Data-driven results for the light-quark connected contribution to the muon g - 2

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based on: 2203.05070 2211.11055 2306.16808 2311.09523 2411.06637 (with e^+e^- data)
and 2605.12205 (tau-based data):

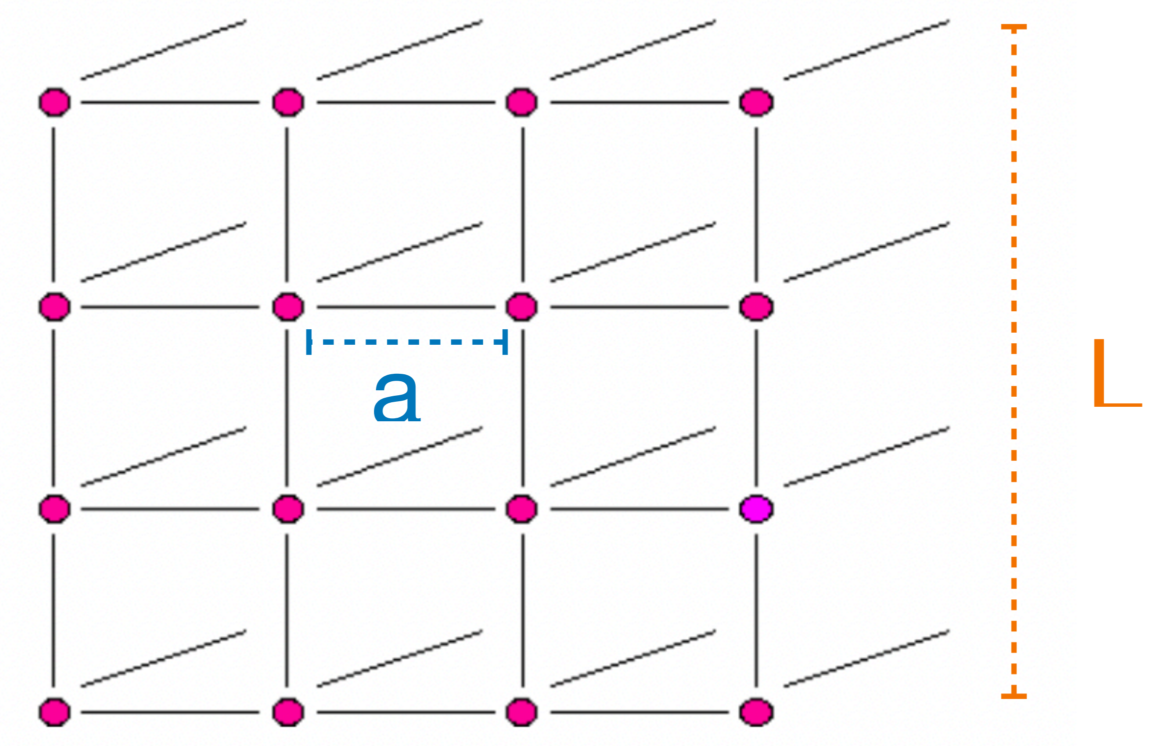
with: N. Allen, G. Benton, M. Golterman, A. Keshavarzi, K. Maltman, L. Mansur, and S. Peris

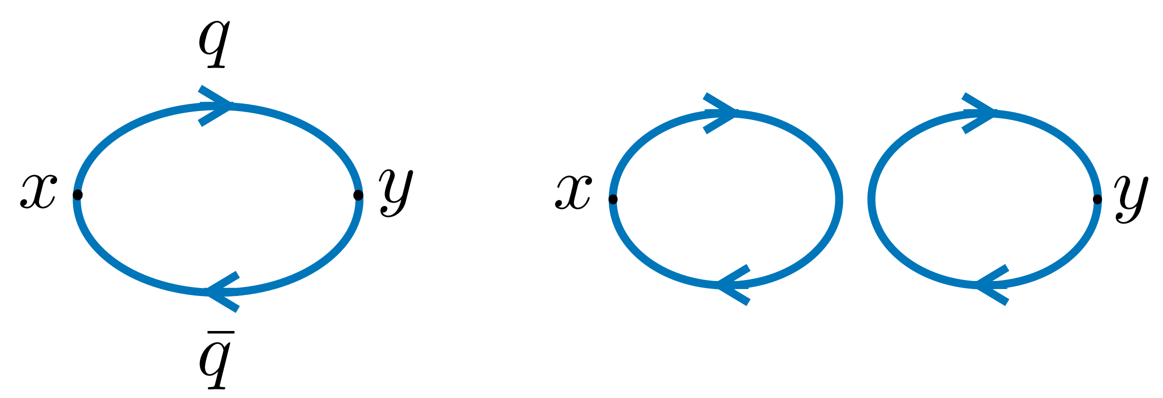
June 8, 2026

Motivation

- Competitive *data-driven* results for the light-quark connected (lqc) and full strange + light-quark disconnected (s+lqd) contributions to $g-2$
- Shed light on the origin of the discrepancies observed in $g-2$ comparing data-driven and lattice QCD results for the HVP contribution

Lattice QCD

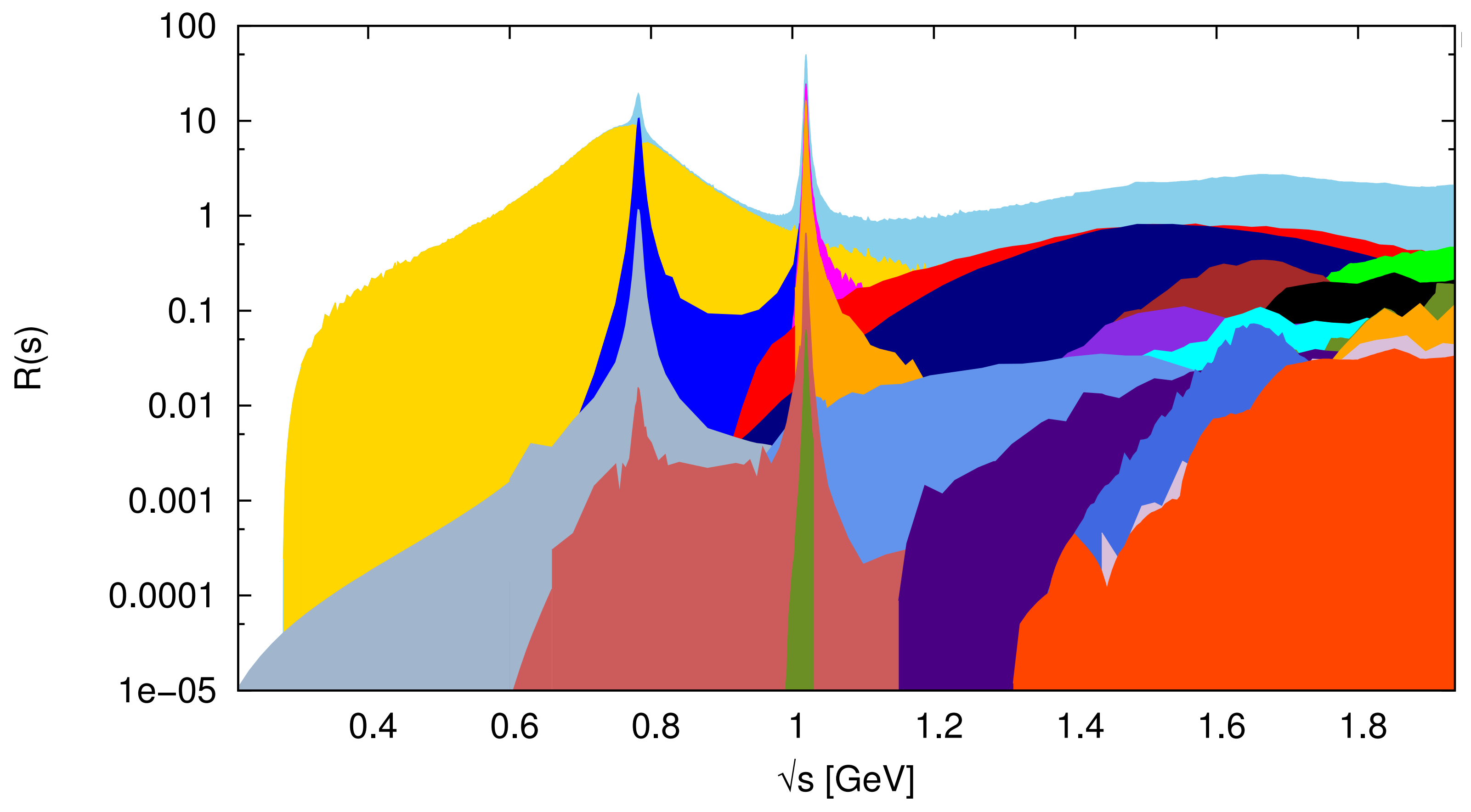




(quark-level) connected/disconnected contributions

Dispersive

Keshavarzi, Nomura, Teubner '19 (KNT19)



