# Towards an improved understanding of $a_1$ and $a_2$ transition form factors

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in collaboration with

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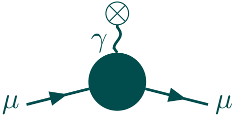
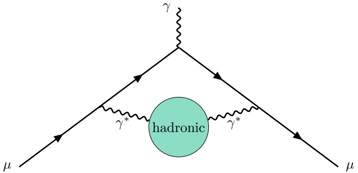
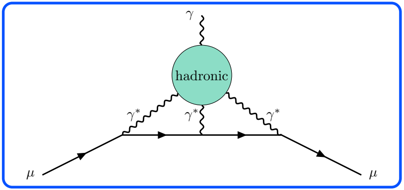
# Motivation: $(g-2)_\mu$

- anomalous magnetic moment of the $\mu$:
  $\vec{\mu} = -g\frac{e}{2m_\mu}\vec{s}$




- search for discrepancy between theoretical SM prediction and experimental result of $(g-2)_\mu$ Aguillard et al. 2023; Aliberti et al. 2025
- largest uncertainty comes from hadronic vacuum polarisation (HVP) and hadronic light-by-light scattering (HLbL)

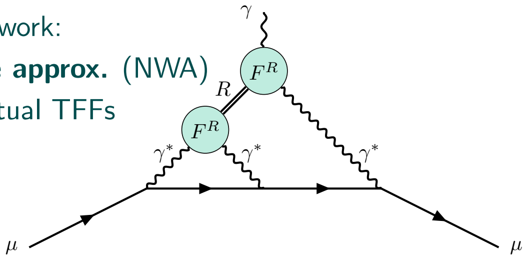



- HLbL: pole contributions of hadronic resonances contribute significantly to the error budget

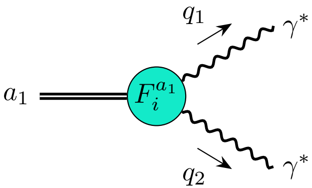
# Motivation: significance of TFFs for $(g-2)_\mu$

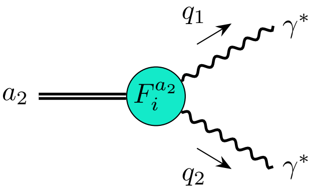
- **axial/tensor meson** intermediate states contribute to HLbL
- in dispersive framework:
  **narrow-resonance approx.** (NWA)
  $\rightarrow$ need doubly-virtual TFFs




- axial-vector poles: $f_1$ analysed Zanke et al. 2021+2023, $a_1$ and $f_1'$ via $U(3)$ symmetry
- tensor poles: HLbL dispersive framework in 4-point kinematics has still kinematic singularities; different hierarchies and results found with different approaches (cf. Eirini's and Emilis' talks)

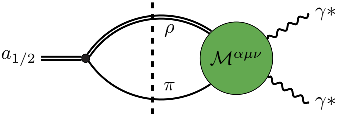
# Axial-vector- and tensor-meson TFFs



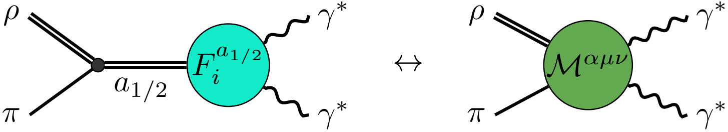

- not sufficiently described by ChPT—incorporate unitarity, crossing symmetry, and analyticity via dispersive approach
- axial-vector mesons: decay to $\gamma\gamma$ forbidden by Landau–Yang theorem Landau 1948; Yang 1950
- scarce experimental data, difficult to measure ($f_1$ with much more data Zanke et al. 2021+2023)
- dominant decay channel: $a_{1/2} \to 3\pi$, isobar model: $a_{1/2} \to \rho\pi$ analogously $f_1' \to K\bar{K}\pi$ (compare $f_2 \to \pi\pi$ Hoferichter and Stoffer 2019; Zillinger 2025)
- use $a_{1/2} \to \rho\pi$ for modelling $a_{1/2} \to \gamma^*\gamma^*$

