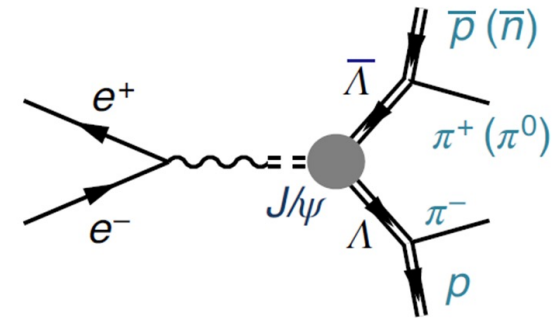


Spin effects in $e^+ e^-$ annihilation to baryon-antibaryon pair

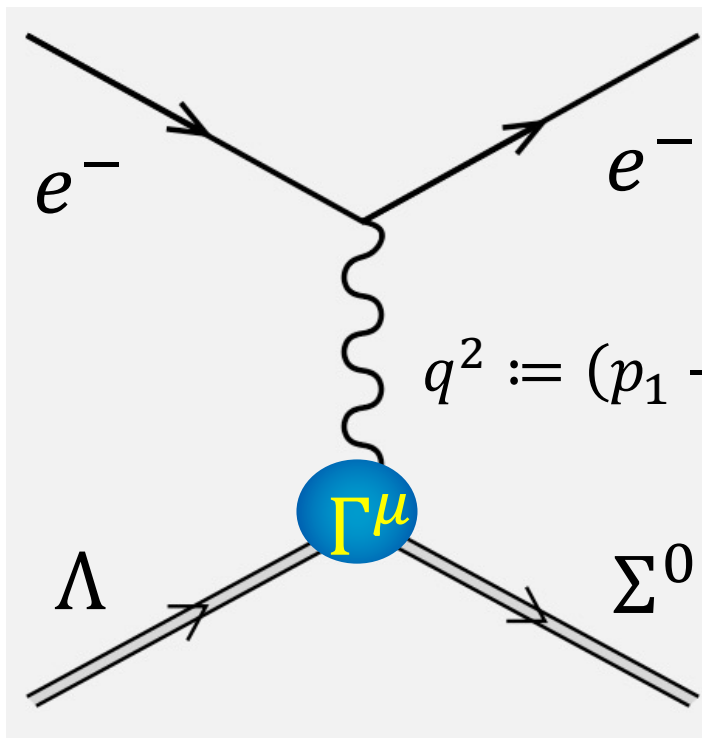
Andrzej Kupsc

$$e^+ e^- \rightarrow B \bar{B}$$

- Baryon form factors
- Spin polarization and correlations
- Polarimeters
- Modular formalism for angular distributions
- Example results



(Transition) Form Factors



$B_{1/2}$

$$q^2 := (p_1 - p_2)^2 < 0$$

$$\Gamma_\mu = -ie \left[F_1(q^2) \gamma_\mu + F_2(q^2) \frac{i\sigma_{\mu\nu} q^\nu}{M_\Lambda + M_\Sigma} \right]$$

F_1 (Dirac) and F_2 (Pauli) form factors

Sachs Form Factors (FFs) \Leftrightarrow helicity amplitudes:

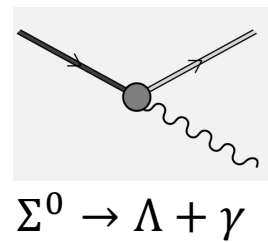
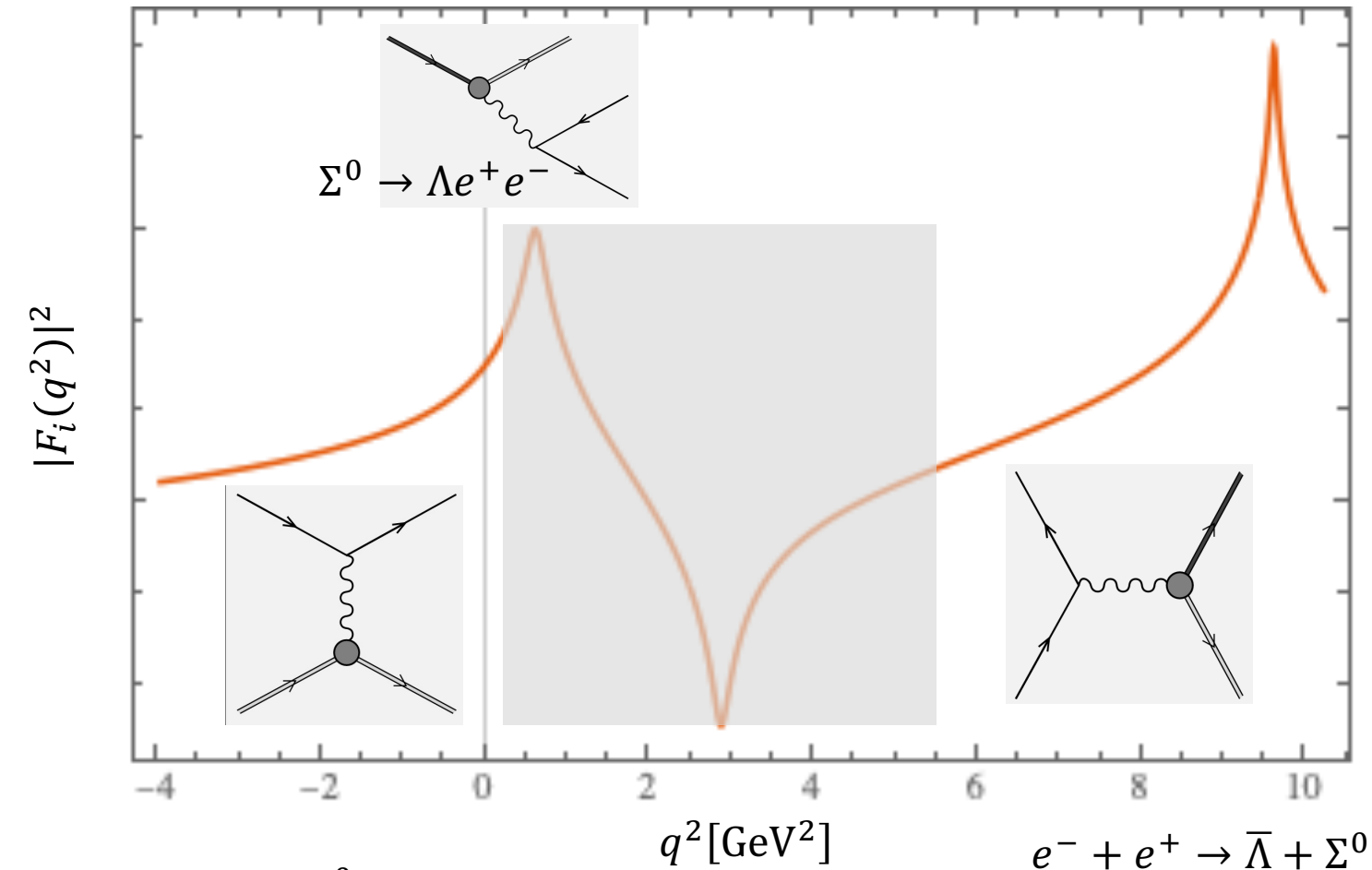
$$G_M(s) = F_1(s) + F_2(s), \quad G_E(s) = F_1(s) + \tau F_2(s)$$

