Hybrid calculation of hadronic vacuum polarization in muon $g - 2$ to 0.48%

Based on ***Nature* 653, 373–377 (2026)**

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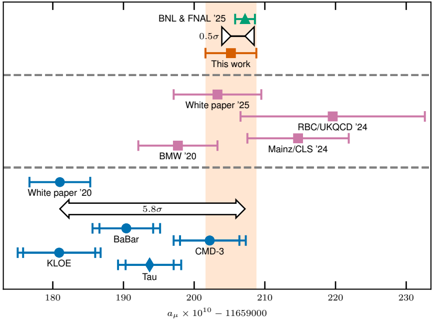
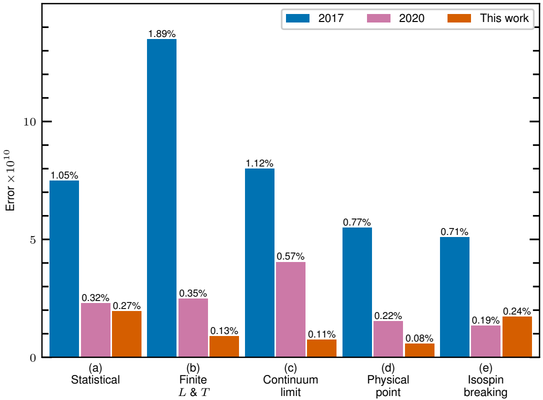
on behalf of BMW and DMZ

Status: experiment and theory

- Until 2020, precision of lattice calculation was not enough to be significant
- Theory results with competing precision were data-driven, leading to WP20.
- 2020: first sub-percent lattice determination [BMW20] casts doubts on the WP20-experiment tension.
- 2024: BMW-DMZ "hybrid" calculation: most precise to date.
- 2024-2025: many LQCD calculation at sub-percent. Theory result of WP25 is fully lattice.



Error budget and improvement



References:

BMW 20 ≡ Borsanyi, S. *et al.* Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature* **593**, 51–55. arXiv: 2002.12347 [hep-lat] (2021).

BMW 17 ≡ Borsanyi, S. *et al.* Hadronic vacuum polarization contribution to the anomalous magnetic moments of leptons from first principles. *Phys. Rev. Lett.* **121**, 022002. arXiv: 1711.04980 [hep-lat] (2018).

Lattice QCD approach

- Time + zero-momentum representation [hep-lat/1107.4388 Meyer, Bernecker]

$$a_\mu^{\mathrm{LO-HVP}} = \alpha^2 \int_0^\infty dt\, K(t) C(t)$$

- Two-point function of electromagnetic currents:

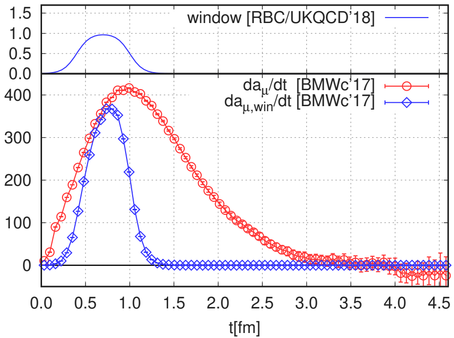
$$C(t) = \frac{1}{3} \sum_{i=1}^{3} \langle J_i(t) J_i(0) \rangle$$

- The kernel K(t), related to the leptonic part, is known

$$K(t) = \int_0^\infty \frac{dQ^2}{m_\mu^2}\, \omega\!\left(\frac{Q^2}{m_\mu^2}\right) \left[t^2 - \frac{4}{Q^2} \sin^2\!\left(\frac{Qt}{2}\right)\right],$$

$$\omega(r) = \left[r + 2 - \sqrt{r(r+4)}\right]^2 / \sqrt{r(r+4)}$$

- Construct and compare window observables [RBC/UKQCD '18] to reduce/enhance certain systematics

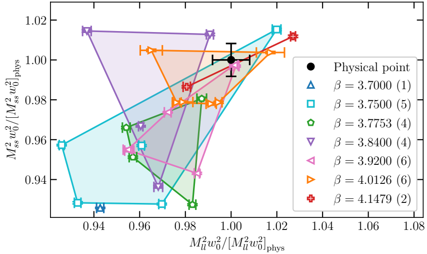


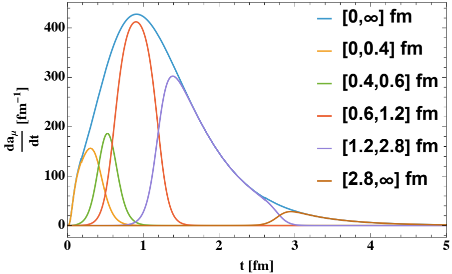
Improvement over BMW20

- 28 large-scale simulations, scattered around the physical point. We use “staggered fermions”.
- Blinded analysis.
- Including **new lattice spacing**, closer to the continuum. $L \sim 6$ fm, $T \sim 9$ fm.

