Calibrated Study of the Correlation between Heavy Quark Masses and HVP observables

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Preliminary results based in a work in collaboration with

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Outline

1) **Motivation**
    - **HVP contribution to g-2**
    - **Relativistic QCD Sum Rules**
2) **Structural correlation: Results**
3) **Conclusions and Outlook**

Motivation: why a_μ^q ?

The problem:

- HVP is dominant theory uncertainty.
- Disagreement between approaches.
- Local quark-hadron duality.
- $\hat{m}_q$ used as external input to a_μ^q .

Our solution:

- Simultaneous determination of $\hat{m}_q$ and a_μ^q within QCD SR.
- Anti-correlation reduces final error budget.
- Promote K(s) into the duality relation

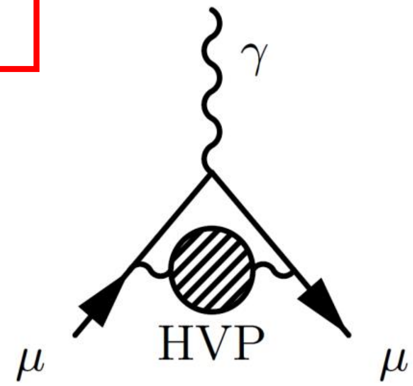
Impact:

- More precise and consistent a_μ^q for heavy quarks.
- Flavor decomposition may help specially to compare with lattice QCD estimates.

The HVP contribution to a_μ

The HVP contribution is accessed through a dispersive relation connecting it to the **hadronic cross section**:

$$a_\mu^{\mathrm{HVP},(i)} = \left(\frac{\alpha\, m_\mu}{3\pi}\right)^2 \int_{s_{th}}^{\infty} ds\, \frac{\hat{K}^{(i)}(s)}{s^2}\, R_{\mathrm{had}}(s)$$



The **challenge** is entirely in $R_{\mathrm{had}}(s)$:

1) Data only available within a **finite energy range.**

2) pQCD fails near threshold (**NP**).

3) Within standard approach, one is forced to switch from data to pQCD at some fixed energy. **Local quark-hadron duality**.

Is the associated uncertainty correctly quantified?
Our approach: QCD Sum Rules within global duality & cross-correlation within different sum rules.

QCD Sum Rules within global duality

The moments of $R_{\text{had}}(s)$ also determines the quark mass $\hat{m}_q$ **[SVZ,'79]**

The correlator of two heavy-quark vector current (calculable in pQCD) obeys:

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{\pi} \int_{4m_q^2}^{\infty} \frac{ds}{s} \frac{\operatorname{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

Using the Optical Theorem,

$$R_q(s) = 12\pi \operatorname{Im}\Pi(s + i\epsilon)$$

For t→0

$$\mathcal{M}_n := \frac{12\pi^2}{n!} \frac{d^n}{dt^n} \hat{\Pi}_q(t)\Big|_{t=0} = \int_{4m_q^2}^{\infty} \frac{\mathrm{ds}}{s^{n+1}} R_q(s)$$

L.H.S.: theory

R.H.S.: data

LHS from theory:

$$\Pi_q(t) = Q_q^2 \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n \left(\frac{t}{4\hat{m}_q^2}\right)^n$$

[Maier et al. '08]
[Chetyrkin, Steinhauser'06]
[Kiyo et al.'09]
[Greynat et al. '09]

