Optimal rate of convergence to asymptotic profiles for fast diffusion in bounded domains

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Abstract

In this talk, we review recent developments in the asymptotic analysis of energy solutions to the Cauchy–Dirichlet problem for the fast diffusion equation on bounded domains. It is well known that every energy solution vanishes in finite time and that, after a suitable rescaling, the solution converges to an asymptotic profile, which is a nontrivial solution of the Emden–Fowler equation. Bonforte and Figalli (CPAM, 2021) were the first to determine an exponential rate of convergence of nonnegative rescaled solutions to nondegenerate positive asymptotic profiles in a weighted L^2 norm on smooth bounded domains, by developing the so-called nonlinear entropy method. Subsequently, the speaker (ARMA, 2023) developed an energy method based on a *quantitative gradient inequality* and proved the convergence at the same exponential rate in the Sobolev norm on bounded $C^{1,1}$ domains. Through an analysis of a formal linearized problem associated with, but distinct from, the original nonlinear problem, Bonforte and Figalli predicted that the rate should be sharp. The optimality of the rate was later proved by the speaker and Maekawa (JFA, 2026), who also extended the exponential convergence result to possibly sign-changing solutions on general bounded domains by further developing the energy method based on the quantitative gradient inequality.

Most existing results, however, assume that the asymptotic profile is *nondegenerate*, meaning that the corresponding linearized operator has trivial kernel. Although such nondegeneracy is known to hold for generic domains, least-energy solutions of the Emden–Fowler equation on thin annuli are known to be nonradial. Owing to rotational invariance, these solutions are degenerate and therefore lie beyond the scope of previous results. We establish the optimal exponential convergence to such degenerate asymptotic profiles. This part of the talk is based on recent joint works with Norihisa Ikoma (Keio University) and Yasunori Maekawa (Kyoto University).