Novel Azimuthal Observables from Two-Photon Collision at e^+e^- Colliders

Yajin Zhou (Shandong University)
周雅瑾（山东大学）

Based on: *Phys.Rev.Lett.* 134 (2025), Yu Jia, Jian Zhou, YZ

2026.06.8-11

PhiPsi26@Pisa, Italy

Outline

- Introduction

  Equivalent photon approximation, linearly polarized gluon/photon, TMD factorization…

- Azimuthal modulations in $\gamma\gamma \rightarrow \pi\pi$ at e^+e^- colliders

  1. In chiral perturbative theory
  2. Data-driven method

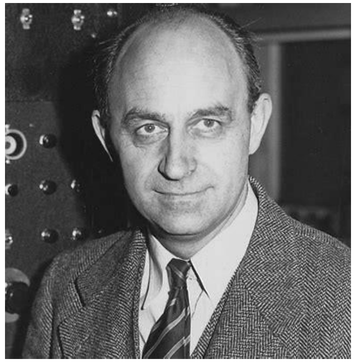
- summary

Equivalent photon approximation (EPA)

Over 100 years

1. *1924, Fermi; Z.Phys.* 29 (1924) 315-327

"...consider that when a charged particle passes near a point, ... it is equivalent to the electric field at the same point if it were struck by light with an appropriate continuous distribution of frequencies."



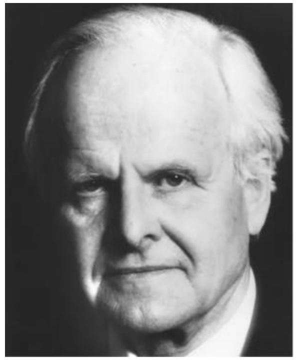
Nuovo Cim. 2 (1925) 143-158, *arXiv:hep-th/0205086*

"one of Fermi's favorite ideas and that he often used it in later life."

2. *1930's, Weizäscker and Williams (individually);*

developed to relativistic charged particles,
method of virtual quanta.
EM field almost transvers, quasi-real photon

photon flux: $$n(\omega) = \frac{4Z^2\alpha_e}{\omega}\int \frac{d^2k_\perp}{(2\pi)^2}k_\perp^2\left[\frac{F(k_\perp^2+\omega^2/\gamma^2)}{(k_\perp^2+\omega^2/\gamma^2)}\right]^2$$



Linearly polarized photon

Low electron energy, Sommerfeld, 1931

relativistic energies, May, Wick, 1951

ANNALEN DER PHYSIK

5. FOLGE, 1931, BAND 11, HEFT 3

Über die Beugung und Bremsung der Elektronen

Von A. Sommerfeld

(Mit 12 Figuren)

Übersicht: Der I. Teil bildet eine systematische Einleitung zu der Behandlung des kontinuierlichen Röntgenspektrums im II. Teil. Der I. Teil geht nur in der Methode, nicht in den Resultaten über die Arbeiten von Gordon, Mott, Temple hinaus. Der II. Teil setzt, im Gegensatz zu Arbeiten von Oppenheimer und Sugiura den Endzustand des gebremsten Elektrons als ebene, durch Beugung modifizierte Welle an. Polarisation und Intensität im kontinuierlichen Spektrum werden nach der Methode der Matrixelemente berechnet. Um die azimutale Verteilung der Intensität, insbesondere die Voreilung des Maximums zu erhalten, muß die Rechenmethode verfeinert werden durch Berücksichtigung der Retardierung. Die Resultate werden mit Messungen von Kulenkampff verglichen.

Detection of Gamma-Ray Polarization by Pair Production*

G. C. Wick
Radiation Laboratory, University of Californ
December 12, 1950

IT has been pointed out by Yang,[1] th
provide a method for detecting the p
the high energy range: $h\nu \gg mc^2$ (*m* be
where the usual Compton recoil method b
idea is to utilize the azimuthal dependenc
cross section $d\sigma$, the azimuth ϕ being
direction **k** of the incident quantum an
taining **k** and the electric polarization ve

On the Production of Polarized High Energy X-Rays

M. May and G. C. Wick
Department of Physics, University of California, Berkeley, California
December 12, 1950

THE purpose of this note is to examine the possibility of experiments with *polarized* x-rays of high energy, the polarization being obtained by using only a portion of the x-ray beam emitted (by a betatron, synchrotron, or linear-accelerator target) at an angle θ to the direction of the electron beam. The optimum angle,

PHYSICAL REVIEW VOLUME 84, NUMBER 2 OCTOBER 15, 1951

On the Polarization of High Energy Bremsstrahlung and of High Energy Pairs

Michael M. May
Physics Department, University of California, Berkeley, California
(Received July 2, 1951)

The polarization of bremsstrahlung due to electrons with initial energies much larger than $137Z^{-\frac{1}{3}}\ mc^2$ is calculated under relativistic, small angles approximations. The cross section for photons polarized normally to the plane containing the initial direction of the electron and the direction of the photon is found to be larger than for photons polarized in that plane. A similar calculation shows that the plane containing one of a pair produced by a polarized photon together with the direction of that photon tends to lie parallel to the plane of polarization rather than normal to it, except for one special case. The effect of the deviation due to multiple scattering of electrons in the target upon the angular dependence of the polarization is considered.

Linearly polarized photon

- In EPA (Equivalent photon approximation), photons induced by relativistically moving charged particles are linearly polarized due to their transverse momentum

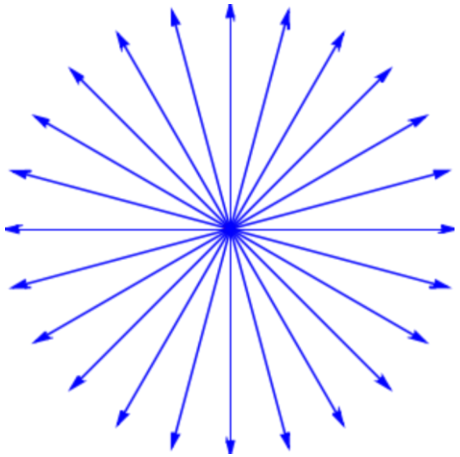
In position space

static electric field

Lorentz boost

side view

beam view



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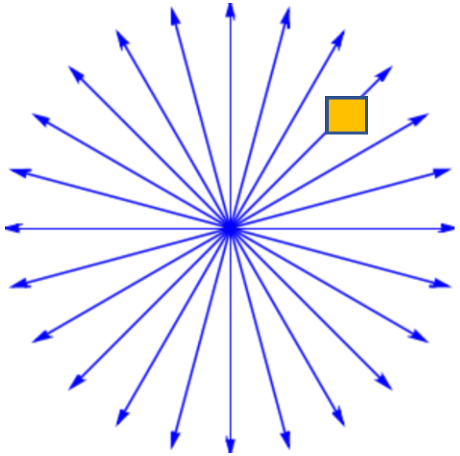

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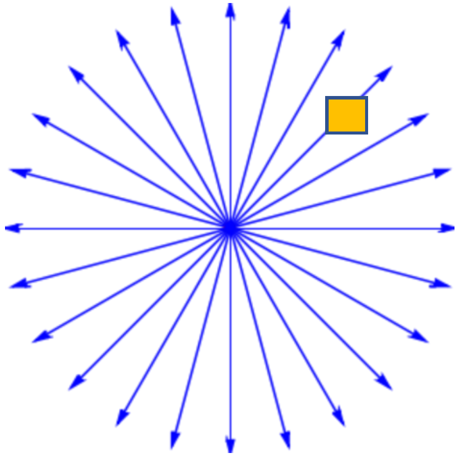

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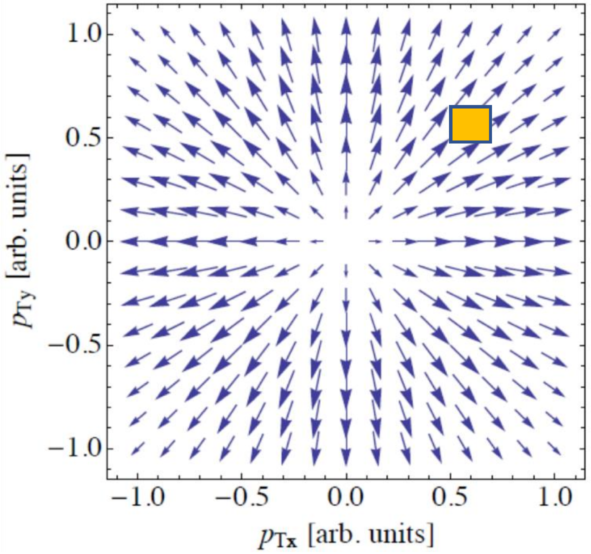
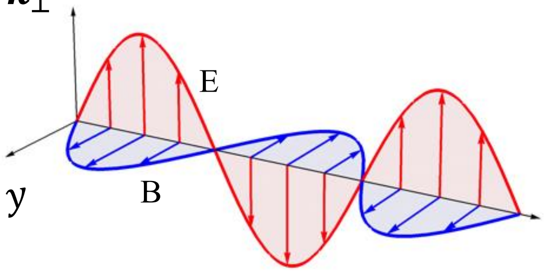




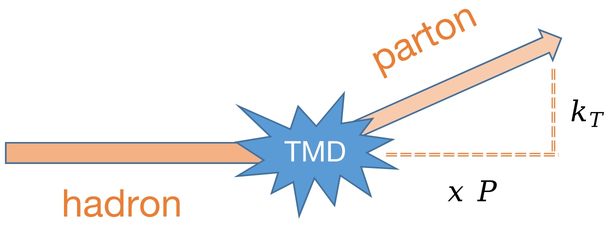
$\epsilon_{\perp} // \boldsymbol{k}_{\perp}$

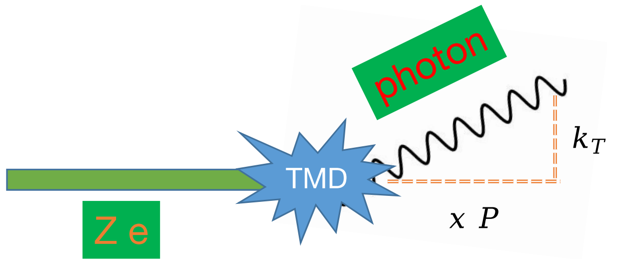
Linearly polarized

Linearly polarized photon/gluon, formalism in the framework of TMD (transverse-momentum-dependent) factorization

EPA photons induced by relativistic charged particles are linearly polarized due to transverse momentum

- gluon/photon TMD factorization:





$$\int \frac{2dy^- d^2y_\perp}{xP^+(2\pi)^3} e^{ik\cdot y} \langle P|F_+^\mu(0)F_+^\nu(y)|P\rangle\Big|_{y^+=0} = \delta_\perp^{\mu\nu} f_1(x,k_\perp^2) + \left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \delta_\perp^{\mu\nu}\right) h_1^\perp(x,k_\perp^2),$$

P. Mulders, J. Rodrigues, PRD63(2001)

> the numbers of gluons with opposite circular polarizations in a longitudinally (transversely) polarized nucleon. The off-diagonal function $H^\perp$ also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, $H^\perp$ flips the polarization.

