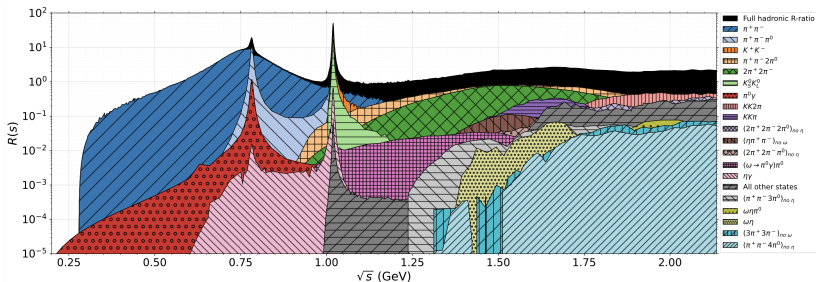


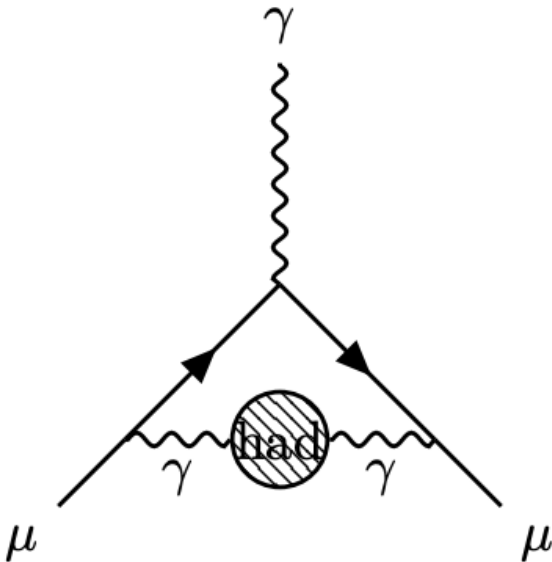
Hadronic Contributions to Muon $g - 2$ (and Other Precision Observables) in the Dispersive Approach

Aidan Wright



LEVERHULME
 TRUST





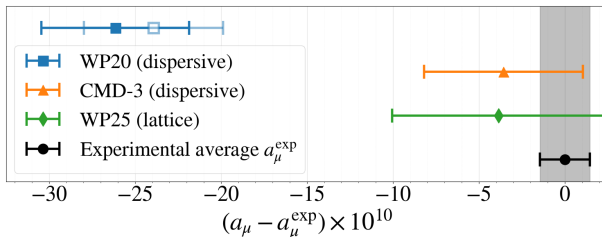


Hadronic Vacuum Polarisation

Problem: QCD is **non-perturbative** at low \sqrt{s} .

Implication: HVP of photon cannot be calculated in loop integrals etc.

- Solution: discretise spacetime and perform **lattice QCD** calculation.
- Solution: Directly measure **t -channel $e - \mu$ scattering**.
 - Upcoming MUonE experiment.
- Solution: **dispersion integral** over the $e^+e^- \rightarrow \text{hadrons}$ cross section.
 - Dispersive/data-driven methods...
 - The main focus of this presentation – including **KNTW** data compilation.



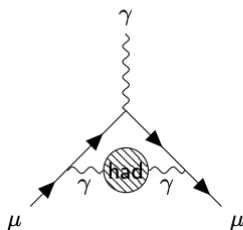


Dispersive Approach for a_{μ}^{HVP}

Problem: QCD is **non-perturbative** at low \sqrt{s} .

Implication: HVP of photon cannot be calculated in loop integrals etc.

