

Dispersive description of the $\gamma^* T \rightarrow \pi\pi$ subprocess and its role in $(g - 2)_\mu$

Emilis Kaziukėnas^{1,2}

with

Massimiliano Procura¹, Peter Stoffer^{3,4}, Eleftheria Solomonidi^{3,4}, Eirini Lymperiadou^{3,4}
and Jan-Niklas Toelstede^{3,4}

¹University of Vienna, Austria, ²Vienna Doctoral School in Physics, Austria

³University of Zurich, Switzerland, ⁴Paul Scherrer Institute, Villigen, Switzerland

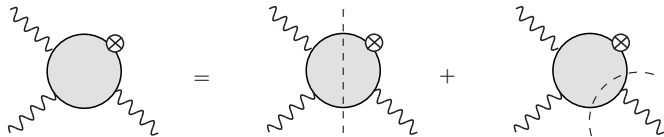
14th International Workshop on e^+e^- collisions from Phi to Psi
June 9 2026, Scuola Normale Superiore di Pisa

Overview

- 1 Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.
- 2 $\gamma^* T \rightarrow \pi\pi$, as an ingredient of tensor TFFs.
- 3 Glimpse at the construction $(g - 2)_\mu$: $f_2(1270)$ pole pieces.

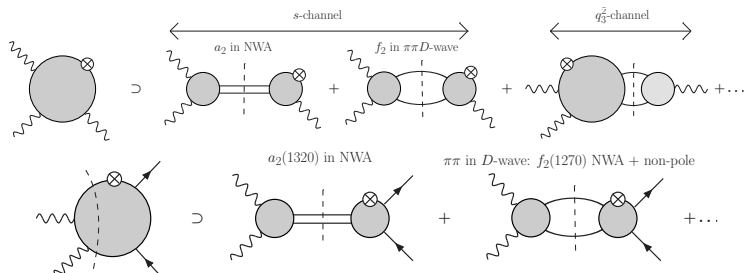
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1) The road to spin-2: $\gamma^* T \rightarrow \pi^+ \pi^-$ and $T \rightarrow \gamma^* \gamma^*$ appear!



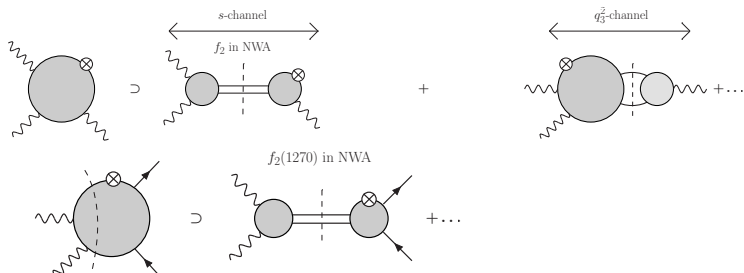
⇒ **Eirini's talk!** What do we need for spin-2 resonances?

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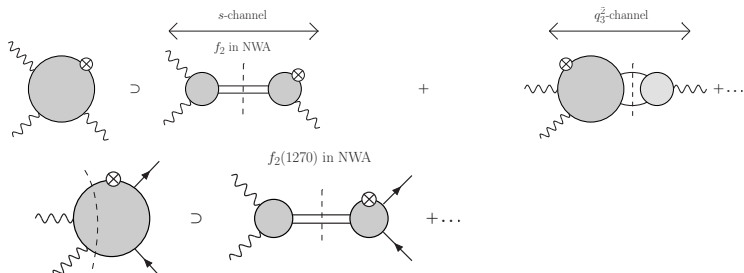
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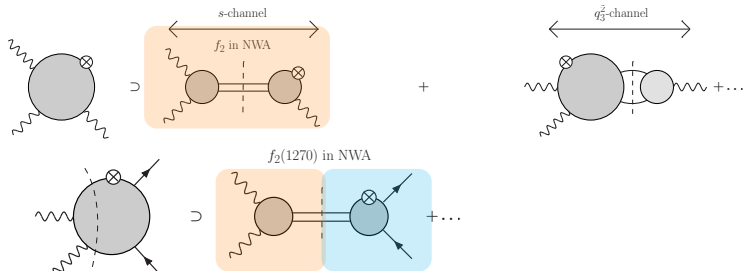


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 q_3^2 -Cut : TFFs \mathcal{F}_i^T and $\gamma^* T \rightarrow \pi^+ \pi^-$ - new!