- direct loop calculation not possible,

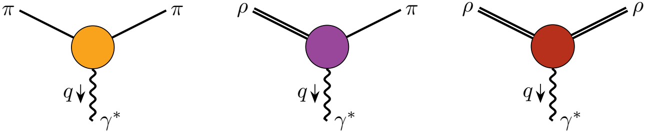



ambiguous

- match $a_{1/2}$ NWA exchange in $\rho\pi \to \gamma^*\gamma^*$ to $\mathcal{M}^{\mu\nu\alpha}$ Zillinger 2025

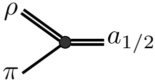



- extract $a_1/a_2$ TFFs, relate them to $\rho\pi$ rescattering and known form factors $F_\pi^V$, $F_{\rho\pi}$, $G_1$, $G_2$, $G_3$ via $\rho\pi \to \gamma^*\gamma^*$ system



## Ingredients needed

- vertex $a_{1/2} \to \rho\pi$ from symmetry considerations

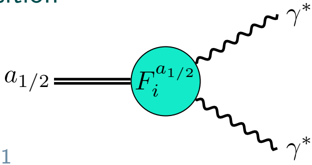


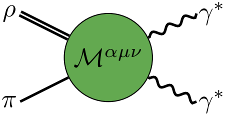

- TFF definition and tensor decomposition

$$\mathcal{M}^{\mu\nu\beta}_{a_1\gamma^*\gamma^*} = \frac{\mathrm{i}}{M^2_{a_1}} \sum_{i=1}^{3} F_i^{a_1}\, \tilde{T}_i^{\mu\nu\beta},$$

$$\mathcal{M}^{\mu\nu\alpha\beta}_{a_2\gamma^*\gamma^*} = \sum_{i=1}^{5} \frac{1}{M^{n_i}_{a_2}} F_i^{a_2}\, \tilde{T}_i^{\mu\nu\alpha\beta}$$

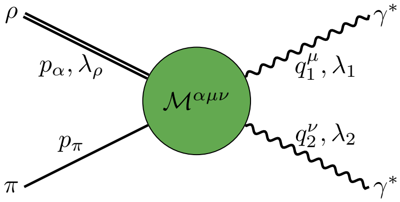
Hoferichter and Stoffer 2020; Zanke et al. 2021




- description of 

respecting all **symmetries**, including **rescattering** and **crossed-channel** terms; tensor decomposition $\mathcal{M}^{\mu\nu\alpha} = \sum_i \mathcal{F}_i T_i^{\mu\nu\alpha}$ free of kinematic singularities

# $\rho\pi \to \gamma^*\gamma^*$ system: tensor decomposition




$$\langle \rho(p, \lambda_\rho)\pi(p_\pi)|\gamma^*(q_1, \lambda_1)\gamma^*(q_2, \lambda_2)\rangle \sim \epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\mathcal{M}^{\mu\nu\alpha}(q_1, q_2, p)$$

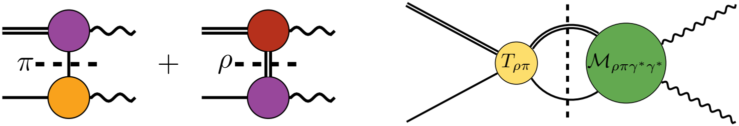
$$\mathcal{M}^{\mu\nu\alpha}(q_1, q_2, p) = \sum_{i=1}^{13} \mathcal{F}_i(s, t, u, q_1^2, q_2^2)\, T_i^{\mu\nu\alpha}(q_1, q_2, p)$$

- **gauge-invariant** description of $\rho\pi \to \gamma^*\gamma^*$
- scalar functions $\mathcal{F}_i$ **free of kinematic zeros and singularities** $\to$ Bardeen–Tung–Tarrach (BTT) procedure to find basis for $T_i^{\mu\nu\alpha}$ Bardeen and Tung 1968; Tarrach 1975
- constructed basis, identified redundancies HS 2025

# $\rho\pi \to \gamma^*\gamma^*$ system: unitarity and crossing symmetry

- different bases (BTT, helicity, angular mom.) have different advantages
- dispersion relation for BTT scalar functions (no kin. sing.)