- Small x (dipole) gluons/photons are highly linearly polarized

A. Metz, J. Zhou, PRD84(2011)

C. Li, J. Zhou, YZ, PLB795(2019)

$$f_1(x,k_\perp^2) = h_1^\perp(x,k_\perp^2)$$

Linearly polarized photon/gluon, formalism in the framework of TMD (transverse-momentum-dependent) factorization

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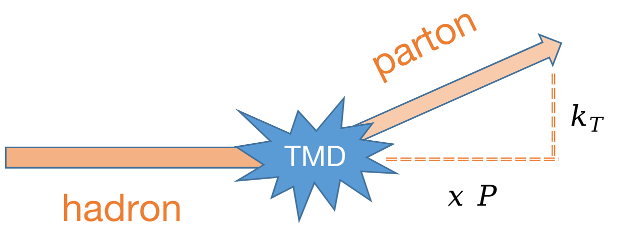

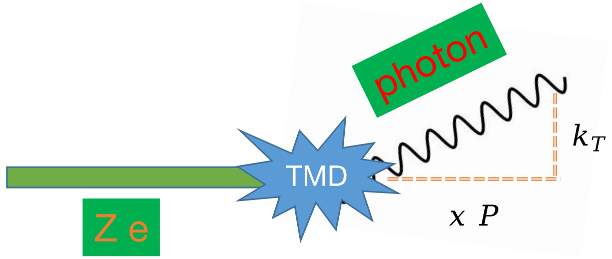

$$\int \frac{2dy^- d^2 y_\perp}{xP^+(2\pi)^3} e^{ik\cdot y} \langle P|F_+^\mu(0) F_+^\nu(y)|P\rangle\Big|_{y^+=0} = \delta_\perp^{\mu\nu} f_1(x, k_\perp^2) + \left(\frac{2k_\perp^\mu k_\perp^\nu}{k_\perp^2} - \delta_\perp^{\mu\nu}\right) h_1^\perp(x, k_\perp^2),$$

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A. Metz, J. Zhou, PRD84(2011)

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$$f_1(x, k_\perp^2) = h_1^\perp(x, k_\perp^2)$$

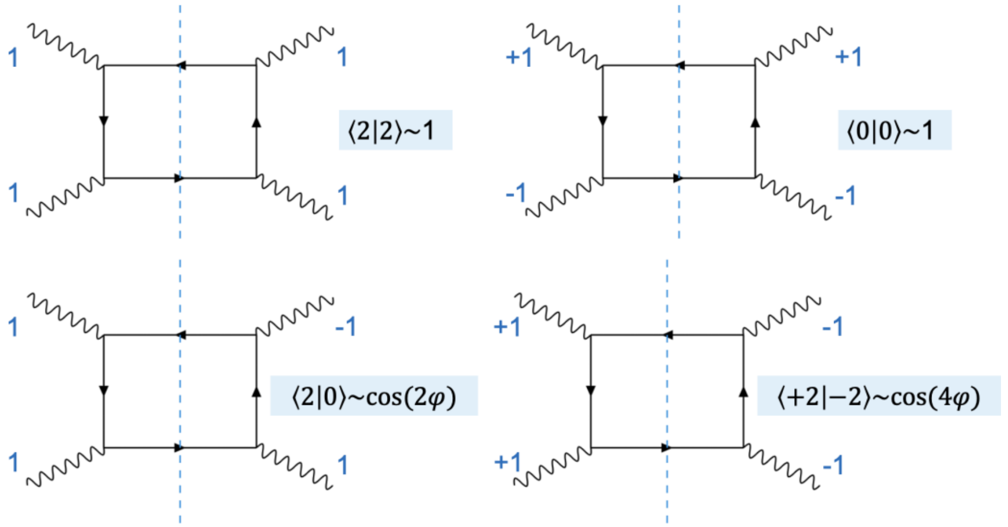
Collinear factorization: $\sigma \sim$ PDFs(x) $\otimes$ hard part
TMD factorization: $\sigma \sim$ TMDs(x,kt) $\otimes$ hard part

Dilepton production in Ultraperipheral collision (UPC)

$$\gamma(x_1 P + k_{1\perp}) + \gamma(x_2 \bar{P} + k_{2\perp}) \rightarrow l^+(p_1) + l^-(p_2)$$

$$\frac{d\sigma}{d^2\mathbf{p}_{1\perp} d^2\mathbf{p}_{2\perp} dy_1 dy_2} = \frac{2\alpha_e^2}{Q^4}\left[\mathcal{A} + \mathcal{B}\cos 2\phi + \mathcal{C}\cos 4\phi\right]$$

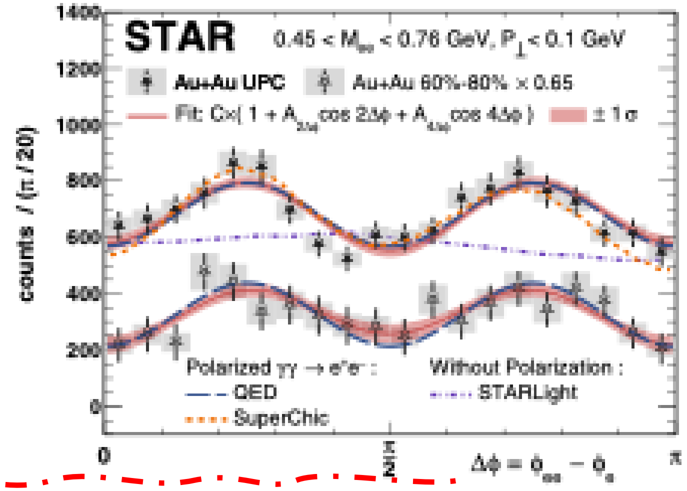
$$f_1^{\gamma}(x_1, k_{1\perp}^2) h_1^{\perp\gamma}(x_2, k_{2\perp}^2) \qquad h_1^{\perp\gamma}(x_1, k_{1\perp}^2) h_1^{\perp\gamma}(x_2, k_{2\perp}^2)$$



	Measured	QED calculation
Tagged UPC	16.8%±2.5%	16.5%
60%-80%	27%±6%	34.5%

C. Li, J. Zhou and YZ, PLB795, 576(2019)

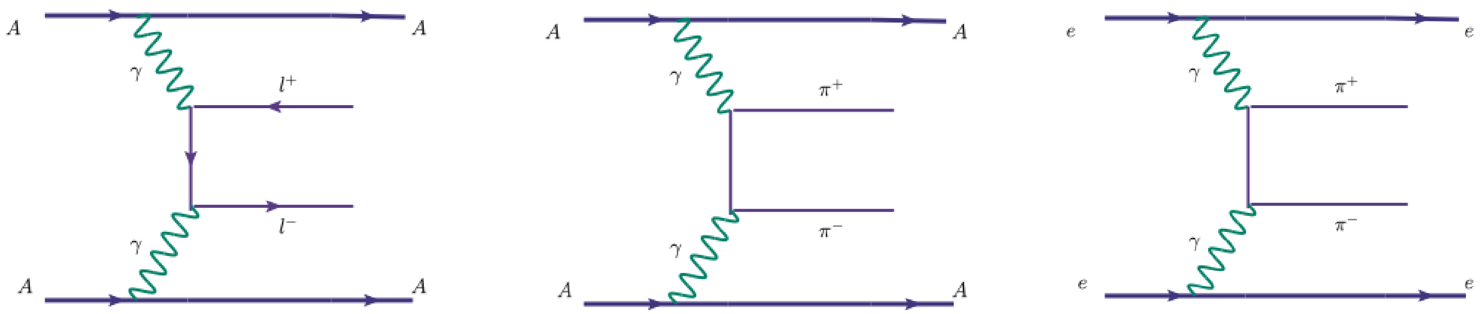
C. Li, J. Zhou and YZ, PRD101, 034015(2020)



STAR collaboration, PRL127, 052302 (2021)

We predicted the azimuthal asymmetries in dilepton@UPC, later verified by STAR

Why azimuthal physics is "new" in e⁺e⁻ two-photon



similar to $\gamma\gamma \to l^+l^-$, we can also study $\gamma\gamma \to M\bar{M}$ process, either in UPCs or at e^+e^- colliders, more complicated.

- Standard two-photon analyses: EPA + QED collinear factorization for $\gamma\gamma \to M\bar{M}$
- Classic observables: invariant mass / rapidity distributions of the meson pair
- Realistic e^+e^- kinematics: mesons are nearly but not exactly back-to-back in the transverse plane → photon $k_\perp$ should not be neglected in the correlation limit ($p_{M\bar{M}} \ll p_M$), TMD factorization
- Then linear polarization of the photon becomes a first-class effect → azimuthal modulations in the lab.