helicity non-flip

helicity flip

$$\tau = \frac{q^2}{(M_\Lambda + M_\Sigma)^2}$$

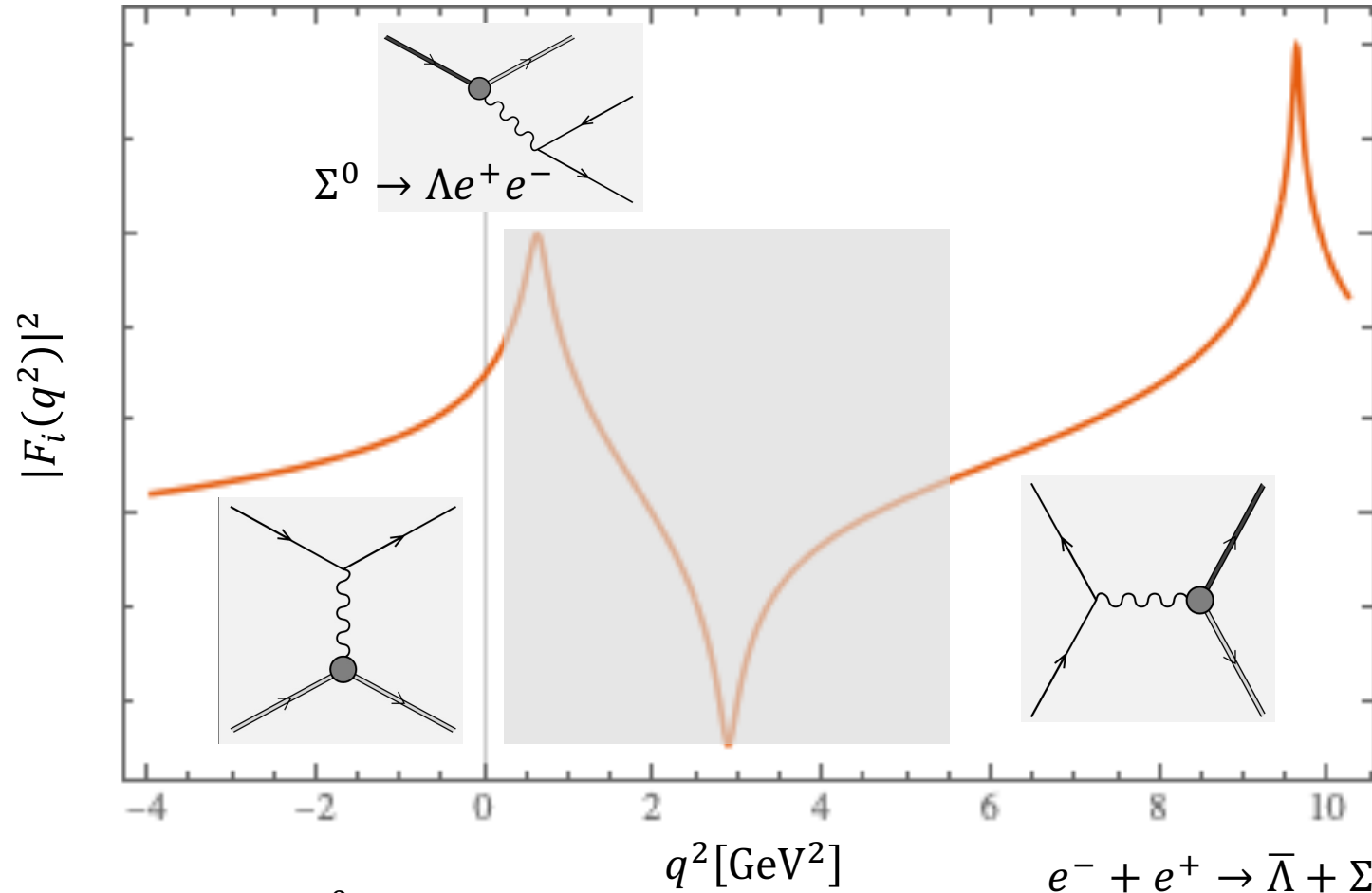
(Transition) Form Factors



$$F_i(q^2) \approx F_i(0) \left[1 + \frac{1}{6} \langle r_i^2 \rangle q^2 \right]$$

$$\langle r^2 \rangle = 6 \left. \frac{d \ln F_i(q^2)}{dq^2} \right|_{q^2=0}$$

(Transition) Form Factors

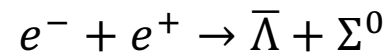
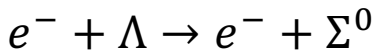


BESIII

Nature Commun.
15 (2024) 8812

$$\left| \frac{G_E}{G_M} \right| = 0.86(3)$$

$$\text{Arg} \frac{G_E}{G_M} = 58(5)^\circ$$



$F_i(q^2)$ complex functions for $q^2 > 4m_\pi^2$
Observables: $|F_i(q^2)|^2$ and $\text{Arg}(F_1(q^2)F_2^*(q^2))$

$$R = \left| \frac{G_E}{G_M} \right| \quad \left(\alpha_\psi = \frac{\tau - R^2}{\tau + R^2} \right)$$

$$G_E = R G_M e^{i\Delta\Phi}$$

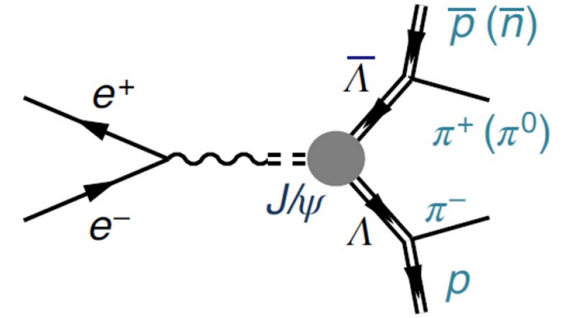
Baryon-antibaryon pair production

Thresholds:

$\Lambda\bar{\Lambda}$: 2.231 GeV

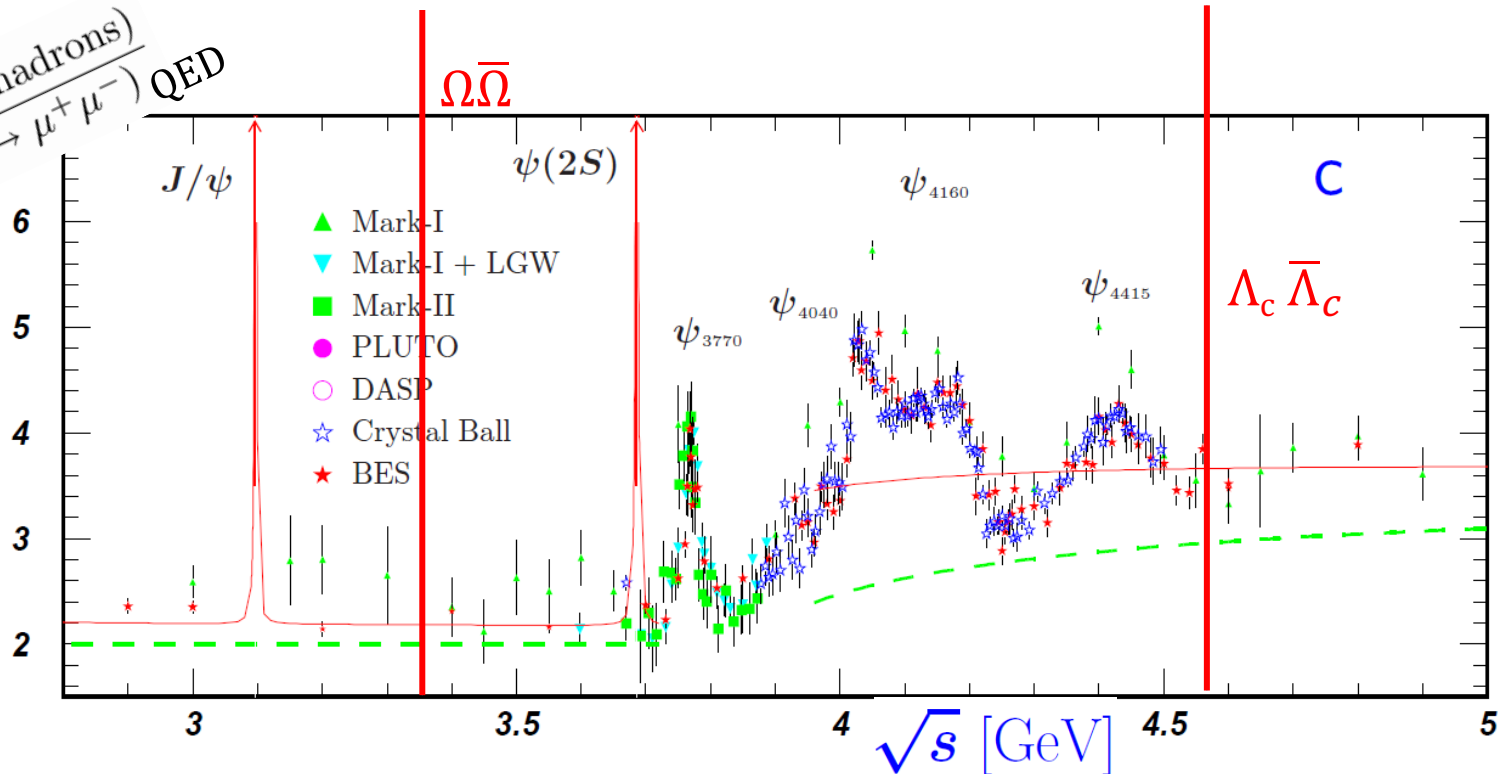
$\Xi^-\bar{\Xi}^+$: 2.643 GeV

$(\Omega\bar{\Omega})$: 3.345 GeV

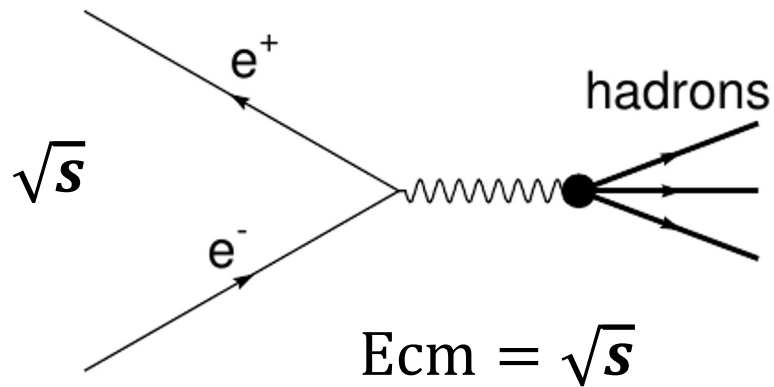


$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \text{QED}$$

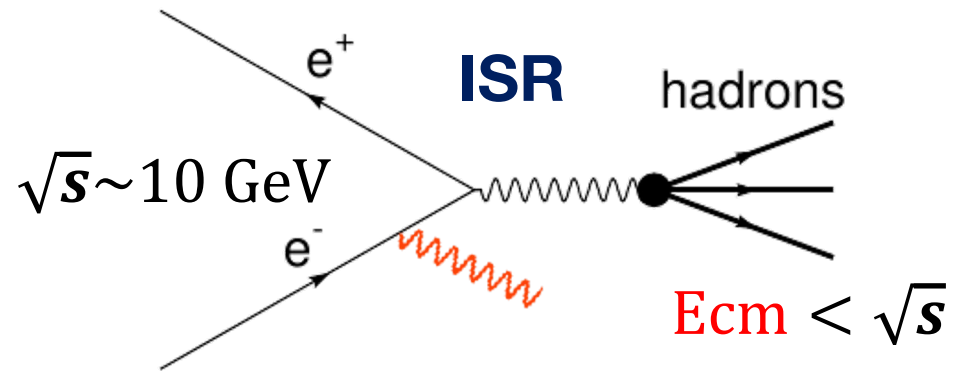
R



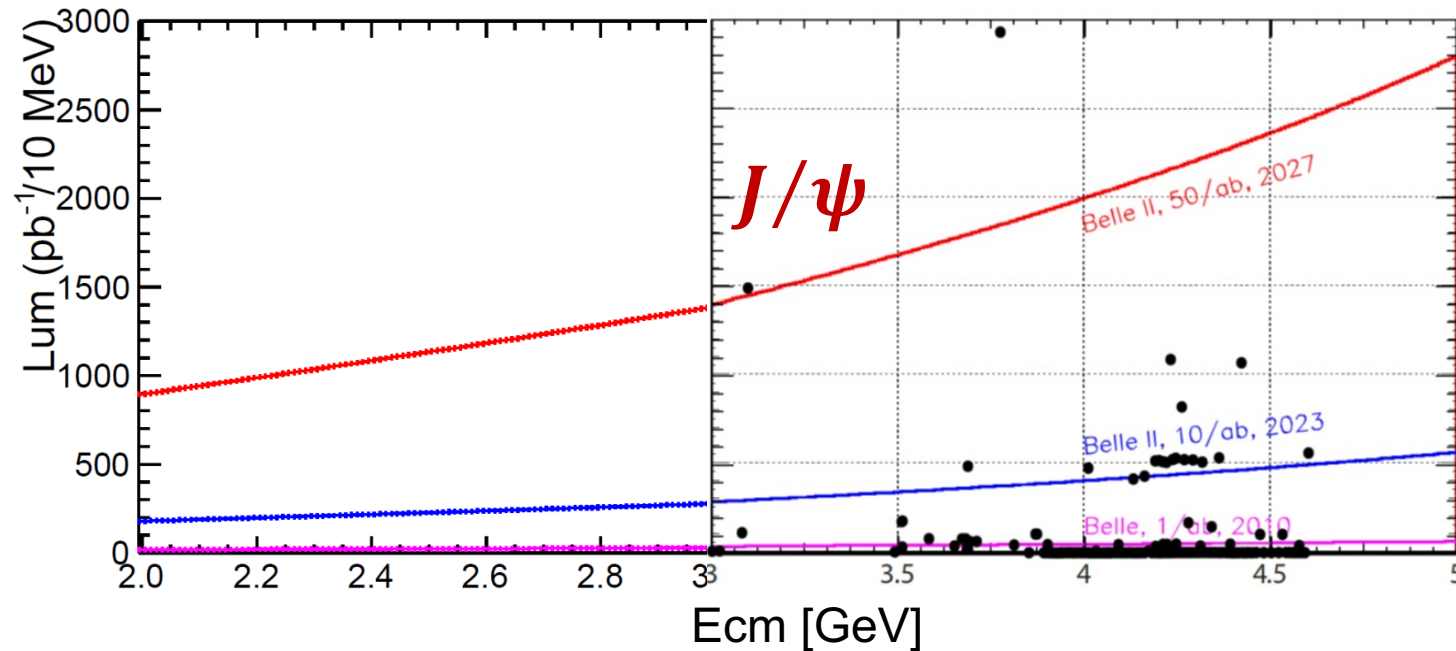
Direct scan BESIII



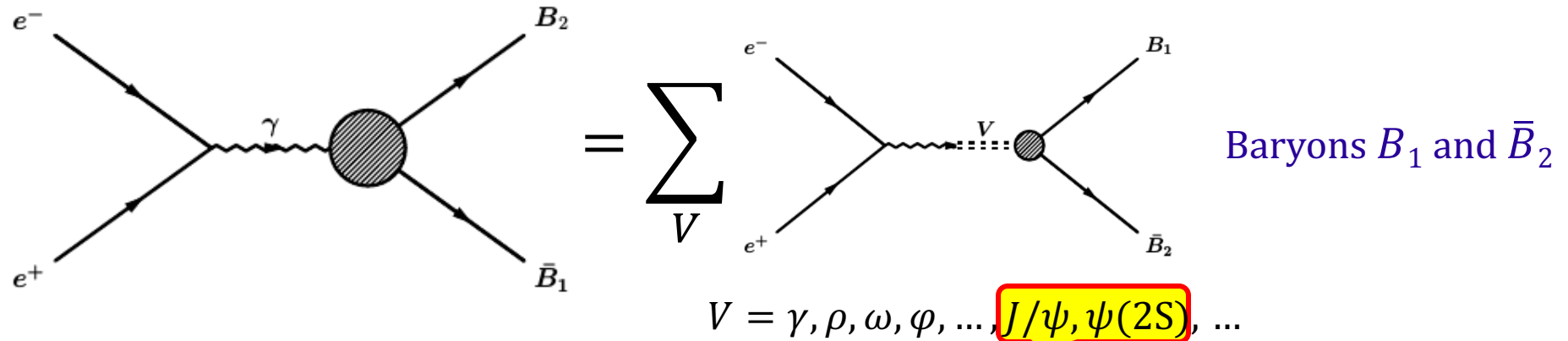
ISR BelleII



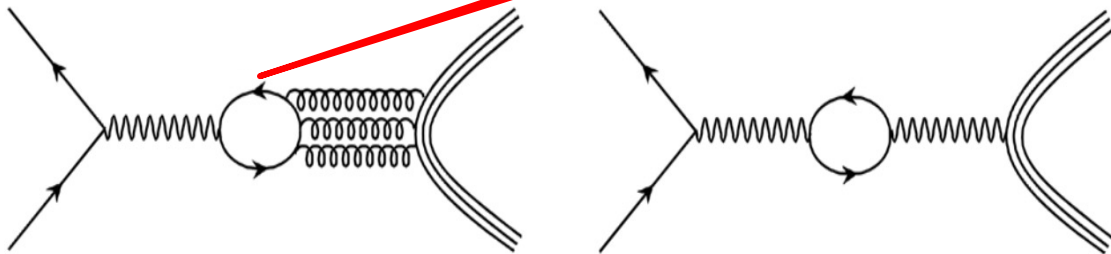
- (very) high luminosity at selected c.m. energies
- better resolution: at J/ψ 0.9 MeV: $10^{10} J/\psi$
- many E_{cm} simultaneously
- reduced point-to-point systematics
- mass resolution limited by detector
- boost of hadronic system may help efficiency



Baryon FFs (continuum):



vs J/ψ decay:



Time like spin $\frac{1}{2}$ baryon FFs:

Dubnickova, Dubnicka, Rekaló

Nuovo Cim. A109 (1996) 241

Gakh, Tomasi-Gustafsson Nucl.Phys. A771 (2006) 169

Czyz, Grzelinska, Kuhn PRD75 (2007) 074026

Fältdt EPJ A51 (2015) 74; EPJ A52 (2016)141

Charmonia decays:

Fältdt, Kupsc PLB772 (2017) 16

$J/\psi, \psi(2S) \rightarrow B\bar{B}$

$$\mathcal{B}(J/\psi \rightarrow p\bar{p}) = (21.21 \pm 0.29) \times 10^{-4}$$

decay mode	$\mathcal{B}(\text{units } 10^{-4})$	α_ψ	eff	events proposal
$J/\psi \rightarrow \Lambda\bar{\Lambda}$	$19.43 \pm 0.03 \pm 0.33$	0.469 ± 0.026	40%	3200×10^3
$\psi(2S) \rightarrow \Lambda\bar{\Lambda}$	$3.97 \pm 0.02 \pm 0.12$	0.824 ± 0.074	40%	650×10^3
$J/\psi \rightarrow \Xi^0\bar{\Xi}^0$	11.65 ± 0.04	0.66 ± 0.03	14%	670×10^3
$\psi(2S) \rightarrow \Xi^0\bar{\Xi}^0$	2.73 ± 0.03	0.65 ± 0.09	14%	160×10^3
$J/\psi \rightarrow \Xi^-\bar{\Xi}^+$	10.40 ± 0.06	0.58 ± 0.04	19%	810×10^3
$\psi(2S) \rightarrow \Xi^-\bar{\Xi}^+$	2.78 ± 0.05	0.91 ± 0.13	19%	210×10^3