QCD Sum Rules within global duality

RHS parametric model: **[Erler, Masjuan, Spiesberger '16 & '22]**

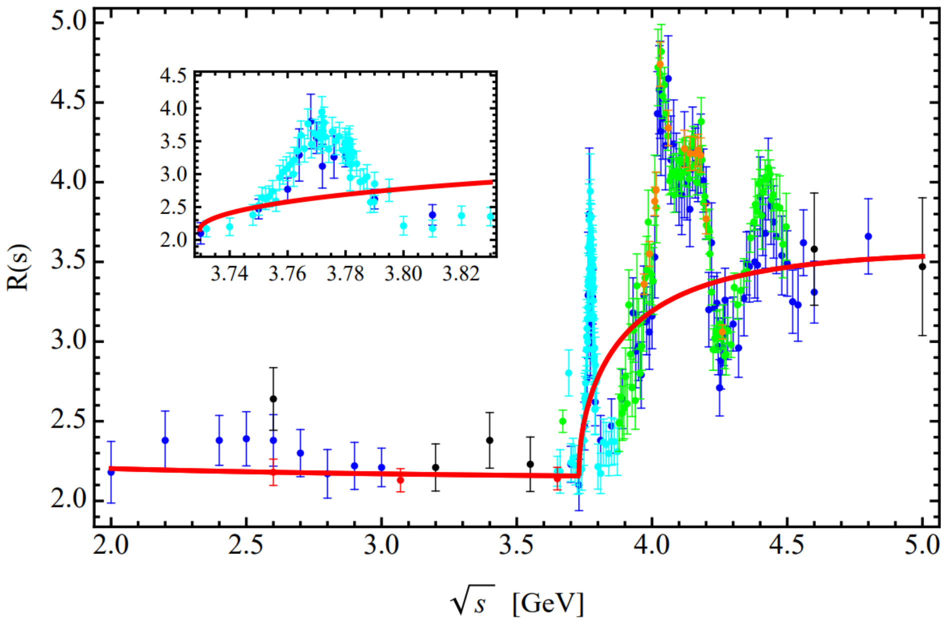
$$R_q^{\text{cont}}(s) = 3Q_q^2\lambda_1^q(s)\sqrt{1-\frac{4\,\hat{m}_q^2(2M)}{s'}}\left[1+\lambda_3^q\frac{2\,\hat{m}_q^2(2M)}{s'}\right]$$

$$R_q(s) = R_q^{\text{res}}(s) + R_q^{\text{cont}}(s)$$

This avoid local duality assumptions by construction.

Standard procedure: $\mathcal{M}_n^{\text{pQCD}} = R_q^{\text{Res}} + \int_{D\bar{D}}^{s_0}\frac{ds}{s^{n+1}}R_q(s) + pQCD(\mu)$

We really want: $\mathcal{M}_n^{\text{pQCD}} = R_q^{\text{Res}} + \int_{D\bar{D}}^{\infty}\frac{ds}{s^{n+1}}R_q(s)$



QCD Sum Rules within global duality

One can also define a negative Sum Rule,

$$\mathcal{M}_0 := -\lim_{t\to\infty}\left[\hat{\Pi}(-t) - \hat{\Pi}^{\infty}(-t)\right] = \int_{\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s}\left[R_q(s) - R_q^{\infty}(s)\right]$$

but has a divergent part

$$\lim_{t\to\infty}\hat{\Pi}_q(-t) \sim \log(t) \longleftrightarrow \int_{4m_q^2}^{\infty}\frac{\mathrm{d}s}{s}R_q(s) \sim \log(\infty)$$

Fortunately the divergent part is given by the zero mass limit of $R_q(s)$ (can be easily subtracted).

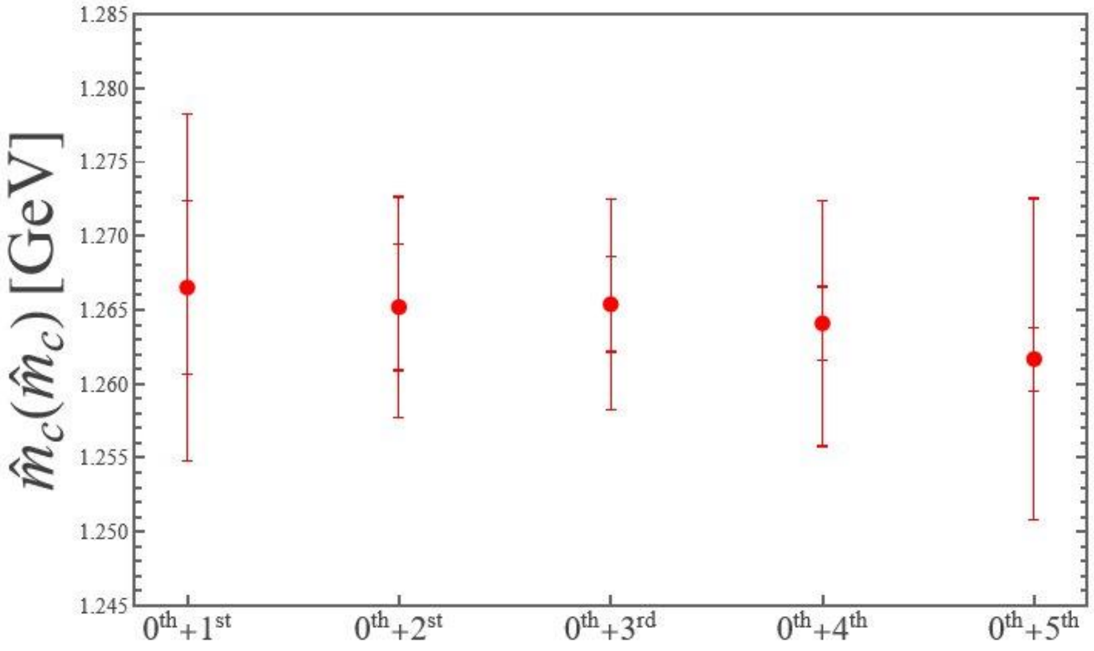
Including the zeroth sum rule will be **crucial**.