Y. Jia, J. Zhou and YZ, PRL.134(2025)141901

measurements

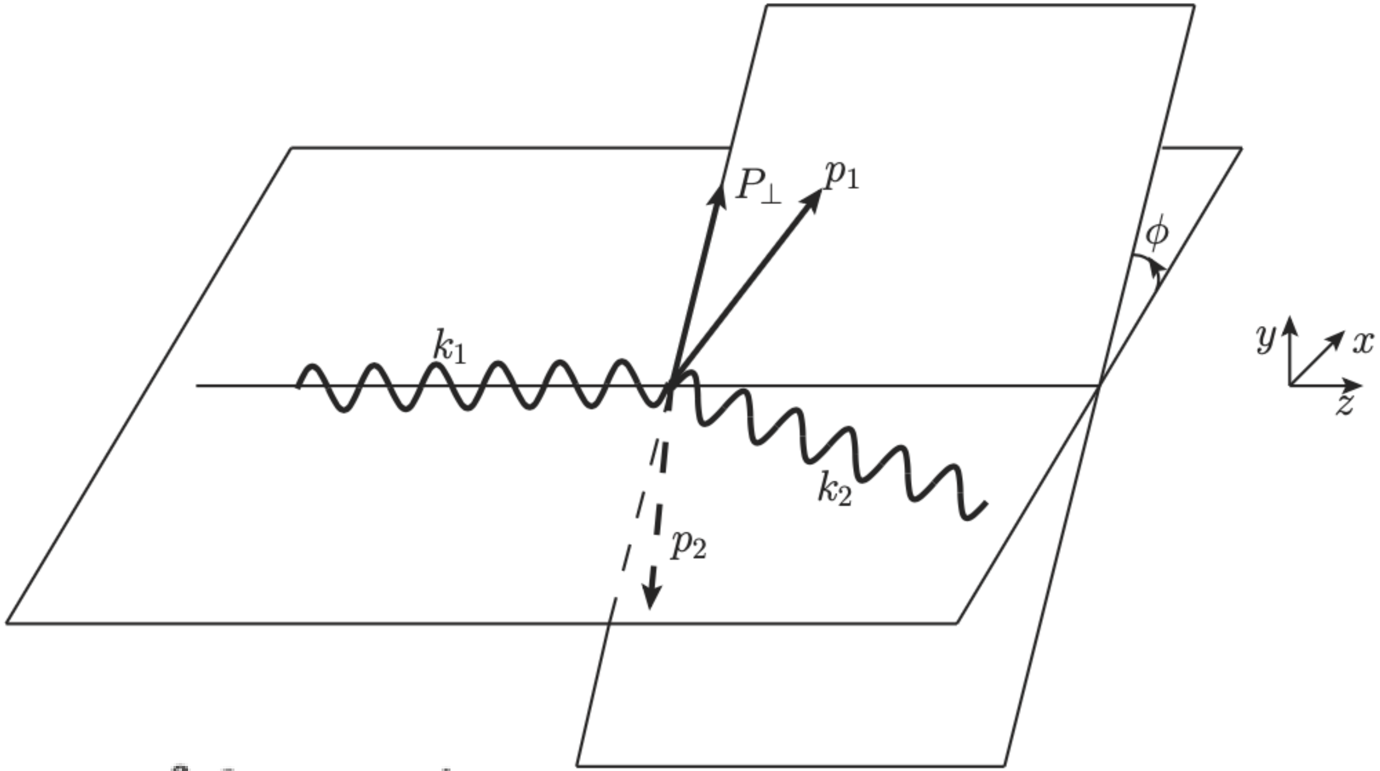
$$\gamma(k_1) + \gamma(k_2) \to \pi^+(p_1) + \pi^-(p_2)$$

Correlation limit: $|\mathbf{q}_\perp| \ll |\mathbf{P}_\perp|,$

$$\vec{P}_\perp = \frac{\vec{p}_{1\perp} - \vec{p}_{2\perp}}{2} \simeq \vec{p}_{1\perp} \simeq -\vec{p}_{2\perp}$$

$$\vec{q}_\perp = \vec{k}_{1\perp} + \vec{k}_{2\perp} = \vec{p}_{1\perp} + \vec{p}_{2\perp}$$

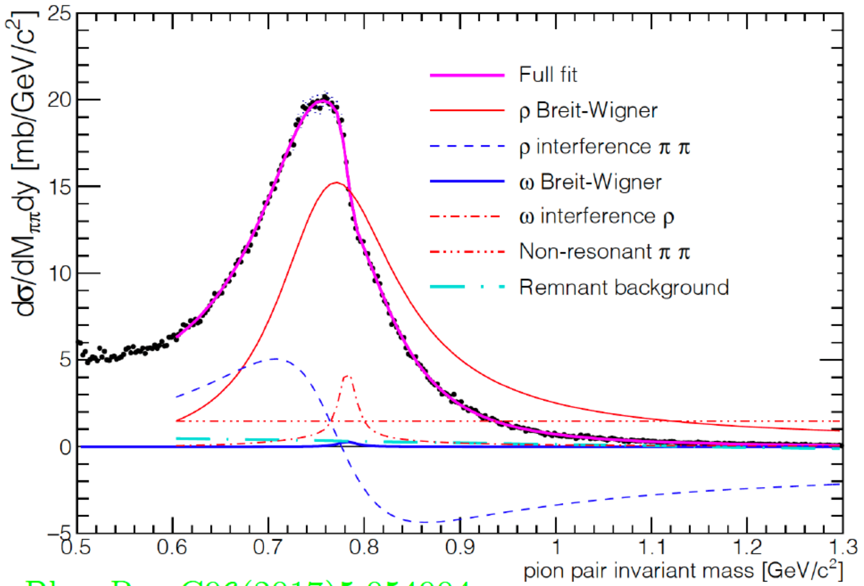
$$\phi = \vec{P}_\perp \wedge \vec{q}_\perp$$



$$\langle \cos(n\phi) \rangle \equiv \frac{\int d\sigma \cos n\phi}{\int d\sigma},$$

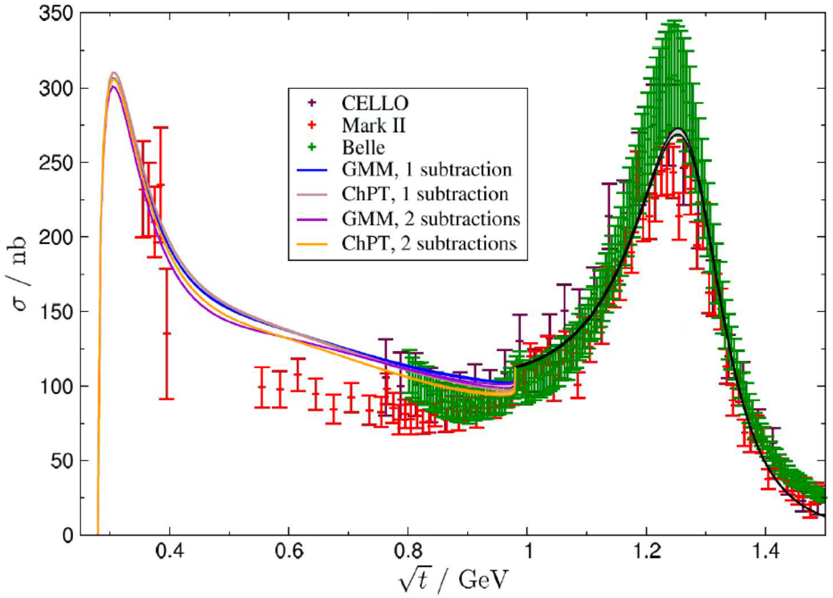
pion pair production, e^+e^- collider vs. UPC

UPC : only low invariant mass is possible



STAR, Phys.Rev.C96(2017)5,054904

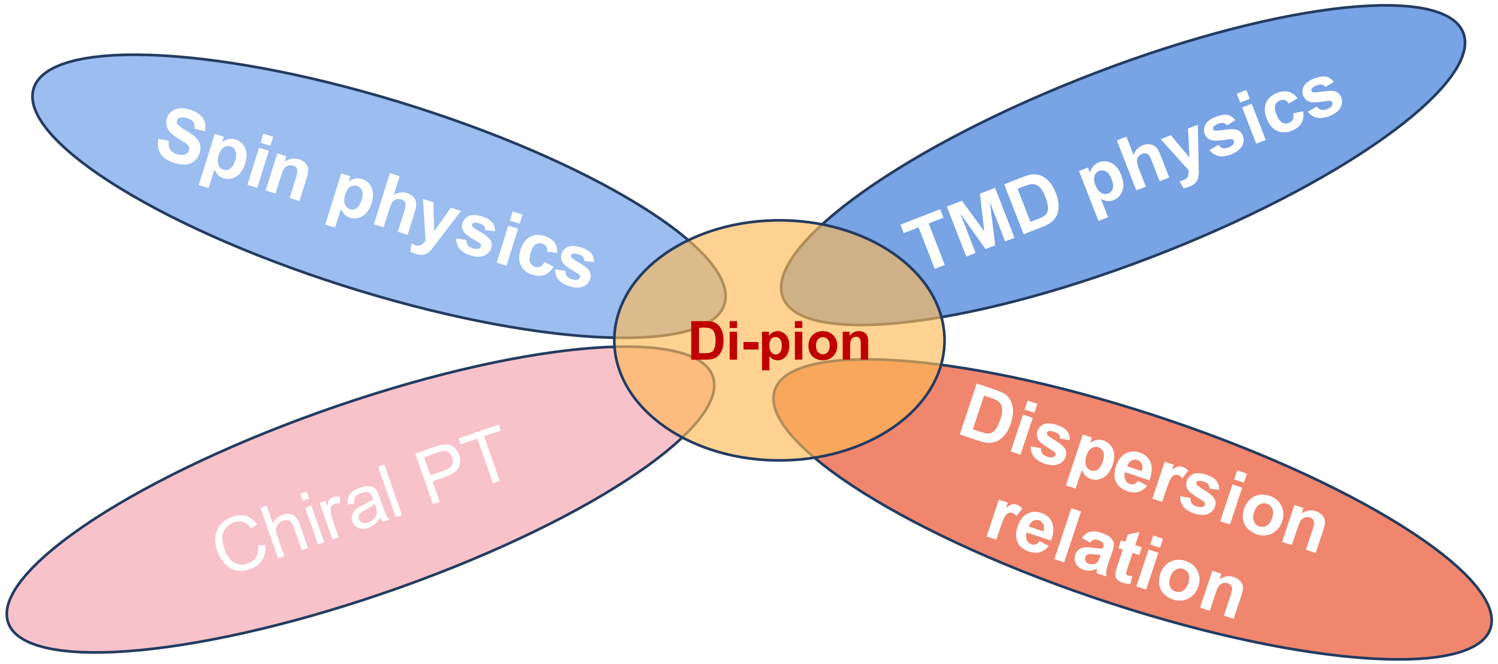
e^+e^- collider, not effected by ρ resonance



