P-wave charmonium contribution to hidden-charm states from reanalysis of lattice QCD data

Pan-Pan Shi (石盼盼)

Instituto de Fisica Corpuscular (IFIC), Spain

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PPS, M. Albaladejo, M.-L. Du, F.-K. Guo, J. Nieves, Phys.Rev.D.111 (2025) 074043

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3. Possible charmonium-liek states
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Structure of charmonium-like states

- $\chi_{cJ}(1P)$

  0^{++}: $M =$3440 (3415.50 ± 0.19) MeV

  1^{++}: $M =$3510 (3510.67 ± 0.05) MeV

  2^{++}: $M =$3555 (3556.17 ± 0.07) MeV

- $\chi_{cJ}(2P)$

  0^{++}: $M =$3920 MeV

  ($\chi_{c0}(3860)$, $\chi_{c0}(3915)$ or $\chi_{c0}(3930)$?)

  1^{++}: $M =$3950 MeV

  ($\chi_{c1}(3872)$, $\chi_{c1}(4010)$, $\chi_{c1}(4140)$?)

  2^{++}: $M =$3980 MeV

  ($\chi_{c2}(3930)$?)

The opening of the charmed-meson channel increases the difficulty to investigate the $2P$ charmonia

There are three hypotheses:

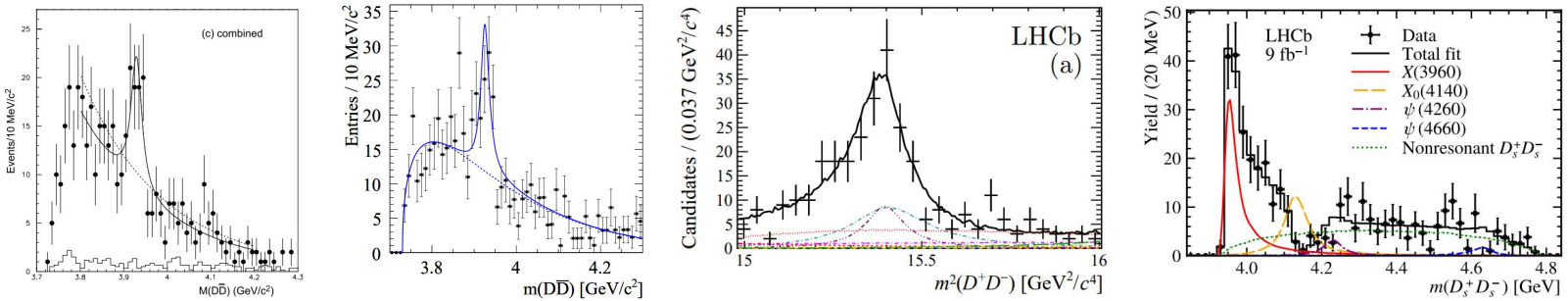
- charmonia below the $D^{(*)}\bar{D}^{(*)}$ threshold, the molecular states near the threshold
- In the Fock space, the wave function is $|\psi\rangle = |c\bar{c}\rangle + |[cq][\bar{c}\bar{q}]\rangle + |D^{(*)}\bar{D}^{(*)}\rangle + \cdots$ ($\mathcal{N}_{\text{Phy.}} = \mathcal{N}_{c\bar{c}}$)
- coexistence of hadronic molecules and quark model state

C. Hanhart, E. Klempt, Int. J. Mod. Phys. A 35(2020)2050019

Possible structures in the $D\bar{D}$–$D_s\bar{D}_s$ system

- Experimental results in the $D\bar{D}$ and $D_s\bar{D}_s$ systems

$\gamma\gamma \to D\bar{D}$ $\gamma\gamma \to D\bar{D}$ $B^+ \to D^+D^-K^+$ $B^+ \to D_s^+D_s^-K^+$ (12σ)



[Belle]PRL96,082003 [Babar]PRD81,092003 [LHCb]PRD102,112003 [LHCb]PRL131.071901

No significant signals observed in the $\eta\eta'$ spectrum in $\psi(3770) \to \gamma\eta\eta'$ (90% CL)