$10^{10} J/\psi$ **BESIII**

$3.2 \times 10^9 \psi(2S)$

$\text{BF}(J/\psi \rightarrow B\bar{B}) \sim 10^{-3}$

Polarization and spin correlations in $e^+e^- \rightarrow B_1\bar{B}_2$

$$V \rightarrow B\bar{B}$$

Helicity bases: $J, \lambda = -J, \dots, +J$

Multipole expansion using Q bases (\Leftarrow real coef. and symmetries):

$$\begin{aligned}
 \rho_{1/2} &= \frac{\mathbb{1}^{[2]}}{2} + \sum_{m=-1}^1 r_m^1 Q_m^{[2]1} \text{ Pauli matrices} \\
 \rho_1 &= \frac{\mathbb{1}^{[3]}}{3} + \sum_{m=-1}^1 r_m^1 Q_m^{[3]1} + \sum_{m=-2}^2 r_m^2 Q_m^{[3]2} \text{ Initial state} \\
 \rho_{3/2} &= \frac{\mathbb{1}^{[4]}}{4} + \sum_{m=-1}^1 r_m^1 Q_m^{[4]1} + \sum_{m=-2}^2 r_m^2 Q_m^{[4]2} + \sum_{m=-3}^3 r_m^3 Q_m^{[4]2} \\
 &\quad \text{vector(3)} \qquad \text{dipole(5)} \qquad \text{octupole(7)}
 \end{aligned}$$

Polarization and spin correlations in $e^+e^- \rightarrow B_1\bar{B}_2$

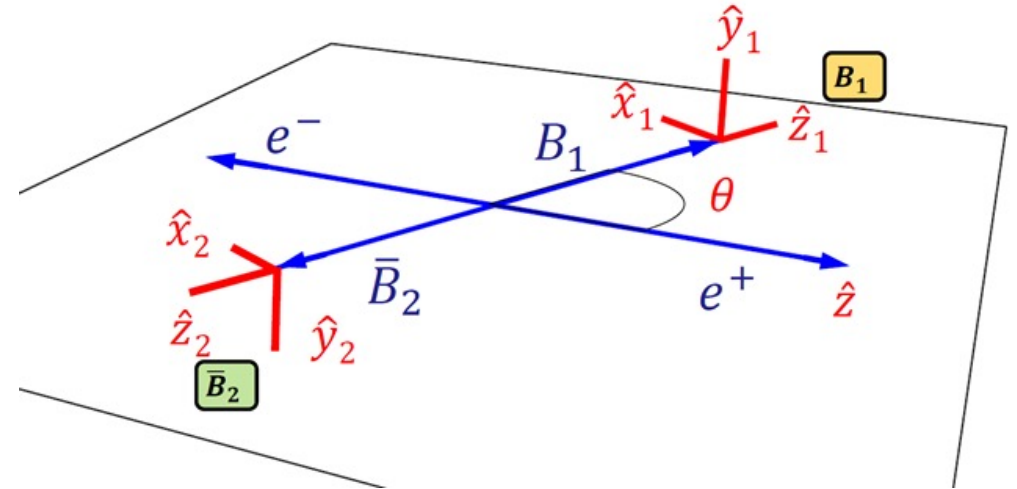
General two spin $\frac{1}{2}$ particle state:

$$\rho_{1/2,1/2} = \frac{1}{4} \sum_{\mu\bar{\nu}} C_{\mu\bar{\nu}} \sigma_{\mu}^{B_1} \otimes \sigma_{\bar{\nu}}^{\bar{B}_2}$$

($\sigma_0 = \mathbf{1}_2, \sigma_1 = \sigma_x, \sigma_2 = \sigma_y, \sigma_3 = \sigma_z$)

Unpolarized electron-positron annihilation \Rightarrow helicity $\pm 1 \Rightarrow \gamma^* / J/\psi$ have dipole polarization

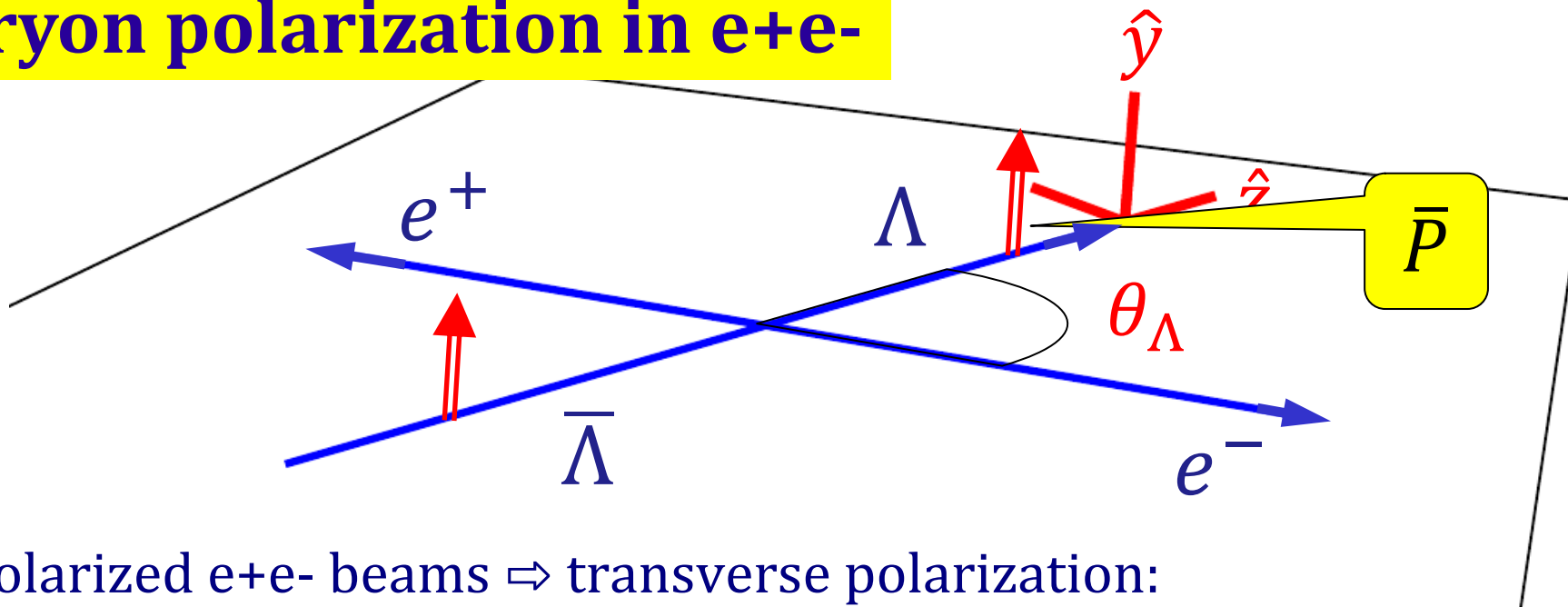
$$C_{\mu\bar{\nu}} = (1 + \alpha_{\psi} \cos^2 \theta) \begin{pmatrix} 1 & 0 & P_y & 0 \\ 0 & C_{xx} & 0 & C_{xz} \\ -P_y & 0 & C_{yy} & 0 \\ 0 & -C_{xz} & 0 & C_{zz} \end{pmatrix}$$



$$C_{\mu\bar{\nu}} = \begin{pmatrix} 1 + \alpha_{\psi} \cos^2 \theta & 0 & \beta_{\psi} \sin \theta \cos \theta & 0 \\ 0 & \sin^2 \theta & 0 & \gamma_{\psi} \sin \theta \cos \theta \\ -\beta_{\psi} \sin \theta \cos \theta & 0 & \alpha_{\psi} \sin^2 \theta & 0 \\ 0 & -\gamma_{\psi} \sin \theta \cos \theta & 0 & -\alpha_{\psi} - \cos^2 \theta \end{pmatrix}$$

$$\beta_{\psi} = \sqrt{1 - \alpha_{\psi}^2} \sin(\Delta\Phi) \quad \gamma_{\psi} = \sqrt{1 - \alpha_{\psi}^2} \cos(\Delta\Phi)$$

Baryon polarization in e+e-

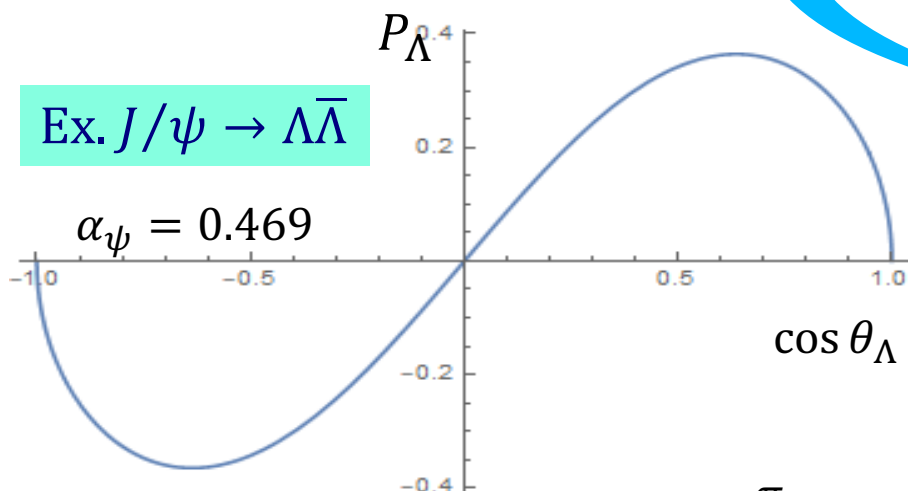


Unpolarized e+e- beams \Rightarrow transverse polarization:

$$P_y(\cos \theta_\Lambda) = \frac{\sqrt{1 - \alpha_\psi^2} \cos \theta_\Lambda \sin \theta_\Lambda}{1 + \alpha_\psi \cos^2 \theta_\Lambda} \sin(\Delta\Phi)$$

