Short-Distance-Free Leptonic $(g-2)$

14th International Workshop on e^+e^- collisions from Phi to Psi 2026

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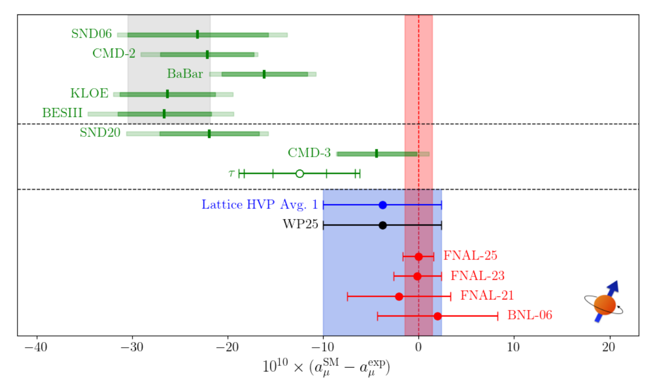
Hadrons CRC 1660 Nuclei

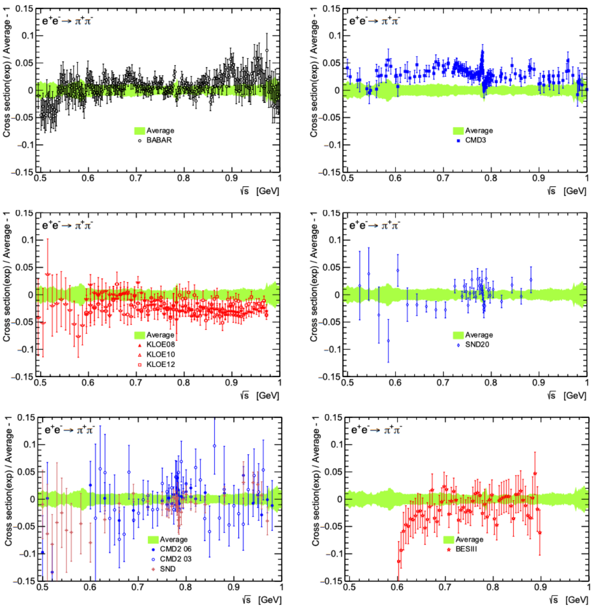
$(g-2)_\mu^{\mathrm{HVP}}$

- ◇ Much measurements has been done for $(g-2)_\ell$ or a_ℓ,
- ◇ the leading-order hadronic vacuum polarization (LO HVP) contribution provides the dominant theoretical uncertainty for a_μ, and inconsistency between data-driven and lattice QCD.



[WP25]

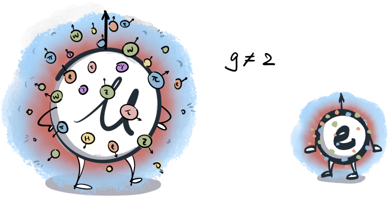
$(g-2)_\mu^{\mathrm{HVP}}$: Tensions within Data-driven



[Eur.Phys.J.C 84 (2024) 7, 721, 2312.02053 [hep-ph]/B. Malaescu, muon g-2 workshop 2024/ WP25]

Can a_e be used to constrain HVP,
and therefore improve the uncertainty of a_μ?

$(g-2)^{\mathrm{HVP}}$: muon & electron



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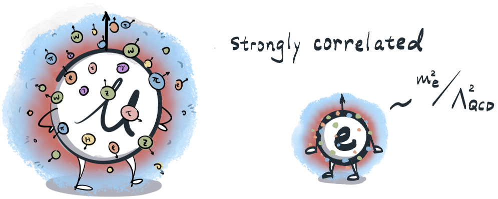