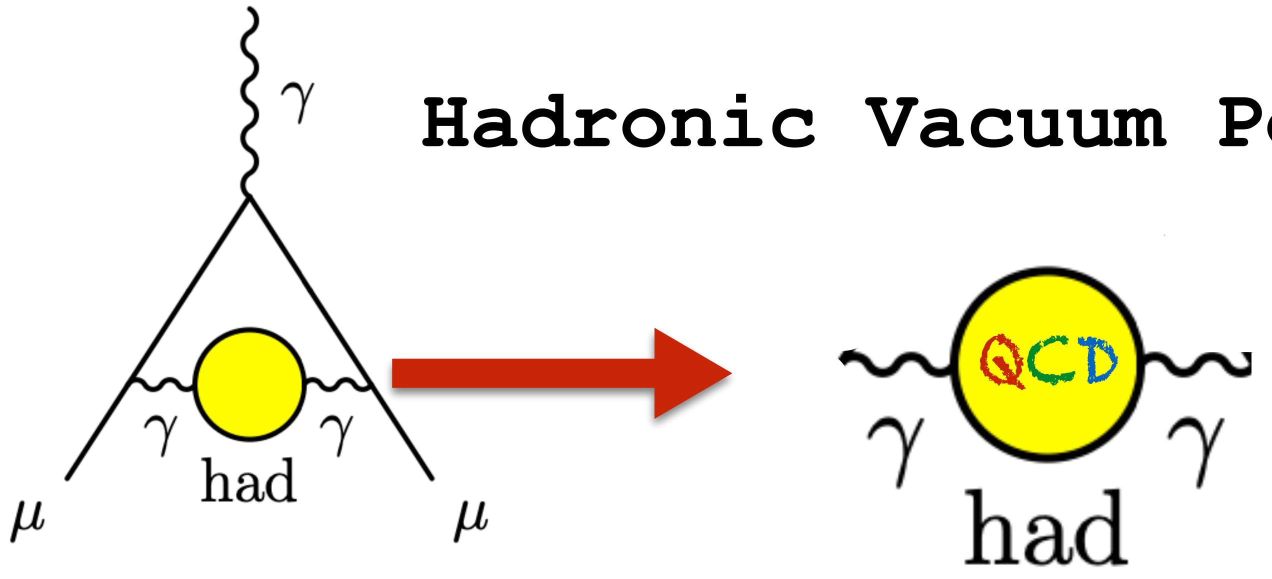
Use isospin symmetry to isolate in the data-driven approach the lqc and s+lqd contributions in order to compare with lattice results [correcting for isospin breaking (IB).] Already published using e^+e^- data.

DB, Golterman, Maltman, Peris '22 '23; Benton, DB, Golterman, Keshavarsi, Maltman, Peris '24

This talk: what do tau-data give for the lqc contribution? Tau-based results (including isospin-breaking corrections) in order to compare with isospin symmetric lattice results

Allen, DB, Golterman, Maltman, Mansur, Peris 2605.12205

Hadronic vacuum polarization



Hadronic Vacuum Polarization (HVP)

$$(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2) = i\int d^4x\, \langle 0|T(j_\mu^{\rm EM}(x) j_\nu^{\rm EM}(0)|0\rangle$$

$$j_\mu^{\rm EM} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$$

Dispersive (data-driven)

Usual dispersive representation

$$\Pi(q^2) - \Pi(0) = q^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{\frac{1}{\pi}{\rm Im}\Pi(s)}{s(s-q^2+i\epsilon)}$$

Optical theorem relates the imaginary part to

$$R(s) = \frac{3s}{4\pi\alpha}\sigma^{(0)}[e^+e^- \to \text{hadrons}(+\gamma)]$$

Leading order contribution to $a_\mu^{\rm HVP}$

$$a_\mu^{\rm HVP} = \frac{4\alpha^2 m_\mu^2}{3}\int_{m_\pi^2}^{\infty} ds\, \frac{\hat{K}(s)}{s^2}\rho_{\rm EM}(s)$$

$$\rho_{\rm EM}(s) = \frac{1}{12\pi^2}R(s)$$

Brodsky & de Rafael, '68
Lautrup & de Rafael '68

Lattice QCD

$$C(t) = \frac{1}{3}\sum_{i=1}^{3}\int d^3x \langle j_i^{\rm EM}(\vec{x},t) j_i^{\rm EM}(0)\rangle$$
$$= \frac{1}{2}\int_{m_\pi^2}^{\infty} ds\,\sqrt{s}\,e^{-\sqrt{s}t}\,\rho_{\rm EM}(s) \quad (t>0)$$

Bernecker and Meyer '11

Leading order contribution to $a_\mu^{\rm HVP}$

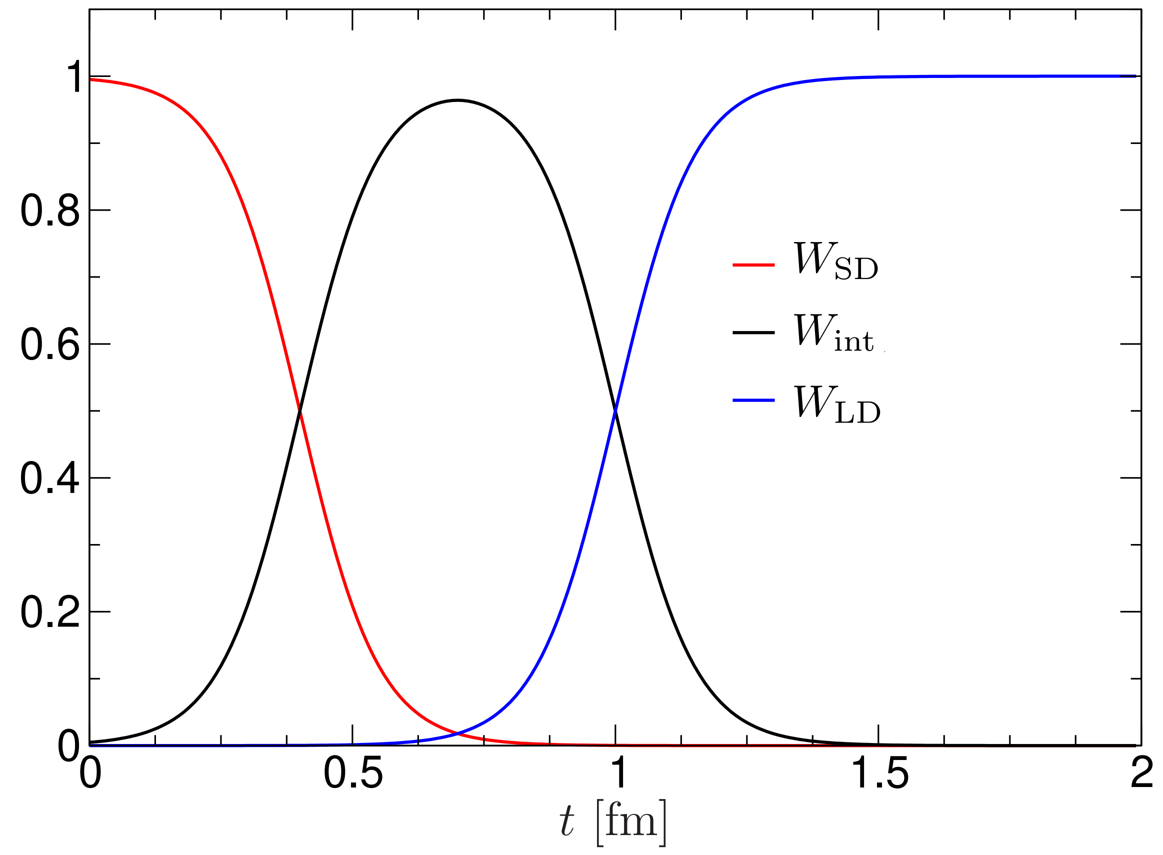
$$a_\mu^{\rm HVP} = 2\int_0^\infty dt\, w(t) C(t)$$

$$\frac{\hat{K}(s)}{s^2} = \frac{3\sqrt{s}}{4\alpha^2 m_\mu^2}\int_0^\infty dt\, w(t)\, e^{-\sqrt{s}t}$$