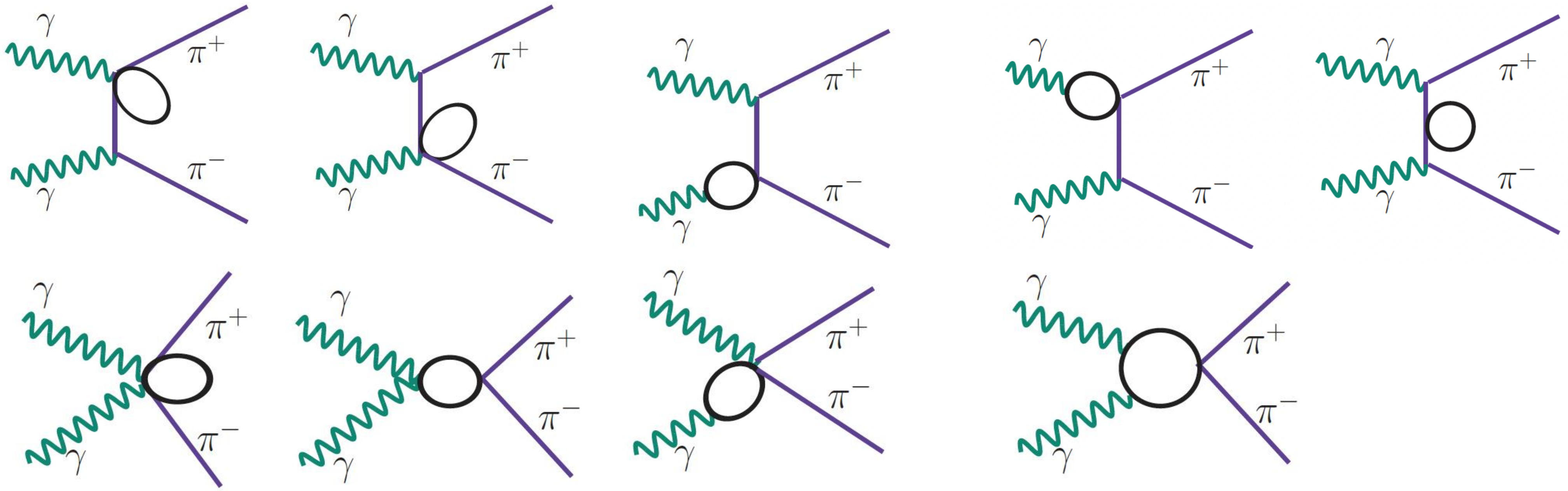
Conclusion: to study the polarization effect of poin pair production from $\gamma\gamma$ fusion,

e^+e^- collider is better

Theoretical setup



$\gamma\gamma \to \pi^+\pi^-$ in Chiral perturbative theory (χPT) at one loop

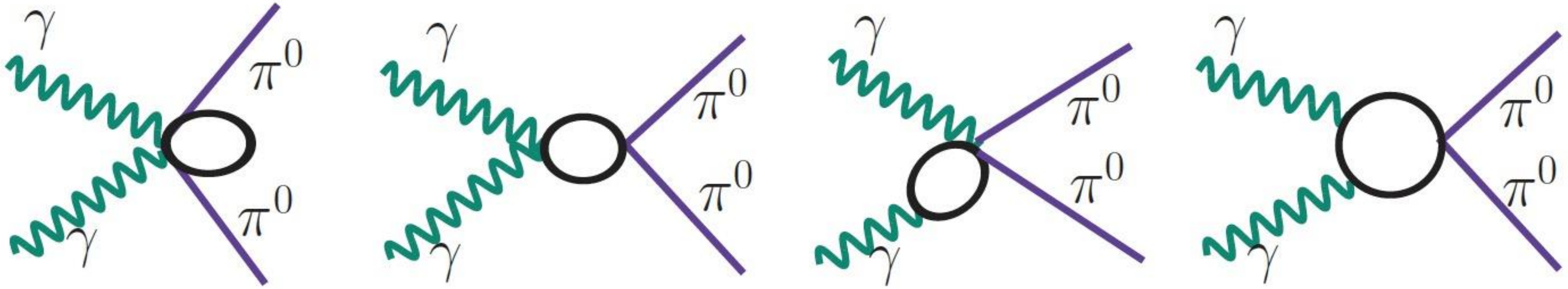


$$\mathcal{M}(\gamma\gamma \to \pi^+\pi^-) = 2ie^2 \left[\mathcal{C}\epsilon(k_1)\cdot\epsilon(k_2) - \frac{2P_\perp^2}{P_\perp^2 + m_\pi^2}(\epsilon(k_1)\cdot\hat{P}_\perp)(\epsilon(k_2)\cdot\hat{P}_\perp) \right],$$

$$\mathcal{C} = 1 + \frac{4Q^2}{f_\pi^2}(L_9^r + L_{10}^r) - \frac{1}{16\pi^2 f_\pi^2}\left(\frac{3}{2}Q^2 + m_\pi^2 \ln^2 g_\pi(Q^2) + \frac{1}{2}m_K^2 \ln^2 g_K(Q^2)\right)$$

J.BIJNENS and F. CORNET, NPB296(1988)557-568

$\gamma\gamma \to \pi^0\pi^0$ in χPT at one loop

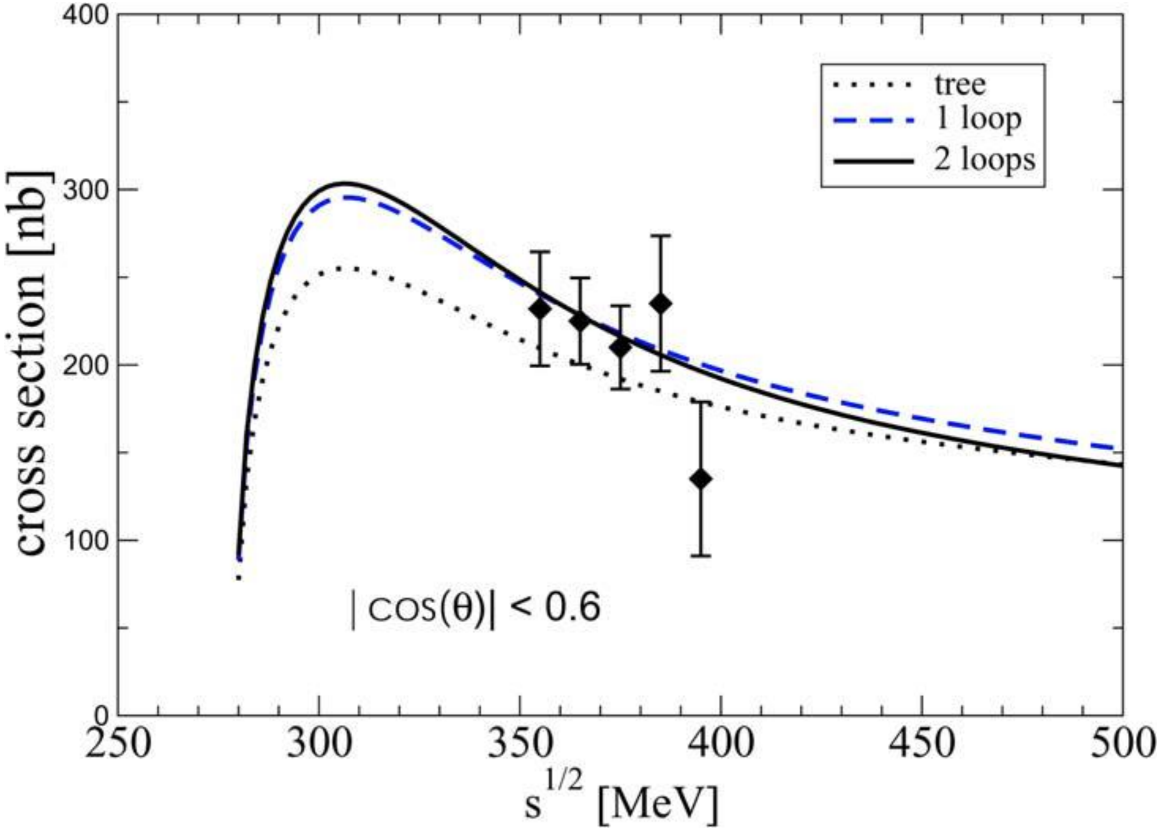


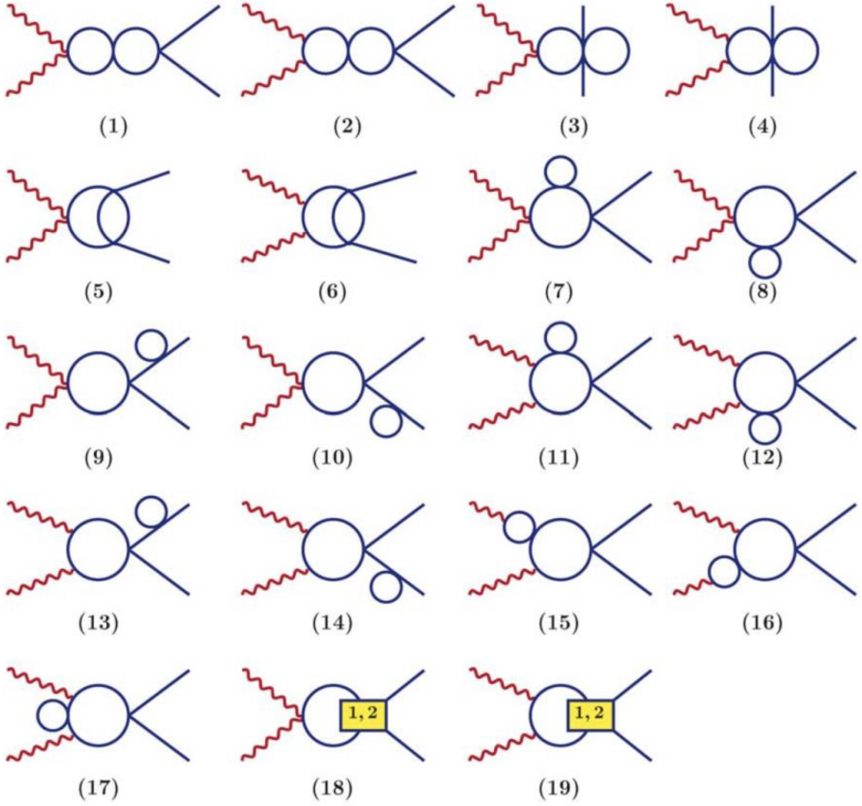
$$\mathcal{M}(\gamma\gamma \to \pi^0\pi^0) = i\mathcal{D}4e^2\epsilon(k_1)\cdot\epsilon(k_2),$$

$$\mathcal{D} = \frac{Q^2}{16\pi^2 f_\pi^2}\left[\left(1-\frac{m_\pi^2}{Q^2}\right)\left(1+\frac{m_\pi^2}{Q^2}\ln^2 g_\pi(Q^2)\right)-\frac{1}{4}\left(1+\frac{m_K^2}{Q^2}\ln^2 g_K(Q^2)\right)\right]$$