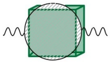
$$a = 0.064\text{fm}\ [96^3 \times 144] \longrightarrow \mathbf{0.048\ fm}\ [128^3 \times 192]\,.$$

- Break up calculation into optimised set of 5 windows.
- Hybrid approach: **bulk of the value from Lattice QCD+QED** in $[0, 2.8]$ fm, 96.1% of total a_μ.
- **Data-driven tail**: $[2.8, \infty]$ fm (3.9%) from low-energy region of e^+e^- annihilation data.

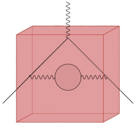




- Typical lattice volumes have size $L_{\mathrm{ref}} \sim 6$ **fm**: it contains $\sim 98\%$ of a_μ



- The remaining 2% needs to be estimated precisely: **need 0.2% total uncertainty to match experiment**



· ChPT: low-energy (**long-range**) effective theory of QCD. **Three loop** in progress: infinite volume calculation finished
[Lellouch, Lupo, Sjö, Szabo, Vanhove '25]

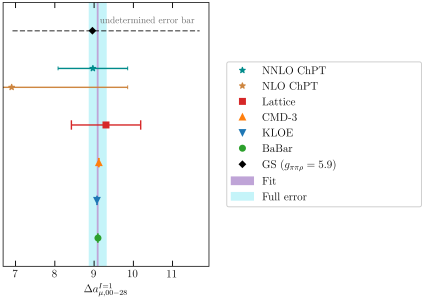
· Turn e^+e^- **data** into finite-volume physics
[Lüscher '91, Lellouch-Lüscher '01, H.B.Meyer '11], Hansen-Patella ['19, '20]

- BMW20: $L_{\mathrm{big}} \sim 11$fm box which contains almost entirely a_μ.

- BMW/DMZ-24: e^+e^- tail reduces the part of the calculation that is in a finite volume

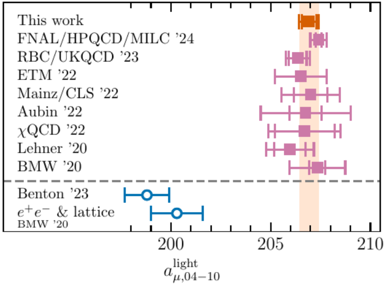
$$18.5(2.5) \rightarrow 9.3(9)$$

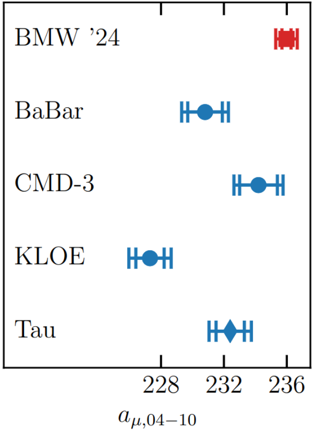
- BMW/DMZ-24 uses lattice in the $L_{\mathrm{big}} = 11$fm box, and checks consistency with other methods (Fig. $\rightarrow$)



Intermediate window

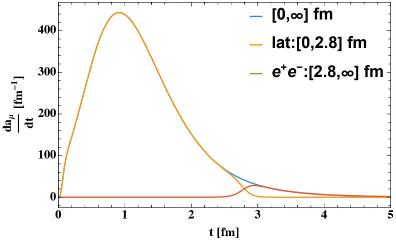
- Euclidean window [RBC'18] corresponding to **[0.4, 1] fm**
- Contains the bulk of the ρ-resonance contributions to a_μ
- Cleanest LQCD observable, and shows **clear tensions with data-driven *minus* CMD3**. Consistent between LQCD calculations.

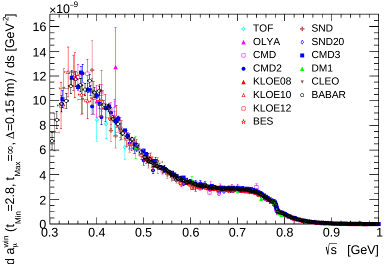




Data-driven tail

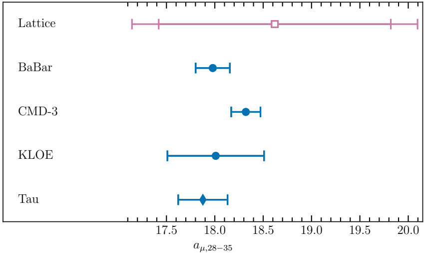
- Consider the long-distance tail of the correlator beyond 2.8 fm, $\sim 4\%$ **of** a_μ.
- In the tail, LQCD has larger uncertainty because of statistical errors and enhanced finite-volume effects.
- On the other hand, data agrees for the tail.
- Dominated by low-energy spectrum: ρ-region heavily suppressed.