Solution: **dispersion integral** over the $e^+e^- \rightarrow \text{hadrons}$ cross section.



$$\propto \Pi_{\mu\nu}^{\text{had.}}(q^2)$$

by definition

$$\propto \Pi^{\text{had.}}(q^2)$$

due to gauge invariance

$$\propto \int \frac{\text{Im} \{ \Pi^{\text{had.}}(s) \}}{s(s - q^2 - i\epsilon)} ds$$

by analyticity and Cauchy's theorem

$$\propto \int \frac{\sigma_{\text{had.}}^0(s)}{s - q^2 - i\epsilon} ds$$

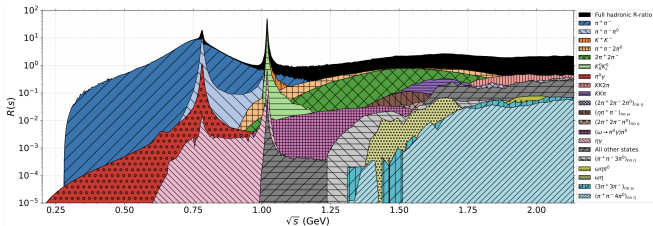
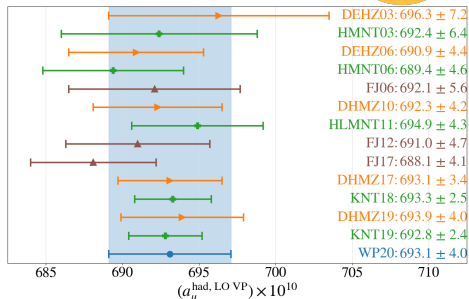
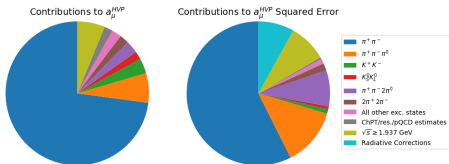
by unitarity \implies the Optical Theorem

$$\left(\text{Im} \left[\text{Diagram 1} \right] \propto \left| \text{Diagram 2} \right|^2 \right)$$



Hadronic Data

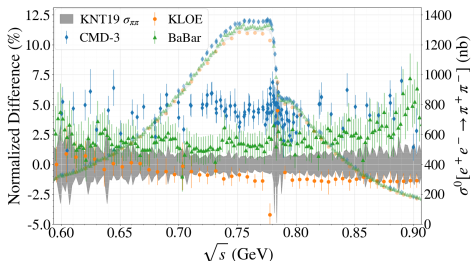
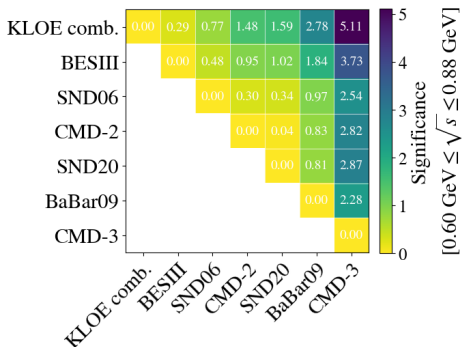
- > 11 200 data points in > 50 hadronic channels.
- Dominated (> 70%) by $e^+e^- \rightarrow \pi^+\pi^-$.





Tensions in $\pi^+\pi^-$

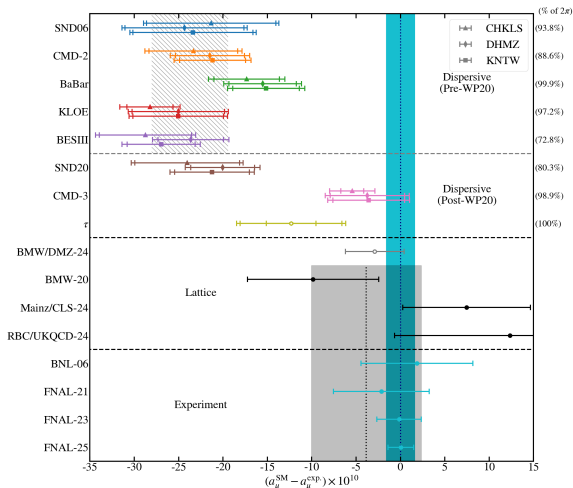
- **Historic Problem:** $\sim 2.5\sigma$ KLOE/BaBar tension.
- **Historic Solution:** local error inflation; additional 'ad-hoc' systematic.
- Not ideal, but gave a_μ^{HVP} to $< 1\%$.
- Need to improve to match experiment!
- **Current Problem:** $> 5\sigma$ KLOE/CMD-3 tension, $\sim 2.5\sigma$ BaBar/CMD-3 tension!
 - CMD-3 'corroborated' by new SND preliminary.
 - BaBar confirm their earlier result – remarkable agreement with improved systematics.
- **Current Solution:** None as yet...
 - Nothing suggests earlier data is defective.
 - Dispersive method is robust.