J.BIJNENS and F. CORNET, NPB296(1988)557-568

Two-loop result for $\gamma\gamma \to \pi^+\pi^-$ in χPT





One-loop is good enough for our calculation

J. Gasser, M.A. Ivanov, M.E. Sainio, Nucl. Phys. B745 (2006)

$\gamma\gamma \to \pi^+\pi^-$ at e^+e^- collider, 1-loop, χPT

- cross section with linearly polarized photon:

$$
\begin{aligned}
\frac{d\sigma_{e^+e^-\to\gamma\gamma\to\pi^+\pi^-}}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2} &= \frac{4\alpha_e^2}{Q^4}\int d^2k_{1\perp}d^2k_{2\perp}\delta^2(q_\perp - k_{1\perp} - k_{2\perp})x_1x_2 \\
&\times\Bigg\{\left|\mathcal{C}\,\hat{k}_{1\perp}\cdot\hat{k}_{2\perp} - \frac{2P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{1\perp}\cdot\hat{P}_\perp)(\hat{k}_{2\perp}\cdot\hat{P}_\perp)\right|^2 h_1^\perp(x_1,k_{1\perp}^2)\,h_1^\perp(x_2,k_{2\perp}^2) \\
&+\left[\frac{1}{2}|\mathcal{C}|^2 + \frac{2P_\perp^4(\hat{k}_{1\perp}\cdot\hat{P}_\perp)^2}{(P_\perp^2+m_\pi^2)^2} - \frac{(\mathcal{C}+\mathcal{C}^*)P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{1\perp}\cdot\hat{P}_\perp)^2\right]h_1^\perp(x_1,k_{1\perp}^2)\left(f(x_2,k_{2\perp}^2) - h_1^\perp(x_2,k_{2\perp}^2)\right) \\
&+\left[\frac{1}{2}|\mathcal{C}|^2 + \frac{2P_\perp^4(\hat{k}_{2\perp}\cdot\hat{P}_\perp)^2}{(P_\perp^2+m_\pi^2)^2} - \frac{(\mathcal{C}+\mathcal{C}^*)P_\perp^2}{P_\perp^2+m_\pi^2}(\hat{k}_{2\perp}\cdot\hat{P}_\perp)^2\right]h_1^\perp(x_2,k_{2\perp}^2)\left(f(x_1,k_{1\perp}^2) - h_1^\perp(x_1,k_{1\perp}^2)\right) \\
&+\left[\frac{P_\perp^4}{(P_\perp^2+m_\pi^2)^2} + \frac{1}{2}|\mathcal{C}|^2 - \frac{(\mathcal{C}+\mathcal{C}^*)P_\perp^2}{2(P_\perp^2+m_\pi^2)}\right]\left(f(x_1,k_{1\perp}^2) - h_1^\perp(x_1,k_{1\perp}^2)\right)\left(f(x_2,k_{2\perp}^2) - h_1^\perp(x_2,k_{2\perp}^2)\right)\Bigg\}
\end{aligned}
$$

Y. Jia, J. Zhou and YZ,
PRL.134(2025)141901

χPT limitation: only valid at low-energy

To investigate a broader energy range, we use a data-driven approach

Dipoin production at e^+e^- collider, data-driven method

Ling-Yun Dai and M.R. Pennington, "Comprehensive Amplitude Analysis of $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ and $\overline{K}K$ below 1.5 GeV", PRD90, 036004 (2014)

$$\frac{d\sigma}{d\Omega} = \frac{\rho(s)}{128\pi^2 s}\left[|M_{+-}|^2 + |M_{++}|^2\right],$$

isospin decomposition of the amplitudes:

$$\mathcal{F}_\pi^{+-}(s) = -\sqrt{\frac{2}{3}}\mathcal{F}_\pi^{I=0}(s) - \sqrt{\frac{1}{3}}\mathcal{F}_\pi^{I=2}(s),$$

$$\mathcal{F}_\pi^{00}(s) = -\sqrt{\frac{1}{3}}\mathcal{F}_\pi^{I=0}(s) + \sqrt{\frac{2}{3}}\mathcal{F}_\pi^{I=2}(s),$$

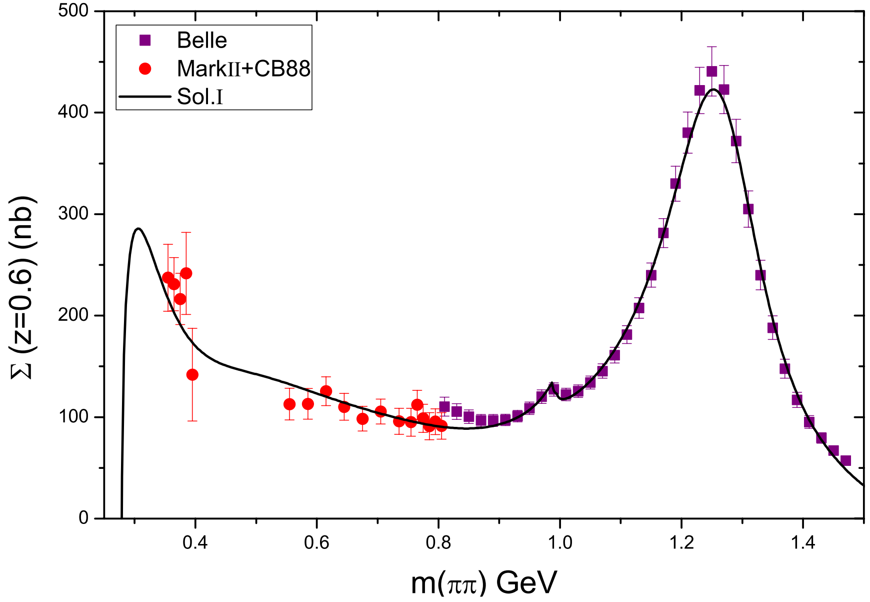
$$\mathcal{F}_K^{+-}(s) = -\sqrt{\frac{1}{2}}\mathcal{F}_K^{I=0}(s) - \sqrt{\frac{1}{2}}\mathcal{F}_K^{I=1}(s),$$

$$\mathcal{F}_K^{00}(s) = -\sqrt{\frac{1}{2}}\mathcal{F}_K^{I=0}(s) + \sqrt{\frac{1}{2}}\mathcal{F}_K^{I=1}(s).$$

partial wave expansions of the amplitudes:

$$M_{++}(s,\theta,\phi) = e^2\sqrt{16\pi}\sum_{J\geq 0} F_{J0}(s) Y_{J0}(\theta,\phi),$$

$$M_{+-}(s,\theta,\phi) = e^2\sqrt{16\pi}\sum_{J\geq 2} F_{J2}(s) Y_{J2}(\theta,\phi).$$



Dipoin production at e^+e^- collider, data-driven method

- cross section in helicity amplitude form with linearly polarized photon:

$$\frac{d\sigma}{d^2p_{1\perp}d^2p_{2\perp}dy_1dy_2} = \frac{1}{16\pi^2Q^4}\int d^2k_{1\perp}d^2k_{2\perp}$$
$$\times\ \delta^2(q_\perp - k_{1\perp} - k_{2\perp})x_1x_2$$
$$\times\ \left\{\frac{1}{2}\left(|M_{+-}|^2 + |M_{++}|^2\right) f(x_1,k_{1\perp}^2)f(x_2,k_{2\perp}^2)\right.$$
$$-\ \cos(2\phi_1)\mathrm{Re}[M_{++}M_{+-}^*]\, f(x_2,k_{2\perp}^2)h_1^\perp(x_1,k_{1\perp}^2)$$
$$-\ \cos(2\phi_2)\mathrm{Re}[M_{++}M_{+-}^*]\, f(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)$$
$$+\ \frac{1}{2}\left[\cos 2(\phi_1-\phi_2)|M_{++}|^2 + \cos 2(\phi_1+\phi_2)|M_{+-}|^2\right]$$
$$\left.\times\ h_1^\perp(x_1,k_{1\perp}^2)h_1^\perp(x_2,k_{2\perp}^2)\right\}$$

Reduce to collinear factorization after integrating over $k_{1,2\perp}$, → $\frac{1}{2}(|M_{++}|^2 + |M_{+-}|^2)$ff

Contribute to $\cos(2\phi)$ azim. Assym., induced by linearly polarized photon, Involve the relative phase between M_{++} and M_{+-}