[BESIII]PRD108.052012

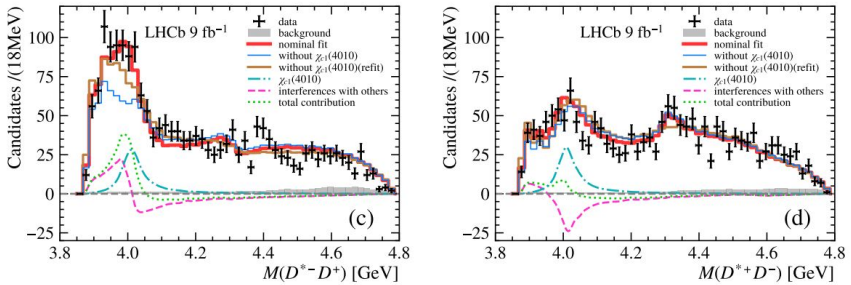
- LQCD results (1) Virtual state: $c\bar{c}, D\bar{D}, J/\psi\omega$ with $m_\pi \simeq 266, 156$ MeV C.B.Kabg, et al, JHEP09.089

  (2) two shallow states below $D\bar{D}$ and $D_s\bar{D}_s$ thresholds, and a broad resonance: $c\bar{c}, D\bar{D}, D_s\bar{D}_s, J/\psi\omega$ with $m_\pi \simeq 280$ MeV S. Prelovsek, et al, JHEP06.035

  (3) No signals near $D\bar{D}$ and $D_s\bar{D}_s$ thresholds: $\eta_c\eta, D\bar{D}, D_s\bar{D}_s, J/\psi\omega$ with $m_\pi \simeq 239 - 391$ MeV D.J. Wilson, et al, PRL132.241901,arXiv:2602.09862

Possible structures in the $D\bar{D}^*$ system with $I(J^{PC}) = 0^+(1^{++})$

- Experimental results: $X(3872)$ and a new resonance $\chi_{c1}(4010)$ with significance 16σ in the reaction $B^+ \to D^{*\pm} D^{\mp} K^+$ [LHCb]PRL133.131902



- Lattice results

  (1) A state below $D\bar{D}^*$ threshold (88±26 MeV): $c\bar{c}, D\bar{D}^*$ with $m_\pi \simeq 280$ MeV G.S. Bali, et al, PRD84.094506

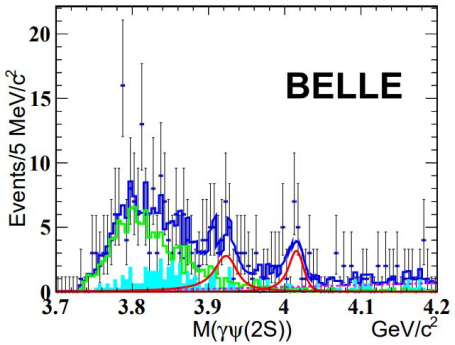
  (2) A bound state (11±7 MeV): $c\bar{c}, D\bar{D}^*$ with $m_\pi \simeq 266$ MeV S. Prelovsek, et al, PRL111.192001

  (3) ΔE (8±15/9±8 MeV): $c\bar{c}, D\bar{D}^*$ (+/- tetraquark operators) with $m_\pi \simeq 280$ MeV M. Padmanath, et al, PRD92.034501

  (4) A shallow bound state and resonance [$\sim \chi_{c1}(4010)$]: $c\bar{c}, D\bar{D}^*, J/\psi\omega$ with $m_\pi \simeq 250 - 417$ MeV H. Li, et al, arXiv:2402.14541

Possible structures in the $D^*\bar{D}^*$ system with $I(J^{PC}) = 0^+(2^{++})$

- Experimental results: considering $\chi_{c2}(3930)$ as $\chi_{c2}(2P)$, the possible hint of the $D^*\bar{D}^*$ molecule X_2 reported in the reaction $\gamma\gamma \to \gamma\psi(2S)$ with significance 2.8σ [Belle]PRD105.112011



Possible search for X_2 in the $\gamma J/\psi$ invariant mass distribution (events > 35) by Belle in the same reaction (only 18 ± 7 event in $\gamma\psi(2S)$ channel) PPS et al, PLB843.137987