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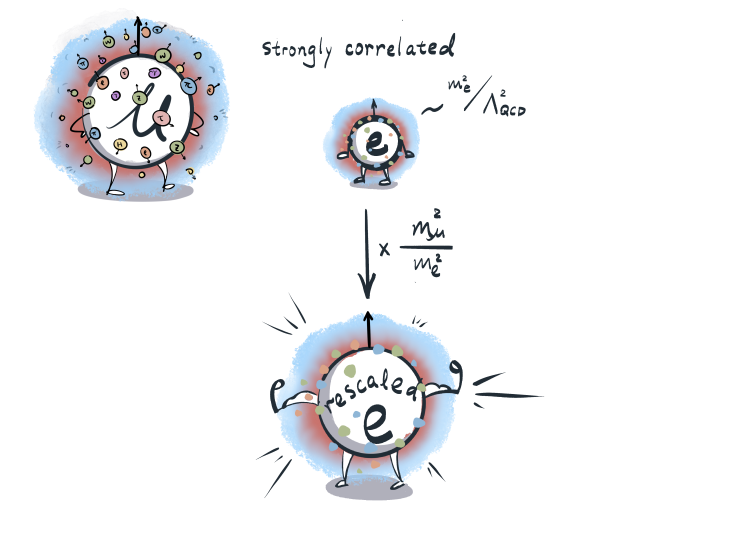

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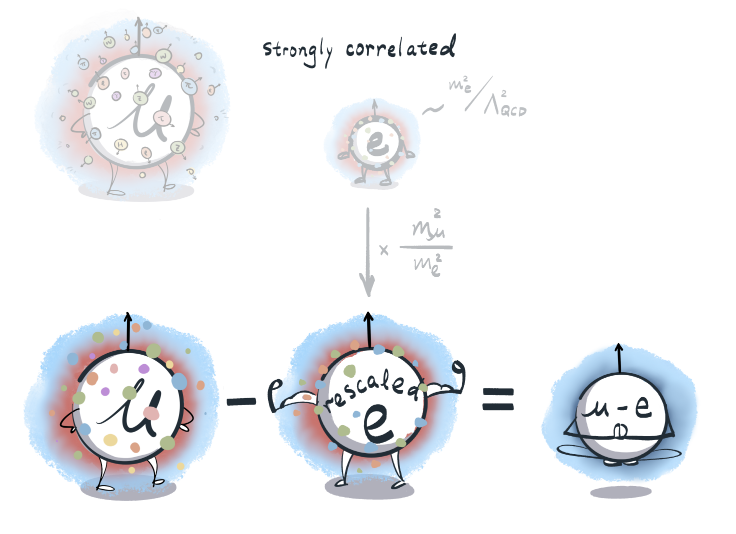

$a_{\mu-e} \equiv a_\mu - \left(m_\mu^2/m_e^2\right) a_e \Rightarrow$ high-energy effects cancellation

$(g-2)^{\mathrm{HVP}}$: muon versus electron

LO HVP	$a_\mu\ (10^{-10})$	$a_e\ (10^{-14})$	$a_{\mu-e}\ (10^{-10})$
lattice QCD	713.2(6.1)[1]	189.3(8.2)[2]	−96.1(29.0)
data-driven	692.8(2.4)[3]	186.10(66)[3]	−102.8(4)
discrepancy	20.4(6.6)	3.2(8.2)	6.7(29.0)

1. [WP25] R. Aliberti et al., Phys. Rept. 1143, 1 (2025), arXiv:2505.21476 [hep-ph].
2. [BMW] S. Borsanyi et al., Phys. Rev. Lett. 121, 022002 (2018), arXiv:1711.04980 [hep-lat].