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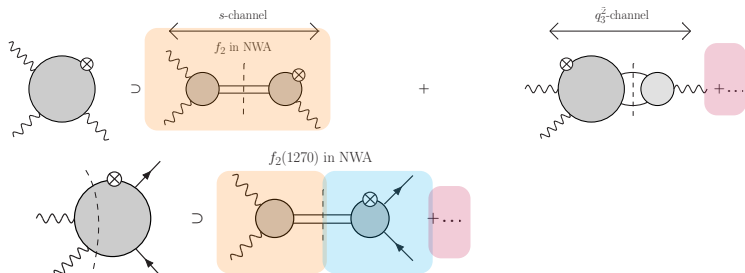
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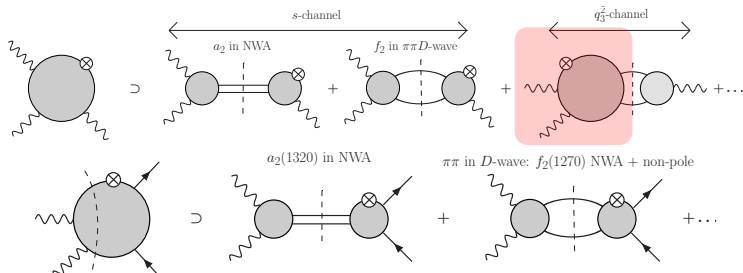
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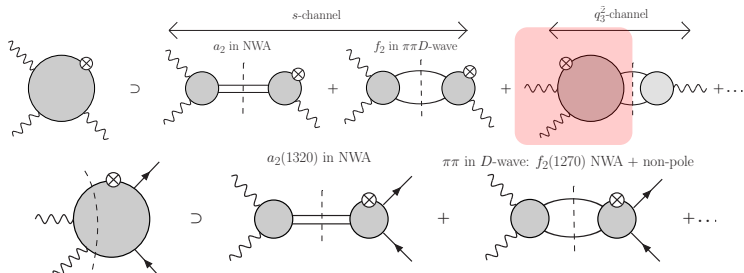
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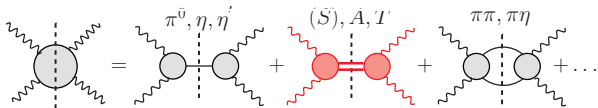
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1) Tensors in HLbL so far: perils of the old approach



- **Old approach:** disperse in s, t, u , then $q_4 \rightarrow 0$ at the end:

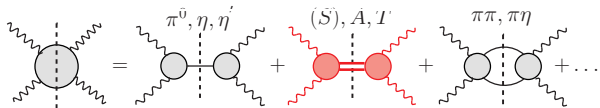
$$a_{\mu}^{\text{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^{\infty} dQ_2 \int_0^{\infty} dQ_1 \int_{-1}^1 d\tau \sqrt{1 - \tau^2} Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau).$$

- **Tensors (and $J \geq 2$ intermediates) in 4 pt. kinematics approach - kinematic singularities in Q_i^2** (due to redundant basis of HLbL) *M. Hoferichter, P. Stoffer, M. Zillinger, arXiv:2402.14060*

$$\Pi_i \supset \frac{1}{Q_1^2 Q_2^2 (Q_1^2 - Q_2^2)} \mathcal{F}_i^T \mathcal{F}_j^T + \dots$$

- **Sum rules remove all kin. singularities**, but only guaranteed for ∞ -tower, not individual intermediates (incl. tensors)! \Rightarrow **Are tensors doomed in the old approach?**

1) Tensors in HLbL so far: perils of the old approach



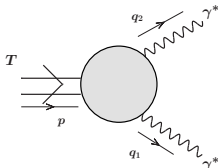
$$\Pi_i \supset \frac{1}{Q_1^2 Q_2^2 (Q_1^2 - Q_2^2)} \mathcal{F}_i^T \mathcal{F}_j^T + \dots$$

Special cases exist, singularities vanish when:

- **1** Only $\mathcal{F}_1^T \neq 0$
 - **2** When pairs $\mathcal{F}_{1,2}^T \neq 0$ or $\mathcal{F}_{1,3}^T \neq 0$.
- Model approaches exist that utilise this.
 - Can not set other TFFs to 0 if they are generated (and they are)!

Triangle kinematics approach resolves this (and more)!

1) Pheno of $f_2(1270)$: introduction



- General $T \rightarrow \gamma^* \gamma^*$ amplitude described by $\times 5$ TFFs:

$$\mathcal{M}^{\mu\nu\alpha\beta} = \sum_{i=1}^5 T_i^{\mu\nu\alpha\beta} \frac{1}{m_T^{2n}} \mathcal{F}_i^T(q_1^2, q_2^2)$$

- Known light-cone OPE (Brodsky-Lepage limit) *Hoferichter, Stoffer, arXiv:2004.06127v2*:

$$\mathcal{F}_1^T(q_1^2, q_2^2) \sim \frac{1}{Q^4} f_1(\omega), \quad \mathcal{F}_{i \neq 1}^T(q_1^2, q_2^2) \sim \frac{1}{Q^6} f_i(\omega),$$

with $Q^2 = 1/2(q_1^2 + q_2^2)$, $\omega = 2/Q^2 \times (q_1^2 - q_2^2)$.

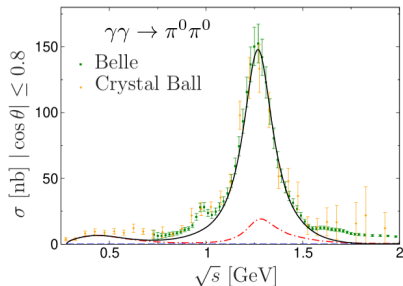
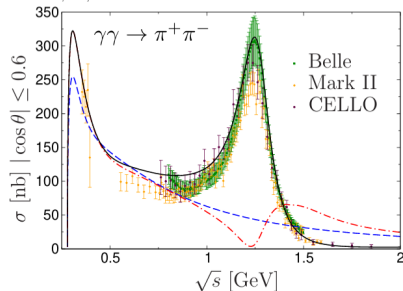
- Quark Model:**

$$\mathcal{F}_1^T(q_1^2, q_2^2) = \frac{\mathcal{F}_1^T(0,0) \Lambda_T^4}{(\Lambda_T^2 - q_1^2 - q_2^2)^2}, \quad \mathcal{F}_{i \neq 1}^T(q_1^2, q_2^2) = 0.$$

\Rightarrow works in the old approach!

