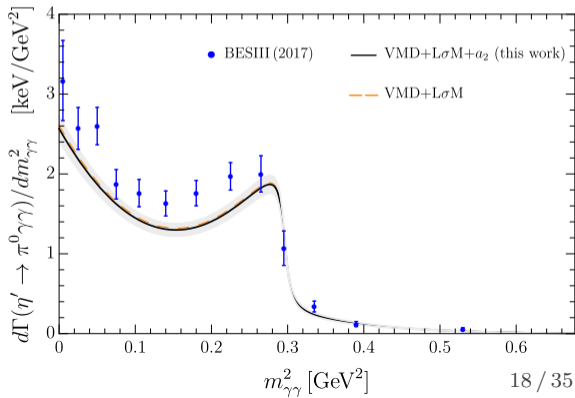
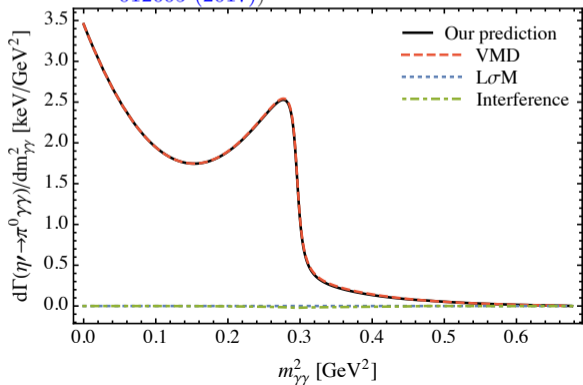
$\eta \rightarrow \pi^0 \gamma \gamma$: status

- Our final prediction including vector, scalar and tensor resonance exchanges is: $\text{BR} = 1.17(17) \times 10^{-4}$ (Escribano, SGS, Royo, Phys.Rev.D 112 (2025) 11, 114009)
- In agreement with KLOE-II: $\text{BR} = 0.98(11)(14) \times 10^{-4}$ (JHEP 01 (2026) 091)
- 5σ below the A2 measurement
 $\text{BR} = 2.54(27) \times 10^{-4}$
(Phys.Rev.C 90, 025206 (2014))
- Theoretical predictions show some scatter



$$\eta' \rightarrow \pi^0 \gamma \gamma$$

- BR = $2.78(21) \times 10^{-3}$ (Escribano, SGS, Royo, Phys.Rev.D 112 (2025) 11, 114009)
 - VMD completely dominates: ω (78%), ρ (5%), ϕ (0%), interference (17%)
 - a_2 tensor meson contribution is negligible
- Compatible with BESIII, BR = $3.20(7)(23) \times 10^{-3}$ (Ablikim et.al. Phys.Rev.D 96, 012005 (2017))



Leptophobic B boson model

- What if a **new force** couples predominantly to quarks?
- **New boson** from a new $U(1)_B$ gauge symmetry (aka B boson, leptophobic Z')

$$\mathcal{L}_{\text{int}} = \left(\frac{1}{3} g_B + \varepsilon Q_q e \right) \bar{q} \gamma^\mu q B_\mu - \varepsilon e \bar{\ell} \gamma^\mu \ell B_\mu,$$

- New gauge coupling: $\alpha_B = g_B^2/4\pi$
- B is a singlet under isospin: $I^G(J^{PC}) = 0^-(1^{--}) \Rightarrow B$ is ω **meson-like**
- Searches depend on the mass m_B and decay channels

	Dark-photon-like	m_B [MeV]		Novel signatures
Decay \rightarrow	$B \rightarrow e^+e^-$	$B \rightarrow \pi^0\gamma$	$B \rightarrow \pi^+\pi^-\pi^0$	$B \rightarrow \eta\gamma$
Production \downarrow	$m_B \sim 1 - 140$ MeV	140 - 620 MeV	620 - 1000 MeV	
$\pi^0 \rightarrow B\gamma$	$\pi^0 \rightarrow e^+e^-\gamma$	-	-	-
$\eta \rightarrow B\gamma$	$\eta \rightarrow e^+e^-\gamma$	$\eta \rightarrow \pi^0\gamma\gamma$	-	-
$\eta' \rightarrow B\gamma$	$\eta' \rightarrow e^+e^-\gamma$	$\eta' \rightarrow \pi^0\gamma\gamma$	$\eta' \rightarrow \pi^+\pi^-\pi^0\gamma$	$\eta' \rightarrow \eta\gamma\gamma$
$\omega \rightarrow \eta B$	$\omega \rightarrow \eta e^+e^-$	$\omega \rightarrow \eta\pi^0\gamma$	-	-
$\phi \rightarrow \eta B$	$\phi \rightarrow \eta e^+e^-$	$\phi \rightarrow \eta\pi^0\gamma$	-	-

- Searches in meson factories are gaining attention
 - $\eta \rightarrow \gamma B \rightarrow \gamma\gamma\pi^0$ (JEF), $\phi \rightarrow \eta B \rightarrow \eta\pi^0\gamma$ (KLOE-II), $\eta \rightarrow B\gamma \rightarrow \pi^+\pi^-\gamma$ (Belle-II)

Previous limits on α_B and m_B

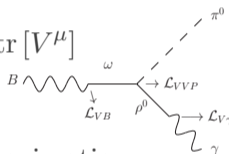
- New boson from a new $U(1)_B$ gauge symmetry

$$\mathcal{L}_{\text{int}} = \left(\frac{1}{3} g_B + \varepsilon Q_q e \right) \bar{q} \gamma^\mu q B_\mu - \varepsilon e \bar{l} \gamma^\mu l B_\mu,$$

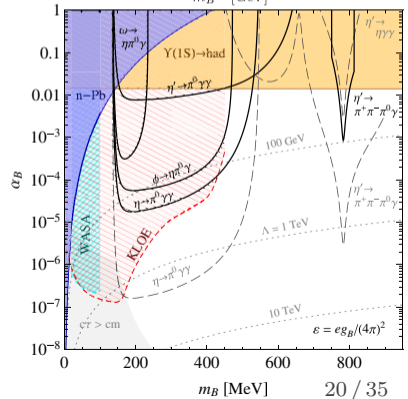
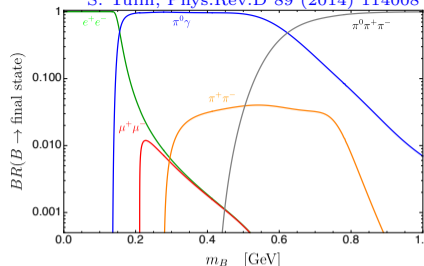
- Conventional VMD picture:

$$\mathcal{L}_{V\gamma} \rightarrow \mathcal{L}_{VB} = -2 \frac{1}{3} g_B g f_\pi^2 B^\mu \text{tr} [V^\mu]$$

$$A^\mu \rightarrow B^\mu, e \rightarrow g_B$$



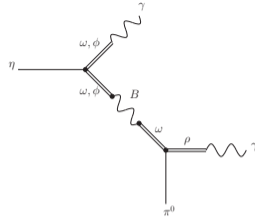
- Assuming **Narrow-Width Approximation**:
 $\text{BR}(\eta \rightarrow \pi^0 \gamma \gamma)_{\text{Bboson}} = \text{BR}(\eta \rightarrow B \gamma) \times \text{BR}(B \rightarrow \pi^0 \gamma)$
- Assuming **no SM contribution**
- $\text{BR}(\eta \rightarrow \pi^0 \gamma \gamma)_{\text{Bboson}} < \text{BR}_{\text{exp}}$ at 2σ
 - $\text{BR}(\eta \rightarrow \pi^0 \gamma \gamma)_{\text{exp}} = 2.21(53) \times 10^{-4}$
 - $\text{BR}(\eta' \rightarrow \pi^0 \gamma \gamma)_{\text{exp}} < 8 \times 10^{-4}$ (90% C.L.)
 - $\text{BR}(\eta' \rightarrow \eta \gamma \gamma)_{\text{exp}}$ no data