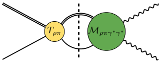
$$\mathcal{F}_i(s,t,u;q_1^2,q_2^2) = \mathcal{F}_i^{\text{Born}}(s,t,u;q_1^2,q_2^2) + \frac{1}{2\pi \mathrm{i}}\int_{s_{\text{thr}}}^{\infty} \mathrm{d}s' \frac{\text{disc}_s^{\rho\pi}\mathcal{F}_i(s',t,u;q_1^2,q_2^2)}{(s'-s-\mathrm{i}\epsilon)}$$




- Born terms: $\rho/\pi$ exchange in $t/u$ channel
- 13 scalar functions and helicity amplitudes, partial waves $L = 0, 2$ corresponds to $J = 1, 2, 3$ due to spin
- need $J = 1, 2$ partial waves for matching to $a_1, a_2$ NWA amplitudes

# $\rho\pi \to \gamma^*\gamma^*$ system: coupled equations

- discontinuity for partial-wave projected helicity amplitudes from unitarity relation (dep. on only one variable)

  $\mathrm{disc}_s^{\rho\pi} m^J_{\lambda_1\lambda_2;\lambda_\rho}(s) \sim \sum_{\lambda_{\mathrm{RH}}} t^{J*}_{\lambda_{\mathrm{RH}}\lambda_\rho}(s) m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s)$ 


  $\to$ need dispersion relations in terms of $m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s)$, $t^J_{\lambda_{\mathrm{RH}}\lambda_\rho}(s)$
- partial-wave projection & expansion of disp. relations for $\mathcal{F}_i$

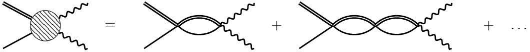
  $\to$ system of coupled equations, cf. $\pi\pi \to \gamma^*\gamma^*$ Hoferichter and Stoffer 2019; Hoferichter et al. 2011

  —but $\rho$ spin complicates things

$$m^J_j(s) = m^{\mathrm{Born},J}_j(s) + \sum_{J',j'} \frac{1}{2\pi i} \int_{s_{\mathrm{thr}}}^{\infty} \mathrm{d}s' K^{jj'}_{JJ'}(s,s') \mathrm{disc}_s^{\rho\pi} m^J_j(s'),$$

  $K^{jj'}_{JJ'}(s)$ kernel functions, to be determined
- find optimised basis for $m^J_j(s)$ (ideally, $K^{jj'}_{JJ'}(s)$ diagonalise)

# Right-hand/unitarity cut: $\rho\pi$ rescattering

- Watson's theorem: imaginary part of $m^J_{\lambda_1\lambda_2;\lambda_\rho}(s)$ in terms of rescattering phase shift,
  $t^J_{\lambda_{\mathrm{RH}}\lambda_\rho}(s) \sim \sin\delta_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s)\exp\left\{\mathrm{i}\delta_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s)\right\}$
- Omnès problem 
  

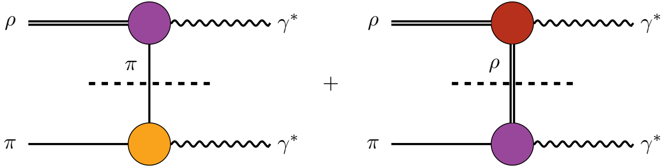
  Muskhelishvili–Omnès solution:

$$m^J_{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s) = \Delta_J^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s) + \frac{\Omega_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s)}{\pi}\times$$
$$\times\int_{s_{\mathrm{thr}}}^{\infty}\mathrm{d}s'\,\frac{\Delta_J^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s')\sin\delta_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s')}{|\Omega_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s')|(s'-s)}$$

  with Omnès functions $\Omega_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s) = \exp\left\{\frac{s}{\pi}\int_{s_{\mathrm{thr}}}^{\infty}\mathrm{d}s'\,\frac{\delta_J^{\lambda_{\mathrm{RH}}\lambda_\rho}(s')}{s'(s'-s)}\right\}$
  and inhomogeneities $\Delta_J^{\lambda_1\lambda_2;\lambda_{\mathrm{RH}}}(s)$ including Born terms
- input for $\rho\pi$ phases? —$a_{1/2}$ NWA or 3-body results...