More X_2 events can be expected in the $\gamma J/\psi$ ($\gamma\psi(2S)$) invariant mass distribution in Super tau-charm Facility PPS et al, CPL41.031301

- Lattice results

  A resonance far below $D^*\bar{D}^*$ threshold ($\chi_{c2}(3930)$): $\eta_c\eta, D\bar{D}, D_s\bar{D}_s, D\bar{D}^*, D_s\bar{D}_s^*, \psi\omega, D^*\bar{D}^*, \psi\phi$ with $m_\pi = 391$ MeV D.J. Wilson et al, PRL132.241901

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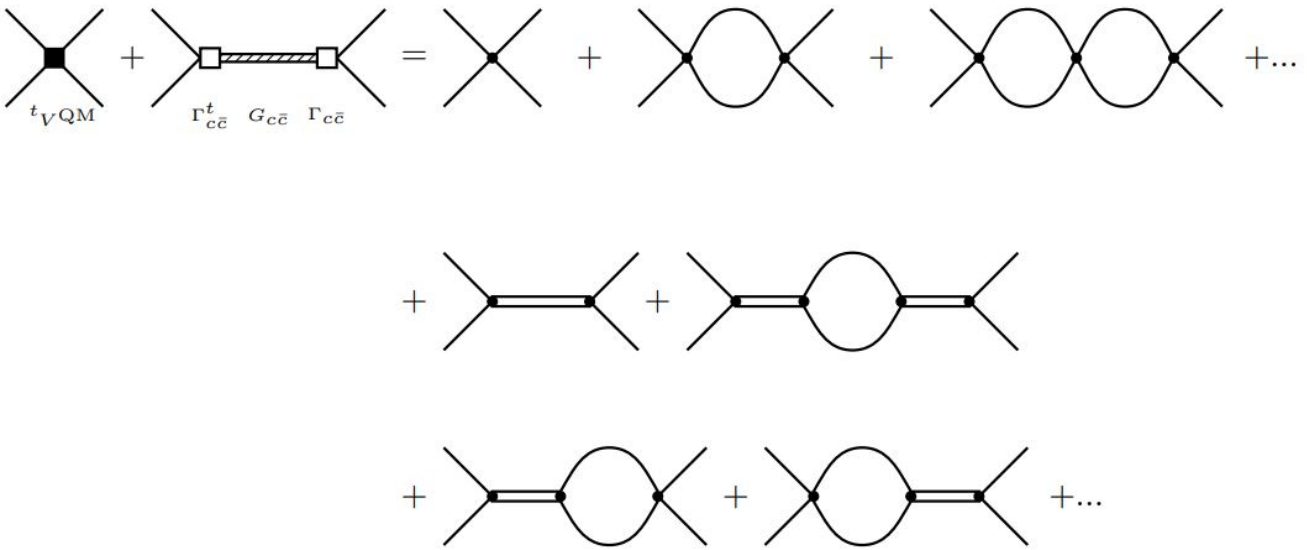
T-matrix in the finite and infinite volumes

- T-matrix in the infinite volume (Lippmann-Schwinger equation)

$$T_{ij}(E) = F_\Lambda(p_i)\left(V(E)^{-1} - G(E)\right)^{-1}_{ij} F_\Lambda(p'_j),$$

where the Gaussian form factor $F_\Lambda(p_i) = \exp(-p_i^2/\Lambda^2)$ for charmed-meson channels is used to ensure unitarity of S-matrix. The Green's functions for the charmed pair and charmonium are

$$G^{ii}_{\rm QM}(E) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{e^{-2\vec{q}^{\,2}/\Lambda^2}}{E - m_{i,1} - m_{i,2} - \vec{q}^{\,2}/2\mu_i + i\epsilon}, \quad G_{c\bar{c}}(E) = \left(E - \mathring{m}_{\chi_{cJ}}\right)^{-1}.$$