3. A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020), arXiv:1911.00367 [hep-ph].

Dispersion Relation & Kernel function

- the dispersion relation for ℓ lepton

$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2} \int_{s_0}^{\infty} \mathrm{d}s \, \frac{\operatorname{Im} \Pi(s)}{s} K_\ell(s)$$

- the kernel function

$$K_\ell(s) = \frac{m_\ell^2}{s} \int_0^1 \mathrm{d}x \frac{x^2(1-x)}{1-x+x^2(m_\ell^2/s)}$$

- with electron approximation
  $m_e^2/s \to 0, K_e(s) \simeq m_e^2/3s$

$$K_{\mu-e}(s) = K_\mu(s) - \frac{m_\mu^2}{m_e^2} K_e(s) \simeq -\frac{m_\mu^4}{s^2} \int_0^1 \mathrm{d}x \frac{x^4}{1-x+x^2(m_\mu^2/s)}$$

- LQCD: time-momentum representation

[Bernecker and Meyer, EPJA 47, 148 (2011), 1107.4388 [hep-lat]]

Dispersion Relation & Kernel function

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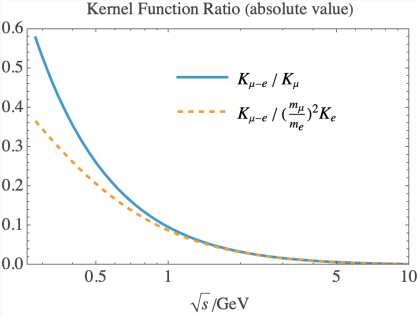

$\Rightarrow$ high-s suppression

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◇ LQCD: time-momentum representation

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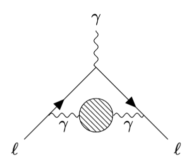
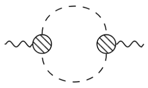
$\pi^+\pi^-$ channel

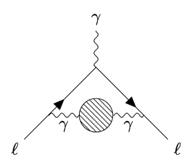


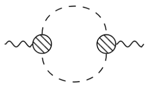


- Theory model:
  vector-meson-dominance (VMD)
  $\operatorname{Im}\Pi^{(\pi\pi)}(s) =$

$$\frac{\alpha}{12}\left(1-\frac{4m_\pi^2}{s}\right)^{\frac{3}{2}}|F_\pi(s)|^2\theta(s-4m_\pi^2)$$

$\pi^+\pi^-$ channel





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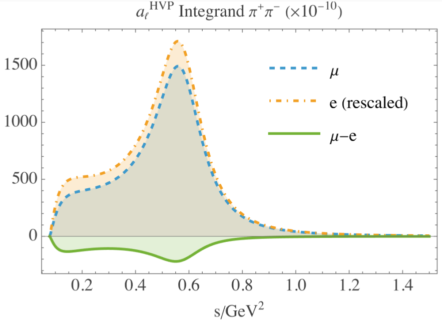
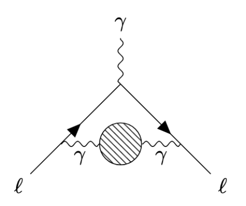


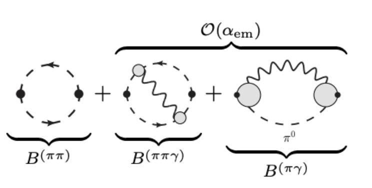
$a_\ell^{\text{HVP,LO}}$	μ ($\times 10^{-10}$)	e ($\times 10^{-14}$)	$\mu - e$ ($\times 10^{-10}$)
VMD model[1]	508.6	140.0	−90.0
KNT eval.[2]	503.46(1.91)	138.59(54)	−89.1(4)

1. V. Biloshytskyi, D. Erb, H. B. Meyer, J. Parrino, and V. Pascalutsa, Eur. Phys. J. C 86, 497 (2026), arXiv:2509.08115 [hep-ph].
2. A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D 101, 014029 (2020), arXiv:1911.00367 [hep-ph].