1) Pheno of $f_2(1270)$: D -wave resonance in $\gamma\gamma \rightarrow \pi\pi$

$I = 0, S, D$ -wave contributions

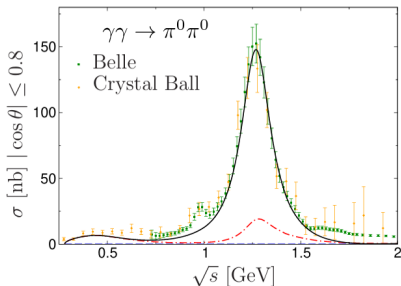
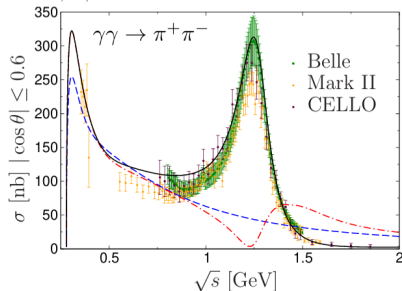


M. Hoferichter, P. Stoffer arXiv:1905.13198

- $\gamma\gamma \rightarrow \pi\pi$: arises in $e^+e^- \rightarrow e^+e^-\pi\pi$.
- $\gamma^*\gamma^* \rightarrow \pi\pi$ (S - and D -wave) calculation - good match with data.
- For $\sqrt{s} > 1\text{GeV}$ - $f_2(1270)$ resonance dominates.

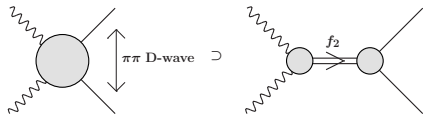
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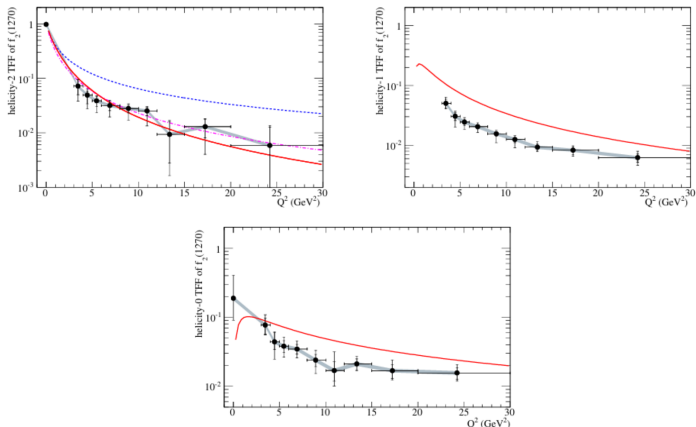
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- **But what about strictly $T \rightarrow \gamma^*\gamma^*$ data?**



- **Source:** ρ, ω, ϕ in LHC of $\gamma\gamma \rightarrow \pi\pi$ D -wave. Different method of TFF extraction \Rightarrow M. Hoferichter, M. Zillinger, in preparation

1) Pheno of $f_2(1270)$: TFF data is sparse...

M. Masuda *et al.* [Belle Collaboration], Phys. Rev. **D93**, 032003 (2016)



- Only **singly-virtual helicity amplitudes** - $e^+e^- \rightarrow e^+e^-\pi^0\pi^0$ data.
- Only consistent with QM for $\lambda = 2$, can only probe $\mathcal{F}_{1,2,5}^T$!
- Also know $\Gamma_{T \rightarrow \gamma\gamma} = 2.6(5)$ keV, $r_h = 1/24 |\mathcal{F}_2^T(0,0)|^2 / |\mathcal{F}_1^T(0,0)|^2 = 0.095(20)$.

1) hQCD prediction: tensions with the QM!

J.Mager, L. Cappiello, J. Leutgeb, A. Rebhan arXiv:2501.19293

$$S = -2k_T \int d^5x \sqrt{g} (\mathcal{R} + 2\Lambda) + \frac{1}{2g_5^2} \text{tr} \int d^5x \sqrt{g} F_{MN} F^{MN}$$

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = -m_T \frac{1}{g_5^2} \text{tr} Q^2 \int \frac{dz}{z} h_n(z) \mathcal{J}(z, Q_1) \mathcal{J}(z, Q_2),$$