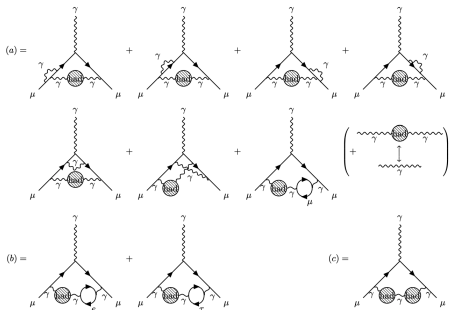
Implications...

- Analysis groups must manage tensions to estimate HVP.
- Tensions \implies WP25 quoted only lattice - **but TI is not finished yet!**
- Need to understand dispersive method results or $g - 2$ understanding incomplete.
- **Goal:** representative and accurate a_μ^{HVP} dispersive prediction to fully understand experiment implications.



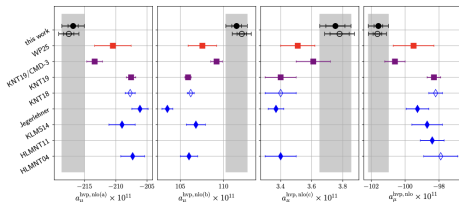


NLO HVP Contributions



- NLO a_{μ}^{HVP} dispersion integrals also known.
- Dispersive contributions separated as above (class (c) involves double integral over s, s' of hadronic bubbles).
- Contributions $\sim 30\times$ smaller than LO.

- Low energies also important here \implies suffers from CMD-3 tension.
- Subleading for a_{μ}^{HVP} uncertainty \implies WP25 took the range with a conservative uncertainty band. (**Not** appropriate for LO.)
- New lattice result – expectedly matches CMD-3.



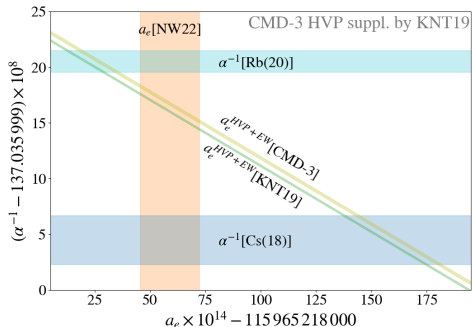
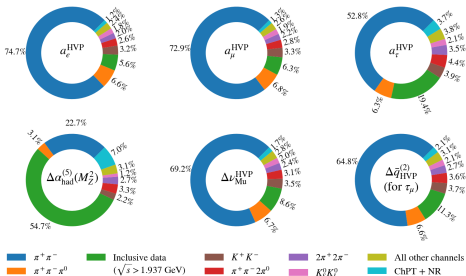
Lattice determination of the higher-order hadronic vacuum polarization contribution to the muon $g - 2$,

Mainz-CLS arXiv:2604.08351



Other Observables in the Dispersive Approach

Contributions to observables ($\sqrt{s} \leq 11.199 \text{ GeV}$)



- Can compute various other observables in dispersive approach.
- Affected to varying extents by CMD-3 ($\pi^+\pi^-$) tensions.
- Most critical for a_μ^{HVP} ...

- ...typically due to other theory uncertainties/tensions.
- Example: CMD-3 discrepancy most significant in HVP for a_e^{HVP} .
- However, a_e prediction limited by $\sim 5\sigma$ between α -evaluations.



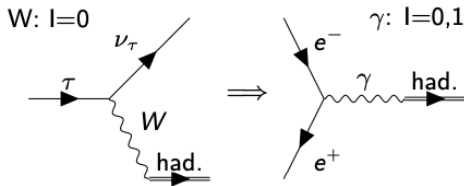
Proposed Solutions - τ Data

The $e^+e^- \rightarrow \pi^+\pi^-$ data are in tension.

The $\tau \rightarrow \nu_\tau \pi^0 \pi$ data are not.

\implies Supplement the former with the latter to get a more consistent average?