$\eta \rightarrow \pi^0 \gamma \gamma$ decays: B boson calculation

- Two diagrams corresponding to the exchange of a B boson

$$\mathcal{A}_{\eta \rightarrow \pi^0 \gamma \gamma}^{B \text{ boson}} = g_{B\eta\gamma}(t) g_{B\pi^0\gamma}(t) \left[\frac{(P \cdot q_2 - m_\eta^2)\{a\} - \{b\}}{m_B^2 - t - i\sqrt{t}\Gamma_B(t)} + \left\{ \begin{array}{l} q_2 \leftrightarrow q_1 \\ t \leftrightarrow u \end{array} \right\} \right],$$

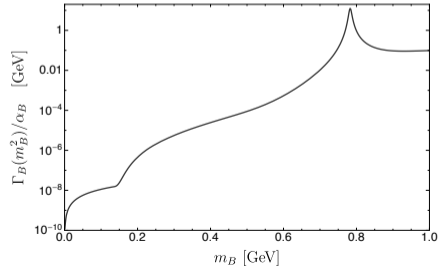


- $g_{BP\gamma}$ couplings:

$$g_{B\pi^0\gamma}(t) = \frac{\sqrt{2}eg_B}{4\pi^2 f_\pi} F_\omega(t), \quad g_{B\eta\gamma}(t) = \frac{eg_B}{12\pi^2 f_\pi \sqrt{3}} \left[(c_\theta - \sqrt{2}s_\theta)F_\omega(t) + (2c_\theta + \sqrt{2}s_\theta)F_\phi(t) \right],$$

- Energy-dependent width

$$\begin{aligned} \Gamma_B(q^2) &= \frac{\gamma_{B \rightarrow \ell^+ \ell^-}(q^2)}{\gamma_{B \rightarrow \ell^+ \ell^-}(m_B^2)} \Gamma_{B \rightarrow \ell^+ \ell^-} \theta(q^2 - 4m_\ell^2) \\ &+ \frac{\gamma_{B \rightarrow \pi^0 \gamma}(q^2)}{\gamma_{B \rightarrow \pi^0 \gamma}(m_B^2)} \Gamma_{B \rightarrow \pi^0 \gamma} \theta(q^2 - m_{\pi^0}^2) \\ &+ \frac{\gamma_{B \rightarrow \pi\pi}(q^2)}{\gamma_{B \rightarrow \pi\pi}(m_B^2)} \Gamma_{B \rightarrow \pi\pi} \theta(q^2 - 4m_\pi^2) \\ &+ \frac{\gamma_{B \rightarrow 3\pi}(q^2)}{\gamma_{B \rightarrow 3\pi}(m_B^2)} \Gamma_{B \rightarrow 3\pi} \theta(q^2 - 9m_\pi^2) \end{aligned}$$



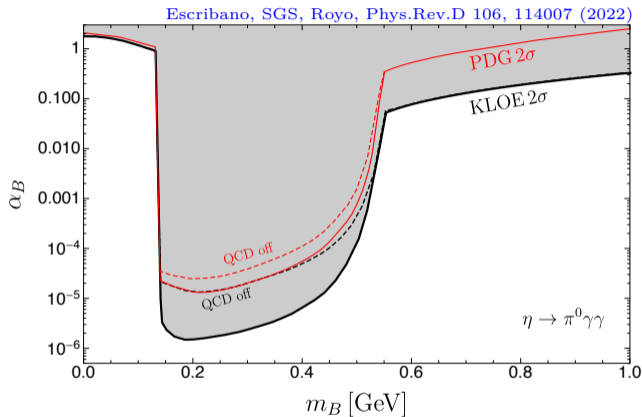
New limits on α_B, m_B

- SM contribution on
- **Not** assuming the NWA
- $\text{BR}_{\text{SM+Bboson}} < \text{BR}_{\text{exp}}$ at 2σ

$$\text{BR}(\eta \rightarrow \pi^0 \gamma \gamma)_{\text{exp}}^{\text{pdg}} = 2.56(22) \times 10^{-4}$$

$$\text{BR}(\eta \rightarrow \pi^0 \gamma \gamma)_{\text{exp}}^{\text{KLOE}} = 1.23(14) \times 10^{-4}$$

B. Cao [KLOE], PoS EPS-HEP2021 (2022) 409



- Limits **strengthened** by one order of magnitude*

*These limits were obtained without the a_2 contribution to the SM.

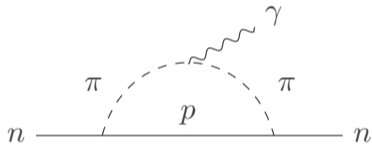
Strong CP problem

- QCD Lagrangian with a θ term:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{quarks}} - \frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu} + \theta \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu},$$

- The θ term implies that QCD violates P and CP
- This CP-violation is measurable: the θ term causes an EDM for the neutron

$$|d_n| \sim \bar{\theta}, \text{ with } \bar{\theta} = \theta + \arg(\det M_q)$$



- Experimental upper limits on the neutron EDM:

$$|d_n| \lesssim 1.8 \times 10^{-26} e \text{ cm (C. Abel et. al., PRL 124, 081803 (2020))}$$

- Constrains $\bar{\theta} \lesssim 10^{-10}$
- Why is θ so small? (one of the open issues of the SM)

The QCD axion

- The Peccei-Quinn solution of the strong CP-problem (Peccei, Quinn'77)
 - New $U(1)_A$ global symmetry (a.k.a $U(1)_{PQ}$),
 - Broken spontaneously at the high energy scale f_a and anomalous
- Nambu-Goldstone boson: the axion a (Weinberg'78; Wilczek'78)

$$\mathcal{L} \supset - \left(\theta + \frac{a}{f_a} \right) \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu}.$$

- Its VEV cancels the θ -term: $\theta + \frac{\langle a \rangle}{f_a} = 0$, and solves the strong CP-problem:

$$a \rightarrow \langle a \rangle + a, \quad \mathcal{L} \supset - \frac{a}{f_a} \frac{g_s^2}{32\pi^2} \tilde{G}_{\mu\nu}^a G^{a,\mu\nu}.$$

- Axion mass: $m_a^2 = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2}$
- The scale f_a is identified with the electroweak symmetry breaking scale v :

$$f_a \sim v = (\sqrt{2}G_F)^{-1/2} \approx 246 \text{ GeV},$$

- The PQWW axion ruled out experimentally

Axion-Like Particles (ALPs)

- “Yukawa basis”: ALP with gluon and quark couplings (*à la* DFSZ)

$$\begin{aligned}\mathcal{L}_{\text{ALP}} = & \mathcal{L}_{\text{QCD}} + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{1}{2} M_{\text{PQ}}^2 a^2 \\ & - Q_G \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} \left(e^{iQ_q \frac{a}{f_a} \gamma^5} \right) q,\end{aligned}$$

M_{PQ} : PQ-breaking contribution to the mass

$Q_{q,G}$: PQ charges

- The heavy-flavor c, b, t quarks contributions have been integrated out
- Equivalent to the “usual” derivative basis (related via chiral rotations of the quarks)*