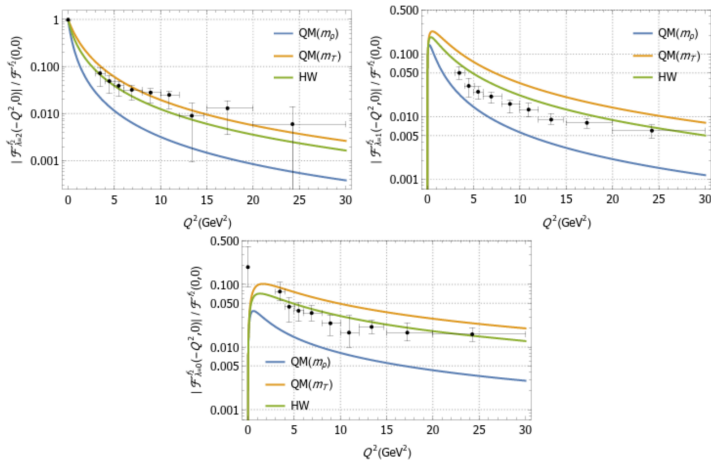
$$\mathcal{F}_3^T(-Q_1^2, -Q_2^2) = -m_T^3 \frac{1}{g_5^2} \text{tr} Q^2 \int \frac{dz}{z} h_n(z) \frac{\partial_z \mathcal{J}(z, Q_1)}{Q_1^2} \frac{\partial_z \mathcal{J}(z, Q_2)}{Q_2^2}.$$

- Class of large- N_c models, relies on **4D gauge - 5D gravity duality**.
- Hard wall (finite 5th dimension) model: predicts **two non-zero TFFs**. \Rightarrow **one of the special combinations**
- 3 free parameters (2 fixed from axial vector model) - **fit from SDCs** (possible because of ∞ -resonance tower predicted).

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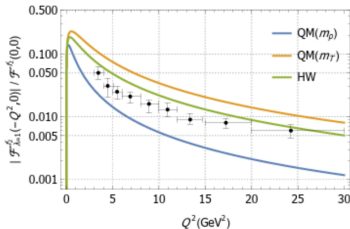
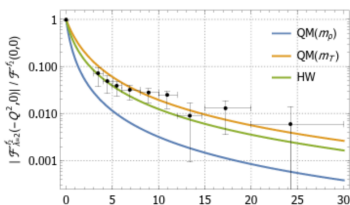
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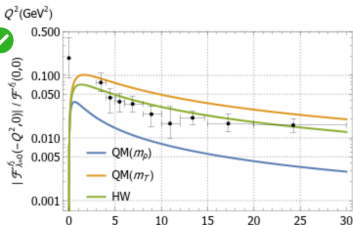
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⇒ Predictions: ✓

$m_T = 1.235 \text{ GeV}$

$\Gamma = 2.3 \text{ keV}$

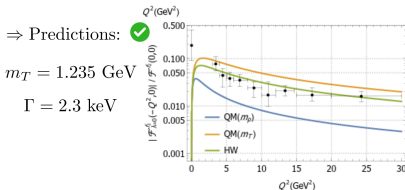
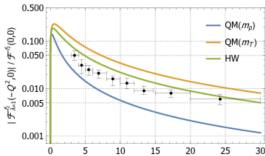
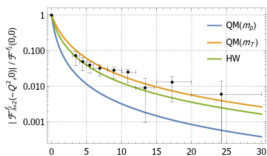


⇒ Drawback: improving uncertainties? ✗

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⇒ T-pole contributions:

$$HW : a_{\mu}(f_2 + a_2 + f_2') = 2.9(4) \times 10^{-11}$$

$$QM : a_{\mu}(f_2 + a_2 + f_2') = -2.5(8) \times 10^{-11}$$

M. Hoferichter, P. Stoffer, M. Zillinger arXiv:2412.00190

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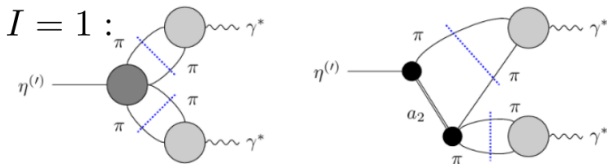
⇒ "sign problem" - disp. evaluation needed to reduce uncertainties!

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2) Gold standard: $\eta^{(\prime)} \rightarrow \gamma^* \gamma^*$ comparison

S. Holz, M. Hoferichter, B.-L. Hoid, B. Kubis arXiv:2412.16281v2



$$F_{\eta^{(\prime)}\gamma^*\gamma^*}(q_1^2, q_2^2) = F^{I=1}(q_1^2, q_2^2) + F^{I=0}(q_1^2, q_2^2) + F^{\text{eff}}(q_1^2, q_2^2) + F^{\text{asym}}(q_1^2, q_2^2)$$

Tensors: **This talk!** **TBD!**

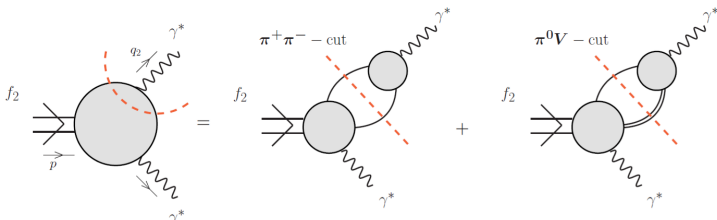
$$\mathcal{F}_i^T(q_1^2, q_2^2) = \mathcal{F}_i^{I=1}(q_1^2, q_2^2) + \mathcal{F}_i^{I=0}(q_1^2, q_2^2) + \mathcal{F}_i^{\text{eff}}(q_1^2, q_2^2) + \mathcal{F}_i^{\text{asym}}(q_1^2, q_2^2)$$

- Both $\eta^{(\prime)}$, $f_2(1270)$ - $I = 0$, same isospin decomposition.
- $I = 0$ construction: different channels - $\eta^{(\prime)} \rightarrow 2(\pi^+\pi^-)$, but $f_2 \rightarrow \pi\pi$.