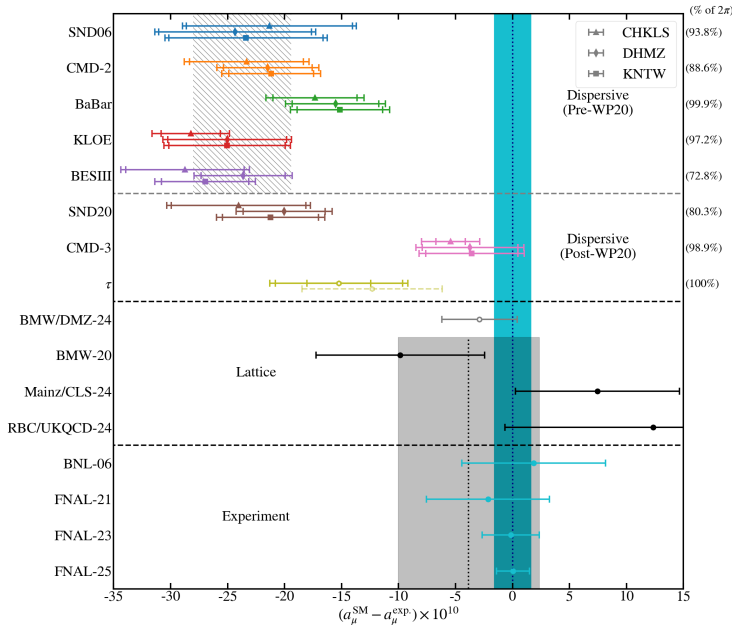
- Requires calculation of **isospin breaking corrections**.
- Not included in WP20 average due to potentially large unknown uncertainties.
- Some progress with model-independent and lattice evaluations.
- Concern:** interference between hadronic channels may not be negligible \implies large previously unaccounted terms.
- Full, assuredly accurate calculation not yet complete.
- Next slide: τ updates following CCHH structure dependent effects studies.



	Refs. [166, 194]	Ref. [209]	Refs. [237, 247]	Our estimate
Phase space	-7.88	-7.52	-	-7.7(2)
S_{EW}	-12.21(15)	-12.16(15)	-	-12.2(1.3)
G_{EM}	-1.92(90)	$(-1.67)_{-1.39}^{+0.60}$	-	-2.0(1.4)
FSR	4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ - ω mixing	4.0(4)	2.87(8)	3.79(19)	3.9(3)
ΔM_ρ	0.20 $^{+27}_{-19}$ (9)	1.95 $^{+1.56}_{-1.55}$	-	-
$\Delta\Gamma_\rho(\Delta M_\rho)$	4.09(0)(7)	3.37	-	-
$\frac{F_V}{F_A}$ (w/o ρ - ω)	$\Delta\Gamma_\rho(\pi\pi\gamma)$ -5.91(59)(48)	-6.66(73)	-	-
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$ -	-	-	-
Total	-1.62(65)(63)	$(-1.34)_{-1.71}^{+1.72}$	-	-1.5(4.7)
Sum	-14.9(1.9)	$(-15.20)_{-2.63}^{+2.26}$	-	-15.0(5.1)

WP25, R. Aliberti et al - Phys. Rept. 1143 (2025),

arXiv:2505.21476v3



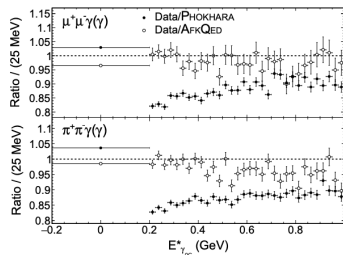


Proposed Solutions - Radiative Correction Explanation

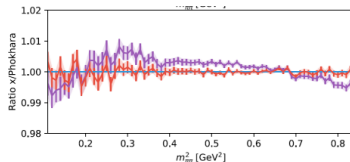
Processes $e^+e^- \rightarrow \pi^+\pi^- + n\gamma$ occur alongside $e^+e^- \rightarrow \pi^+\pi^-$.
Experiments often rely on Monte Carlo to handle additional photons.
 \implies Issues with these Monte Carlos mean old data are defective?