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- Finite-volume energy levels: $\det[\mathbb{I} - V(E_n(L))\widetilde{G}(E_n(L), L)] = 0$

$$\widetilde{G}^{ii,N}_{\rm QM}(E) = \frac{1}{\gamma L^3} \sum_{\vec{n}\in\mathbb{Z}^3} \frac{e^{-2\vec{k}_i^{\,2}/\Lambda^2}}{E - w_i(\vec{k})},$$

where the Lorentz boost factor is $\gamma = \sqrt{E_{\rm cm}^2 + \vec{P}^2}/E_{\rm cm}$ with $\vec{P} = 2\pi\vec{d}/L$, and

$$\vec{k}_i = \frac{2\pi}{L}\vec{\gamma}^{-1}\vec{n}\,' = \frac{2\pi}{L}\vec{\gamma}^{-1}\left[\vec{n} - \frac{\vec{d}}{2}\left(1 + \frac{m_{i,1}^2 - m_{i,2}^2}{E_{\rm cm}^2}\right)\right].$$

Low-energy interaction in the HQSS

At LO, the interactions between charmed mesons are described by the contact term [C.Hidalgo-Duque et al]PRD87.076006; [E.Cincioglu et al]EPJC76.576

- The isoscalar 0^{++} systems: $\{D\overline{D},\ D_s\overline{D}_s, \chi_{c0}(2P)\}$

$$V(0^{++}) = \begin{pmatrix} C_{0a} & \frac{1}{\sqrt{2}}(C_{0a}-C_{1a}) & -\frac{\sqrt{3}}{2}d \\ \frac{1}{\sqrt{2}}(C_{0a}-C_{1a}) & \frac{1}{2}(C_{0a}+C_{1a}) & -\sqrt{\frac{3}{8}}d \\ -\frac{\sqrt{3}}{2}d & -\sqrt{\frac{3}{8}}d & 0 \end{pmatrix},$$

- The isoscalar $1^{++}/2^{++}$ systems: $\{D^*\overline{D}/D^*\overline{D}^*,\ \chi_{c1}(2P)/\chi_{c2}(2P)\}$

$$V(1^+/2^+) = \begin{pmatrix} C_{0X} & d' \\ d' & 0 \end{pmatrix},$$

The LECs are defined as $C_{0a} = 2C_a^{(1)}/3 + C_a^{(8)}/3$, $C_{1a} = C_a^{(8)}$; d and d' are the effective coupling between charmonium and charmed pair.

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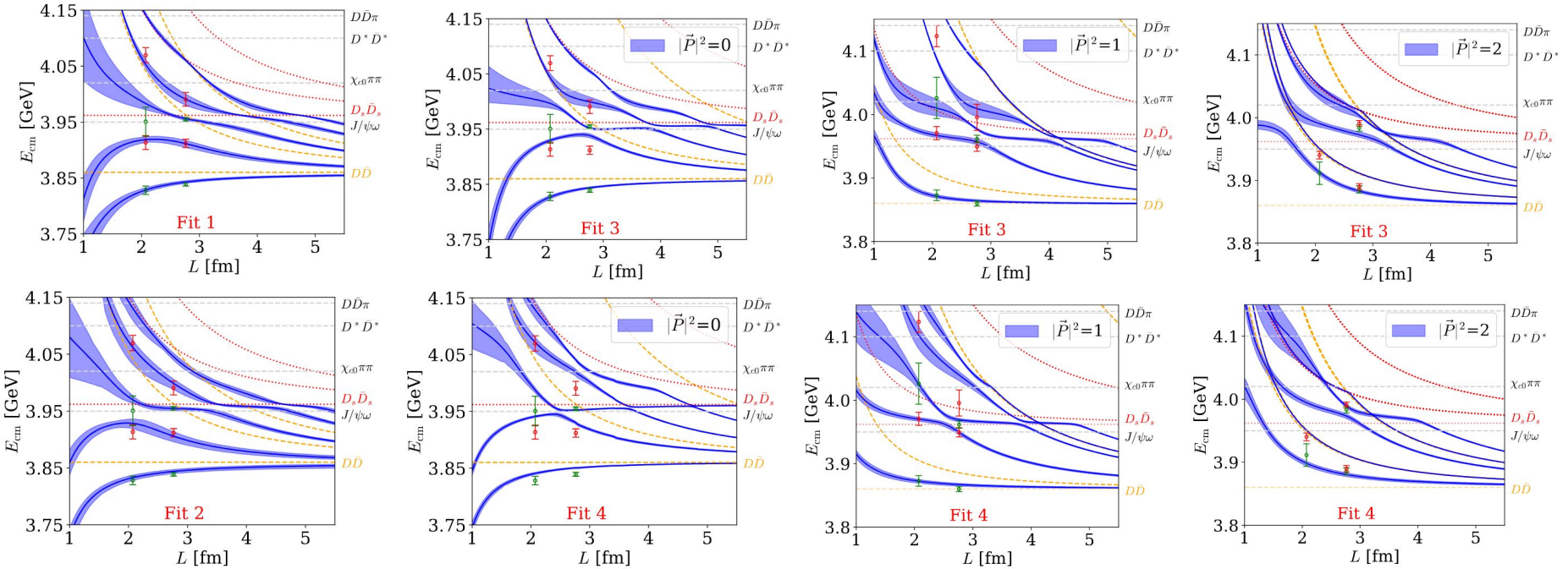
Determination of the LECs in the $D\bar{D}$–$D_s\bar{D}_s$ system