Challenges in T case:

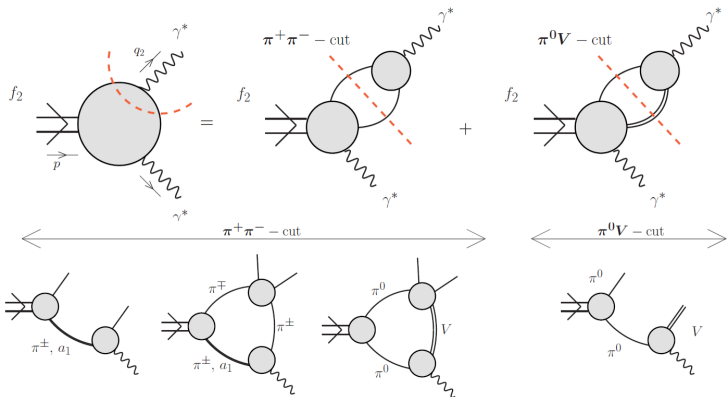
- 1** $I = 0$ case: small for $\eta^{(\prime)} \rightarrow \gamma^* \gamma^*$, VMD. Hard to estimate for $T \rightarrow \gamma^* \gamma^*$ (couplings unavailable).
- 2** No intermediate checks: $\eta^{(\prime)} \rightarrow \pi\pi\gamma$ data available (KLOE, BESIII), but no $T \rightarrow \pi\pi\gamma$.

2) ($I = 1$) TFF construction: death by 1000 cuts



- Requires $\gamma^* T \rightarrow \pi\pi$ (in P -wave) and $\gamma^* T \rightarrow \pi^0 V \Rightarrow$ **generate $\times 5$ TFFs!**
- Need to solve in $q_i^2 < 0$, $q_1^2 = m_T^2$, $q_2 \rightarrow 0$ limits: different analytic behaviour.

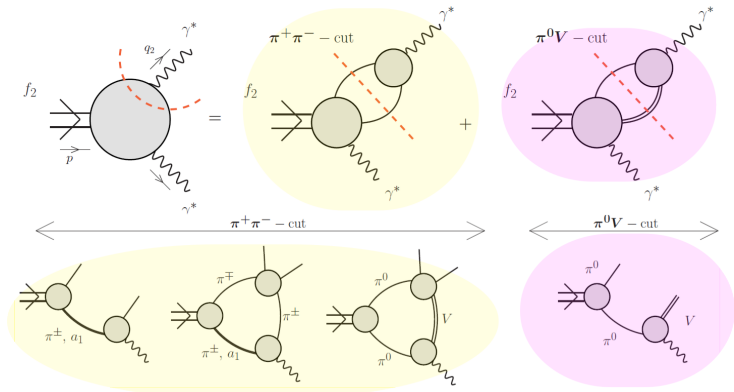
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Input: P -wave phase $\delta_1(s)$, $f_2 \rightarrow \pi\pi$, $V \rightarrow 3\pi$, $V \rightarrow \pi^0\gamma$.
Fit from $\Gamma_{T \rightarrow \gamma\gamma}$ (if needed): $f_2 \rightarrow \pi a_1$.

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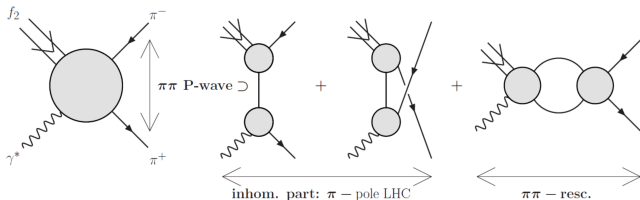


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Fit from $\Gamma_{T \rightarrow \gamma\gamma}$ (if needed): $f_2 \rightarrow \pi a_1$.

- **Status:** $\gamma^* T \rightarrow \pi\pi$ for π, a_1 solved \Rightarrow **Discuss now!**, incorporating V intermediates.

2) The $\gamma^* T \rightarrow \pi\pi$ subprocess: Inhom. Omnès problem

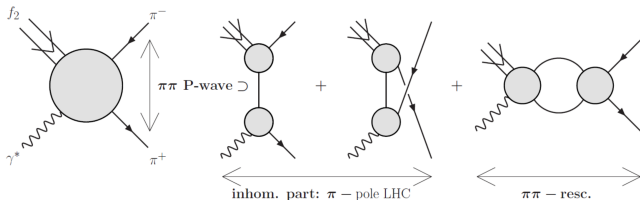


- $\gamma^* T \rightarrow \pi\pi$ definition: $\times 8$ scalar funcs - $\mathcal{M}^{\mu\nu\alpha\beta} = \sum_i T^{\mu\nu\alpha\beta} B_i$.
- Start with π -pole LHC contribution (similar for other intermediates.) and **enforce Watson's theorem (rescattering)**.
- Generally - hyperbolic disp. rels:

$$B_i(s, t, u) = B_i^{\text{Born}}(s, t, u) + \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\text{Im}B_i(s', z')}{s' - s} + \frac{1}{\pi} \int_{9M_\pi^2}^{\infty} dt' \text{Im}B_i(t', u') \left(\frac{1}{t' - t} + \frac{1}{t' - u} - \frac{1}{t' - a} \right).$$

- **Solution setup:** $B_i(s, t, u) \Rightarrow$ partial waves $h_i^J(s) \Rightarrow$ Roy-Steiner equations (coupling of different J -waves) - complicated!