- BaBar study of ISR in $\pi^+\pi^-$:
 - PHOKHARA generator potentially overestimates NLO.
 - Potentially significant NNLO contributions.
 - \implies Inaccuracy of BESIII and KLOE?
- KLOE and BESIII studies find much better agreement.
- Detector effects studies ongoing – some systematics potentially underestimated but nowhere near sufficiently significant to explain differences.
- RadioMonteCarLow2 – improving Monte Carlos for next generation experiments.

Conclusion: Not the full answer, but Monte Carlo upgrades important, see tomorrow...



J.P. Lees et al - arXiv:2308.05233





Proposed Solutions - Lattice Hybrids

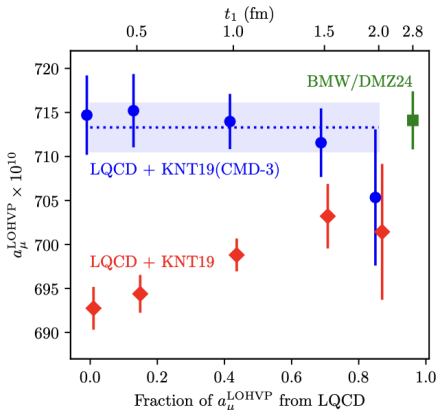
Data are \sim consistent at small \sqrt{s} .

Lattice long distance windows have relatively large uncertainties.

\implies Supplement long distance lattice with low energy dispersive?

- Largely good idea - hybrid likely for next WP to maximise precision.
- Already exists: BMWc-DMZ hybrid with switch at 2.8 fm.
- **However:**
 - Does not touch root cause of tensions.
 - Value highly dependent on lattice/dispersive switchover length.
 - KNT19/CMD-3 tension is still relatively large (2.8σ at 2.8 fm).
 - Still greatly desirable to have two independent theory methods.
- Former - see effects of varying data input right.

Conclusion: not yet viable.





Proposed Solutions - Lattice Hybrids

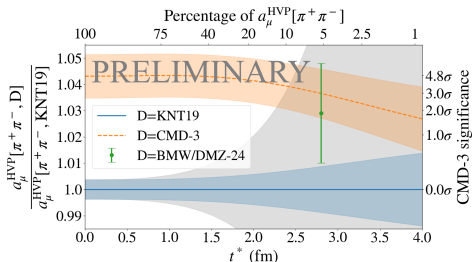
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CMD-3 obtained by substituting into KNT19 where available.
BMW/DMZ-24 est. by subtraction of KNT19 non- 2π in window.
BMW/DMZ-24 from [D. Giusti](#) – FCCP25; The HVP contribution to the muon $g - 2$ from the lattice.



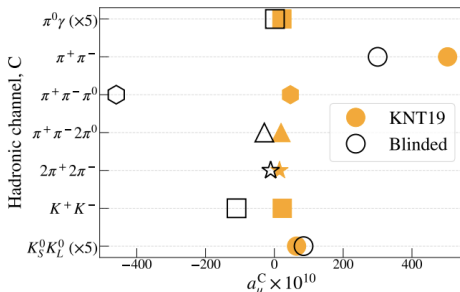
KNTW - Blinding and New Analysis

- Choices during combination:
 - Radiative correction routines;
 - Re-binning procedure;
 - Fitting procedure - correlations;
 - Additional constraints;
 - Error inflation;
 - Interpolation/integration;
 - Additional systematic uncertainties...
- KNTW-DHMZ 1σ difference - these clearly influence the central value!

- Need to mitigate and/or quantify these effects \implies KNTW new analysis.
- Want to avoid biases in updated procedure \implies KNTW **blinded** analysis.

$$a_{\mu}^{\text{blind}}[i] = \frac{1}{4\pi^3} \int_{s_{\text{th}}}^{\infty} ds \left\{ \sigma_i^0(s) K_{\mu}(s) B_i(s) \right\}$$

	DHMZ19	KNT19	Difference
$\pi^+ \pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+ \pi^- \pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+ \pi^- \pi^+ \pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+ \pi^- \pi^0 \pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
$K^+ K^-$	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0 \gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, ∞] GeV	17.15(31)	16.95(19)	0.20
Total $a_{\mu}^{\text{HVP, LO}}$	694.0(1.0)(3.5)(1.6)(0.1) _{(0.7)_{DV+QCD}}	692.8(2.4)	1.2