The LECs C_{0a}, C_{1a}, d and $\mathring{m}_{\chi_{c1}}$ are constrained by the Lattice data in s.

Prelovsek, et al, JHEP06.035



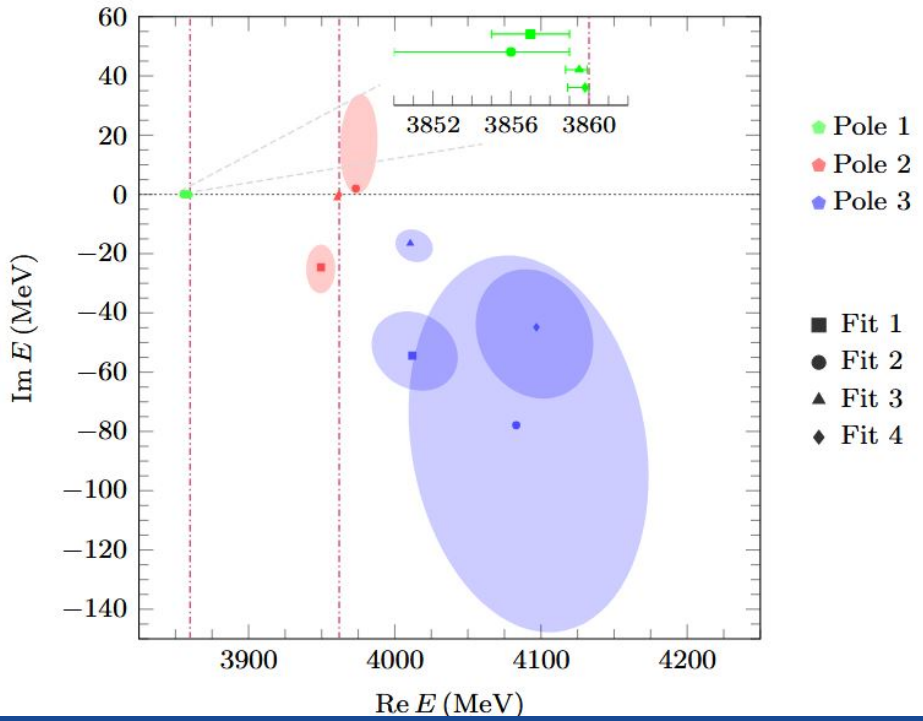
Fit 1: $\Lambda = 0.5$ GeV in the rest frame; Fit 2: $\Lambda = 1.0$ GeV in the rest frame; Fit 3: $\Lambda = 0.5$ GeV in the rest and moving frames; Fit 4: $\Lambda = 1.0$ GeV in the rest and moving frames.