QCD Sum Rules within global duality

Two parameters to determine : m_q , λ_3^q

We use zeroth moment + nth moment

No local experimental data on R(s)



$$\frac{9}{4}Q_q^2\left(\frac{1}{2\hat{m}_q(\hat{m}_q)}\right)^{2n}\bar{C}_n = \sum_{\text{resonances}} \frac{9\pi\Gamma_R^e}{M_R^{2n+1}\hat{\alpha}_{em}^2(M_R)} + \int_{4M^2}^{\infty}\frac{\mathrm{d}s}{s^{n+1}}R_q(s)$$

$$n \geq 1$$

Our approach is **different**:

- We try to avoid local duality, consider **global duality**.
- Quark mass determined from solving two different **moment equations**.
- The corresponding a_μ contribution is **determined simultaneously with the same pair of SR.**
- The **anti**-correlation between these two quantities is powerful: it **partially cancels common sources of uncertainty.**
- The **uncertainty is constraint aswell** by imposing two moments (or derivatives).

Weighted Sum Rules: natural framework

Key observation: both integrals are weighted averages of the same R(s) function:

$$a_\mu^q \sim \int_{s_{th}^2}^{\infty} \frac{\hat{K}(s)}{s^2} R_q(s)\, ds \qquad \longleftrightarrow \qquad \mathcal{M}_n \sim \int_{s_{th}^2}^{\infty} \frac{1}{s^{n+1}} R_q(s)\, ds$$

Why privilege one kernel over the other? Natural generalisation: use the kernel of the observable you care about:

$$\mathcal{M}_n \sim \boxed{\int_{s_{th}^2}^{\infty} \frac{ds}{s^{n+1}} K(s) R_q^{\text{th}}(s) = \int_{s_{th}^2}^{\infty} \frac{ds}{s^{n+1}} K(s) R_q^{\text{had}}(s)}$$

With proper K(s), these are sum rules for the observable itself and "its derivatives".

Weighted Sum Rules: natural framework

The deeper structure: define the Q^2–dependent generalisation of a_μ:

$$\hat{A}_\mu(Q^2) \equiv A_\mu(Q^2) - A_\mu(0) = \sum_{i=1}^{\infty} a_\mu^{(i)} (Q^2)^i$$

Then each coefficient is precisely a weighted moment,

$$\frac{1}{n!} \frac{\partial^n A_\mu(Q^2)}{\partial (Q^2)^n}\bigg|_{Q^2=0} = a_\mu^{(n)} = \int_{s_{th}^2}^{\infty} \frac{ds}{s^{n+1}} K(s) R_q(s)$$