*if weak interactions are neglected

Lagrangian for ALPs coupled to mesons

- Step 1: **map** \mathcal{L}_{ALP} into χPT at leading order:

$$\begin{aligned} \mathcal{L}_{\text{ALP}}^{\chi\text{PT@LO}} &= \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} M_{\cancel{\text{PQ}}}^2 a^2 - \frac{1}{2} m_0^2 \left(\eta_0 - \frac{Q_G f_\pi}{\sqrt{6} f_a} a \right)^2 \\ &+ \frac{f_\pi^2}{4} \text{Tr} \left[\partial_\mu U^\dagger \partial^\mu U \right] + \frac{f_\pi^2}{4} \text{Tr} \left[2B_0 (M_q(a)U + M_q(a)^\dagger U^\dagger) \right], \end{aligned}$$

with the ALP-dependent quark mass matrix:

$$M_q(a) = \text{diag} \left(m_u e^{iQ_u a/f_a}, m_d e^{iQ_d a/f_a}, m_s e^{iQ_s a/f_a} \right),$$

and the representation of the pNGB chiral meson nonet:

$$U = \exp \left(\frac{i\sqrt{2}\Phi}{f} \right), \quad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & K^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}.$$

Diagonalization of the mass matrix

- Step 2: **diagonalization** of the mass matrix ($\phi \equiv (\pi_3, \eta_8, \eta_0, a)$)

$$\mathcal{L}_{\text{ALP}}^{\chi\text{PT@LO}} \supset -\frac{1}{2}\phi^T \widetilde{M}^2 \phi, \quad \widetilde{M}^2 = \begin{pmatrix} m_{\pi_3}^2 & m_{\pi_3\eta_8}^2 & m_{\pi\eta_0}^2 & m_{\pi_3 a}^2 \\ & m_{\eta_8}^2 & m_{\eta_8\eta_0}^2 & m_{\eta_8 a}^2 \\ & & m_{\eta_0}^2 & m_{\eta_0 a}^2 \\ & & & m_a^2 \end{pmatrix},$$

$$\begin{pmatrix} \pi_3 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix} = \begin{pmatrix} & & & \theta_{a\pi} \\ & \mathbb{1}_{3\times 3} & & \theta_{a\eta_8} \\ & & & \theta_{a\eta_0} \\ -\theta_{a\pi} & -\theta_{a\eta_8} & -\theta_{a\eta_0} & 1 \end{pmatrix} \begin{pmatrix} & & & 0 \\ & \mathbb{R}_{3\times 3} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta \\ \eta' \\ a_{\text{phys}} \end{pmatrix},$$

where \mathbb{R} is an orthogonal matrix that diagonalizes of the π^0 - η - η' subsystem

$$\mathbb{R} = \begin{pmatrix} 1 & -\theta_{\pi\eta} & -\theta_{\pi\eta'} \\ (\theta_{\pi\eta} \cos \theta_{\eta\eta'} + \theta_{\pi\eta'} \sin \theta_{\eta\eta'}) & \cos \theta_{\eta\eta'} & \sin \theta_{\eta\eta'} \\ (\theta_{\pi\eta'} \cos \theta_{\eta\eta'} - \theta_{\pi\eta} \sin \theta_{\eta\eta'}) & -\sin \theta_{\eta\eta'} & \cos \theta_{\eta\eta'} \end{pmatrix}.$$

Mixing angles and physical axion mass

- In the PQ-preserving limit, *i.e.* $M_{\text{PQ}} = 0$:

$$\theta_{a\pi}^{(\text{PQ})} = -\frac{f_\pi}{f_a} \frac{1}{(1+\epsilon)} \left(\frac{Q_u m_u - Q_d m_d}{m_u + m_d} + \frac{m_u - m_d}{m_u + m_d} \frac{Q_s + Q_G}{2} + \epsilon \frac{Q_u - Q_d}{2} \right) + \mathcal{O}(f_a^{-2}),$$

$$\theta_{a\eta_8}^{(\text{PQ})} = \frac{f_\pi}{f_a} \frac{\sqrt{3}}{2} \frac{1}{(1+\epsilon)} \left(Q_s + \frac{Q_G}{3} - \epsilon \frac{(Q_u + Q_d + 2Q_G/3) + \frac{2B_0 m_s}{m_0^2} (Q_u + Q_d - 2Q_s)}{1 + \frac{6B_0 m_s}{m_0^2}} \right) + \mathcal{O}(f_a^{-2}),$$

$$\theta_{a\eta_0}^{(\text{PQ})} = \frac{f_\pi}{f_a} \frac{1}{\sqrt{6}} \frac{1}{(1+\epsilon)} \left(Q_G + \epsilon \frac{Q_G - \frac{6B_0 m_s}{m_0^2} (Q_u + Q_d + Q_s)}{1 + \frac{6B_0 m_s}{m_0^2}} \right) + \mathcal{O}(f_a^{-2}),$$

where $\epsilon \equiv \frac{m_u m_d}{m_s (m_u + m_d)} \left(1 + 6 \frac{B_0 m_s}{m_0^2} \right) \approx 0.04$.

- Physical axion mass:

$$(m_{a_{\text{phys}}}^{(\text{PQ})})^2 = (Q_u + Q_d + Q_s + Q_G)^2 \frac{B_0 m_u m_d m_s}{\left(m_u m_d + m_u m_s + m_d m_s + \frac{6B_0 m_u m_d m_s}{m_0^2} \right)} \frac{f_\pi^2}{f_a^2},$$

ALP-meson mixing angles

- In the PQ-breaking limit, *i.e.* $M_{\mathbb{P}\mathbb{Q}} \neq 0$:

$$\theta_{a\pi} = \theta_{a\pi}^{(\text{PQ})} \left(1 + \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_\pi^2 - m_a^2} \right),$$

$$\theta_{a\eta_8} = \theta_{a\eta_8}^{(\text{PQ})} \left(1 + \cos^2\theta_{\eta\eta'} \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_\eta^2 - m_a^2} + \sin^2\theta_{\eta\eta'} \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_{\eta'}^2 - m_a^2} \right) + \theta_{a\eta_0}^{(\text{PQ})} \frac{\sin 2\theta_{\eta\eta'}}{2} \left(\frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_{\eta'}^2 - m_a^2} - \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_\eta^2 - m_a^2} \right),$$

$$\theta_{a\eta_0} = \theta_{a\eta_0}^{(\text{PQ})} \left(1 + \sin^2\theta_{\eta\eta'} \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_\eta^2 - m_a^2} + \cos^2\theta_{\eta\eta'} \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_{\eta'}^2 - m_a^2} \right) + \theta_{a\eta_8}^{(\text{PQ})} \frac{\sin 2\theta_{\eta\eta'}}{2} \left(\frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_{\eta'}^2 - m_a^2} - \frac{M_{\mathbb{P}\mathbb{Q}}^2}{m_\eta^2 - m_a^2} \right).$$

- Valid in the small mixing angle approximation, *i.e.* when $\theta_{a\pi}^{(\text{PQ})}, \theta_{a\eta_8}^{(\text{PQ})}, \theta_{a\eta_0}^{(\text{PQ})} \ll 1$
- Physical ALP mass:

$$m_a^2 \equiv m_{a^{\text{phys}}}^2 = (m_{a^{\text{PQ}}}^{(\text{PQ})})^2 + M_{\mathbb{P}\mathbb{Q}}^2.$$