Theory Initiative White Paper 2020

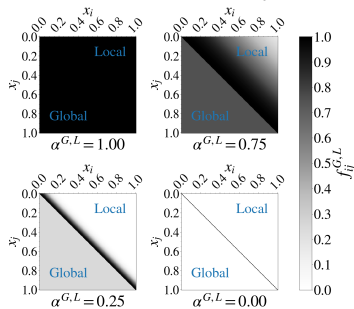
KNTW - arXiv:2409.02827, Phys. Rev. D **111**, L011901



KNTW Correlations Study: Motivation & Setup

- KNTW-DHMZ 1σ attributed to *systematic* correlation use:
 - **KNTW**: systematics are typically normalisations. Correlating fully also most conservative.
 - **DHMZ**: systematic correlations should only be able to influence fit values locally.
- WP20 'KLOE-BaBar systematic' ad-hoc description of:
 - BaBar and KLOE,
 - DHMZ19 and KNT19,
 - Correlated and uncorrelated

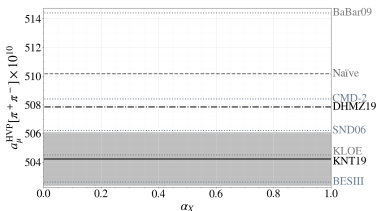
- **KNTW correlations study**: quantify impact of correlations on σ_{had} spectra.
- Use this to quantify an uncertainty.
- Quantify most deviating **spectrum** using measure \mathcal{M} ; adjust correlation through decorrelation procedures $f_{ij}^{G,L}$:



more detail: *Muon $g - 2$: correlation-induced uncertainties in precision data combinations,*

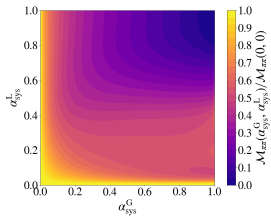
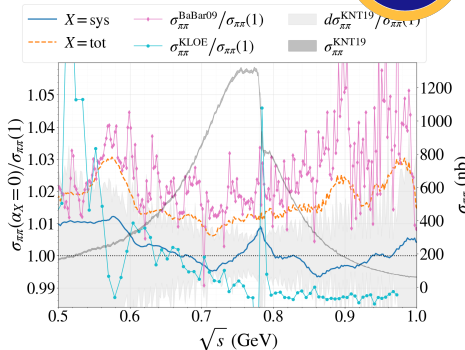
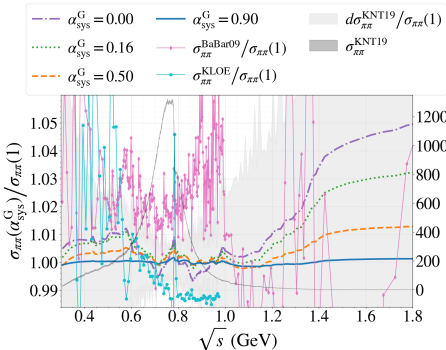
[KNTW arXiv:2604.25004](https://arxiv.org/abs/2604.25004)

result differences.





KNTW Correlations Study: Results & Implications



- max \mathcal{M} (typically) at full decorrelation [left plots, $\pi^+\pi^-$].
- Typically coincides with maximised integrated observable changes, $d^{\rho} a_{\mu}^{\text{HVP}} = 1.08 \times 10^{-10}$.
- Uncertainty \ll KLOE-BaBar, DHMZ/KNTW difference. Could statistics play a role?
- Significant mid-range BaBar statistics permit lower-lying fit. **If they are neglected, higher fit values are possible.**



Conclusions

- Significant tensions remain within dispersive HVP evaluations.
- No (complete) explanation has yet been provided.
- The KNTW new analysis will attempt to accommodate these tensions but is **blinded** and ongoing - so no real results yet!
- Historic (but not current) tensions understood using new procedure.
- The full implications of the result of the FNAL $g - 2$ experiment cannot be known until the dispersive result is known.