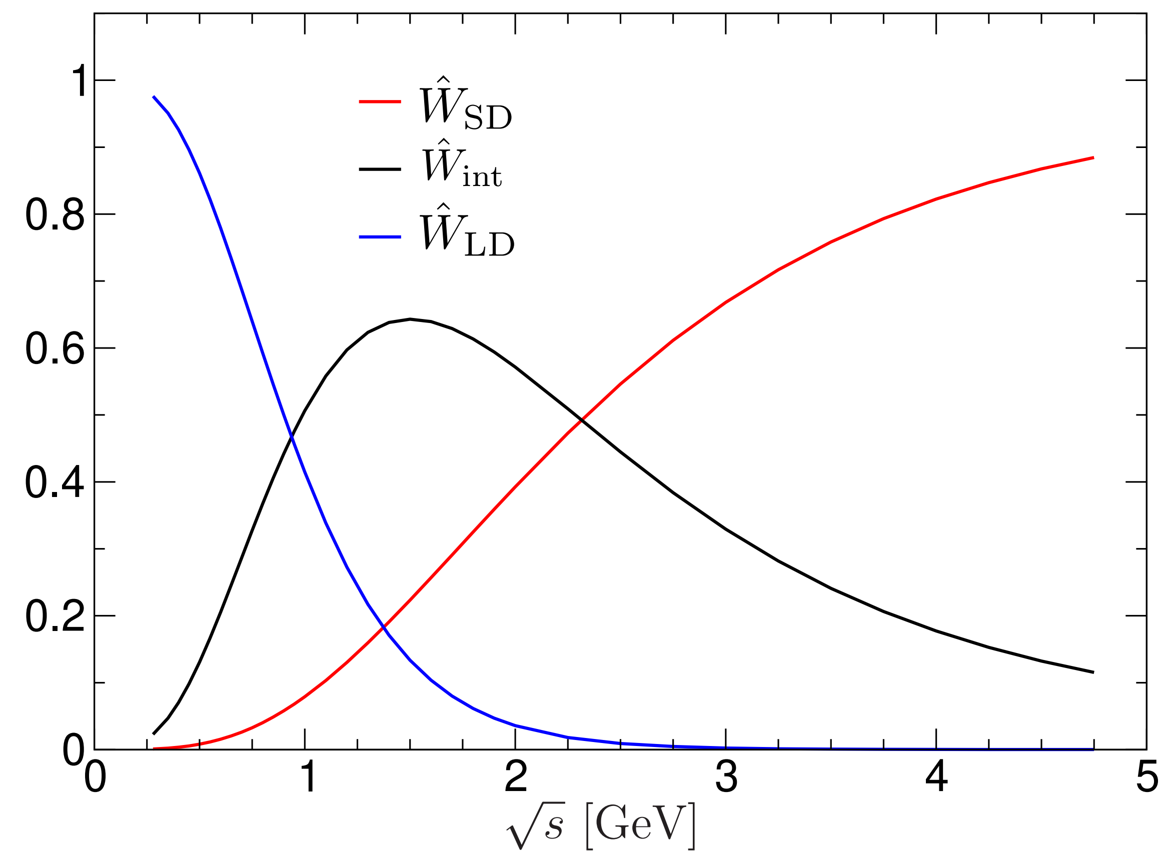
RBC/UKQCD windows

$$a_\mu^W(t_0,t_1;\Delta) = 2\int_0^\infty dt\, W(t;t_0,t_1;\Delta)\, w(t) C(t) = \frac{4\alpha^2 m_\mu^2}{3}\int_{m_\pi^2}^\infty ds\, \frac{\hat{K}(s)}{s^2}\, \hat{W}(s;t_0,t_1;\Delta)\, \rho_{\rm EM}(s)$$

lattice QCD — dispersive



$W(t)$



$\hat{W}(s)$

short-distance — intermediate — long-distance

$$a_\mu^{\rm HVP,LO} = [a_\mu^{\rm HVP,LO}]^{\rm SD} + [a_\mu^{\rm HVP,LO}]^{\rm int} + [a_\mu^{\rm HVP,LO}]^{\rm LD}$$

Blum et al, 1801.07224

Intermediate window:

- Cuts out lattice artifacts at short and long distances (lattice spacing, finite volume). Can be obtained very precisely on the lattice.
- Eight recent high-precision lattice QCD results
- lqc component accounts for about 90% of the total

Short- and long-distance windows:

- More difficult to calculate on the lattice
- Four recent high-precision results for the lqc component of the SD window (BMW, Mainz, ETMC, RBC/UKQCD)
- Four recent results for the LD window
- Data-driven results: this talk

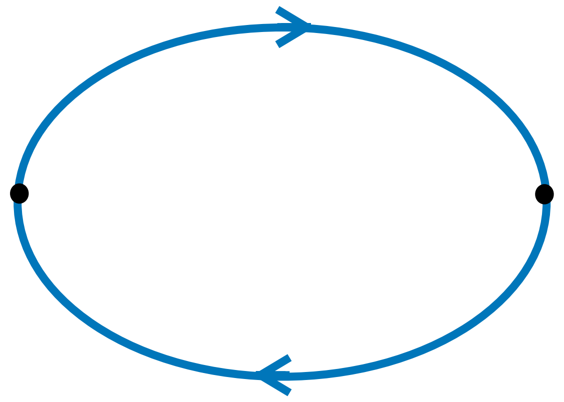
The basic strategy and e^+e^--based results

The basic strategy

EM current: *u*, *d*, and *s* quarks

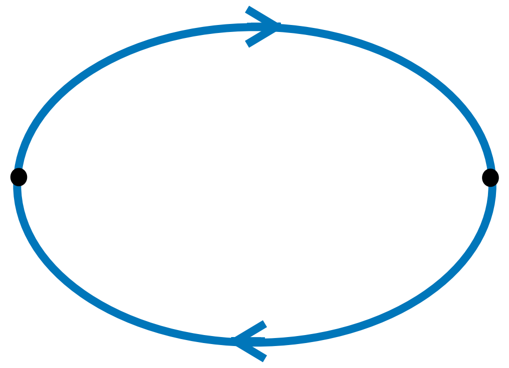
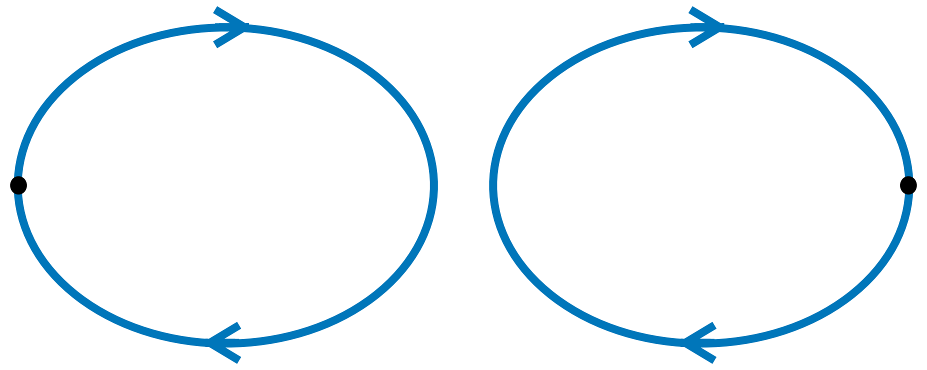
$$j_\mu^{\rm EM} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s = \underbrace{\frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)}_{I\,=\,1} + \underbrace{\frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)}_{I\,=\,0} - \underbrace{\frac{1}{3}\bar{s}\gamma_\mu s}_{s\text{ quark}}$$

Considering the correlator of *I* =1 quark currents, in the isospin limit

$$\frac{1}{4}\langle(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(x)\,(\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d)(y)\rangle = \frac{1}{2}\; x$$  y

isospin 1 is purely light-quark connected

For *I* = 0 light-quark current in the isospin limit, correlator contains connected and disconnected terms

$$\frac{1}{36}\langle(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)(x)\,(\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d)(y)\rangle = \frac{1}{18}\; x$$  $y + \frac{1}{9}\; x$  y

s-quark + light-quark disconnected (s+lqd)

$$\hat{\Pi}^{\rm s+lqd}_{\rm EM} \equiv \hat{\Pi}^{I=0}_{\rm EM} - \frac{1}{9}\hat{\Pi}^{I=1}_{\rm EM}$$

$$a_\mu^{\rm s+lqd} = a_\mu^{I=0} - \frac{1}{9}a_\mu^{I=1}$$

light-quark connected (lqc)

$$\hat{\Pi}^{\rm lqc}_{\rm EM} \equiv \frac{10}{9}\hat{\Pi}^{I=1}_{\rm EM}$$

$$a_\mu^{\rm lqc} = \frac{10}{9}a_\mu^{I=1}$$

Main task: separate the *I* =1/0 components of all exclusive modes

Modes with/without well defined G parity

Unambiguous modes

$$G = (-1)^{I+1}$$

Modes with well defined *G*-parity and $I = 1$ give the dominant contribution to lqc results.