Contribute to $\cos(4\phi)$ azim. Assym., induced by linearly polarized photon

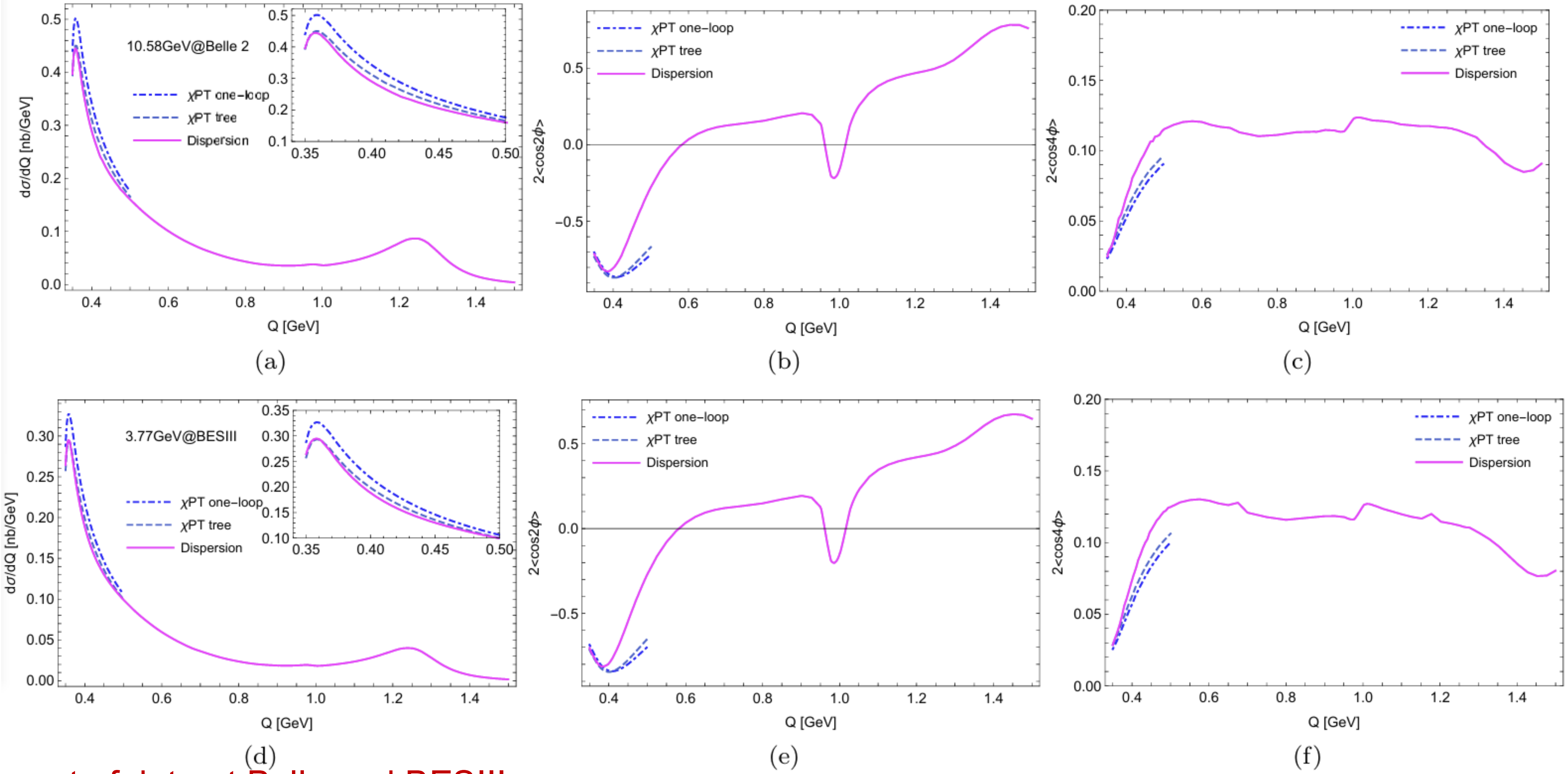
Y. Jia, J. Zhou and YZ, PRL.134(2025)141901

Numerical setup

- Center-of-mass energy: $\sqrt{s}$ = 10.58 GeV (Belle 2) and 3.77 GeV (BESIII)
- Rapidity / polar-angle coverage: $|y_1|, |y_2| \leq 0.38$ (consistent with $|\cos\theta| < 0.6$).
- Correlation cuts: $|P_\perp| > 100$ MeV; integrate $|q_\perp|$ from 0 to 50 MeV.

Numerical results, $\gamma\gamma \to \pi^+\pi^-$

Unpolarized cross section | 2<cos2ϕ> | 2<cos4ϕ>



Great amount of data at Belle and BESIII.

Belle and Belle 2, 1500 fb^{-1}, $4.5*10^7$ $\pi^+\pi^-$ events

Phenomenology impact

- Directly extract the relative phase between M_{++} and M_{+-}
- Azimuthal asymmetries magnify signals, so we can

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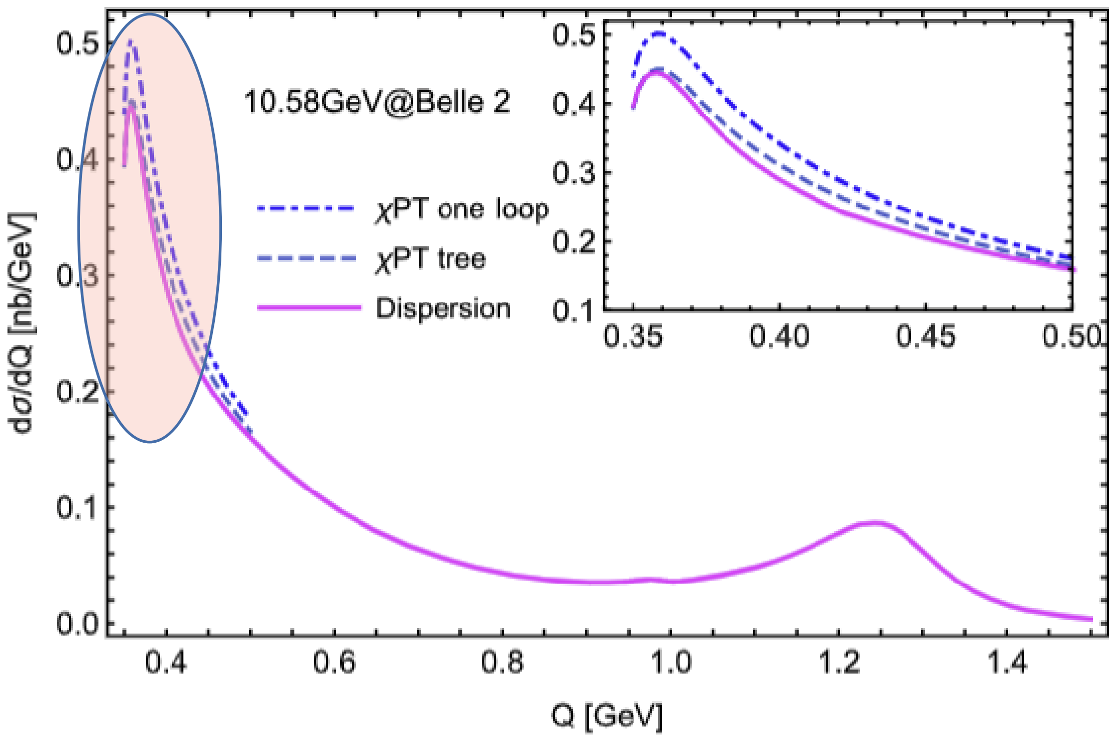