2) The $\gamma^* T \rightarrow \pi\pi$ subprocess: Inhom. Omnès problem



■ **Simplification:** only need $\pi\pi$ P-wave! Mapping $B_i(s, t, u) \Rightarrow$ partial waves $h_i^1(s) \Rightarrow$ back to $B_i^{\text{P-wave}}(s, t, u)$ - no t, u -dep. - single-var. disp. rels!

- Asymptotic ansatz (only $\times 5$ B_i funcs. survive) -unsubtracted disp. rels.:

$$B_{1,2} \asymp \frac{1}{s^2}, \quad B_{3,4,5} \asymp \frac{1}{s},$$

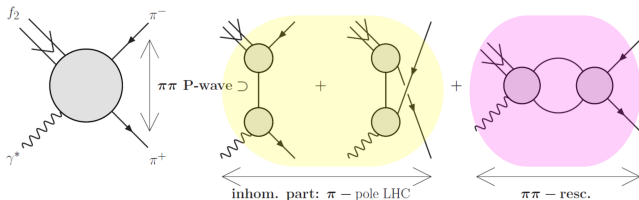
- **Inhomogeneous Omnès problem:**

$$B_i^{\text{P-wave}}(s) = B_{i,\text{Born}}^{\text{P-wave}}(s) + \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{B_{i,\text{Born}}^{\text{P-wave}}(s') \sin \delta_1(s')}{(s' - s - i\epsilon) |\Omega_1(s')|}.$$

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with $s_{R,L} = (m_T \pm \sqrt{q_1^2})^2$ - **soft-singular at $s = m_T^2$ as $q_1^2 \rightarrow 0$. Numerical instabilities - how to deal with them?.**

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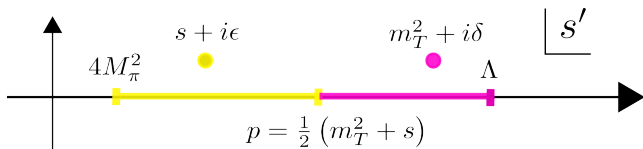
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Inspired by: S. Mutke, M. Hoferichter, B. Kubis arXiv:2406.14608

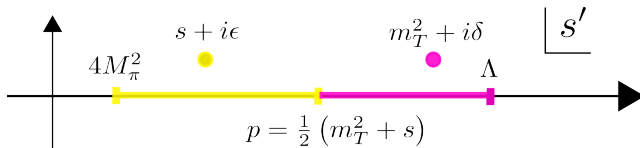


- **ONS limit:** most complicated case (soft- γ^*) singularity. Work with $m_T^2 + i\delta$ prescription:

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Idea: split integration region - isolate singularities, subtract n derivatives at $s' = s$ and $s' = m_T^2$ s.t. integrands finite even in soft limit: $p = s = m_T^2$.

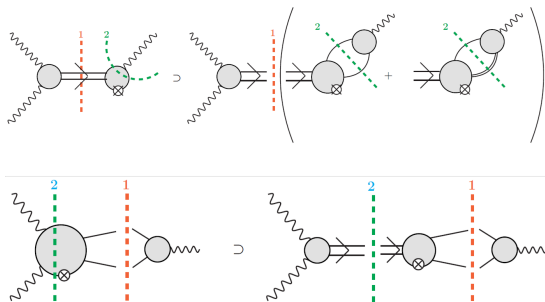
$$B_i^{\text{resc.}}(s) = \text{PV}_s \left(\int_{s_0}^p ds' \frac{T(s') - T_n(s', s)}{(s' - m_T^2)^n (s' - s)} \right) + \text{PV}_{m_T^2} \left(\int_p^\Lambda ds' \frac{T(s') - T_n(s', m_T^2)}{(s' - m_T^2)^n (s' - s)} \right) + K_n(s) + R_n(s),$$

- **Remainders K_n, R_n singular, but evaluated analytically -singularities cancel, stable numerics!** (Other kinematic cases more simple).

Overview

- 1 Hadronic spin-2 resonances: in our construction, phenomenology, current estimates.
- 2 $\gamma^* T \rightarrow \pi\pi$, as an ingredient of tensor TFFs.
- 3 Glimpse at the construction $(g - 2)_\mu$: $f_2(1270)$ pole pieces.

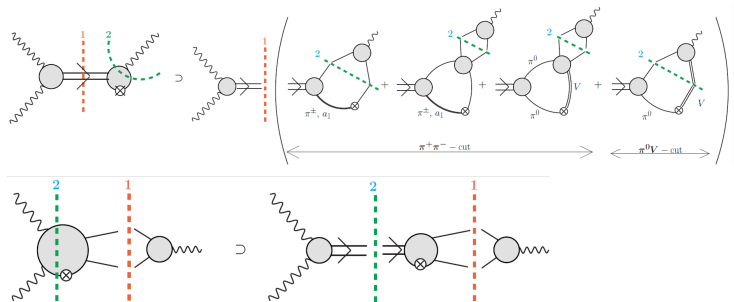
3) A real $f_2(1270)$ contribution to $(g - 2)_\mu$: pole pieces



- Need tensor contribution to $(g - 2)_\mu \in \mathbb{R}$.
- Group subprocesses to c.c. pairs?