$I = 1$ modes X	$[a_\mu^{\rm HVP}]_X \times 10^{10}$
low-s $\pi^+\pi^-$	0.867(18)
$\pi^+\pi^-$	503.5(1.9)
$2\pi^+2\pi^-$	14.87(20)
$\pi^+\pi^-2\pi^0$	19.39(78)
$3\pi^+3\pi^-$ (no ω)	0.231(15)
$2\pi^+2\pi^-2\pi^0$ (no η)	1.35(17)
$\pi^+\pi^-4\pi^0$ (no η)	0.21(21)
$\eta\pi^+\pi^-$	1.340(50)
$\eta 2\pi^+2\pi^-$	0.076(11)
$\eta\pi^+\pi^-2\pi^0$	0.119(20)
$\omega(\to \pi^0\gamma)\pi^0$	0.882(19)
$\omega(\to$ npp$)3\pi$	0.168(32)
$\omega\eta\pi^0$	0.242(53)
Total ($I = 1$)	543.2(2.1)

KNT19 combination

Ambiguous modes

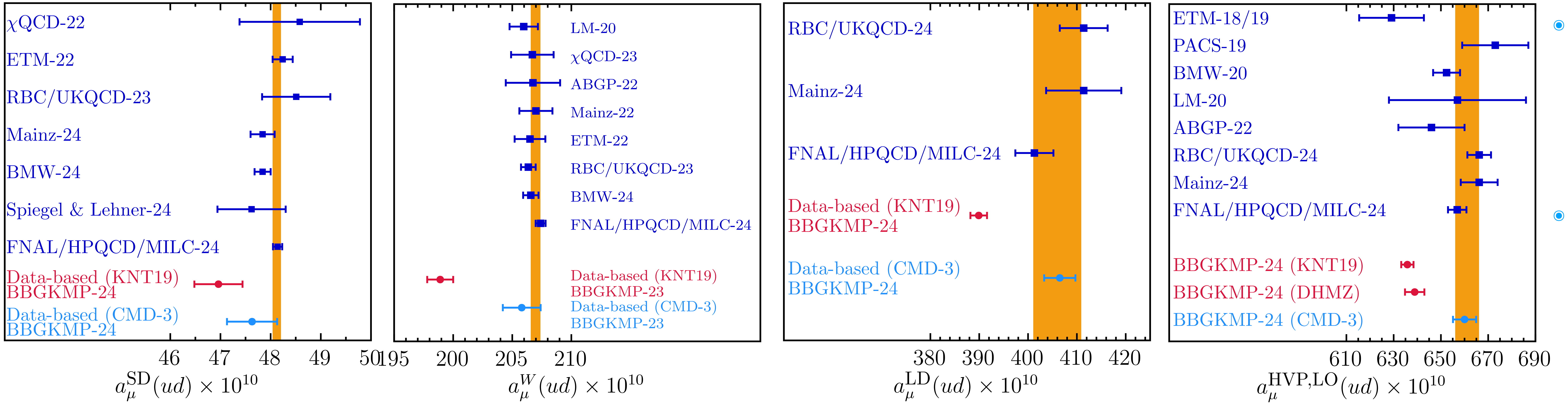
Several ambiguous modes, but external information can help separating $I = 1/0$ in the most relevant channels

- $K\bar{K}$: tau decay data [BaBar]
  $K\bar{K}\pi$: Dalitz plot analysis [BaBar]
- $\pi^0\gamma$ and $\eta\gamma$ $I = 1/0$/MI separation from VMD representation ($\rho + \omega + \phi$ contributions saturate exp. cross sections)
- Other ambiguous modes (very small contributions), maximally conservative separation: 50/50 with 100% uncertainty
  $(K\bar{K}2\pi)_{\rm no-\phi}, K\bar{K}3\pi, n\bar{n}, p\bar{p}, \ldots$

Other contributions

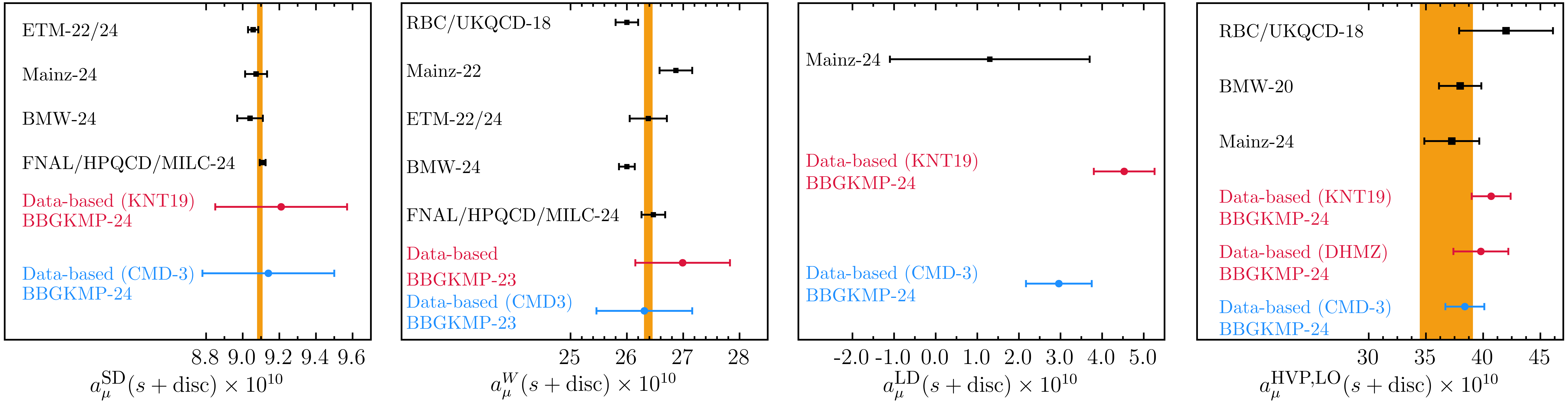
- For energies above $\sqrt{s} > 1.937$ GeV use **perturbative QCD** + duality violations
- Small **isospin-breaking** contributions have to be subtracted to compare with lattice isospin-symmetric results (dispersive analyses for strong IB and lattice results for EM corrections).

e^+e^--based results



- lqc contributions with pre-CMD-3 results show tension with the lattice
- CMD-3 data for 2π contribution fixes the results

- s+lqd contribution in good agreement with lattice
- larger errors

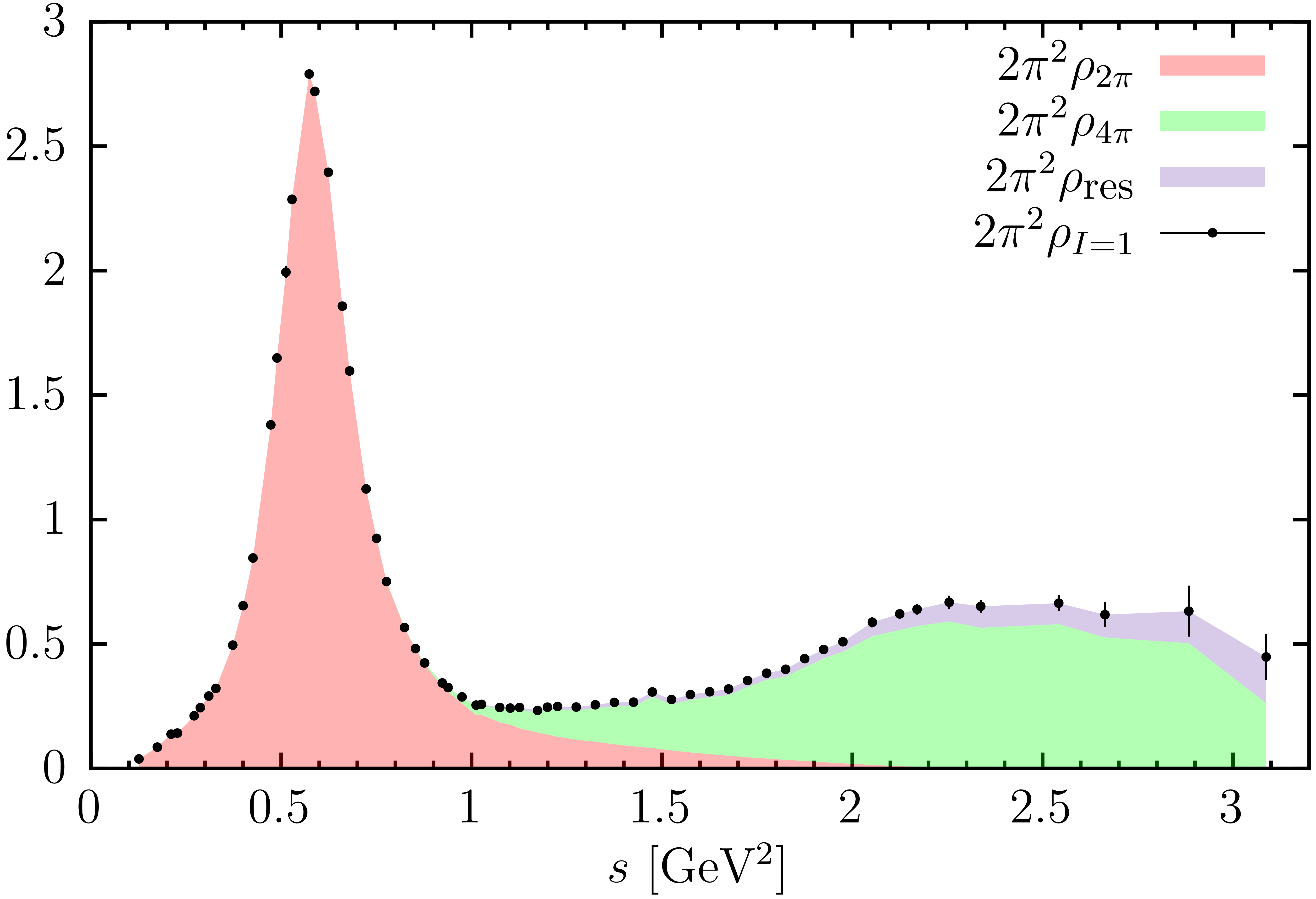


Benton, DB, Golterman, Keshavarsi, Maltman, Peris 2411.06637

tau-based results

tau-based $I = 1$ spectral function

DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147



- inclusive tau-based I=1 spectral function
- combination of ALEPH, OPAL and Belle data for the 2π channel
- combination of ALEPH and OPAL data for 4π channels
- residual modes obtained mostly from e+e- results using conserved-vector current (tiny IB corrections)
- talk by Lucas Mansur in this workshop

- Version 1: use the tau-based $I = 1$ spectral function with pt. QCD after $s_{\rm switch} = (1.76\ \text{GeV})^2 = 3.08\ \text{GeV}^2$.
- Version 2: replace 2π and/or 4π channels with tau-based results (rest from e+e-) with $s_{\rm switch} = (1.937\ \text{GeV})^2 = 3.75\ \text{GeV}^2$.