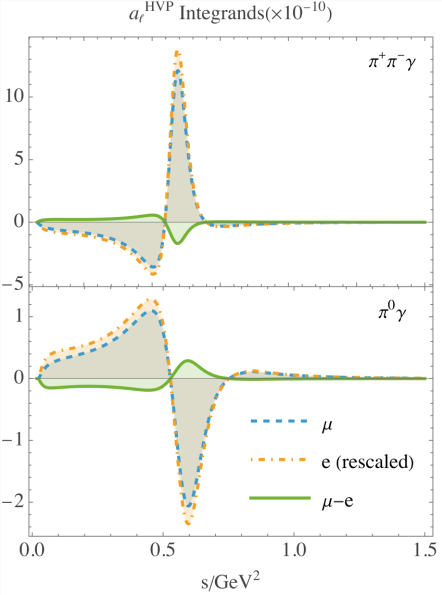
Sub-leading channels

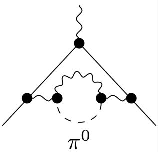


- Sub-leading channels' contribution



[2509.08115 [hep-ph]]

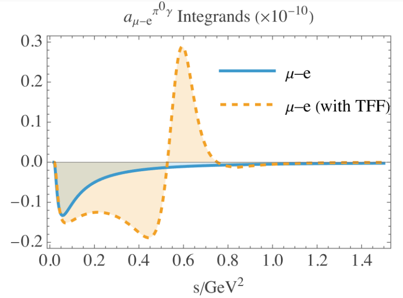


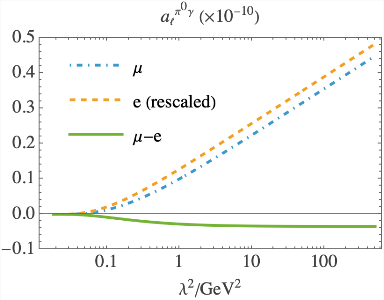


$$\mathcal{L}_{\text{Wess-Zumino-Witten}} = -\frac{\alpha}{4\pi f_\pi}\pi^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- ◇ UV-divergence controlled by a pion transition form factor (TFF) in $\pi\gamma\gamma$ vertices
- ◇ UV-divergence cancels in $a_{\mu-e}$
  $\Rightarrow$ model-independent result without TFF

$$a^{\pi^0\gamma}_{\mu-e}(\text{with TFF}) \simeq -0.45 \times 10^{-11}$$
$$a^{\pi^0\gamma}_{\mu-e}(\chi\text{PT}) = -0.34(5) \times 10^{-11}$$





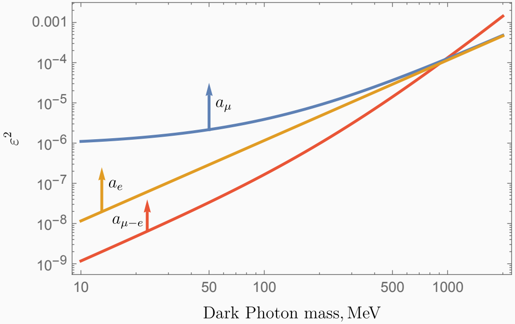
New Physics

- $(g-2)_\ell$ has been treated as a potential probe to BSM effects.
- In supersymmetric models, new physics corrections to a_ℓ (Λ being mass scale of the heavy new particle)

$$\Delta a_{\mu-e}^{\mathrm{BSM}} = m_\mu^2 \left(\frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} \right).$$

  disadvantage: flavour universality $\Delta a_\ell \propto m_\ell^2 \Lambda^{-2}$.

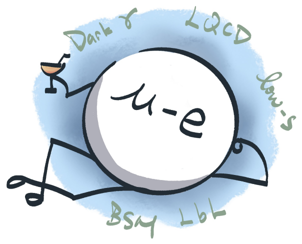
- Dark photon (A'): at low-energy limit, correction to a_ℓ at one-loop level
  $\Delta a_\ell^{A'} \to \mathrm{Im}\,\Pi \propto \epsilon^2 \delta(s - m_{A'}^2)$, ϵ being the mixing angle of SM and A'.