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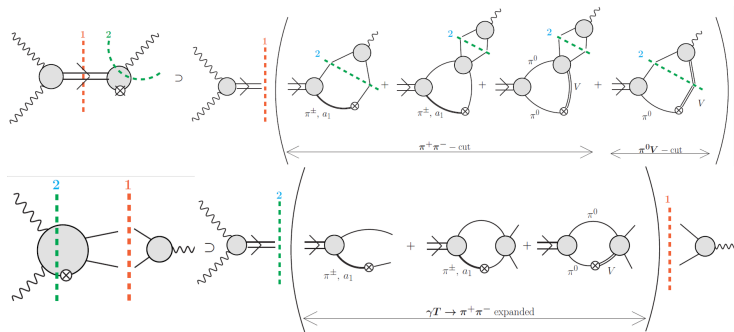
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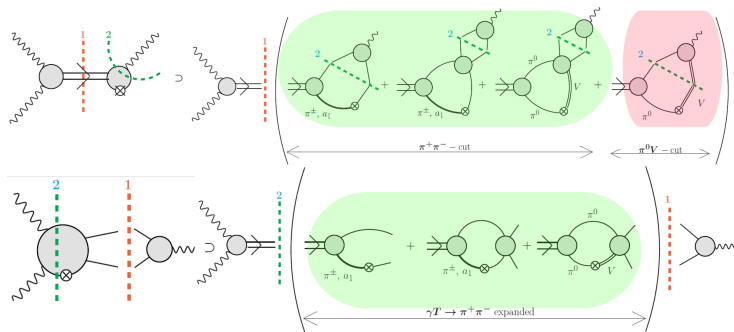
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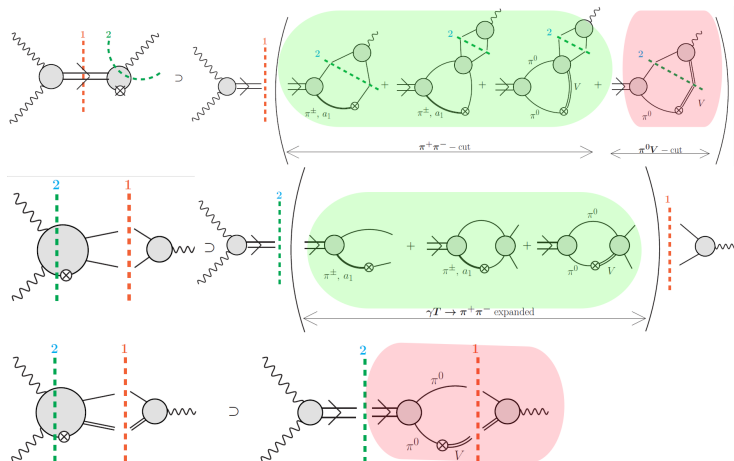


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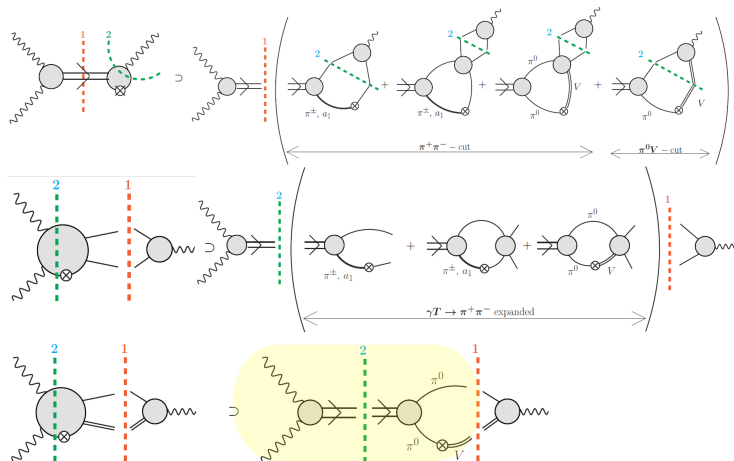
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3) A real $f_2(1270)$ contribution to $(g - 2)_\mu$: pole pieces



- **Problem:** unpaired subprocess in π^0V secondary cut!
- **Solution:** also consider a π^0V intermediate in q_3^2 cut!
- Now need $3\gamma \rightarrow \pi^0V$ - doable in soft limit. **Even more complex for full $3\gamma \rightarrow \pi\pi$, $\gamma\gamma \rightarrow \pi\pi$ in D -wave.**

Summary: tensor resonances \nearrow , $(g - 2)_\mu$ uncertainty \searrow !

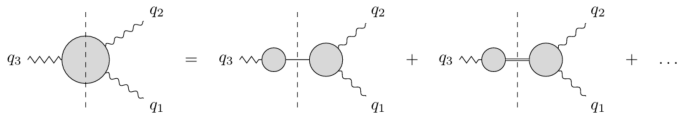
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- **Gold standard:** similarities with $\eta^{(\prime)}$ $\rightarrow \gamma^* \gamma^*$ exist - $l = 0$ contributions, effective poles, matching - TBD.
- **Replicating $f_2(1270)$ pheno:** $l = 1$ part - need V -intermediates, $\gamma^* T \rightarrow \pi\pi$ with π -pole LHC **solved** - approach adaptable for the rest of contributions.