- Step 3: re-express $\mathcal{L}_{\text{ALP}}^{\chi^{\text{PT@LO}}}$ in terms of the **physical states**

$$\pi_3 \rightarrow \pi^0 + \theta_{a\pi} a^{\text{phys}}, \quad \eta_8 \rightarrow \cos\theta_{\eta\eta'}\eta + \sin\theta_{\eta\eta'}\eta' + \theta_{a\eta_8} a^{\text{phys}}, \quad \eta_0 \rightarrow -\sin\theta_{\eta\eta'}\eta + \cos\theta_{\eta\eta'}\eta' + \theta_{a\eta_0} a^{\text{phys}},$$

$\eta/\eta' \rightarrow \pi\pi a$ decay amplitudes at LO

$$\mathcal{A}(\eta \rightarrow 2\pi^0 a) = 2! \frac{m_\pi^2}{f_\pi^2} (\cos \theta - \sqrt{2} \sin \theta) \left[\frac{f_\pi}{2\sqrt{3}f_a} \frac{Q_u m_u + Q_d m_d}{m_u + m_d} - \frac{1}{2\sqrt{3}} \frac{m_d - m_u}{m_u + m_d} \theta_{\pi_3 a} + \frac{1}{6} \theta_{\eta_8 a} + \frac{\sqrt{2}}{6} \theta_{\eta_0 a} \right],$$

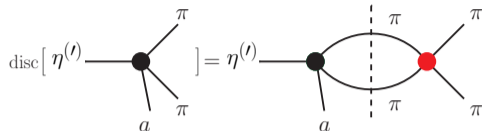
$$\mathcal{A}(\eta \rightarrow \pi^+ \pi^- a) = \frac{m_\pi^2}{f_\pi^2} (\cos \theta - \sqrt{2} \sin \theta) \left[\frac{f_\pi}{\sqrt{3}f_a} \frac{Q_u m_u + Q_d m_d}{m_u + m_d} - \frac{1}{3\sqrt{3}} \frac{m_d - m_u}{m_u + m_d} \theta_{\pi_3 a} + \frac{1}{3} \theta_{\eta_8 a} + \frac{\sqrt{2}}{3} \theta_{\eta_0 a} \right],$$

$$\mathcal{A}(\eta' \rightarrow 2\pi^0 a) = 2! \frac{m_\pi^2}{f_\pi^2} (\sqrt{2} \cos \theta + \sin \theta) \left[\frac{f_\pi}{2\sqrt{3}f_a} \frac{Q_u m_u + Q_d m_d}{m_u + m_d} - \frac{1}{2\sqrt{3}} \frac{m_d - m_u}{m_u + m_d} \theta_{\pi_3 a} + \frac{1}{6} \theta_{\eta_8 a} + \frac{\sqrt{2}}{6} \theta_{\eta_0 a} \right],$$

$$\mathcal{A}(\eta' \rightarrow \pi^+ \pi^- a) = \frac{m_\pi^2}{f_\pi^2} (\sqrt{2} \cos \theta + \sin \theta) \left[\frac{f_\pi}{\sqrt{3}f_a} \frac{Q_u m_u + Q_d m_d}{m_u + m_d} - \frac{1}{3\sqrt{3}} \frac{m_d - m_u}{m_u + m_d} \theta_{\pi_3 a} + \frac{1}{3} \theta_{\eta_8 a} + \frac{\sqrt{2}}{3} \theta_{\eta_0 a} \right],$$

Effects of pion-pion final-state interactions (FSI)

- Unitarity:



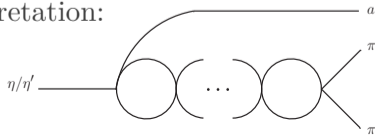
$$\text{disc}\mathcal{A}(s) = 2i\mathcal{A}(s)\sigma_{\pi}(s)T_0^{0*}(s) = 2i\mathcal{A}(s)\sin\delta_0^0(s)e^{-i\delta_0^0(s)},$$

$$\mathcal{A}(s) = \frac{1}{2i\pi} \int_{4M_{\pi}^2}^{\infty} ds' \frac{\text{disc}\mathcal{A}(s')}{s' - s - i\varepsilon}$$

- Analytic solution:

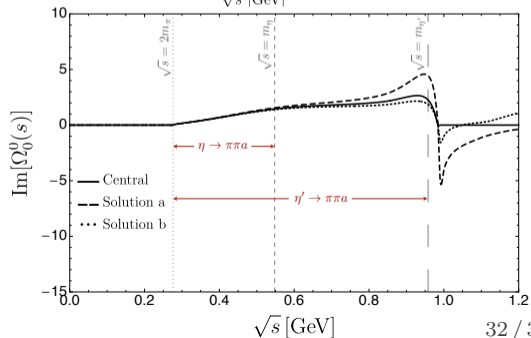
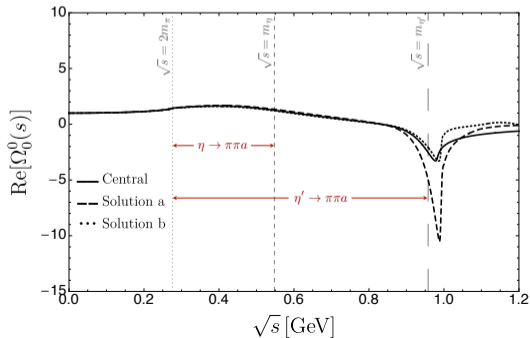
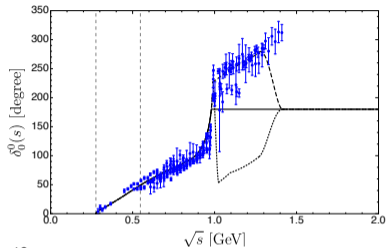
$$\mathcal{A}(s) = \mathcal{A}(\eta \rightarrow 2\pi a)|_{\text{LO}} \times \Omega_0^0(s), \quad \Omega_0^0(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_{\pi}^2}^{\infty} ds' \frac{\delta_0^0(s')}{s'(s' - s - i\varepsilon)} \right\},$$

- Diagrammatic interpretation:



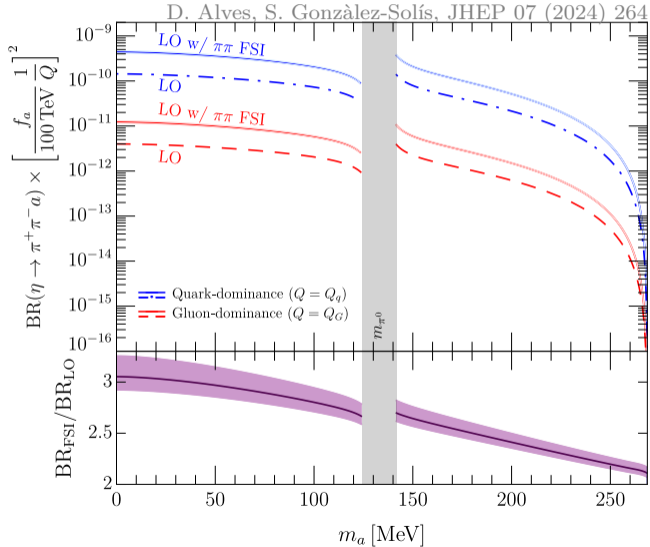
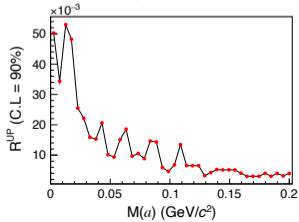
Solution of the Omnès function $\Omega_0^0(s)$

$$\Omega_0^0(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_0^0(s')}{s'(s' - s - i\epsilon)} \right\},$$