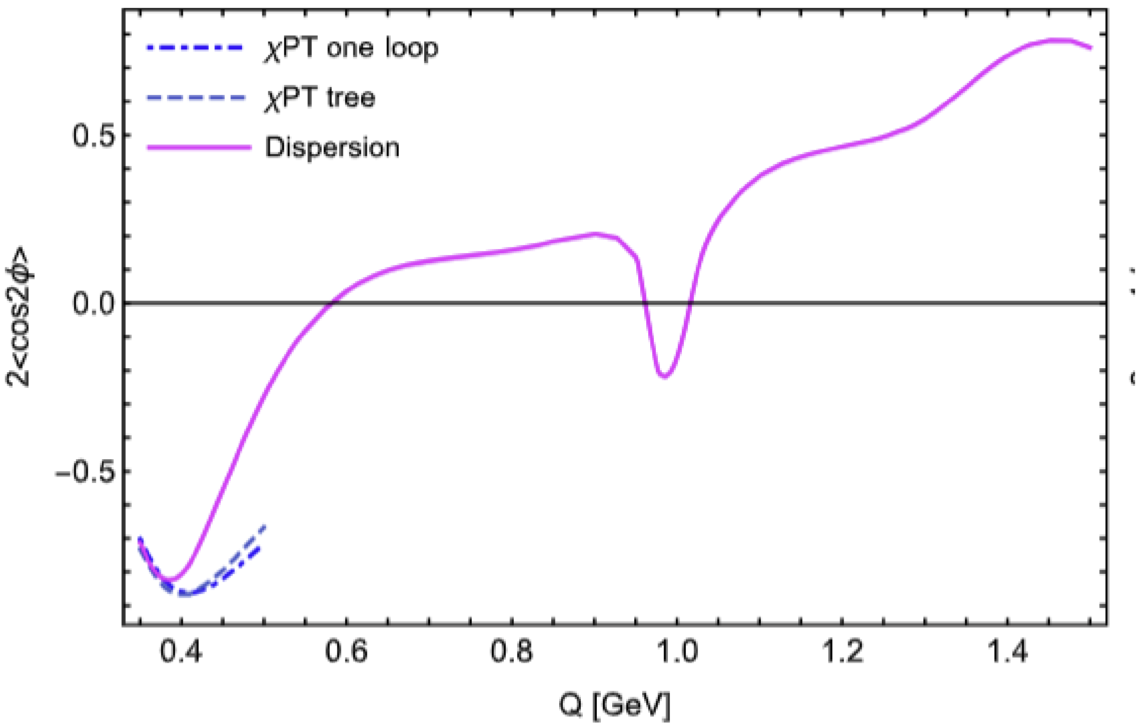

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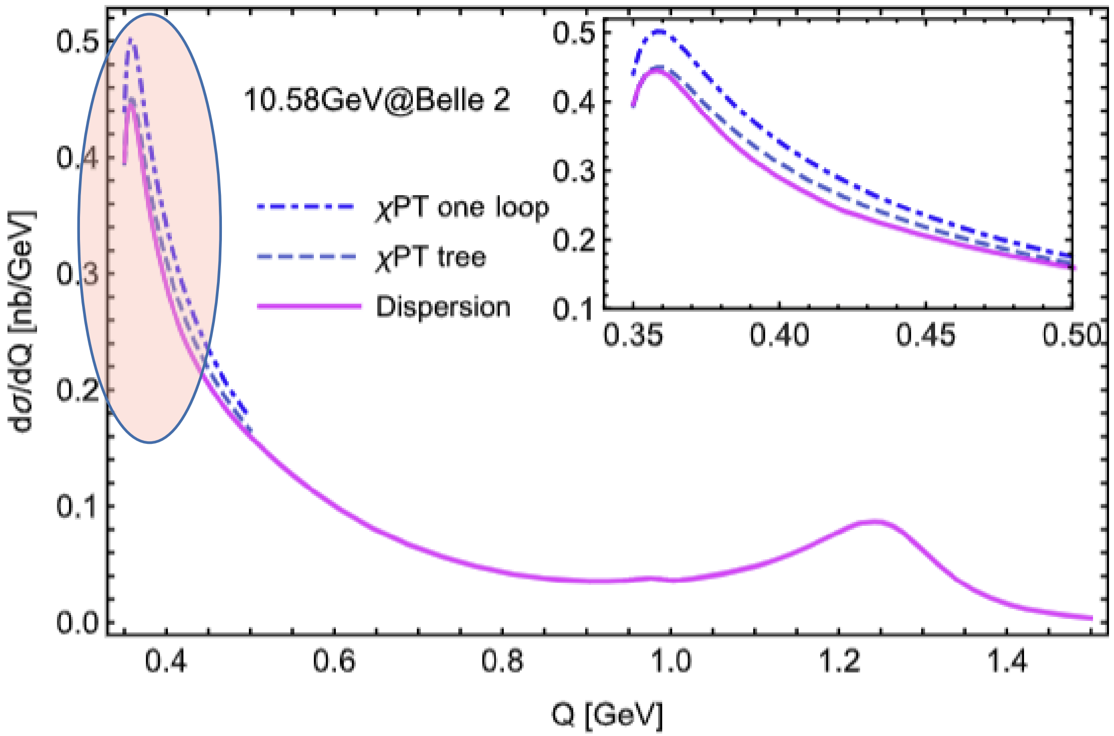

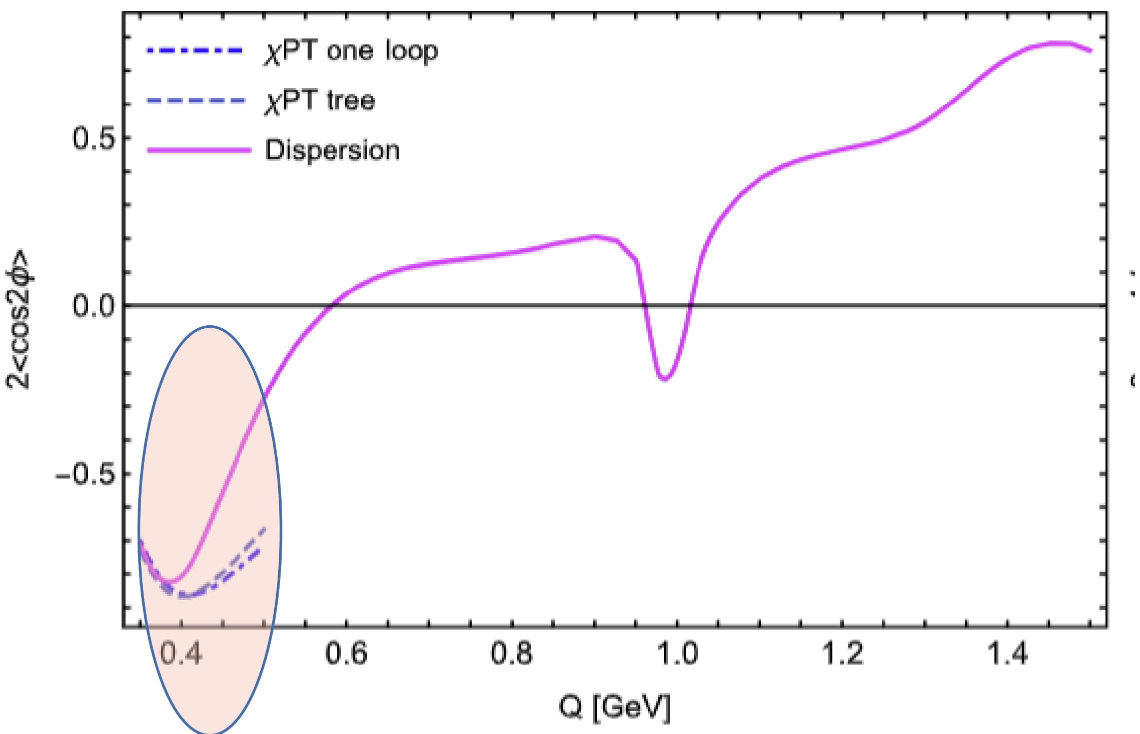

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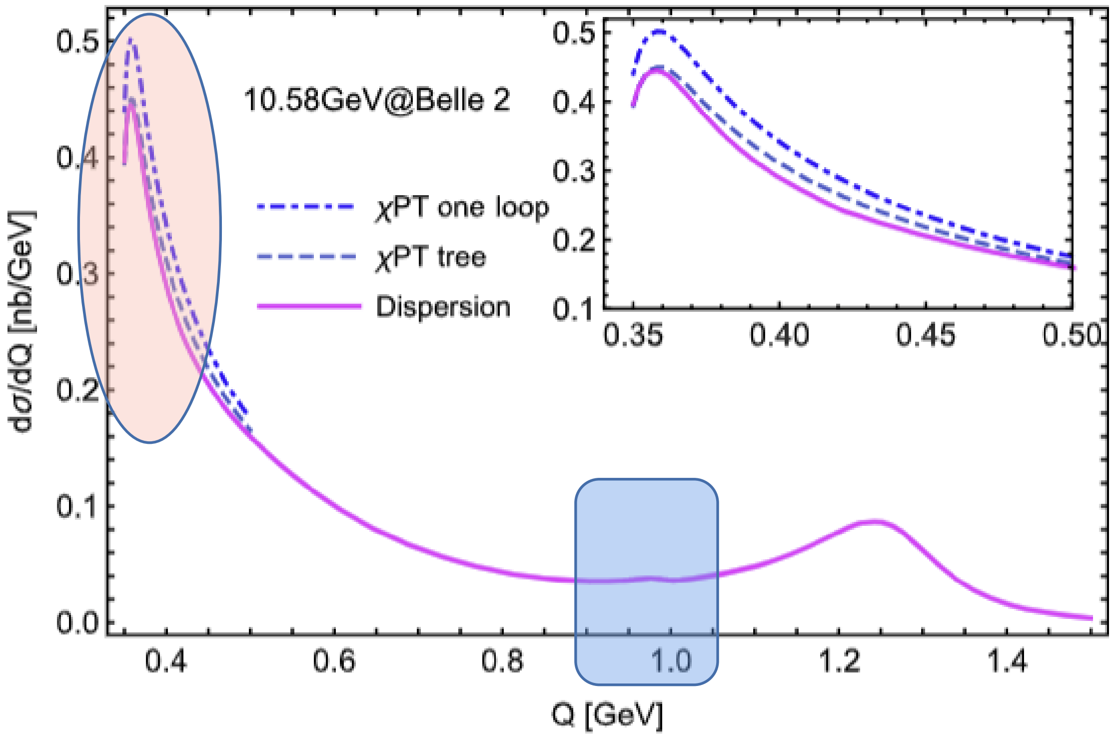