Ex. $J/\psi \rightarrow \Lambda \bar{\Lambda}$

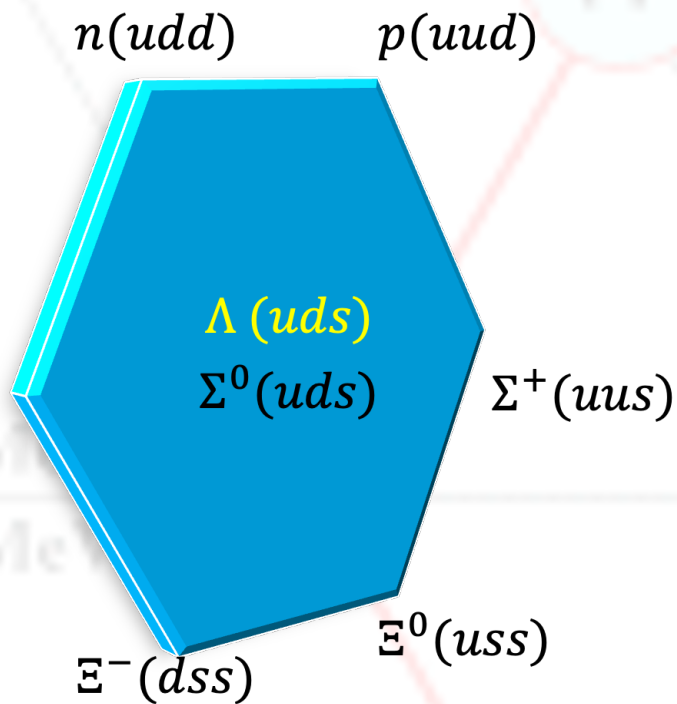
$\alpha_\psi = 0.469$



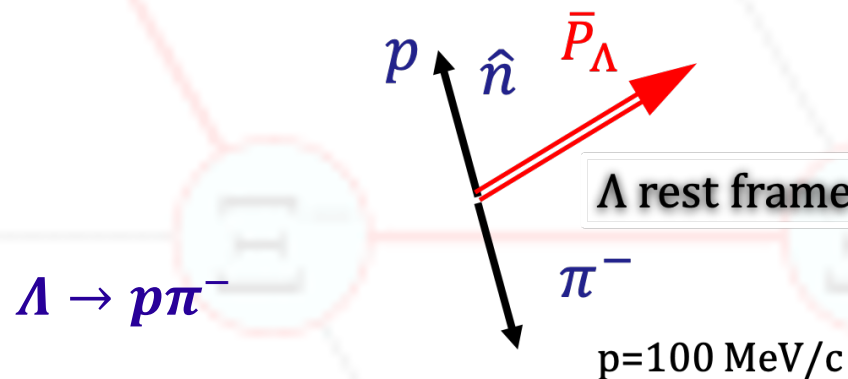
for $\Delta\Phi = \frac{\pi}{2}$

$\Delta\Phi \neq 0$

Spin polarization measurement: hyperons



hyperon	Mass [GeV/c ²]	$c\tau$ [cm]	decay (BF)
$\Lambda(uds)$	1.116	7.9	$p\pi^-$ (63.9%) $n\pi^0$ (35.8%)
$\Sigma^-(dds)$	1.197	4.4	$n\pi^-$ (99.8%)
$\Sigma^+(uus)$	1.189	2.4	$p\pi^0$ (51.6%) $n\pi^+$ (48.3%)
$\Xi^0(uss)$	1.315	8.7	$\Lambda\pi^0$ (99.5%)
$\Xi^-(dss)$	1.321	5.1	$\Lambda\pi^-$ (99.8%)



$$\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi} (1 + \alpha_- \hat{n} \cdot \bar{P}_\Lambda)$$

Spin polarization measurement: $\Lambda \rightarrow p\pi^-$

$$B \rightarrow B_f \pi$$

$$\mathcal{F}(\xi; \vec{P}) = f_0(\xi) + P_x f_x(\xi) + P_y f_y(\xi) + P_z f_z(\xi) \quad \text{p.d.f. of auxiliary variable } \xi$$

$$\mathcal{F}^{B \rightarrow B_f \pi}(\varphi_f, \vartheta_f) = \frac{1}{4\pi} \sum_{\mu=0}^3 P_\mu \sum_{\rho=0}^3 \mathcal{R}_{\mu\rho}^{(4)}(\varphi_f, \vartheta_f) b_{\rho 0}^{B \rightarrow B_f \pi} = \frac{1}{4\pi} \sum_{\mu=0}^3 P_\mu \sum_{\mu'=0}^3 a_{\mu,0}^{B \rightarrow B_f \pi}$$

$\Lambda \rightarrow p\pi^-$

Decay matrices:

$$b_{\mu\nu}^{B \rightarrow B_f \pi} \propto \begin{pmatrix} 1 & 0 & 0 & \alpha_B \\ 0 & \gamma_b & -\beta_B & 0 \\ 0 & \beta_B & \gamma_B & 0 \\ \alpha_B & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma(\vec{P}) \propto \frac{1}{A_N \sqrt{N}} = \frac{1}{\alpha_B \sqrt{N}}$$

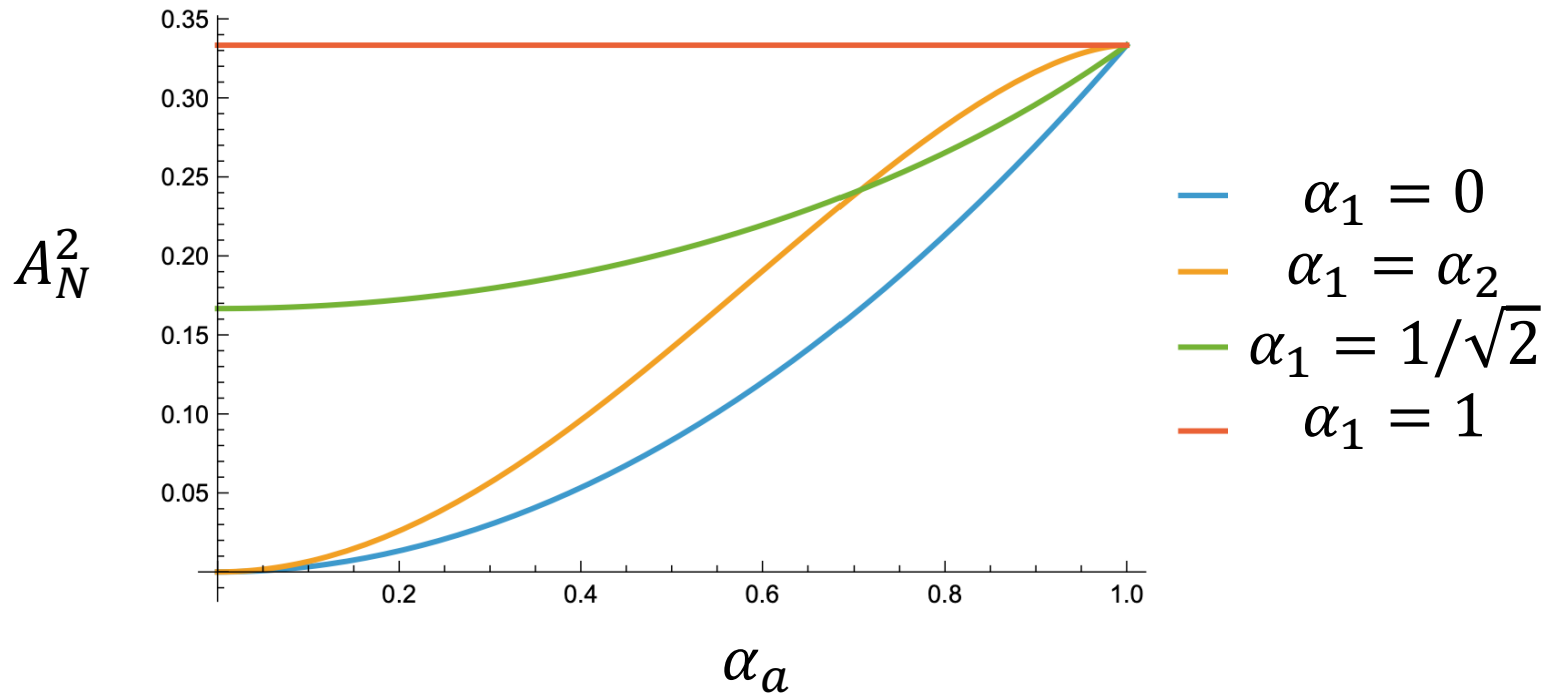
$$\beta = \sqrt{1 - \alpha^2} \sin \phi$$

$$\gamma = \sqrt{1 - \alpha^2} \cos \phi$$

Spin polarization measurement: $\Xi^- \rightarrow \Lambda \pi^- \rightarrow p \pi^- \pi^-$

$$B(P_z) \rightarrow B_1 \pi_1 \rightarrow B_a \pi_a \pi_1$$

$$b_{\rho\nu}^{B \rightarrow a} = \sum_{\rho', \rho''=0}^3 b_{\rho\rho'}^{B \rightarrow 1} \mathcal{R}_{\rho'\rho''}^{(4)}(0, \vartheta_a) b_{\rho''\nu}^{1 \rightarrow a}$$