Poles in the $I(J^{PC}) = 0(0^{++})$ system

- Resulting parameters

	$\mathring{m}_{\chi_{c0}}$[GeV]	d[fm$^{1/2}$]	C_{0a}[fm^2]	C_{1a}[fm^2]	d.o.f.	χ^2/d.o.f.
Fit 1	3.94(2)	0.88(18)	−0.23(52)	−0.64(28)	4	0.86
Fit 2	4.03(5)	0.58(28)	−0.44(24)	−0.41(13)	4	1.63
Fit 3	3.99(1)	0.53(22)	−1.14(29)	−1.03(19)	18	3.26
Fit 4	4.08(4)	0.44(15)	−0.47(10)	−0.55(6)	18	3.27

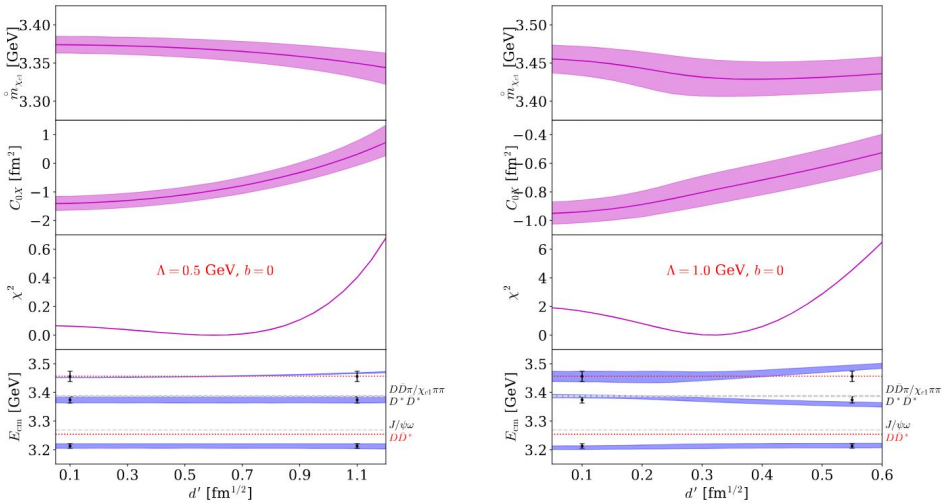
- Pole positions



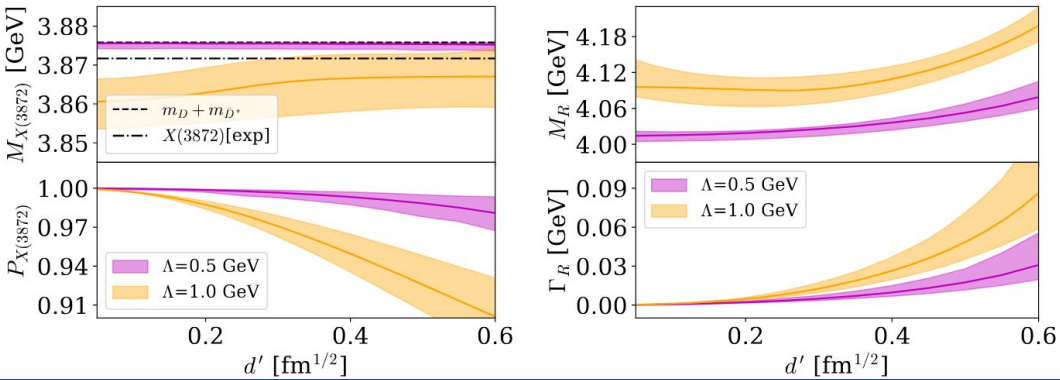
- Pole 1: a bound (Fit 1-3)/virtual (Fit 4) state with the molecular component $P_{D\bar{D}} > 80\%$
- Pole 2: On the $(+,-)$ RS for Fit 2 and $(-,+)$ RS for other cases; relevant to $X(3960)$
- Pole 3: a broad resonance on $(-,-)$ RS, relative to $\chi_{c0}(2P)$