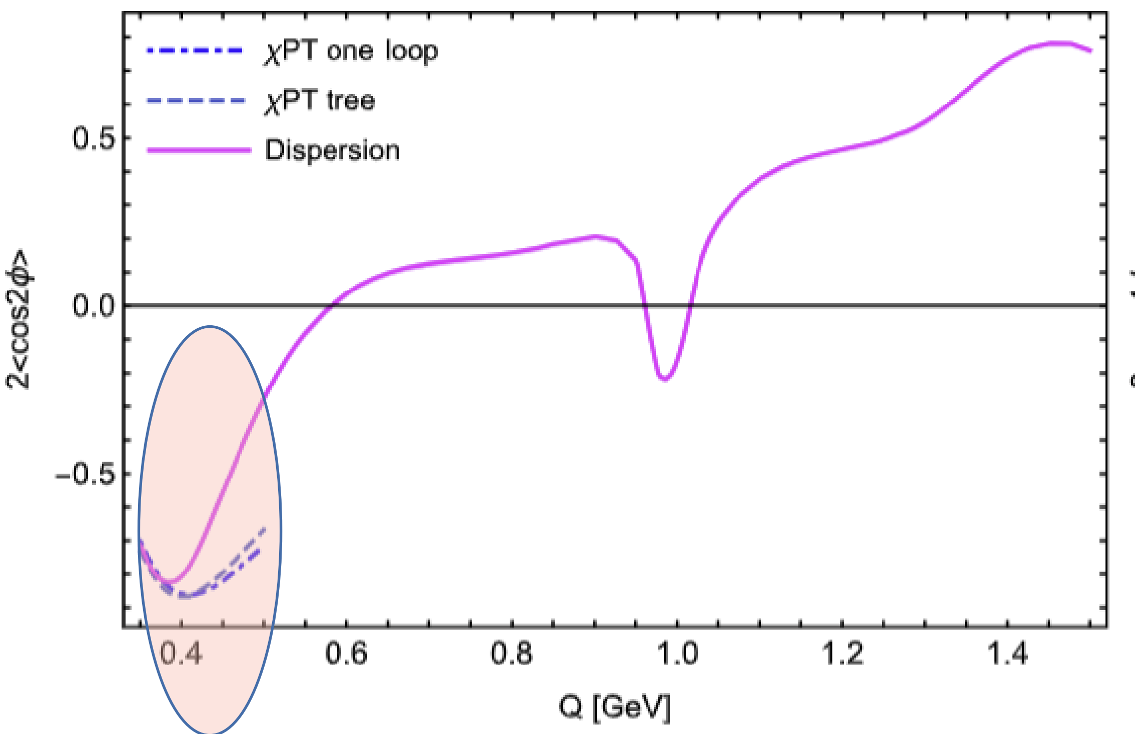

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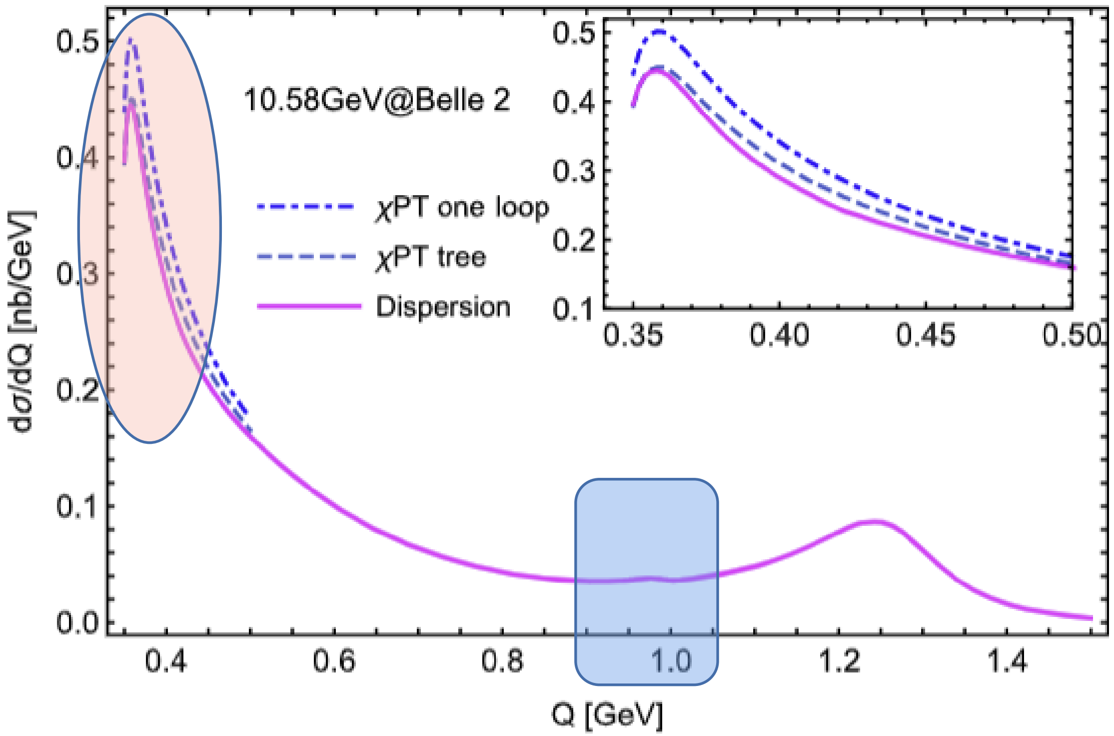
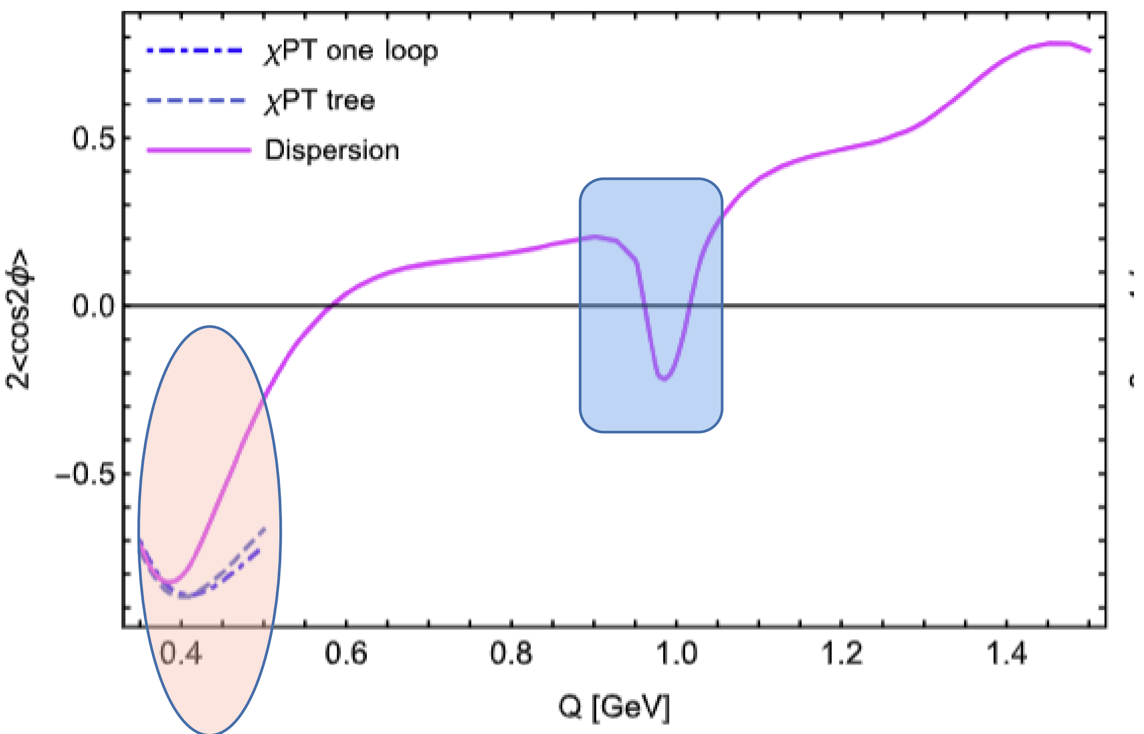




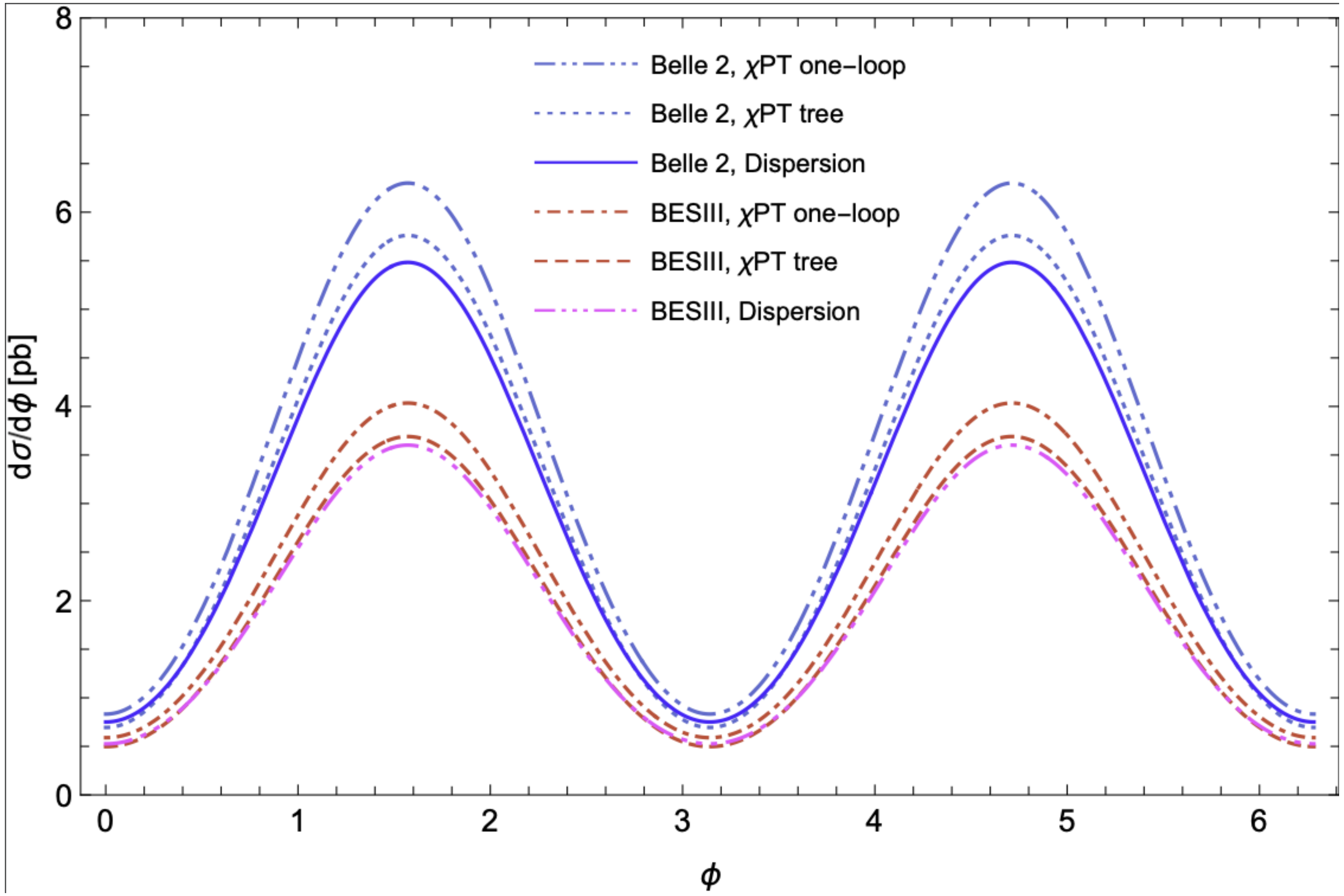
Summary

- Linearly polarized quasi-real photons at e^+e^- colliders open a new window on azimuthal modulations in exclusive meson pair production.
- We predicts pronounced $\langle \cos 2\phi \rangle$ (up to ~40%) and $\langle \cos 4\phi \rangle$ (~6%) asymmetries at Belle 2 and BESIII kinematics, complementary to traditional collinear two-photon analyses.
- Experimental analyses are already underway at Belle II; we eagerly anticipate confrontation with data and continued collaboration to explore these novel observables.

Thank you for your attention!

Backups

Numerical results, $\gamma\gamma \to \pi^+\pi^-$



Numerical results, $\gamma\gamma \to \pi^0\pi^0$