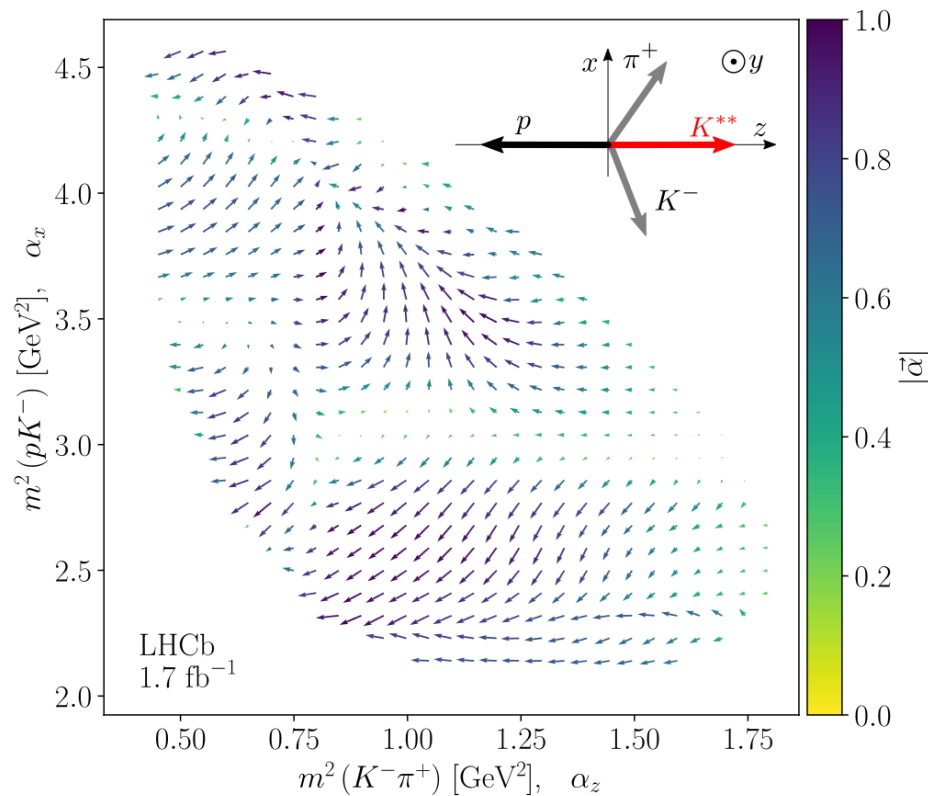


$$A_N^2 = \frac{1}{3} - \frac{(1 - \alpha_1^2)(1 - \alpha_a^2) \operatorname{atanh}(\alpha_1 \alpha_a)}{3\alpha_1 \alpha_a}$$

Spin polarization measurement: charmed baryons



$$\mathcal{F}(s, t, \phi_2, \theta_2, -\chi_2) = \sum_{\mu=0}^3 P_\mu \sum_{\kappa=0}^3 \mathcal{R}_{\mu\kappa}^{(4)}(\phi_2, \theta_2, -\chi_2) b_{\kappa 0}(s, t)$$



$$b_{k0}(s, t) = \begin{bmatrix} 1 \\ \alpha_x(s, t) \\ \alpha_y(s, t) \\ \alpha_z(s, t) \end{bmatrix}$$

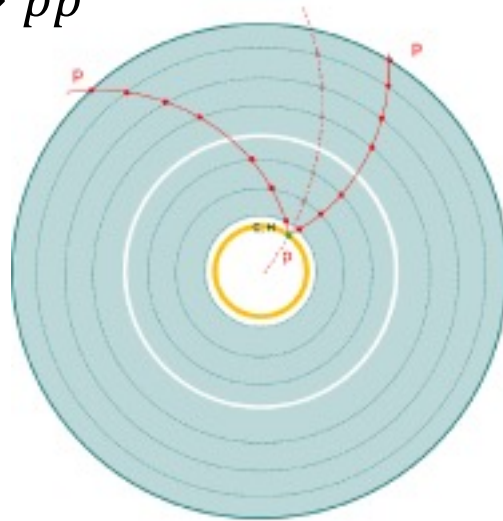
$$A_N^2 = \alpha_x^2 + \alpha_y^2 + \alpha_z^2$$

$$A_N = 0.67(2)$$

Spin polarization measurement: protons

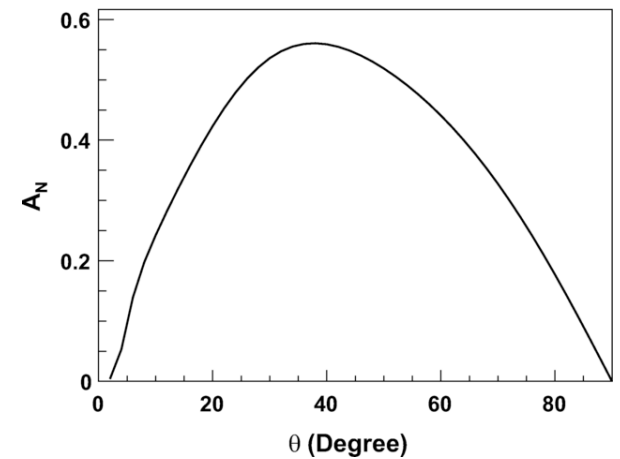
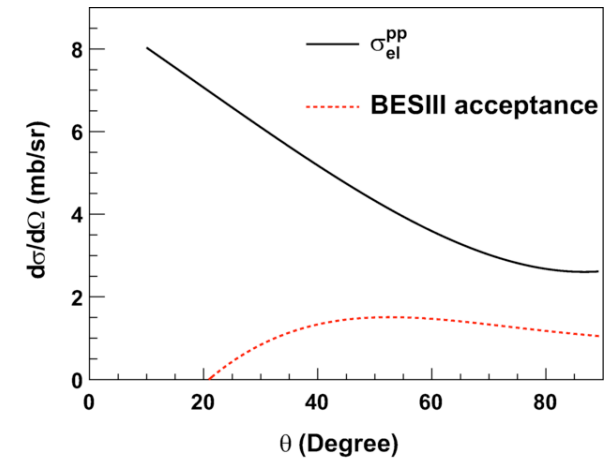
Secondary scattering feasibility at BESIII

Example: $J/\psi \rightarrow p\bar{p}$



$$\frac{d\sigma}{d\varphi d\cos\vartheta} = \frac{1}{2\pi} \frac{d\sigma_0}{d\cos\vartheta} [1 + P_x A_N(\vartheta) \cos\varphi + P_y A_N(\vartheta) \sin\varphi]$$

probability of the proton elastically scattering off a proton in the oil layer of 0.8 mm is ca 1.2×10^{-4}



How to determine nucleon polarization at existing collider experiments?
Y.Liang, X.Lv, AK, B. Gou, and H. Li PRD **112**, L031502 (2025)

Decay angular distributions

$$e^+ e^- \rightarrow (\Lambda \rightarrow p\pi^-)\bar{\Lambda}$$

$$\frac{d\Gamma}{d\cos\theta_\Lambda d\Omega_1} \propto (1 + \alpha_\psi \cos^2\theta_\Lambda) \{1 + \alpha_\Lambda P_y n_{1,y}\}$$

$$\Lambda \rightarrow p\pi^-: \hat{\mathbf{n}}_1 \rightarrow \Omega_1 = (\cos\theta_1, \phi_1) : \alpha_\Lambda$$

⇒ Determine product: $\alpha_\Lambda P_y \sim \alpha_\Lambda \sin(\Delta\Phi)$

$$e^+ e^- \rightarrow (\Lambda \rightarrow p\pi^-)(\bar{\Lambda} \rightarrow \bar{p}\pi^+)$$

$$W = \text{Tr}\rho_{p,\bar{p}} = \sum_{\mu,\bar{\nu}=0}^3 C_{\mu\bar{\nu}} a_{\mu,0}^\Lambda a_{\bar{\nu},0}^{\bar{\Lambda}}$$

⇒ Parameters: 2 production + 2 for decays
($\alpha_\psi, \Delta\Phi, \alpha_\Lambda, \bar{\alpha}_\Lambda$)

BESIII breakthrough measurements

$$e^+ e^- \rightarrow J/\psi \rightarrow \Lambda \bar{\Lambda}$$

nature
physics

LETTERS

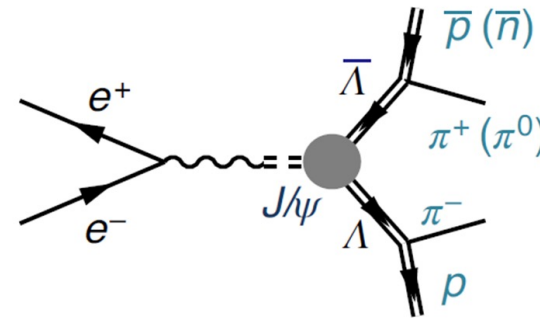
<https://doi.org/10.1038/s41567-019-0494-8>