*conditions:

1. experimental improvement to a_e ($\sim 10^{-13}$) and α ($\sim 10^{-14}$)
2. theoretical improvement of higher-order QED,EW
3. assuming $\delta a_{\mu-e}^{\mathrm{HVP}} \simeq 0.1 \delta a_\mu^{\mathrm{HVP}}$

Conclusions



- Introduction of a sub-Gev window quantity of

$$a_{\mu-e} = a_\mu - (m_\mu^2/m_e^2)\, a_e$$

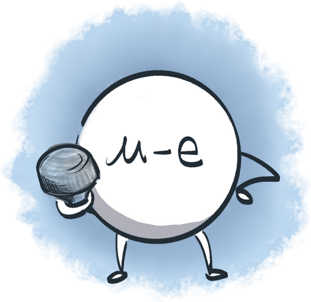
  that utilize the strong correlation of $(g-2)_\ell$ HVP contribution at high-energies
- Formulated as a modification of the kernel function in the dispersion relation
- Largely reduces sensitivity at HVP energy regime where lattice-QCD and data-driven approach presents persistant tension
- A powerful probe to low-energy hadronic effects
- Potential advantages in the field of: χPT, LbL, BSM etc.

arXiV: 2606.08329 [hep-ph]

"Lepton $g-2$ non-universality of hadronic contributions and a sub-GeV window to New Physics"

Thank you!

Questions?



Backup Slide: Kernel function approximation

AMM dispersion relation and a_e approximation:

$$a_\ell^{\mathrm{HVP}} = \frac{\alpha}{\pi^2}\int_{s_0}^{\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,\Pi(s)}{s} K_\ell(s)\,,\; K_\ell(s) = \frac{m_\ell^2}{s}\int_0^1 \mathrm{d}x\, \frac{x^2(1-x)}{1-x+x^2(m_\ell^2/s)}$$

$$a_e^{\mathrm{HVP}} \cong \frac{\alpha m_e^2}{3\pi}\Pi'(0) \equiv \frac{\alpha}{3\pi^2} m_e^2 \int_{s_0}^{\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,\Pi(s)}{s^2}.$$

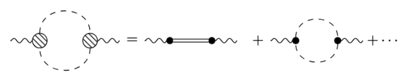
Kernel function expression comes from:

$$\begin{aligned} K_{\mu-e}(s) &= K_\mu(s) - \frac{m_\mu^2}{m_e^2} K_e(s) \\ &\cong -\frac{m_\mu^4}{s^2}\int_0^1 \mathrm{d}x\, \frac{x^4}{1-x+x^2(m_\mu^2/s)} \\ &= K_\mu(s) - \frac{m_\mu^2}{3s}. \end{aligned}$$

Backup Slide: $\pi^+\pi^-$ channel spectral function

$$\operatorname{Im}\Pi^{(\pi\pi)}(s) = \frac{\alpha}{12}\left(1-\frac{4m_\pi^2}{s}\right)^{\frac{3}{2}}|F_\pi(s)|^2\theta(s-4m_\pi^2)$$

The generic spectral function: the one-loop scalar-QED expression (cf., Peskin) augmented with the pion electromagnetic form factor, $F_\pi(q^2)$. In the VMD picture, the latter is essentially described by the ρ-meson exchange.



Two-pion contribution to HVP. The blobs represent the VMD form factor of the pion, and dashed lines are pion propagators. The double line represents ρ-meson propagator.