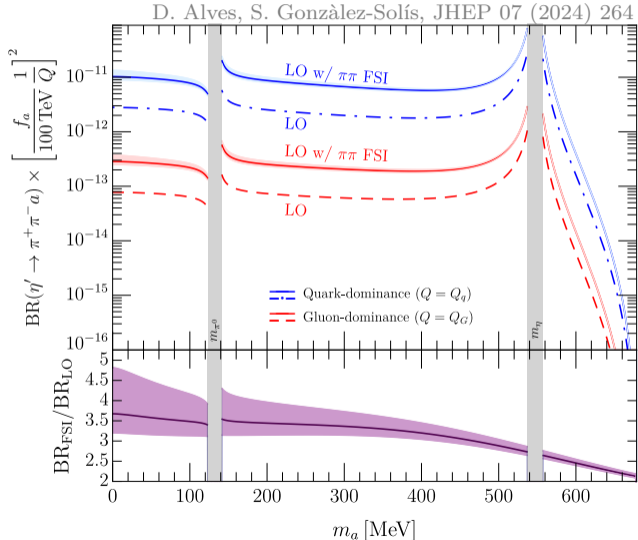
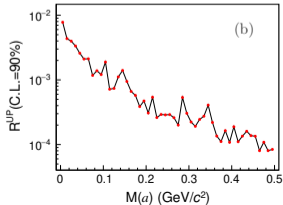
Branching ratio predictions for $\eta \rightarrow \pi^+ \pi^- a$

- Two scenarios:
 - Quark-dominance ($Q_G = 0$)
 - Gluon-dominance ($Q_q = 0$)
- Experimental searches in $\eta \rightarrow \pi\pi a \rightarrow \pi\pi\{\gamma\gamma, l^+l^-\}$ (CMS, JEF, KLOE, REDTOP, HADES)
- First upper bounds from BESIII (2501.10130)

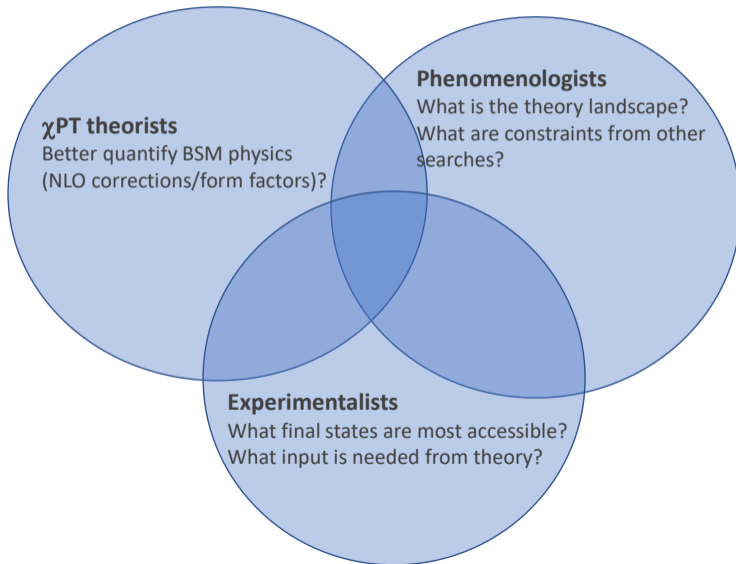


Branching ratio predictions for $\eta' \rightarrow \pi\pi a$

- Two scenarios:
 - Quark-dominance ($Q_G = 0$)
 - Gluon-dominance ($Q_q = 0$)
- Experimental searches in $\eta' \rightarrow \pi\pi a \rightarrow \pi\pi\{\gamma\gamma, \ell^+\ell^-\}$ (CMS, JEF, KLOE, REDTOP, HADES)
- First upper bounds from BESIII (JHEP 07 (2024) 135)



Conclusions



Rich physics program at η, η' factories

Standard Model highlights

- Theory input for light-by-light scattering for $(g-2)_\mu$
- Extraction of light quark masses
- QCD scalar dynamics

Fundamental symmetry tests

- P,CP violation
- C,CP violation

[Kobzarev & Okun (1964), Prentki & Veltman (1965), Lee (1965), Lee & Wolfenstein (1965), Bernstein et al (1965)]

Dark sectors (MeV—GeV)

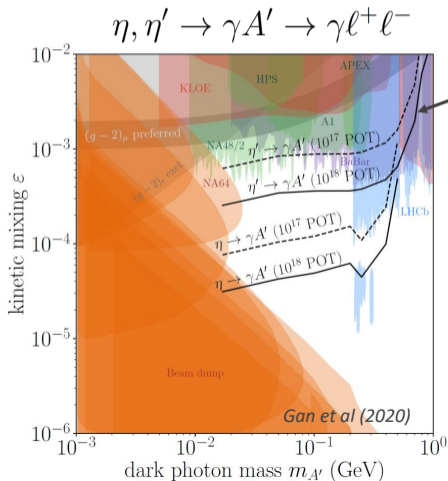
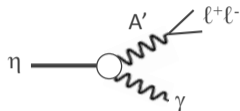
- Vector bosons (dark photon, B boson, X boson)
- Scalars
- Pseudoscalars (ALPs)

(Plus other channels that have not been searched for to date)

Channel	Expt. branching ratio	Discussion
$\eta \rightarrow 2\gamma$	39.41(20)%	chiral anomaly, η - η' mixing
$\eta \rightarrow 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \rightarrow \pi^0 \gamma \gamma$	$2.56(22) \times 10^{-4}$	χ PT at $\mathcal{O}(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \rightarrow \pi^0 \pi^0 \gamma \gamma$	$< 1.2 \times 10^{-3}$	χ PT, axion-like particles (ALPs)
$\eta \rightarrow 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [52]
$\eta \rightarrow \pi^+ \pi^- \pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \rightarrow \pi^+ \pi^- \gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \rightarrow \pi^+ \pi^- \gamma \gamma$	$< 2.1 \times 10^{-3}$	χ PT, ALPs
$\eta \rightarrow e^+ e^- \gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \rightarrow \mu^+ \mu^- \gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \rightarrow e^+ e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \rightarrow \mu^+ \mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \rightarrow \pi^0 \pi^0 \ell^+ \ell^-$		C/CP violation, ALPs
$\eta \rightarrow \pi^+ \pi^- e^+ e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow \pi^+ \pi^- \mu^+ \mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \rightarrow e^+ e^- e^+ e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \rightarrow e^+ e^- \mu^+ \mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \mu^+ \mu^- \mu^+ \mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \rightarrow \pi^+ \pi^- \pi^0 \gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \rightarrow \pi^+ e^- \bar{\nu}_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \rightarrow \pi^+ \pi^-$	$< 4.4 \times 10^{-6}$ [53]	P/CP violation
$\eta \rightarrow 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation Gan, Kubis, Passemar, ST (2020)
$\eta \rightarrow 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation (2020)

Dark photon searches

- **Broad** worldwide effort to search for dark photons (A')
- Most searches are for A' coupling to **leptons**, $\mathcal{L}_{\text{int}} = -e\epsilon j_{\text{em}}^\mu A'_\mu$



REDTOP sensitivities projected for FNAL/BNL (10^{18}) or CERN (10^{17}) POT

Gatto (2019)