IB for tau-based results

Isospin breaking (IB): taking physical $I = 1$ results to the isospin-symmetric world

Isospin limit of QCD defined as one in which all pions have a mass equal to the physical neutral-pion mass.

To first order in IB, corrections are purely electro-magnetic (EM) in origin. The reason is that the $I = 1$current is G-parity positive, while the strong IB mass operator is G-parity negative (two insertions are needed).

Situation is slightly less complicated than the usual corrections necessary to compare tau-based results with e+e- results (no $\rho - \omega$ interference, for example, since $I = 1$).

We assume that the IB correction is dominated by the 2π channel (ρ resonance enhancement plus this channel strongly dominates the data part)

IB for tau-based results

IB factor:

$$F_{\rm IB}(s) = \frac{S_{\rm EW}}{S_{\rm EW}^{\pi\pi}} \frac{1}{G_{\rm EM}(s)} \frac{\beta^3_{\pi^0\pi^0}(s)}{\beta^3_{\pi^-\pi^0}(s)} \frac{|f_+^{\rm model}(s)[m_{\pi^-} \to m_{\pi^0};\ m_{K^-,K^0} \to m_{K^{\rm iso}}]|^2}{|f_+^{\rm model}(s)[\text{real world}]|^2}$$

phase space

$G_{\rm EM}(s)$: Radiative corrections. Recent dispersive results. Colangelo, Cottini, Hoferichter, Holz '26

$S_{\rm EW}$: Inclusive short-distance electroweak correction.

$S_{\rm EW}^{\pi\pi}$: Short-distance electroweak correction to $\tau \to \pi^-\pi^0\nu_\tau$ channel.

$f_+^{\rm model}(s)$: ChPT inspired mode (used in many papers before).
Cirigliano, Ecker, Neufeld '01, Guerrero & Pich '97, Castro, Miranda & Roig '25

$$f_+^{\rm model}(s) = \frac{m_\rho^2}{m_\rho^2 - s - i m_\rho \Gamma_\rho(s)} \exp[2\tilde{H}_{\pi^-\pi^0}(s) + \tilde{H}_{K^-K^0}(s)] \qquad \Gamma_\rho(s) = \frac{m_\rho s}{96\pi f_\pi^2}\left(\beta^3_{\pi^-\pi^0}(s)\theta(s-(m_{\pi^+}+m_{\pi^0})^2) + \frac{1}{2}\beta^3_{K^-K^0}(s)\theta(s-(m_{K^-}+m_{K^0})^2)\right)$$

IB correction uncertainties: EM corrections to the ρ width (see WP25) , uncertainty on $G_{\rm EM}(s)$, plus 50% of the total IB correction (which covers corrections to the ρ mass). Final uncertainty in IB corrections ~ 80%.

tau-based $I = 1$ spectral function

- Version 1: use the tau-based $I = 1$ spectral function with pt. QCD after $s_{\rm switch} = (1.76\ {\rm GeV})^2 = 3.08\ {\rm GeV}^2$.

Mode	SD	W1	LD	HVP
$I = 1$ τ-based data	25.61(28)	192.36(88)	406.2(2.5)	624.2(3.2)
PT+DV	22.26(21)	14.58(39)	0.669(38)	37.52(63)
EM IB	−0.13(11)	−1.40(1.1)	−2.2(1.9)	−3.7(3.1)
lqc total	47.74(37)	205.55(1.5)	404.7(3.1)	657.9(4.5)
WP25	48.123(83)	206.97(41)	406.0(4.9)	659.5(4.7) (Avg. A) 661.1(5.0) (Avg. B)

- Version 2: replace 2π and/or 4π channels with tau-based results (rest from e+e-) with $s_{\rm switch} = (1.937\ {\rm GeV})^2 = 3.75\ {\rm GeV}^2$.

Mode	SD	W1	LD	HVP
τ-based 2π+4π	24.49(28)	189.17(87)	405.4(2.5)	619.1(3.2)
KNT19 2π tail	0.0535(40)	0.0964(72)	0.0090(67)	0.159(12)
KNT19 4π tail	1.177(24)	2.122(44)	0.1981(42)	3.498(73)
Unamb. non $2\pi/4\pi$	1.383(98)	3.17(19)	0.578(23)	5.13(32)
Amb. modes	0.75(32)	1.88(61)	0.613(91)	3.2(1.0)
PT+DV	20.28(10)	11.06(16)	0.346(11)	31.68(28)
EM IB	−0.13(11)	−1.4(1.1)	−2.2(1.9)	−3.7(3.1)
lqc total	48.00(46)	206.1(1.5)	405.0(3.1)	659.1(4.5)

tau-based light-quark connected results

Light-quark connected contributions to the HVP

Allen, DB, Golterman, Maltman, Mansur, Peris 2605.12205
Benton, DB, Golterman, Keshavarsi, Maltman, Peris 2411.06637
lattice averages (vertical bands) from g-2 TI WP26 2505.21476

- tau-based results in very good agreement with the lattice results as well as with CMD-3-based e+e- results
- Small differences in the different tau-based results stem mainly from discrepancies in the 4π channels between tau and e+e- results (Pais relations)

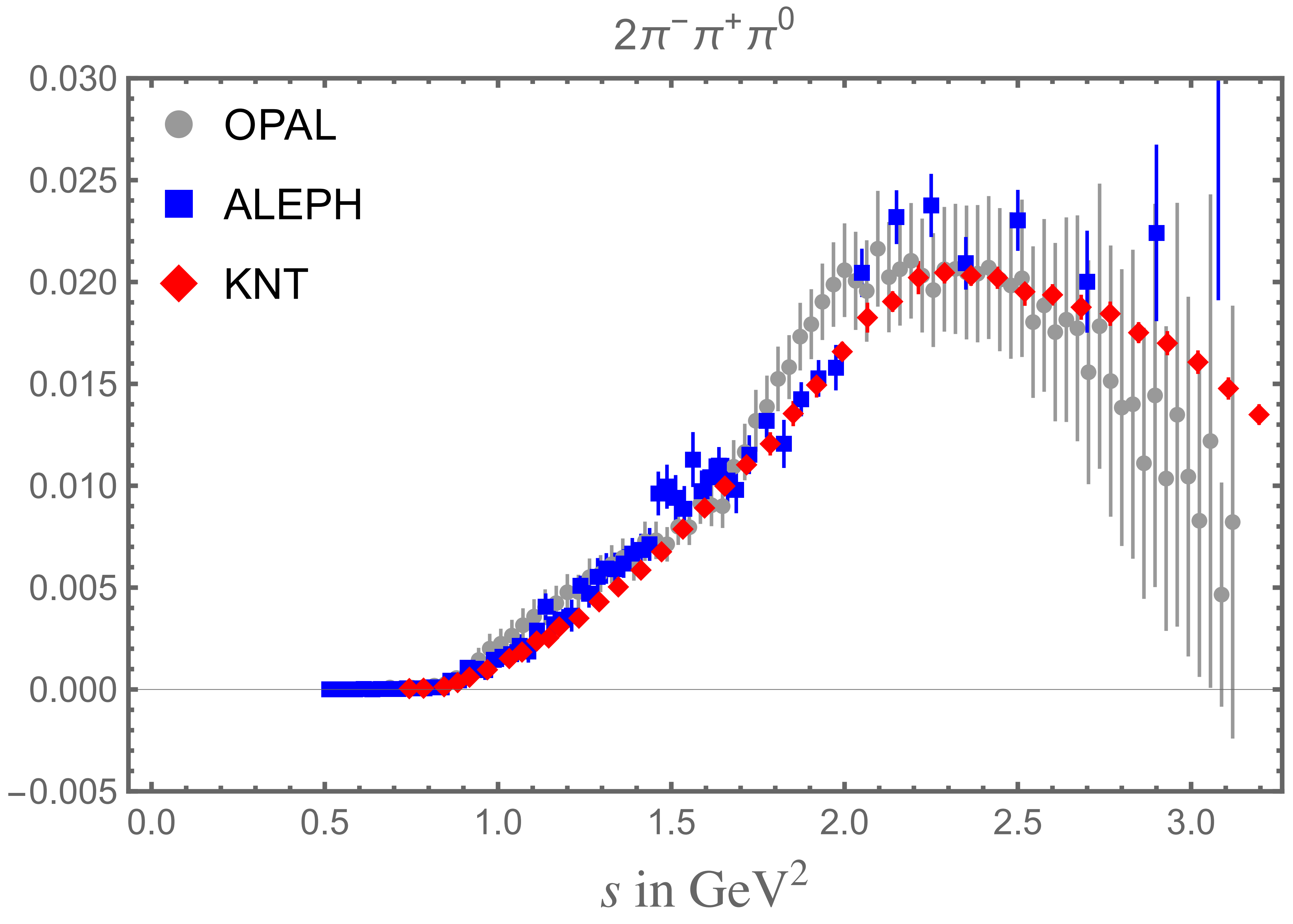
tau-based results: Pais relations for the 4π channels