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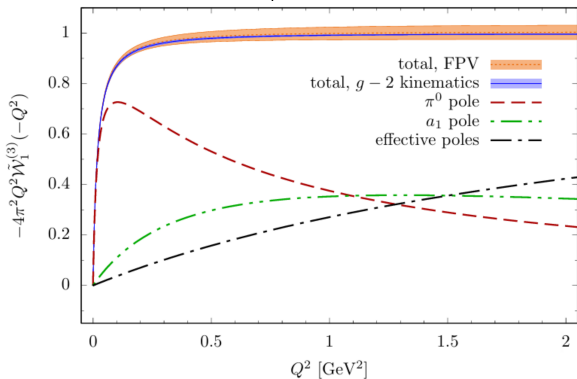
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Thank you for your attention!

Appendix: VVA correlator - testing ground for HLbL



Normal kinematics, then $q_2 \rightarrow 0$

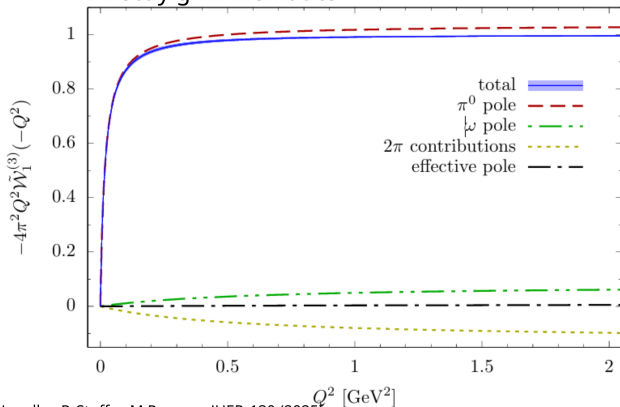


J. Luedke, P. Stoffer, M. Procura, JHEP, 130 (2025)

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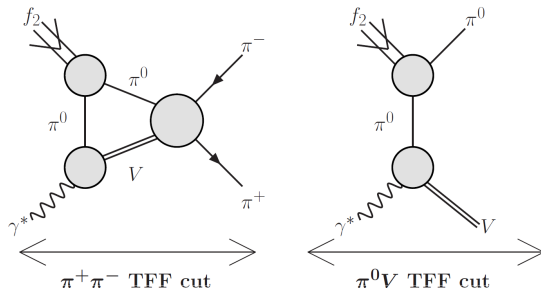
$$\Delta \left[\begin{array}{c} q_2 \\ \text{---} \otimes \text{---} \\ q_3 \text{ ---} \text{---} \text{---} \text{---} \text{---} q_1 \end{array} \right] = \left(\begin{array}{c} q_2 \\ \text{---} \otimes \text{---} \\ q_3 \text{ ---} \text{---} \text{---} \text{---} \text{---} q_1 \end{array} \right)^* + \begin{array}{c} q_2 \\ \text{---} \otimes \text{---} \\ q_3 \text{ ---} \text{---} \text{---} \text{---} \text{---} q_1 \end{array}$$

Directly g-2 kinematics



J. Luedke, P. Stoffer, M. Procura, JHEP, 130 (2025)

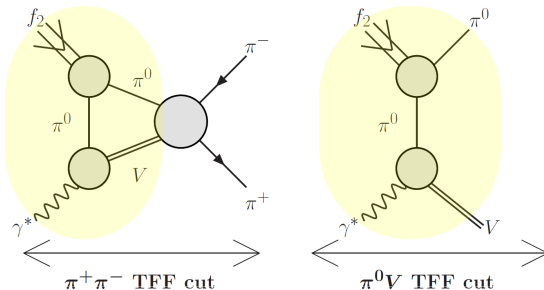
Appendix: ($I = 1$) TFF construction: the $\gamma^* T \rightarrow \pi^0 V$ contributions



- Need dispersive π^0 -pole description in $\gamma^*(q_1) T(q_2) \rightarrow V(q_3) \pi^0(q_4)$, with 23 non-redundant structures (# of hel. amplitudes).
- Basis decomposition - 300+ rank-7 structures T_i :

$$\{L_i^{\mu\alpha\beta\gamma}\} = \{\epsilon_{abcd} T_i^{\mu\alpha\beta\gamma abcd}\},$$

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- Basis decomposition - 300+ rank-7 structures T_i :

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- **BTT+imposing γ^* , T , V symmetries:** 75 (physical) + 73 (unphysical) strucs. \Rightarrow getting to 23?
- **Schouten redundancies- $D = 4$:** $\epsilon^{[\mu\nu\rho\sigma} g^{\alpha]\beta} = 0$. Final result:

$$N^{\mu\alpha\beta\gamma}(g_1, g_2, g_3) = -(g_1 - g_2)^\alpha (g_1 - g_3)^\beta \epsilon^{\mu\gamma\rho\sigma} g_\rho g_\sigma \frac{\mathcal{F}_0^\Gamma f_{V\pi^0}^*(q_1^2)}{f_{V\pi^0}^*(q_1^2)}$$