Many other experiments targeting same dark photon parameter space

Worthwhile to also consider

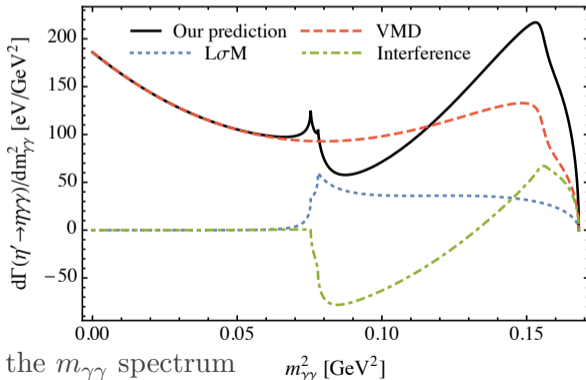
$$\eta' \rightarrow \pi^+ \pi^- A' \rightarrow \pi^+ \pi^- l^+ l^-$$

since $\mathcal{B}(\eta' \rightarrow \pi^+ \pi^- \gamma) \approx 10 \times \mathcal{B}(\eta' \rightarrow \gamma \gamma)$

$\eta' \rightarrow \eta\gamma\gamma$ predictions

- 1st BR measurement by BESIII, $BR = 8.25(3.41)(0.72) \times 10^{-5}$ or $BR < 1.33 \times 10^{-4}$ at 90% C.L. (Ablikim *et. al.* Phys.Rev.D 100, 052015 (2019))
- Our theoretical predictions $BR = 1.17(8) \times 10^{-4}$
(R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))

- VMD predominates (91% of the signal)
- Substantial scalar meson effects (16%)
- Interference between scalar and vector mesons (7%)

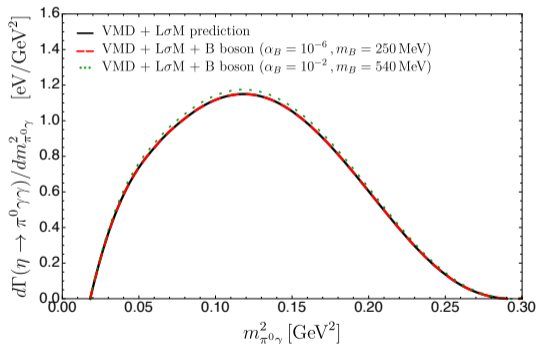
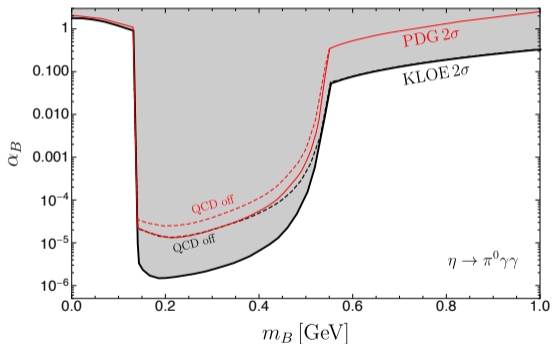


- We look forward to the release of the $m_{\gamma\gamma}$ spectrum

$\pi^0\gamma$ mass distribution

- These constraints would make a B boson signature suppressed

$$\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) \propto \int \frac{\alpha_B^2 dt}{|\mathcal{D}_B(t)|^2} \rightarrow \frac{\alpha_B^2 \pi}{m_B \Gamma_B(m_B^2)} .$$



- Bump-hunt search: accumulate statistics in a certain energy range
- Experimental $\pi^0\gamma$ distribution will be very welcome (JEF?)

New limits on α_B and m_B

- Not assuming the NWA
- QCD contribution on
- $BR < BR_{\text{exp}}$ at 2σ

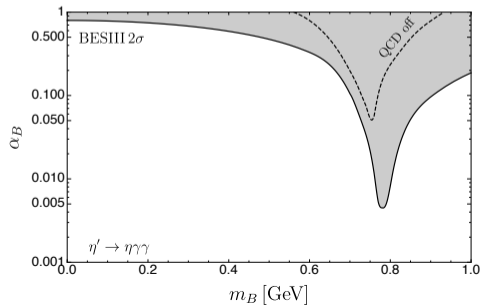
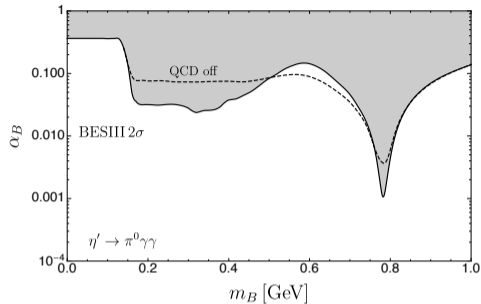
— $BR(\eta' \rightarrow \pi^0 \gamma \gamma)_{\text{exp}} = 3.20(7)(23) \times 10^{-3}$

M. Ablikim *et.al* [BESIII], *Phys.Rev. D* 96 (2017) 012005

— $BR(\eta' \rightarrow \eta \gamma \gamma)_{\text{exp}} = 8.25(3.41)(72) \times 10^{-5}$

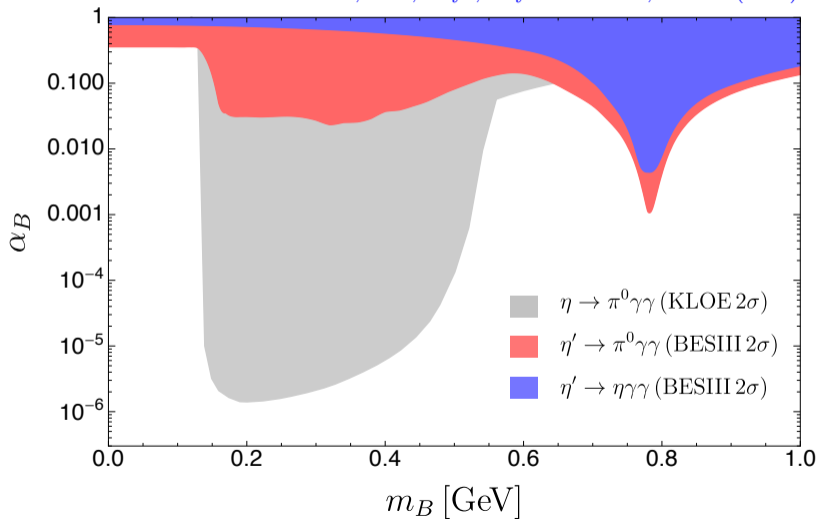
M. Ablikim *et.al* [BESIII], *Phys.Rev. D* 100 (2019) 052015

- Sharp dip when $m_B \sim m_\omega$
- Bounds 4 orders of magnitude weaker than $\eta \rightarrow \pi^0 \gamma \gamma$



New limits on α_B and m_B

Escribano, SGS, Royo, Phys.Rev.D 106, 114007 (2022)