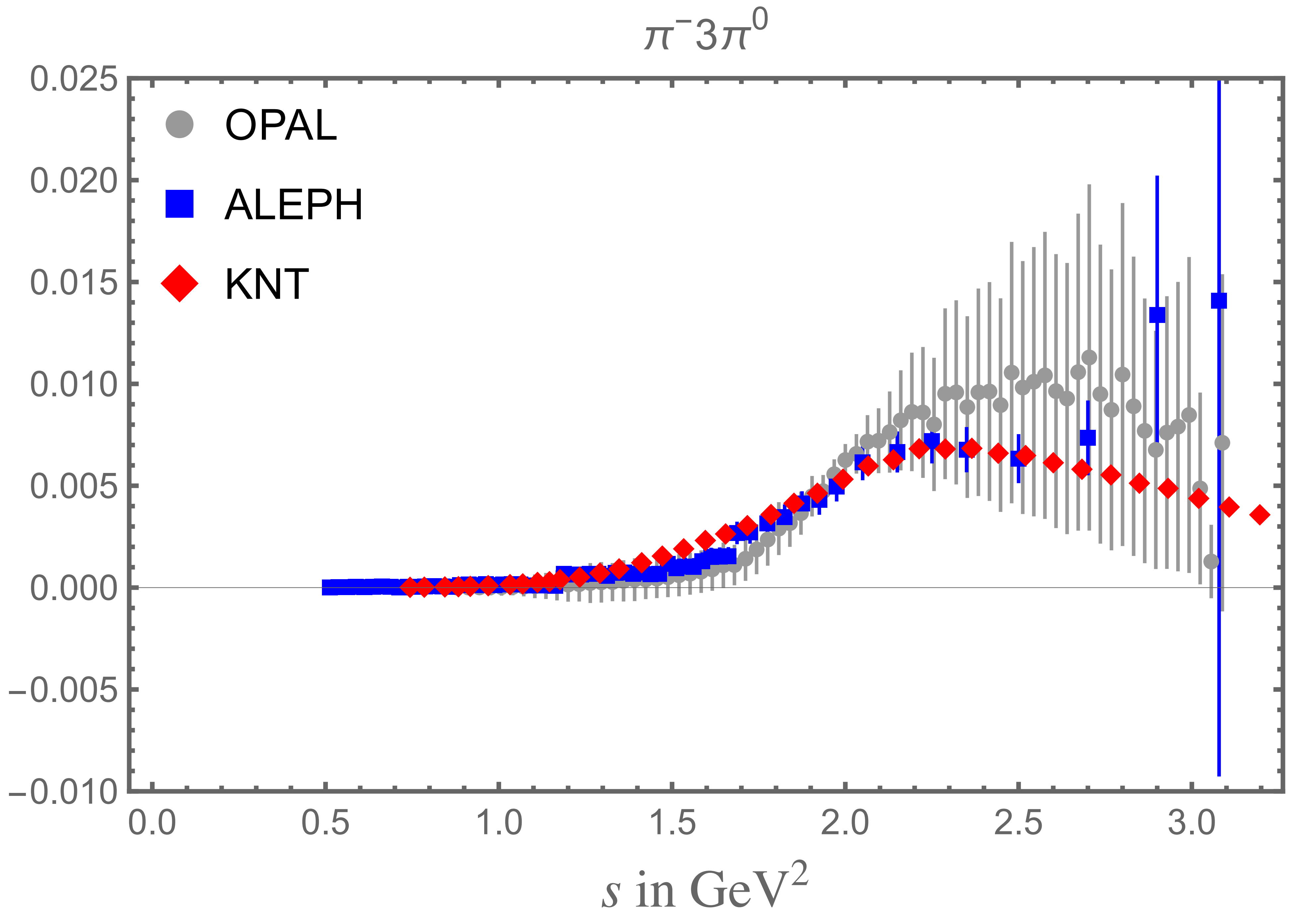
Isospin relation between the 4π channels in tau and e+e-

$$\rho_V[\tau, 2\pi^-\pi^+\pi^0](s) = \frac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s) + \rho_V[\mathrm{EM}, \pi^-\pi^+2\pi^0](s)$$

$$\rho_V[\tau, \pi^-3\pi^0](s) = \frac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s)$$

Deviations expected to be *O*(~1%).





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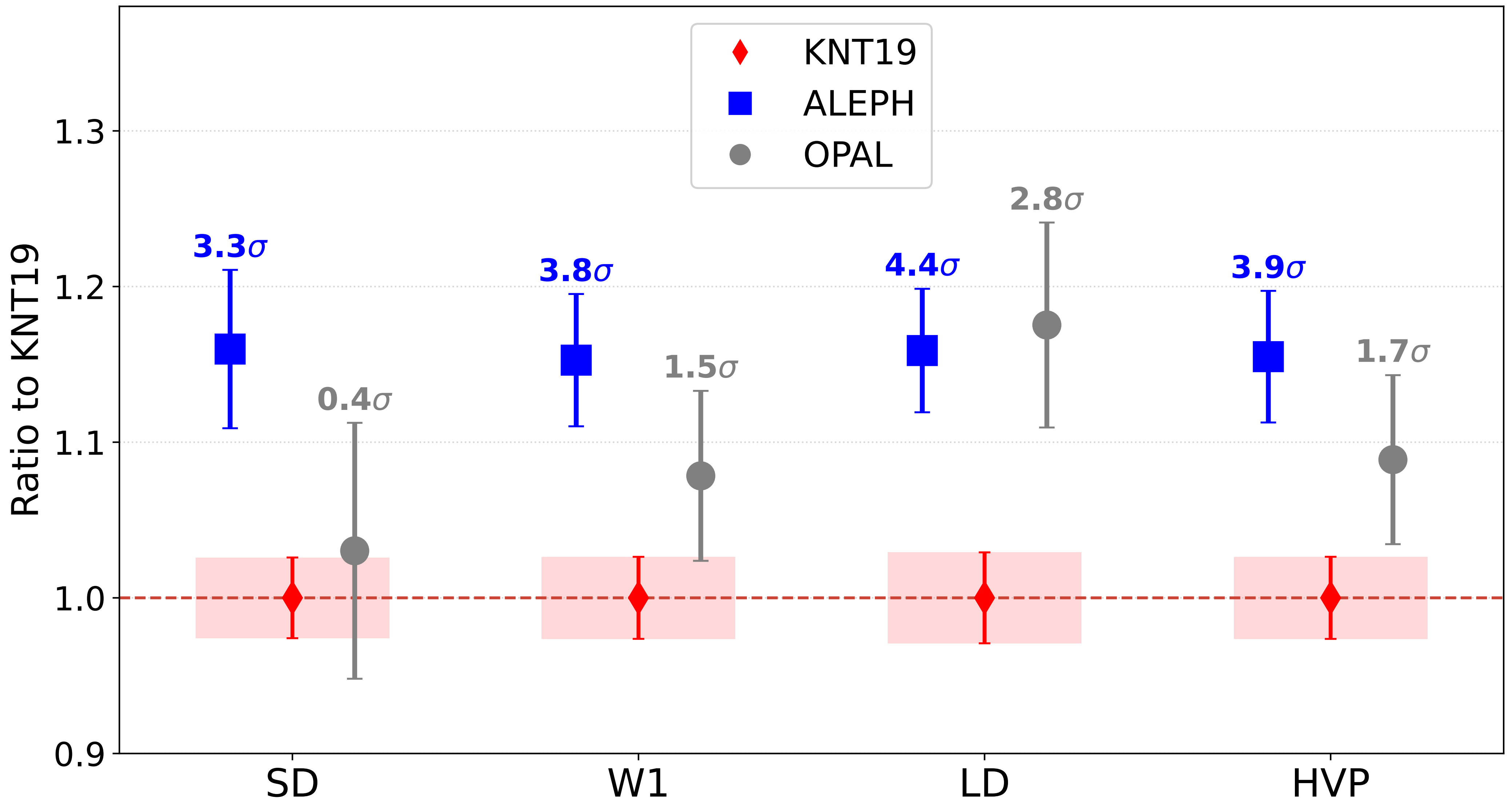
tau-based results: Pais relations for the 4π channels