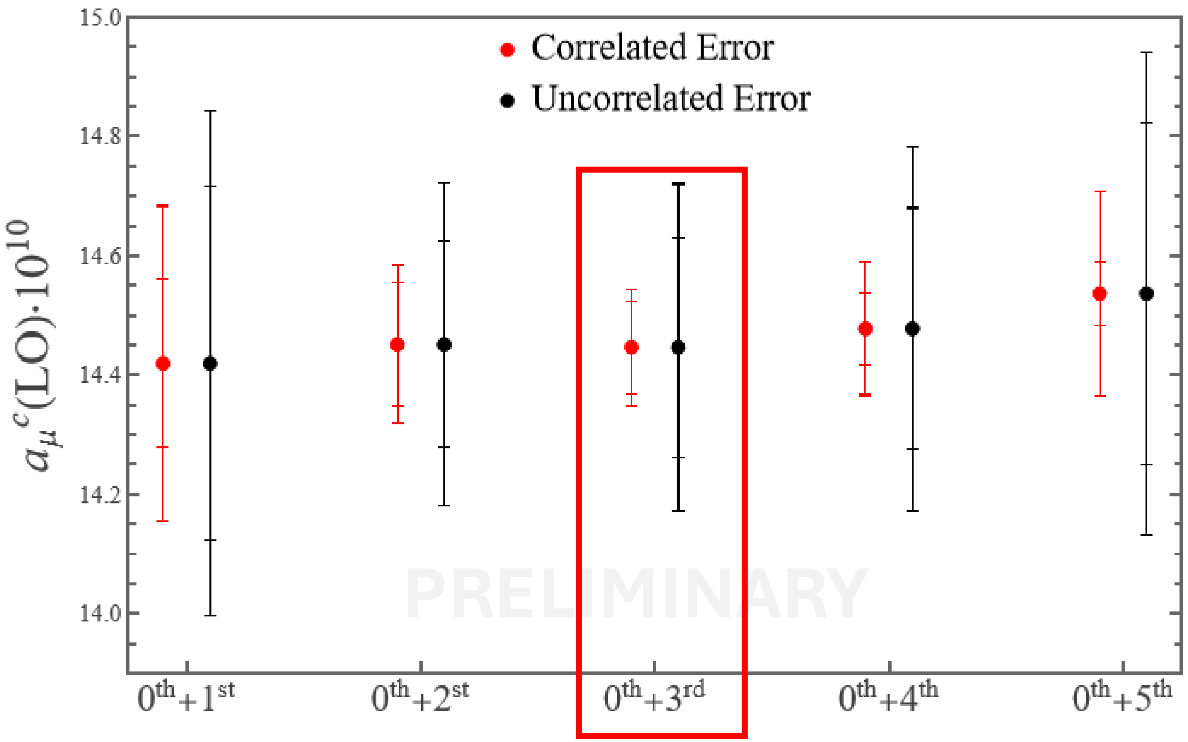
The correlation emerges explicitly! $\hat{m}_q$ and a_μ^q not independent. They are encoded in $\hat{A}_\mu(Q^2)$

CHARM CASE

Charm to a_μ charm

Uncorrelated error: mass as an external input for a_μ. Then propagate the full error.

Correlated error: When jointly determined, the common sources partially cancel.



Red points:

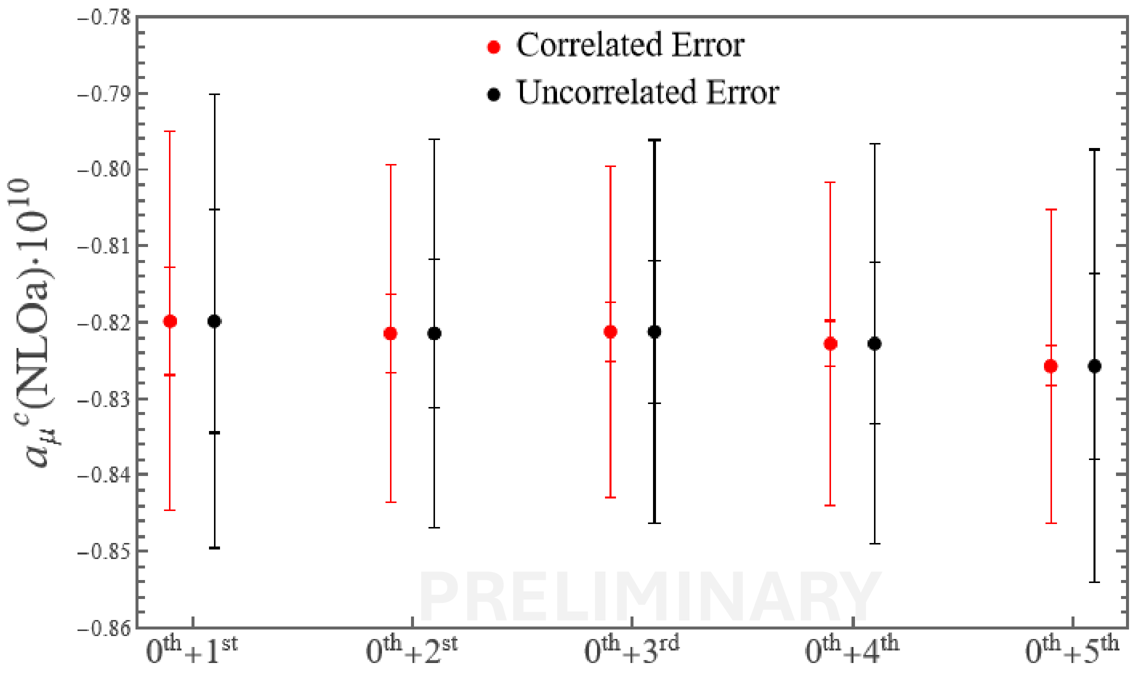
	$(\mathcal{M}_0,\mathcal{M}_1)$	$(\mathcal{M}_0,\mathcal{M}_2)$	$(\mathcal{M}_0,\mathcal{M}_3)$	$(\mathcal{M}_0,\mathcal{M}_4)$	$(\mathcal{M}_0,\mathcal{M}_5)$
$\hat{m}_c(\hat{m}_c)$	1266.5	1265.2	1265.4	1264.1	1261.7
λ_3^c	0.7306	0.7418	0.7402	0.7511	0.7718
$\lambda_3^{c,\mathrm{exp}}$	1.34(17)	1.34(17)	1.34(17)	1.33(17)	1.33(17)
a_μ^c(LO)	14.419	14.451	14.446	14.478	14.537
Statistical	0.141	0.104	0.078	0.060	0.052
NW	0.006	0.008	0.009	0.009	0.009
Truncation	0.003	0.020	0.046	0.087	0.150
$\Delta C_{n,k}^{(i)}$	0.013	0.013	0.013	0.029	0.065
$\lambda_3^q \neq \lambda_3^{q,\mathrm{exp}}$	0.196	0.069	0.029	0.013	0.006
$\Delta\lambda_3^{q,\mathrm{exp}}$	0.106	0.039	0.016	0.008	0.004
Total	0.264	0.133	0.098	0.111	0.172

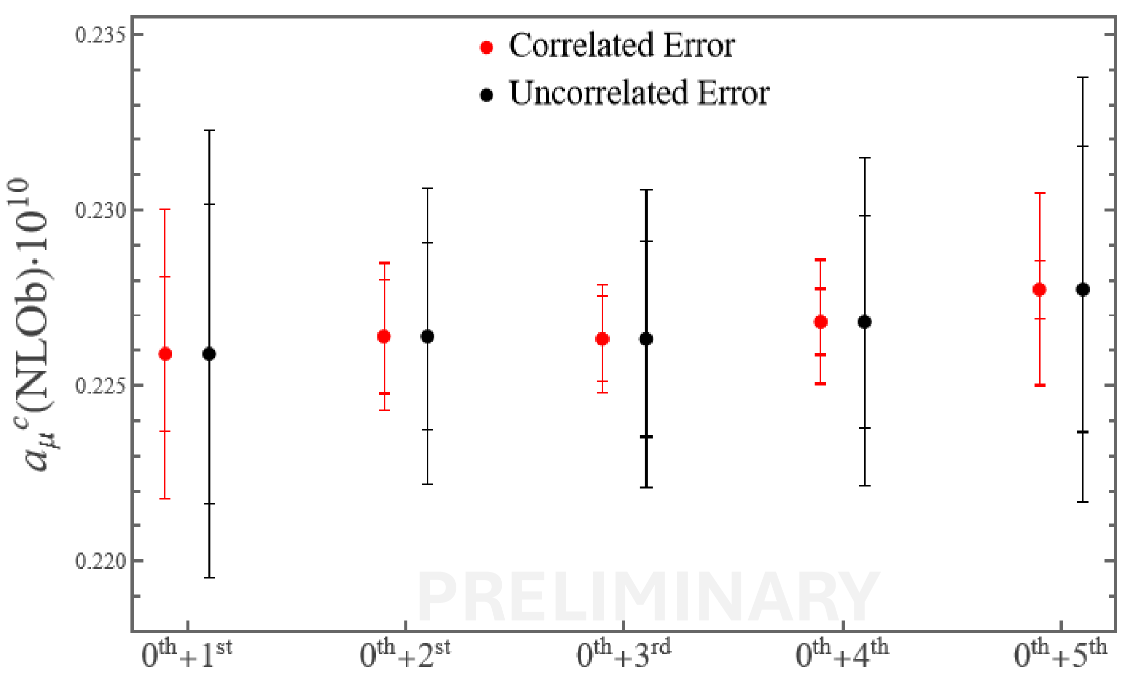
$$\hat{m}_c(\hat{m}_c) = \big(1265 + 2615\,\Delta\hat{\alpha}_s \pm 7\big)\ \mathrm{MeV}$$