Polarization and entanglement in baryon-antibaryon pair production in electron-positron annihilation

Nature Phys. 15 (2019) 631

The BESIII Collaboration*

BESIII

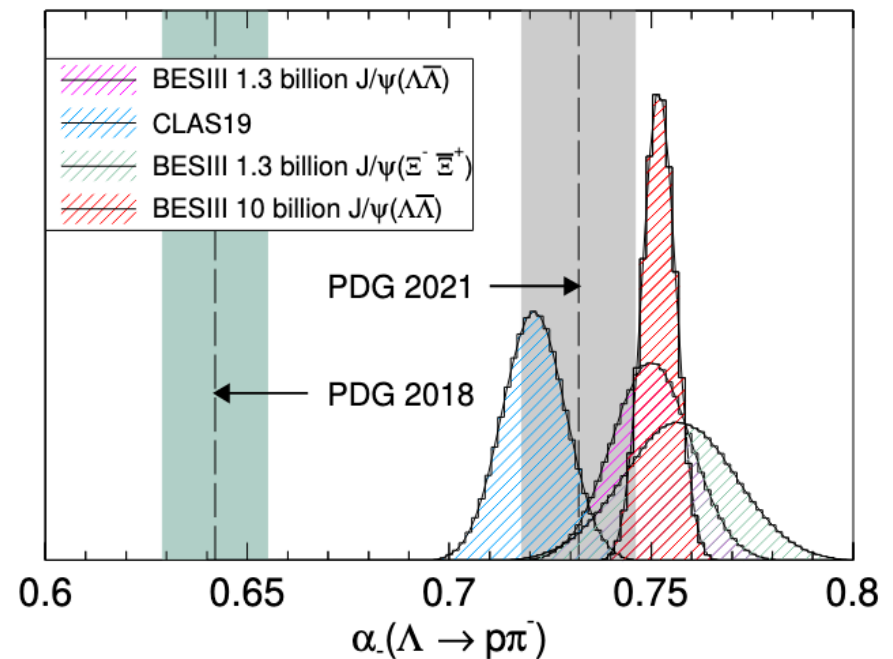


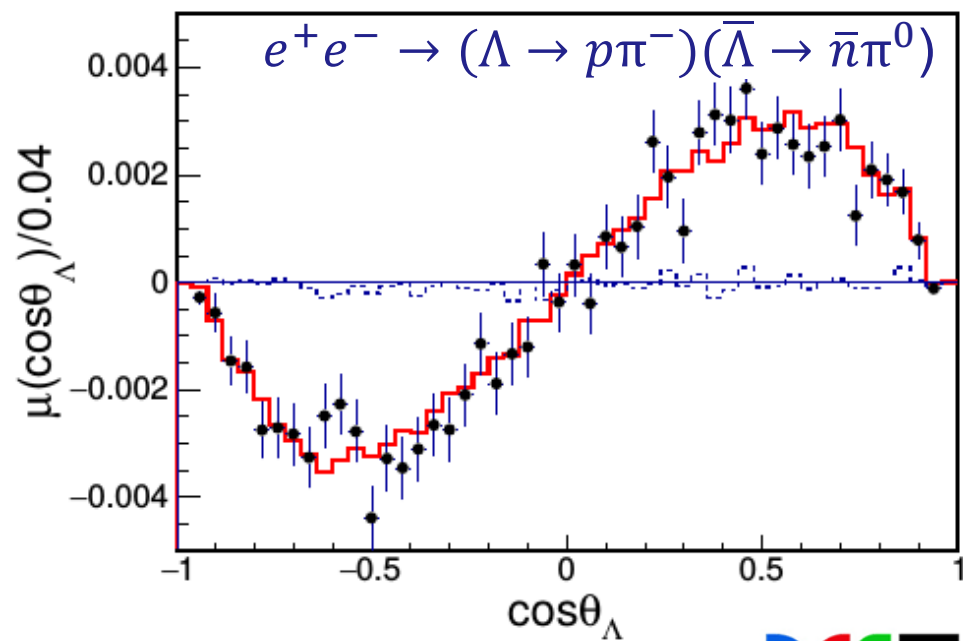
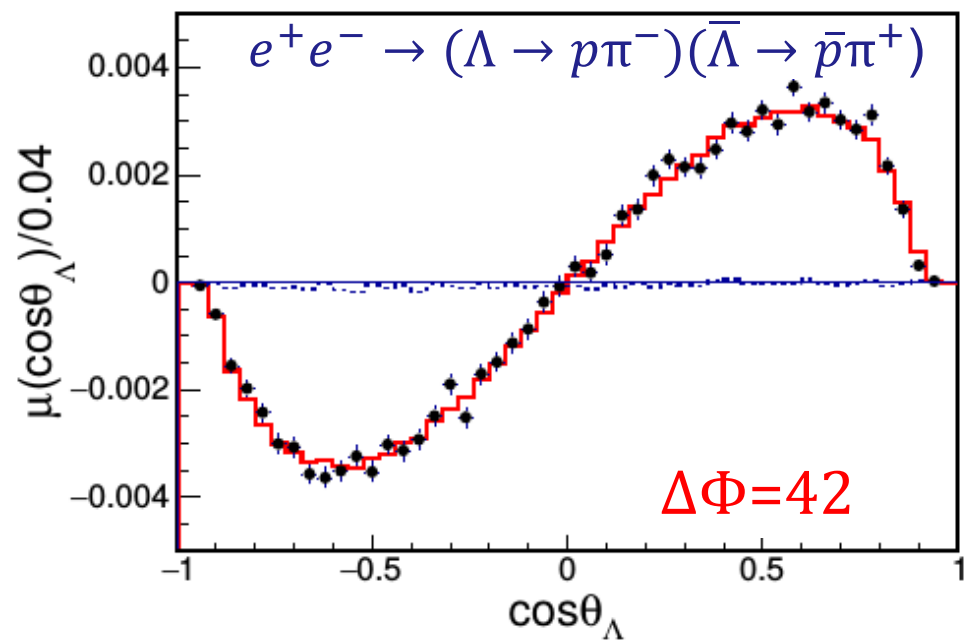
Probing CP symmetry and weak phases with entangled double-strange baryons

[The BESIII Collaboration](#)

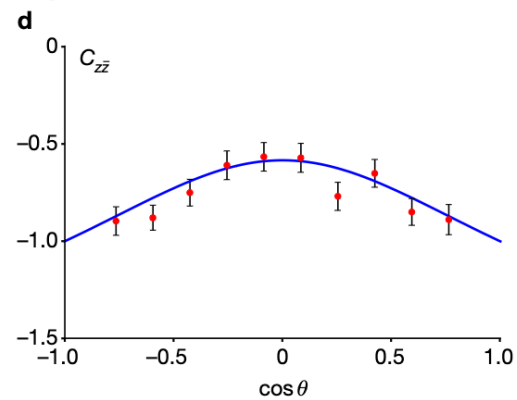
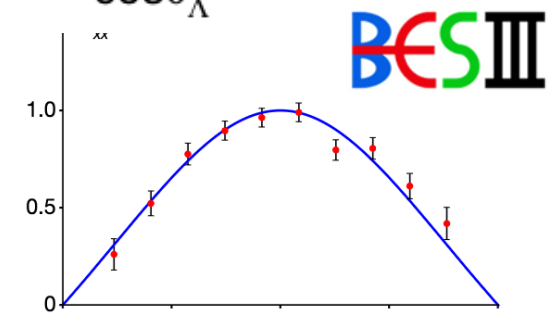
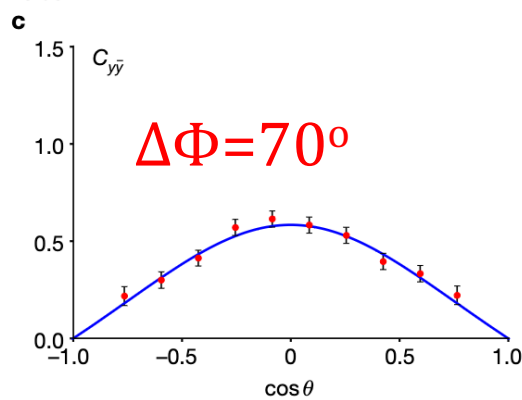
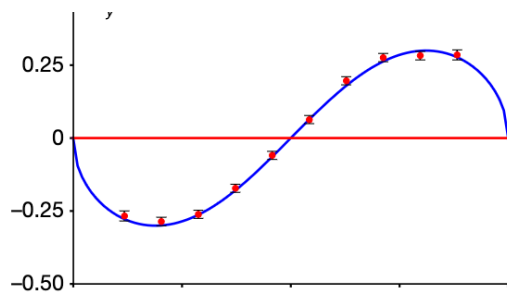
[Nature](#) 606, 64–69 (2022)

$$e^+ e^- \rightarrow J/\psi \rightarrow \Xi^- \bar{\Xi}^+$$

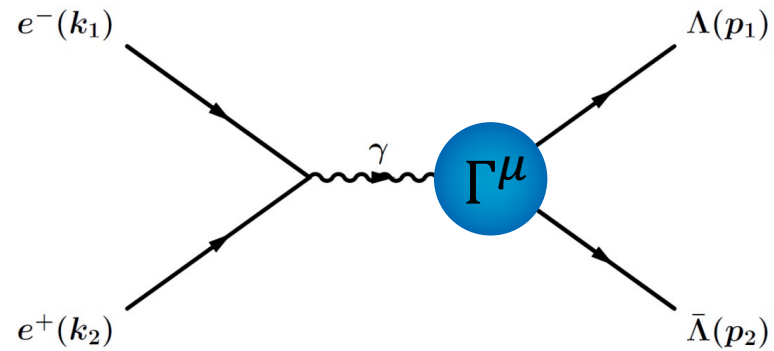




BESIII



$e^+ e^- \rightarrow \gamma^*, Z^* \rightarrow B\bar{B}$ (spin 1/2)



$$\Gamma^\mu = \Gamma_V^\mu + \Gamma_A^\mu \gamma_5$$

$$\Gamma_l^\mu = \gamma^\mu F_1^l(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{m_1 + m_2} F_2^l(q^2) + \frac{q_\mu}{m_1 + m_2} F_3^l(q^2), \quad l = V, A$$