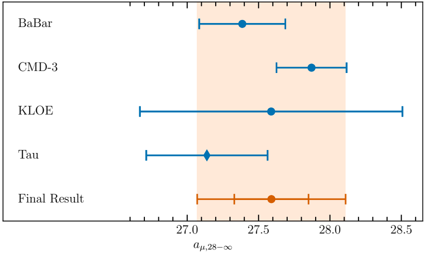




Hybrid approach

- Hybrid approach: use lattice below 2.8, dispersive above. Calculation is mostly lattice.
- Hybrid strategy first proposed in RBC'18.
- **All datasets and lattice agree in this region**
- Offers significant reduction in error (statistics and finite-volume).
- For central value, we consider datasets with full $\sqrt{s}$ coverage of the dominant $\pi^+\pi^-$ contribution.





Uncertainties on the data-driven tail

- To be very conservative we include a number of additional systematics.

1. Ordering of operations: averaging the spectra, and integrating into a_μ.
2. Other experiments with less coverage.
3. Removing/including CMD3.
4. Removing BaBar/KLOE.

Integral of avg.	0.12
Other exp.	0.16
CMD3	0.31
BaBar/KLOE	0.26

- **Negligible impact on full result**.
- $a_{\mu,28-\infty} = 27.59(26)(45)[52]$.

On the claim that the hybrid approach is “not yet viable”

Points raised in A. Wright’s talk on Monday to motivate the statement:

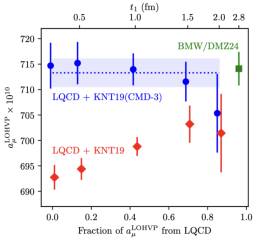
- ? Does not touch root cause of tensions.
- Obviously, does not intend to. Provides a high-precision value for a_μ within the current landscape of lattice/data, in a way that avoids tensions.

- ? Value dependent on switchover time.
- An obvious feature: we ensure that **only data free from tensions enter the hybrid calculation**.

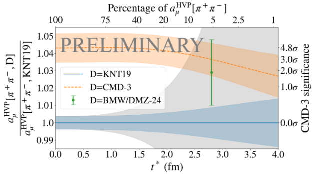
- ? KNT19/CMD3 tension still large at 2.8 fm.
- Removal of CMD3 accounted for in our error, as is removal of BaBar/KLOE.
- Notice that our value for the tail agrees with **all individual datasets** we consider: any further difference is due data combination procedure.

- ? Still desirable to have two independent theory methods.
- Of course!

Figures from A. Wright’s talk on Monday



C.T.H. Davies et al. arXiv:2410.23882



CMD-3 obtained by substituting into KNT19 where available.
BMW/DMZ-24 est. by subtraction of KNT19 non-2π in window.
BMW/DMZ-24 from D. Giusti – FCCP25; The HVP contribution to the muon $g-2$ from the lattice.

Conclusions

With BMW+DMZ-24:

- Hybrid approach provides the most precise value, to date:

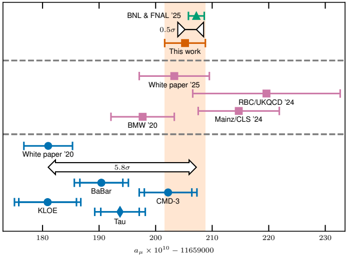
$$a_\mu^{LO-HVP} = 715.1(2.5)(2.3)[3.4]\,.$$

Still work to be done:

- Story far from over until tensions in data-driven are resolved.
- Matching precision of BNL & FNAL experiments, to which the hybrid method provides a possible path.

And of course we wait with excitement:

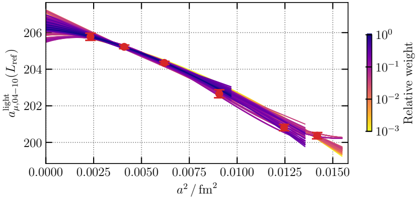
- new e^+e^- analyses.
- New experimental setup at JPARC.
- MUonE for spacelike HVP.



Backup...

Full estimate of systematics

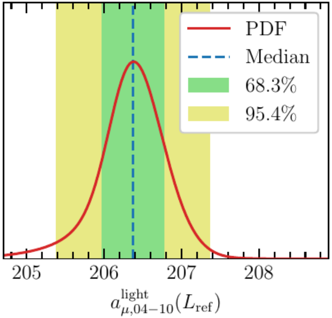
- Thousands of analysis variations to account for systematics: e.g. extrapolations from lattice to continuum (different functional forms, remove coarse lattices, etc.) combined using the **Akaike Information Criterion** (AIC).



- Statistical error from jackknife.

- Each fit is assigned a weight, used to build histograms,

$$w = \exp\left[-\frac{1}{2}\left(\chi^2 + 2n_{\text{par}} - n_{\text{data}}\right)\right] .$$



- Take one (two) sigma confidence band.

- **Total error** from half (quarter, if larger) of the width.

Scale setting

- Scale setting's uncertainty enters in all aspects of the calculation.
- We set the scale in two ways to cross-check
- $a = aM_\Omega / M_\Omega^{\mathrm{phys}}$
- $a = af_\pi / f_\pi^{\mathrm{phys}}$
- Blinded analysis returned compatible values.