Backup Slide: Sub-leading channel results

Channel	$a_\mu^{\mathrm{HVP,LO}}$ $(\times 10^{-10})$	$a_e^{\mathrm{HVP,LO}}$ $(\times 10^{-14})$	$a_{\mu-e}^{\mathrm{HVP,LO}}$ $(\times 10^{-10})$
$\pi^+\pi^-\gamma$	0.711	0.237	-0.304
$\pi^0\gamma$	0.103	0.034	-0.044

Table 1: LO HVP contributions to the muon and electron anomalous magnetic moment and $(\mu - e)$ from sub-leading $\pi^+\pi^-\gamma$ and $\pi^0\gamma$ channels (with TFF).

$$
\begin{aligned}
a_{\mu-e}^{\pi^0\gamma}&(\text{without TFF})\\
=&\frac{\alpha^3 m_{\pi^0}^2}{1728\,\pi^5 f_\pi^2\, r^4}\left[r^2\left(4r^4+81r^2+66\right)-\frac{6}{\sqrt{4r^2-1}}\left(64r^4+28r^2-11\right)\arccos\frac{1}{2r}\right.\\
&\left.-\,6(2r^6+9r^4-11)\log r+18(6r^2+1)\left(\log^2 r+\arccos^2\frac{1}{2r}\right)\right]\\
=&-0.34(5)\times 10^{-11}\,.
\end{aligned}
$$

Backup Slide: Sensitivity to the fine structure constant α

Validity condition for $a_{\mu-e}$:

$$\delta\left(m_\mu^2/m_e^2 \times a_e^{\mathrm{had,exp}}\right) \leq \delta a_\mu^{\mathrm{had,theo}}$$

To satisfy the validity condition, the precision of the fine-structure constant, which is calculated based on theoretical uncertainties from data-driven and lattice method respectively, must be improved to

$$\delta\alpha \leq \begin{cases} 5.39 \times 10^{-14} & \text{dispersive} \\ 8.65 \times 10^{-14} & \text{lattice} \end{cases}$$

where the lepton mass ratio [PDG], experimental measurement of a_e [Phys. Rev. Lett. 130, 071801 (2023)], α measurement using Cs atoms [Science 360, 191 (2018)] and Rb atoms [Nature 588, 61 (2020)], the weak contribution [Rev. Mod. Phys. 97, 025002 (2025)] and QED contributions of a_e^{QED} [WP25, Atoms 7 28 (2019), Phys. Rev. D 97, 036001 (2018)] (where α is introduced).

Current α measurement uncertainty $\delta\alpha$(Cs, Rb) $\sim 10^{-12}$.

Backup Slide: New Physics

flavor-universal new physics, parametrized by a contact dimension-5 operator,

$$\mathcal{L}_{\text{universal}} = \frac{1}{\Lambda^2} \sum_{\ell=e,\mu} m_\ell \bar{\psi}_\ell \sigma_{\mu\nu} F_{\mu\nu} \psi_\ell .$$

In supersymmetric models, flavour universality does not have to hold exactly, giving two different effective values of Λ for the muon and electron:

$$a_{\mu-e}^{\mathrm{BSM}} = m_\mu^2 \left(\frac{1}{\Lambda_\mu^2} - \frac{1}{\Lambda_e^2} \right).$$

The ultimate sensitivity to $\Lambda_\mu^{-2} - \Lambda_e^{-2}$ can be better than to Λ_μ^{-2} and Λ_e^{-2} separately.

Dark photon model at low-energy limit:

$$\mathcal{L}_{d.\,ph.} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(F'_{\mu\nu})^2 + \frac{1}{2} m_{A'}^2 A'^2_\mu + \sum_\ell \bar{\psi}_\ell \gamma_\mu (i\partial_\mu - eA_\mu - e\varepsilon A'_\mu)\psi_\ell .$$

Corrections to the leptonic $a_{\mu(e)}$ can be written as [Phys. Rev. D 80, 095002 (2009)]

$$a_\ell^{A'} = \frac{\alpha}{2\pi} \times \varepsilon^2 \int_0^1 \mathrm{d}x \frac{2m_\ell^2 x^2 (1-x)}{m_\ell^2 x^2 + m_{A'}^2 (1-x)} \Rightarrow \operatorname{Im} \Pi(s) \propto \varepsilon^2 \delta(s - m_{A'}^2).$$