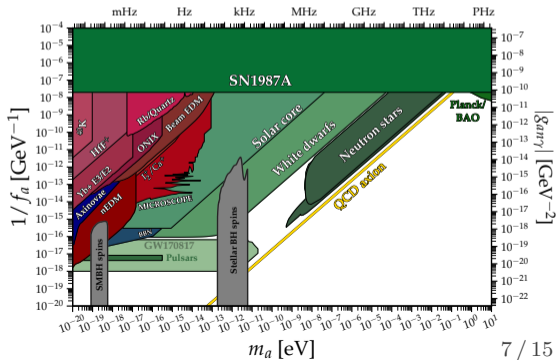


Invisible axion models

- *Invisible* models were developed to make the axion weakly coupled ($f_a \gg v$):
 - Dine-Fischler-Srednicki-Zhitnisky (DFSZ) (PLB 104, 199 (1981), SJNP 31, 260 (1980))
SM quarks carry PQ charges
 - Kim-Shifman-Vainshtein-Zakharov (KSVZ) (PRL 43, 103 (1979), NPB 166, 493 (1980))
SM quarks uncharged under the PQ symmetry
- Axion decay constant window from astrophysical and cosmological data:

$$10^8 \text{ GeV} \lesssim f_a \lesssim 10^{18} \text{ GeV},$$

(for compilations of various constraints,
see: <https://cajohare.github.io/AxionLimits/>)



Lagrangian for ALPs coupled to QCD

- “Derivative basis”: ALPs with gluon and derivative couplings

$$\mathcal{L}_{\text{ALP}} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{1}{2} M_a^2 a^2 - \left(Q_G + \sum_{q=u,d,s} Q_q \right) \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{\partial_\mu a}{f_a} \sum_{q=u,d,s} \frac{Q_q}{2} \bar{q} \gamma^\mu \gamma^5 q,$$

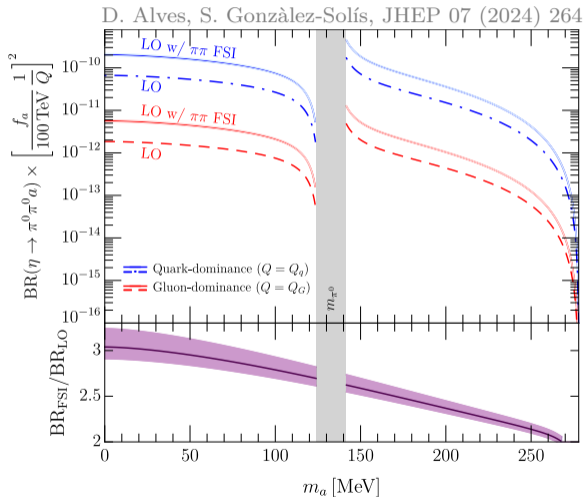
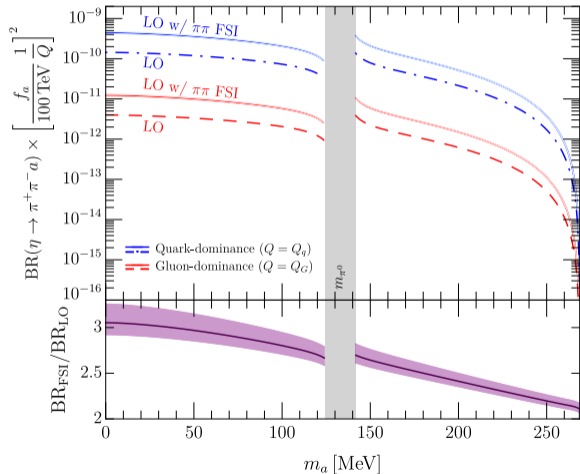
M_a^2 : PQ contribution to the mass, f_a : axion decay constant, $Q_{q,G}$: PQ charges

- “Yukawa basis” (this work, at GeV scale): ALP with gluon and mass couplings

$$\mathcal{L}_{\text{ALP}} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} (\partial_\mu a) (\partial^\mu a) - \frac{1}{2} M_a^2 a^2 - Q_G \frac{\alpha_s}{8\pi} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu} + \sum_{q=u,d,s} m_q \bar{q} \left(e^{i Q_q \frac{a}{f_a} \gamma^5} \right) q,$$

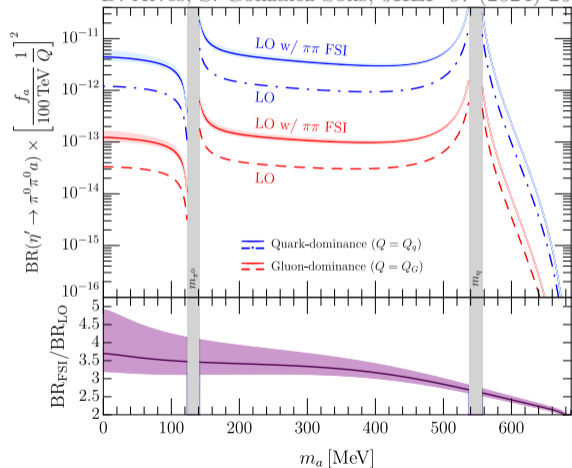
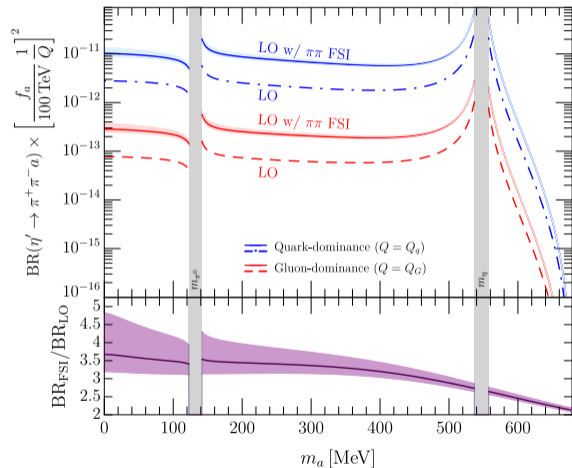
- Equivalent bases (related via chiral rotations of the quarks) if weak interactions are neglected
- The heavy-flavor c, b, t quarks contributions are absorbed in $Q_G \rightarrow Q_G + Q_{t,b,c}$

Branching ratio predictions for $\eta \rightarrow \pi\pi a$ ($\pi\pi = \pi^+\pi^-, \pi^0\pi^0$)



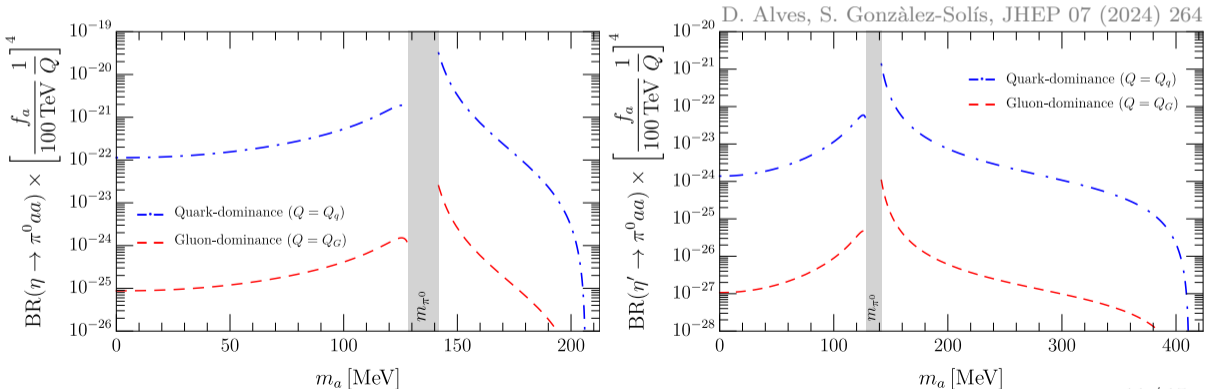
Branching ratio predictions for $\eta' \rightarrow \pi\pi a$