Poles in the $I(J^{PC}) = 0(1^{++})$ system

- Parameters S. Prelovsek, et al. PRL111.192001



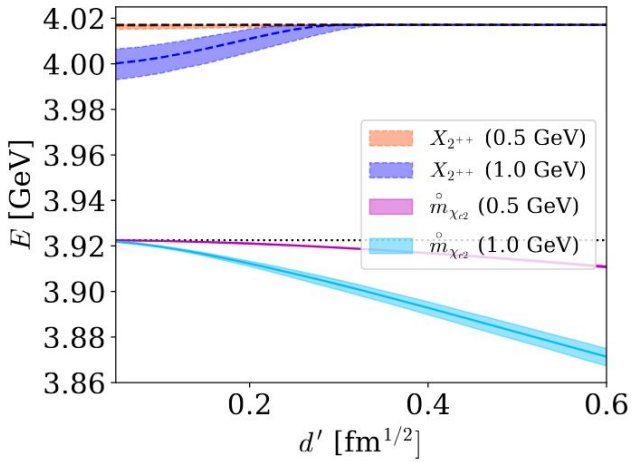
- Pole position in the physical pion mass; shift $\mathring{m}_{\chi_{c1}}$ to $\mathring{m}_{\chi_{c1}} + m_{\rm av}^{\rm exp} - m_{\rm av}^{\rm L}$ [640(20) MeV] with $m_{\rm av}$ the averaged mass of J/ψ and η_c

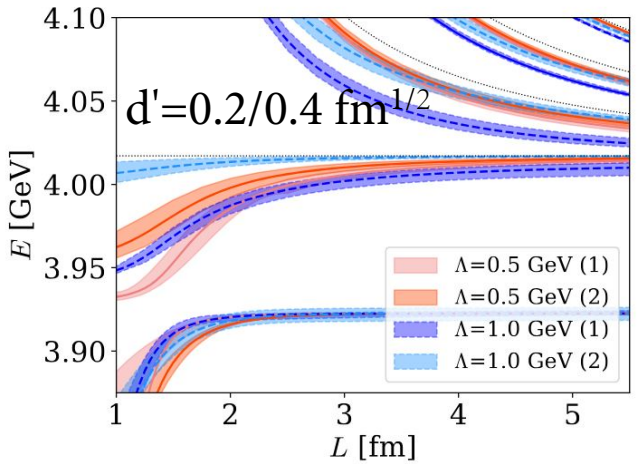


- Pole 1: $X(3872)$ with molecular component $P_{D\bar{D}^*} > 90\%$
- Pole 2: $\chi_{c1}(4010)$ is a candidate of $\chi_{c1}(2P)$ with $M = 4012.5^{+3.6+4.1}_{-3.9-3.7}$ MeV and $\Gamma = 62.7^{+7.0+6.4}_{-6.4-6.6}$ MeV

Poles in the $I(J^{PC}) = 0(2^{++})$ system

The LEC C_{0X} is fixed to be the same as that in $D\bar{D}^*$ system and bare mass $\mathring{m}_{\chi_{c2}}$ is tuned to reproduce the physical $\chi_{c2}(3930)$.





- The $D^*\bar{D}^*$ bound state is significant close to the $D^*\bar{D}^*$ threshold in the large d' region
- The mass of $\chi_{c2}(2P)$ is smaller the $D^*\bar{D}^*$ bound state
- $m_{\chi_{c2}(2P)} < m_{\chi_{c1}(2P)}$: the coupled-channel effect pushes the heavier state up and the lighter state down

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Summary

Based on the reanalysis of the LQCD data of $D\bar{D}$–$D_s\bar{D}_s$ coupled-channel and $D\bar{D}^*$ systems, the possible isoscalar charmonium-like states with J^{PC} the 0^{++}, 1^{++}, and 2^{++} are predicted:

- 0^{++}: two molecules close to $D\bar{D}$ and $D_s\bar{D}_a$ thresholds; a broad resonance above $D_s\bar{D}_s$ threshold identified as $\chi_{c0}(2P)$
- 1^{++}: the $D\bar{D}^*$ bound state $X(3872)$ with compositeness exceeding 90%; the charmonium $\chi_{c1}(2P)$ corresonds to $\chi_{c1}(4010)$
- 2^{++}: a bound state near $D^*\bar{D}^*$ threshold; the $\chi_{c2}(3930)$ is considered as the charmonium $\chi_{c2}(2P)$

Our results are compatible with the hypothesis that coexistence of hadronic molecules and charmonia

Thanks for your attention!

Backup

To check the effect of the energy dependence, we verify C_{0X} to $C_{0X} + bp^2/(2\mu)^2$