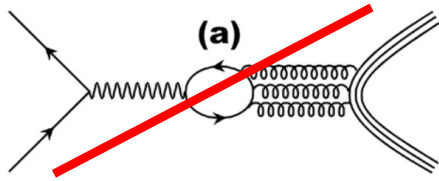
$$G_E = F_1^V + \tau F_2^V \quad \text{and} \quad G_M = F_1^V + F_2^V$$

$$\left. \begin{aligned} {}^3S &\propto 2G_M + \frac{1}{\sqrt{\tau}} G_E \\ {}^3D &\propto \sqrt{2}G_M - \sqrt{\frac{2}{\tau}} G_E \end{aligned} \right\} J^P = 1^-$$

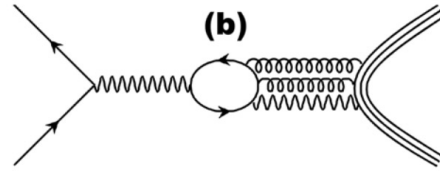
$$\left. \begin{aligned} {}^1P &= \sqrt{2Q_+} \left(\frac{m_1 - m_2}{m_1 + m_2} \sqrt{\frac{1}{\tau}} F_1^A - \sqrt{\tau} F_2^A \right) \\ {}^3P &= 2\sqrt{Q_+} \left(-F_1^A + \frac{m_1 - m_2}{m_1 + m_2} F_2^A \right) \end{aligned} \right\} J^P = 1^+$$

Extracting the femtometer structure of strange baryons using the vacuum polarization effect

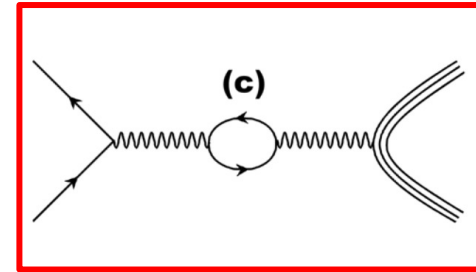
BESIII



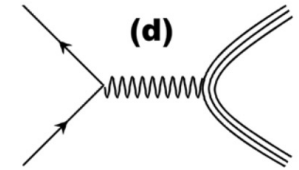
$\Delta I \neq 0$



$gg\gamma^*$



HVP



(d)

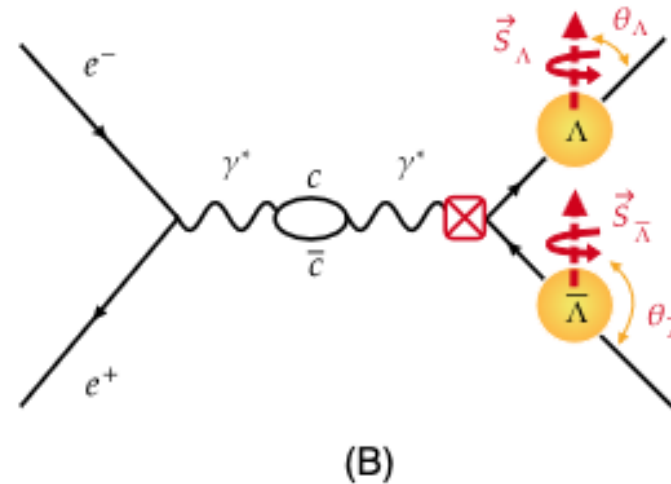
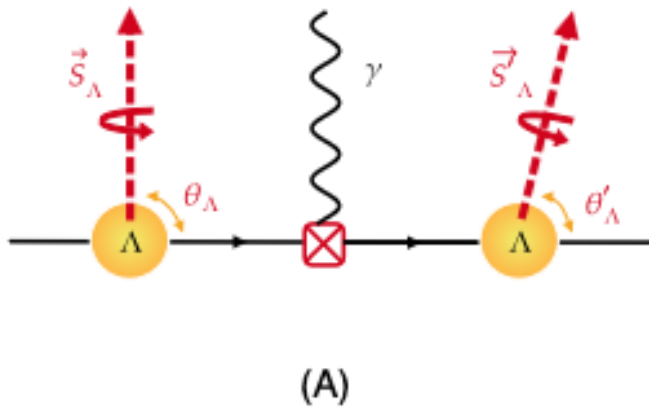
$$\sqrt{s} = m_{J/\psi}$$

$$e^- + e^+ \rightarrow \bar{\Lambda} + \Sigma^0$$

$$\left| \frac{G_E}{G_M} \right| = 0.86(3) \quad \text{Arg} \frac{G_E}{G_M} = 58(5)^\circ$$

Measurement of the Λ Electric Dipole Form Factor at $\sqrt{s} = m_{J/\psi}$

2506.19180 [hep-ex]



$$|d_\Lambda| < 6.5 \times 10^{-19} e \text{ cm.}$$

X. He et al. Phys.Lett.B 839 (2023) 137834

$$F_2^A(m_{J/\psi}^2) = \frac{2e}{3m_{J/\psi}^2} g_V d_\Lambda$$

$Re(d_\Lambda)$	$(-3.1 \pm 3.2 \pm 0.5) \times 10^{-19} e \text{ cm}$
$Im(d_\Lambda)$	$(2.9 \pm 2.6 \pm 0.6) \times 10^{-19} e \text{ cm}$

Spin density matrix for $e^+e^- \rightarrow \Omega^-\Omega^+$

$$\rho_{3/2, \overline{3/2}}^{\lambda_1 \lambda_2, \lambda_1' \lambda_2'} = \sum_{\kappa=\pm 1} D_{\kappa, \lambda_1 - \lambda_2}^{1*}(0, \theta_\Omega, 0) D_{\kappa, \lambda_1' - \lambda_2'}^1(0, \theta_\Omega, 0) A_{\lambda_1 \lambda_2} A_{\lambda_1' \lambda_2'}^*$$

$$A = \begin{pmatrix} \mathbf{h}_4 & \mathbf{h}_3 & 0 & 0 \\ \mathbf{h}_3 & \mathbf{h}_1 & \mathbf{h}_2 & 0 \\ 0 & \mathbf{h}_2 & \mathbf{h}_1 & \mathbf{h}_3 \\ 0 & 0 & \mathbf{h}_3 & \mathbf{h}_4 \end{pmatrix}$$

(Complex) Form Factors

$$\mathbf{h}_k \rightarrow h_k \exp(i\phi_k)$$

$$\rho_{3/2, \overline{3/2}} = \sum_{\mu=0}^{15} \sum_{\bar{\nu}=0}^{15} C_{\mu, \bar{\nu}} Q_\mu \otimes Q_{\bar{\nu}}$$

$$\frac{3}{4} Q_M^L \rightarrow Q_\mu, \mu = 1, \dots, 15$$

$$Q_0 = \frac{1}{4} I \quad \rho_{3/2} = \sum_{\mu=0}^{15} r_\mu Q_\mu$$

Single tag $e^+e^- \rightarrow \Omega^- \bar{\Omega}^+$

Single 3/2-spin baryon density matrix is

$$\rho_{3/2} = \sum_{\mu=0}^{15} r_{\mu} Q_{\mu} = \sum_{\mu=0}^{15} C_{\mu,0} Q_{\mu}$$

Angular distribution (using decay matrices in helicity frames):

$$W = \sum_{\mu=0}^{15} \sum_{\kappa=0}^3 C_{\mu,0} b_{\mu,\kappa}^{\Omega} a_{\kappa,0}^{\Lambda}$$

decay 3/2- \rightarrow 1/2 0 ($\Omega \rightarrow \Lambda K$)

decay 1/2- \rightarrow 1/2 0 ($\Lambda \rightarrow p\pi$)

E.Perotti, G.Faltdt, AK, S.Leupold, JJ.Song PRD99 (2019)056008

Model-Independent Determination of the Spin of the Ω^- and Its Polarization Alignment in $\psi(3686) \rightarrow \Omega^- \bar{\Omega}^+$

BESIII Collaboration • Medina Ablikim (Beijing, Inst. High Energy Phys.) [Show All\(511\)](#)

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Conclusions

Hyperon decays, CP violation, QM tests
using BESIII data samples at J/ψ and $\psi(2S)$

Baryon form factors at BESIII, BelleII, STCF using scan and ISR

Prospects: charmed baryons, proton polarization

Inclusion of higher order em effects will be needed for precision
experiments