## Left-hand cuts: Born terms

- unitarity relations for crossed $t/u$-channels of $\rho\pi \to \gamma^*\gamma^*$, dispersive reconstruction from 1-particle intermediate states




- $\epsilon_\mu^{\lambda_1 *}\epsilon_\nu^{\lambda_2 *}\epsilon_\alpha^{\lambda_\rho}\mathcal{F}_i T_i^{\mu\nu\alpha} = \frac{1}{\pi}\sum_{j=\pi,\rho} \frac{\hat{\rho}_j^t}{t'-t} + \frac{\hat{\rho}_j^u}{u'-u}$,

  $\hat{\rho}_{\pi/\rho}$ related to $F_\pi^V(q^2)$, $G_1(q^2)$, $G_2(q^2)$, $G_3(q^2)$, $F_{\rho\pi}(q^2)$
- project $\mathcal{F}_i$ according to BTT projection of $T_i^{\mu\nu\alpha}$ HS 2025
- include as Born terms in the coupled equations/as inhomogeneities in the Muskhelishvili–Omnès solution
- additional LHCs → modified MO solution

# Conclusion and outlook

- goal: extract $a_{1/2}$ TFFs from matching narrow-resonance amplitude to $\rho\pi \to \gamma^*\gamma^*$ system
- have understood BTT decompositions, helicity amplitudes, Born terms
- work in progress: partial-wave amplitudes, $\rho\pi$ rescattering, system of coupled equations for $\rho\pi \to \gamma^*\gamma^*$
- to do: Muskhelishvili–Omnès solution, matching, extraction of TFFs, inclusion into HLbL framework
- since we are analysing $\rho\pi \to \gamma^*\gamma^*$: any other ideas for this system? Connection to 3-body ansätze, to lattice results?

Thank You!

# Appendix

# References I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# Implementation BTT basis

- find 13 independent gauge-invariant structures (matches # helicity amplitudes), "basis" $\{T_i^{\mu\nu\alpha}\}_{i=1}^{13}$ HS 2025
- problem: Tarrach structures $T_{14}, T_{15}$, not a basis in all kinematic limits

$$(p\cdot q_1)\big(T_9^{\mu\nu\alpha}-T_{11}^{\mu\nu\alpha}\big)-q_1^2\big(T_{10}^{\mu\nu\alpha}-T_{12}^{\mu\nu\alpha}\big)=2(q_1\cdot q_2)T_{14}^{\mu\nu\alpha},$$
$$(p\cdot q_2)\big(T_9^{\mu\nu\alpha}+T_{11}^{\mu\nu\alpha}\big)-q_2^2\big(T_{10}^{\mu\nu\alpha}+T_{12}^{\mu\nu\alpha}\big)=2(q_1\cdot q_2)T_{15}^{\mu\nu\alpha}$$

- need to include $T_{14}^{\mu\nu\alpha}, T_{15}^{\mu\nu\alpha}$ into generating set
- project scalar functions to "basis" $\{\mathcal{F}_i\}_{i=1}^{13}$, manually shuffle parts of $\mathcal{F}_9, \mathcal{F}_{10}, \mathcal{F}_{11}, \mathcal{F}_{12}$ to $\mathcal{F}_{14}, \mathcal{F}_{15}$ (no double counting)
- additionally, need relations like $t-M_\rho^2=q_1^2-2(p\cdot q_1)$ (implies $M^2_{\rho/\pi}$ in LHC $=p^2_{\rho/\pi}$ in intermediate state)