Isospin relation between the 4π channels in tau and e+e-

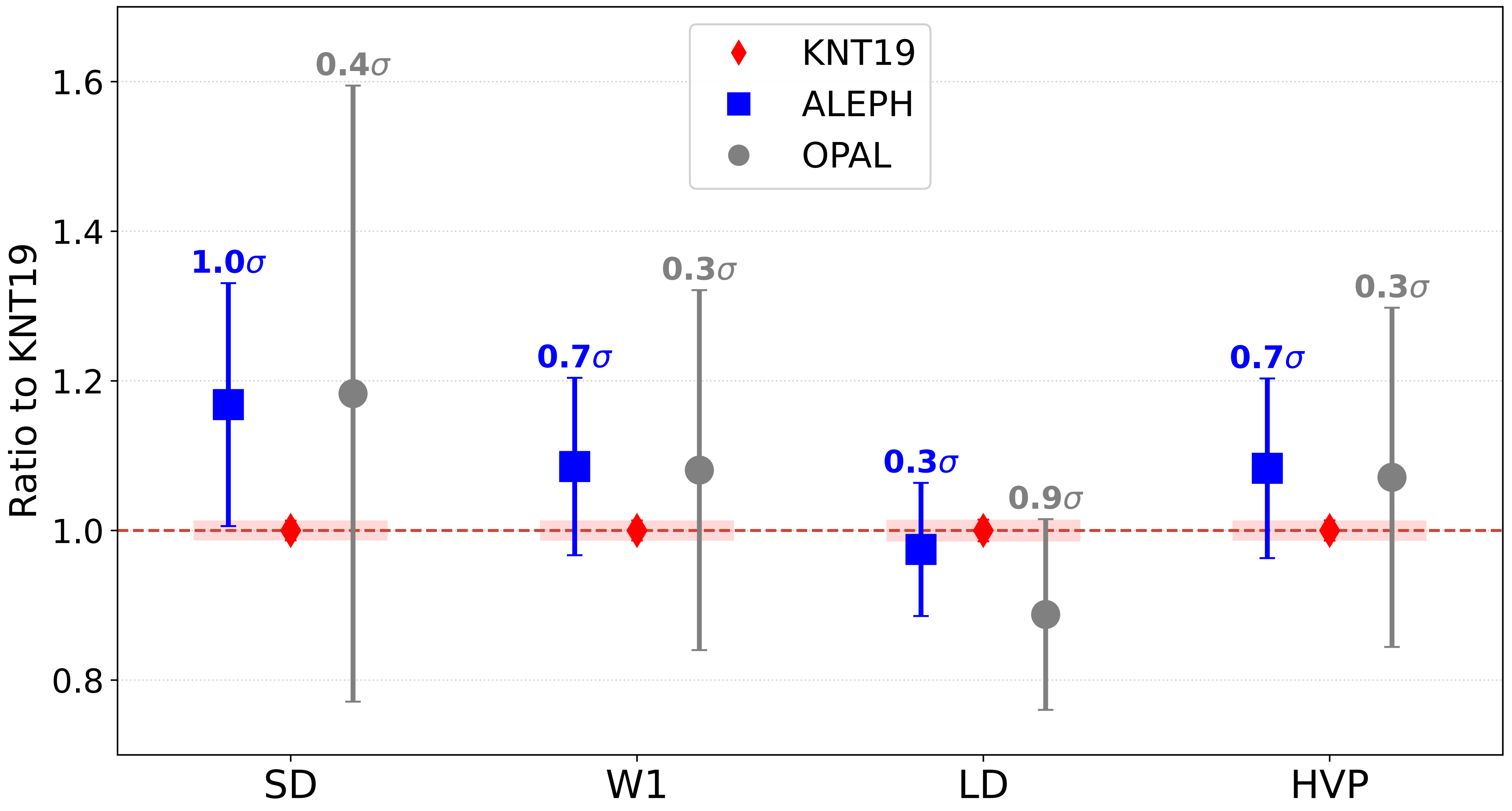
$$\rho_V[\tau, 2\pi^-\pi^+\pi^0](s) = \frac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s) + \rho_V[\mathrm{EM}, \pi^-\pi^+2\pi^0](s)$$
$$\rho_V[\tau, \pi^-3\pi^0](s) = \frac{1}{2}\,\rho_V[\mathrm{EM}, 2\pi^-2\pi^+](s)$$

Integrated ALEPH and OPAL 4π results (normalized to KNT19-based results)

$\tau \to 2\pi^-\pi^+\pi^0\nu_\tau$



$\tau \to \pi^-3\pi^0\nu_\tau$



Allen, DB, Golterman, Maltman, Mansur, Peris 2605.12205

Discrepancies in integrated results from ALEPH and the respective Pais-relation implied e+e- combination for the 1π^0 channel cannot be explained by IB (uncertainties are too large in 3π^0).

Conclusions

- Several recent, precise, lattice QCD results for the RBC/UKQCD windows
- Recent lattice results are, in general, in very good agreement with each other
- Large discrepancy between lattice and KNT19-based data-driven lqc window results (> 5σ for W1)
- Discrepancies driven by the 2π channel: use of CMD-3 2π results eliminates the discrepancy between data driven and lattice-QCD lqc window results
- No statistically meaningful discrepancy between lattice and data-driven results for the s+lqd components (but larger errors on data-driven side)
- IB corrections needed to relate $I = 1$ and isospin-symmetric results are EM only (to first order in IB)
- tau-based results in excellent agreement with the lattice and with e+e- results that use CMD-3 2π cross-sections
- Sizeable discrepancies between tau-based $\tau \to 2\pi^-\pi^+\pi^0\nu_\tau$ and Pais-relation implied e+e- combination (cannot be explained by IB)

extra

Isospin breaking (e+e-)

Isospin breaking (IB) contributions must be subtracted to compare with isospin symmetric lattice results.

To $O(\alpha, m_u - m_d)$, pure *I* = 0/1 EM only, mixed isospin (*ab* = 38) is a combination of EM + strong IB

Isospin limit of QCD defined as one in which all pions have a mass equal to the physical neutral-pion mass

Mixed-isospin (MI) contribution (strong IB + EM)

Expected to be dominated by $\rho - \omega$ interference

Results for the dominant 2π and 3π channels obtained from fits to data (VMD or dispersive).

Hoferichter, Colangelo, Hoid, Kubis, Ruiz de Elvira, Schuh, Stamen, Stoffer, 2307.02532

O(1%) estimate for other, subdominant, channels used as an additional IB uncertainty.

We can safely ignore IB corrections to the already small contributions in the inclusive region (except for the lqc SD window: 1% of pt. QCD added as an additional (small) uncertainty)

Pure *I* = 0,1 Electromagnetic (EM) IB contributions

Inclusive. Extracted from (a combination of) BMW results.

The only (small) lattice input to our final lqc results. Tiny for s+lqd (not included in final numbers).

Isospin breaking (e+e-)

Pure $I = 0,1$ Electromagnetic (EM) IB contributions

Inclusive. Extracted from (a combination of) BMW results. Borsanyi et al., 2002.12347; Boccaletti et al. 2407.10913

The only (small) lattice input to our final lqc results. Tiny for s+lqd (not included in final numbers).

From BMW20 we have the lqc EM contribution to the intermediate window [—0.035(59)] and to the total HVP [—1.57(55), BMW24]

(in units of 10^{-10})

Assumption: remaining —1.54(55) units assigned to the lqc LD window and negligible for the SD window [consistent with Mainz24 IB SD result: 0.15(15)% Kuberski et al, 2401.11895].

Smallness of EM IB in SD window also corroborated by *i)* perturbative SD results, *ii)* potential cancellations similar to those observed in the int. window results in Hoferichter, Colangelo, Hoid, Kubis, Ruiz de Elvira, Schuh, Stamen, Stoffer, 2307.02532

We add a large uncertainty to the SD lqc result to account for this assumption ($\alpha_{\rm EM}$ times the total)

An example of ambiguous mode: $K\bar{K}$ channel

Example of treatment of an ambiguous mode (expected to be dominated by $I = 0$): $K\bar{K}$ channel

From the data combination of KNT19 we have the following total $I = 1+0$ contribution:

$$[a_\mu^{\rm LD}]_{K\bar{K}} = 13.55 \pm 0.11$$

(in units of 10^{-10})

conservative 50/50 separation →

$$[a_\mu^{\rm LD}]_{K\bar{K}}\big|_{I=1,0} = 7 \pm 7$$

✗ maximally conservative separation is not good enough!