$$a_\mu^{c,\mathrm{LO}} = \big(14.456 + 72\,\Delta\hat{\alpha}_s \pm 0.098\big) \times 10^{-10}$$

PRELIMINARY

a_μ charm NLO

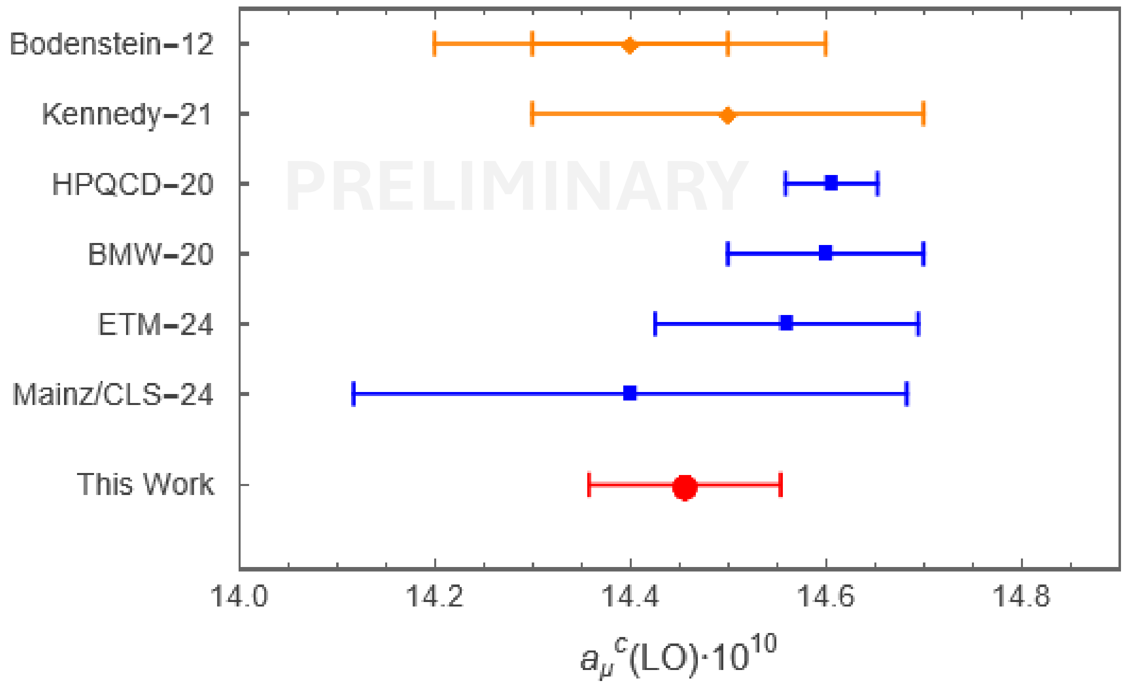


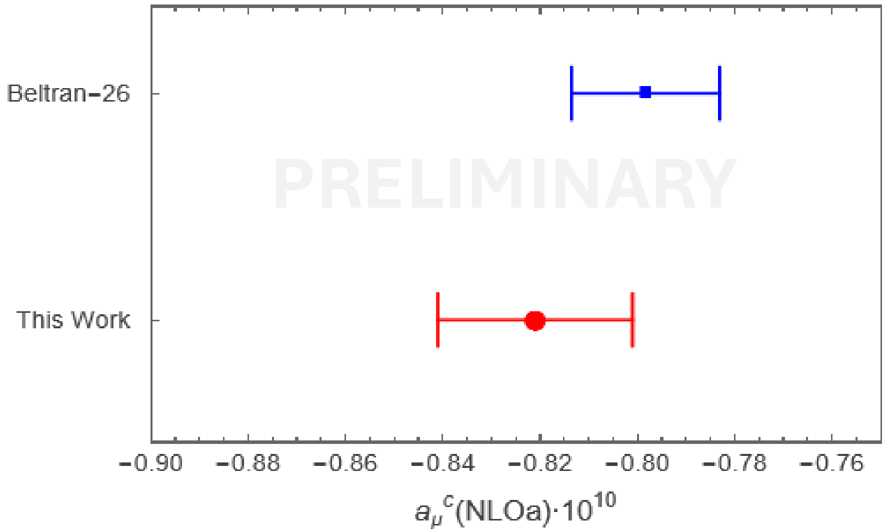


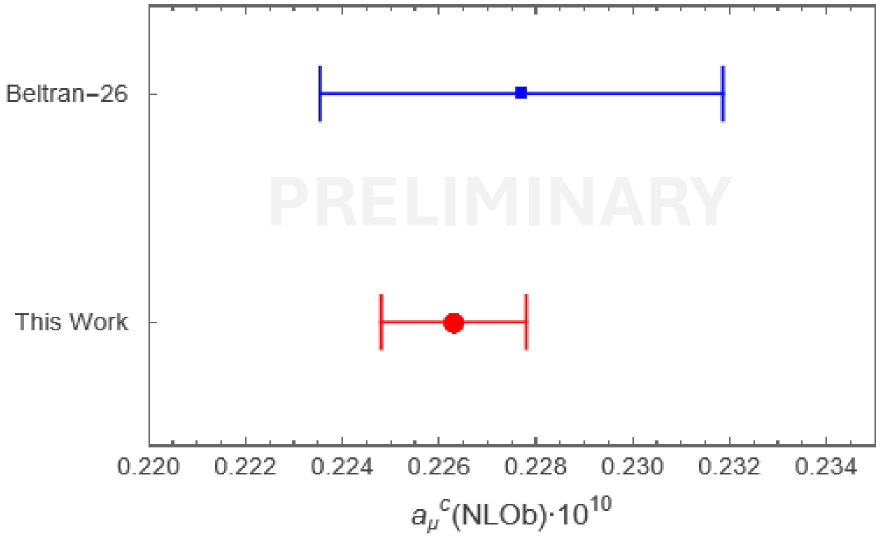
$$a_\mu^{c,\mathrm{NLO}_a} = \left(-0.821 - 5.96\,\Delta\hat{\alpha}_s \pm 0.020\right) \times 10^{-10}$$
$$a_\mu^{c,\mathrm{NLO}_b} = \left(0.2263 + 1.12\,\Delta\hat{\alpha}_s \pm 0.0015\right) \times 10^{-10}$$

PRELIMINARY

Final results







Final results in **[Beltran, Masjuan, AR '26]** (to be submitted to arXiv)

Bottom case : the procedure is the same.

Conclusions

1) We have obtained m_q and a_μ for charm (and bottom) at LO and NLO with **unprecedented precision**.

2) Observation: **correlation is not optional**. Treating them as independent is **inconsistent** at the current level of precision.

3) **Two descriptions of the same observable**: pQCD prediction is not complete in isolation, it is the interplay with the non-perturbative hadronic input that gives the full picture (within **QCD Sum Rules**). Our framework makes this **dialogue** explicit.

4) **Consistency** among different moments and tracking of correlated uncertainties.

5) **The error sources are understood**: resonance parameters, perturbative trunction, continum shape, etc. Clear roadmap for **future improvements**.

THANKS!