D. Alves, S. González-Solís, JHEP 07 (2024) 264

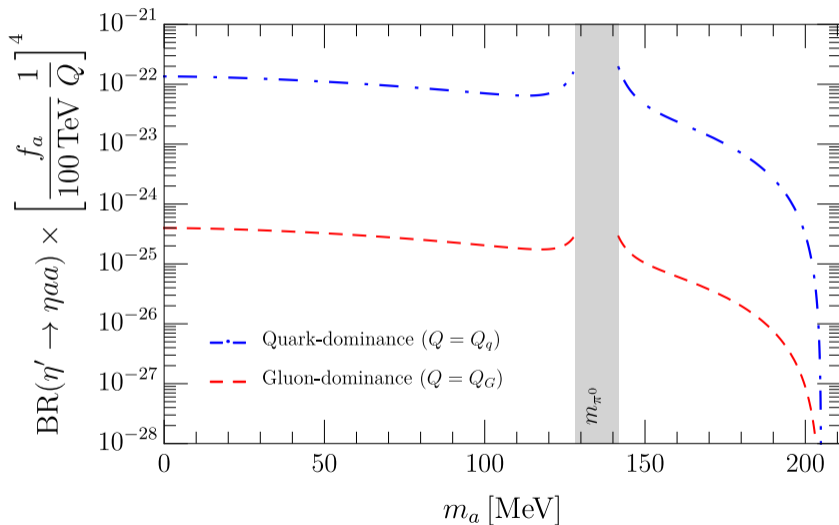


Double production of ALPs in η/η' decays

- $\eta/\eta' \rightarrow \pi^0 aa$ decays
- One extra power of $1/f_a$ suppression, $\text{BR} \sim \mathcal{O}(1/f_a^4)$
- $f_a \sim \mathcal{O}(1 - 10)$ GeV to be sensitive probes of ALPs

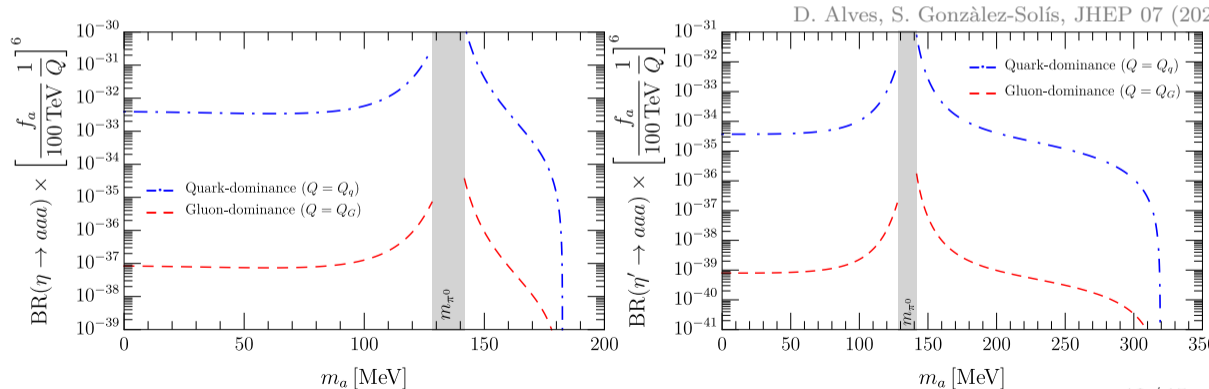


Double production of ALPs in η/η' decays

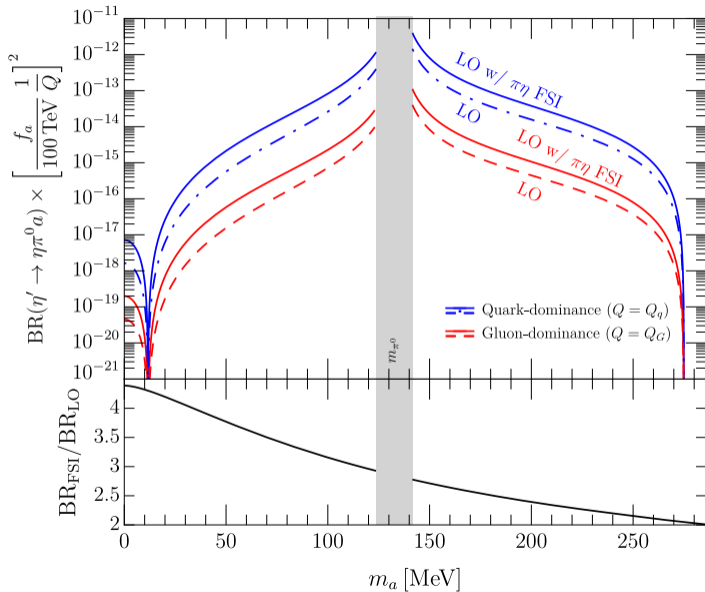


Triple production of ALPs in η/η' decays

- $\eta/\eta' \rightarrow aaa$ decays
- $\text{BR} \sim \mathcal{O}(1/f_a^6)$
- $f_a \sim \mathcal{O}(1)$ GeV to be sensitive probes of ALPs



$$\eta' \rightarrow \eta\pi^0 a$$



Other meson decays

BSM particle	Decay mode	Signal channel	Search strategy
ALPs (a)	$K^\pm \rightarrow \pi^\pm a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell\ell}$
	$K^\pm \rightarrow \pi^\pm \pi^0 a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell, \ell}$
	$K_L \rightarrow \pi^0 a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell\ell}$
	$K_L \rightarrow \pi^0 \pi^0 a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell\ell}$
	$K_L \rightarrow \pi^+ \pi^- a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell\ell}$
	$B^\pm \rightarrow \pi^\pm a$	$a \rightarrow \ell^+\ell^-, 3\pi, \eta\pi\pi, KK\pi$	Higher ALP masses
	$B^\pm \rightarrow K^\pm a$	$a \rightarrow \ell^+\ell^-, 3\pi, \eta\pi\pi, KK\pi$	Higher ALP masses
	$B \rightarrow K^* a$	$a \rightarrow \ell^+\ell^-, 3\pi, \eta\pi\pi, KK\pi$	Higher ALP masses
	$\omega/\phi/J/\psi \rightarrow \pi^0 \pi^0 a$	$a \rightarrow \gamma\gamma, \ell^+\ell^-$ ($\ell = e, \mu$)	Bump-hunt in $d\Gamma/dm_{\gamma\gamma, \ell\ell}$
	$\omega/\phi/J/\psi \rightarrow \pi^0 \pi^0 a$	$a \rightarrow \pi^+ \pi^- \gamma, 3\pi$	
Dark photon (A')	$\pi^0 \rightarrow \gamma A'$	$A' \rightarrow e^+ e^-$	$e^+ e^-$ resonance
	$\pi^0 \rightarrow \gamma^* A'$	$\gamma^* \rightarrow e^+ e^-, A' \rightarrow e^+ e^-$	$e^+ e^-$ resonance
	$\omega/\phi/J/\psi \rightarrow \pi^0 A'$	$A' \rightarrow \ell^+ \ell^-$ ($\ell = e, \mu$)	$\ell^+ \ell^-$ resonance
	$\omega/\phi/J/\psi \rightarrow \pi^0 A'$	$A' \rightarrow \pi^+ \pi^-$	$\pi^+ \pi^-$ resonance
Leptophobic boson (B)	$\omega/\phi \rightarrow \eta B$	$B \rightarrow \gamma \pi^0$	Enhancement in $m_{\pi^0 \gamma}$