But BaBar has measured the (purely $I = 1$) spectrum of $\tau \to K\bar{K}\nu_\tau$

With CVC we have a determination of $I = 1$ $e^+e^- \to K\bar{K}$ up to $s = 2.76$ GeV^2.

$$[a_\mu^{\rm LD}]_{K\bar{K}}\big|_{I=1}(s < 2.76\ {\rm GeV}^2) = 0.1743(84)$$

Using KNT19 results for $s > 2.76$ GeV^2 with the maximally conservative separation, we then find

$$[a_\mu^{\rm LD}]_{K\bar{K}}\big|_{I=1} = 0.181(11)$$

$$[a_\mu^{\rm LD}]_{K\bar{K}}\big|_{I=0} = 13.37(11)$$

enormous reduction in the uncertainty, from 7 to 0.11 units

Data-driven light-quark connected results

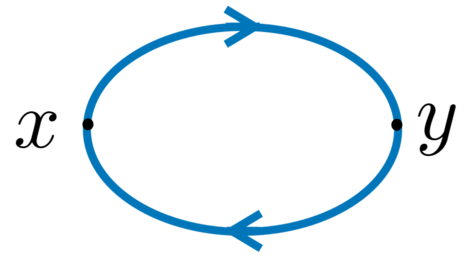
Benton, DB, Golterman, Keshavarzi, Maltman, Peris, PRD (2025).

Short-distance window, light-quark connected

$$a_\mu^{\rm lqc,SD} = 25.99(21) + 0.75(32) + 20.28(10) + 0.00(34) - 0.07(10) = \mathbf{46.96(58)}$$

25.99(21)	0.75(32)	20.28(10)	0.00(34)	0.07(10)
55% (32% 2π)	1.6%	43.2%	—	-0.1%

Intermediate window, light-quark connected



(in units of 10^{-10})

$$a_\mu^{\rm lqc,int} = 186.94(80) + 1.88(61) + 11.06(16) + 0.035(59) - 0.96(0.30) = \mathbf{199.0(1.1)}$$

unambiguous modes $I = 1$	ambiguous modes	pt.QCD + DVs	EM IB	MI IB	total
186.94(80)	1.88(61)	11.06(16)	0.035(59)	0.96(0.30)	199.0(1.1)
94% (80% 2π)	0.9%	5.6%	0.02%	-0.48%	

Long-distance window, light-quark connected

$$a_\mu^{\rm lqc,LD} = 390.6(1.6) + 0.613(91) + 0.346(11) + 1.54(55) - 3.19(15) = \mathbf{389.9(1.7)}$$

390.6(1.6)	0.613(91)	0.346(11)	1.54(55)	3.19(15)
100.2% (88% 2π)	0.16%	0.088%	0.4%	-0.8%

Unambiguous modes: lqc

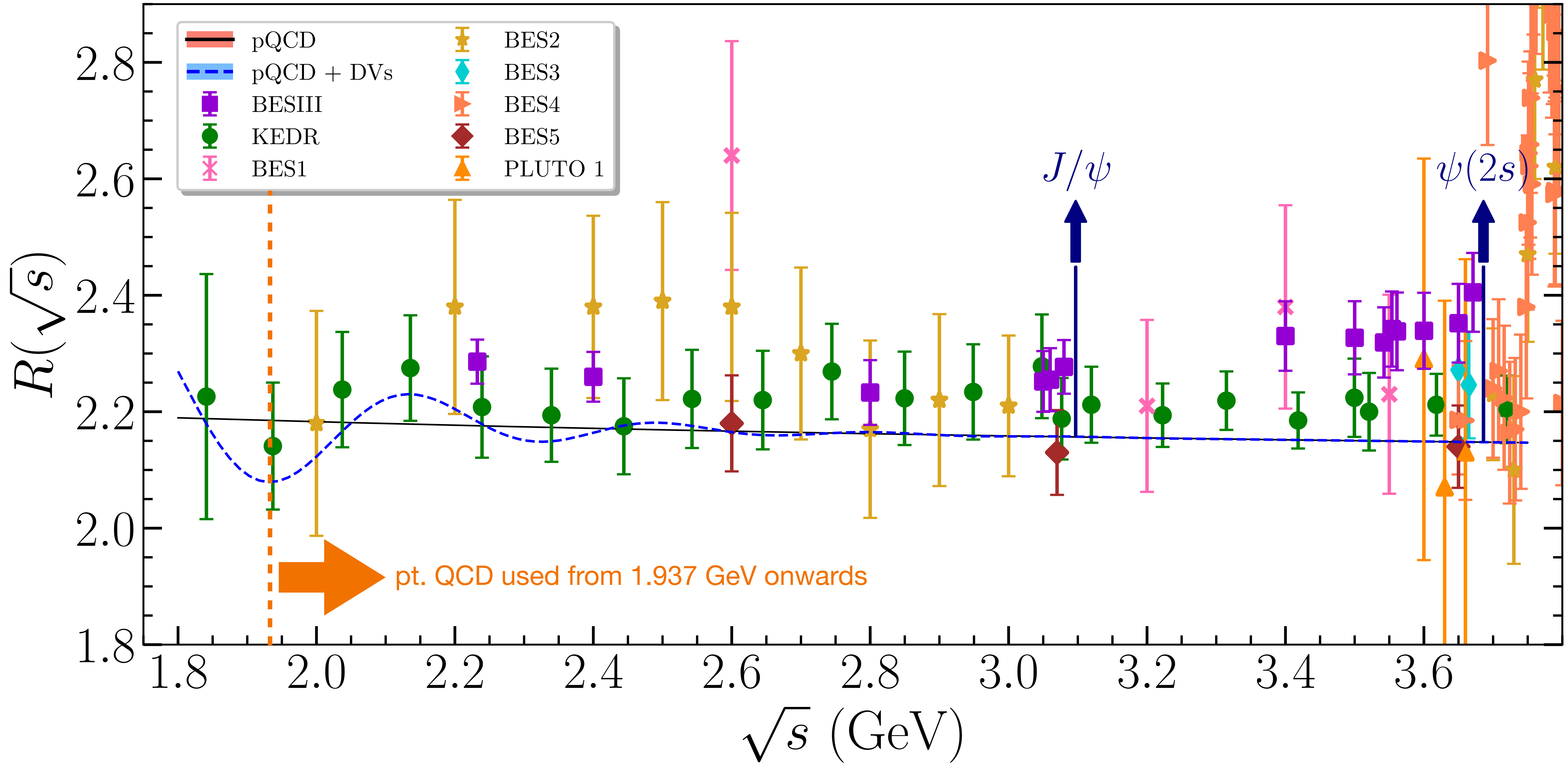
$$\hat{\Pi}^{\text{lqc}}_{\text{EM}} \equiv \frac{10}{9}\hat{\Pi}^{I=1}_{\text{EM}}$$

$$a^{\text{lqc}}_{\mu} = \frac{10}{9} a^{I=1}_{\mu}$$

$I=1$ modes X	$[a^{\text{int}}_{\mu}]_X \times 10^{10}$	$[a^{\text{SD}}_{\mu}]_X \times 10^{10}$	$[a^{\text{LD}}_{\mu}]_X \times 10^{10}$	$[a^{\text{tot}}_{\mu}]_X \times 10^{10}$
low-s $\pi^+\pi^-$	0.02(00)	0.0010(00)	0.842(18)	0.867(18)
$\pi^+\pi^-$	144.13(49)	14.927(52)	344.4(1.4)	503.5(1.9)
$2\pi^+2\pi^-$	9.29(13)	3.239(43)	2.334(34)	14.87(20)
$\pi^+\pi^-2\pi^0$	11.94(48)	3.98(16)	3.46(14)	19.39(78)
$3\pi^+3\pi^-$ (no ω)	0.14(01)	0.0746(47)	0.01485(95)	0.231(15)
$2\pi^+2\pi^-2\pi^0$ (no η)	0.83(11)	0.426(52)	0.092(13)	1.35(17)
$\pi^+\pi^-4\pi^0$ (no η)	0.13(13)	0.067(67)	0.014(14)	0.21(21)
$\eta\pi^+\pi^-$	0.85(03)	0.333(12)	0.1594(63)	1.340(50)
$\eta 2\pi^+2\pi^-$	0.05(01)	0.0239(33)	0.00547(98)	0.076(11)
$\eta\pi^+\pi^-2\pi^0$	0.07(01)	0.0407(66)	0.0065(11)	0.119(20)
$\omega(\to\pi^0\gamma)\pi^0$	0.53(01)	0.1469(34)	0.2014(42)	0.882(19)
$\omega(\to \text{npp})3\pi$	0.10(02)	0.0529(99)	0.0116(23)	0.168(32)
$\omega\eta\pi^0$	0.15(03)	0.081(18)	0.0144(29)	0.242(53)
Total ($I=1$)	168.24(72)	23.40(19)	351.6(1.4)	543.2(2.1)
Total (lqc)	186.94(80)	25.99(21)	390.6(1.6)	603.6(2.3)

Perturbative QCD

QCD perturbation theory is used in the inclusive region



Some tension between pt. QCD and recent BES-III results (in purple) BES-III, 2112.11728 Boito & Caram, '25

We add duality violation (DV) contributions and significantly enlarge the pt. QCD error by taking the DVs as the uncertainty DB, Golterman, Maltman, Peris, 1805.08176, DB, Eiben, Golterman, Maltman, Mansur, and Peris, 2502.08147