

Bootstrapping rho meson scattering at large N

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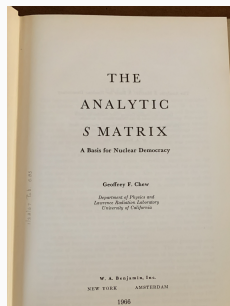
Prin meeting “Carving out the landscape of QFT” 2026

Based on **work in progress** with J. Albert, L. Rastelli and A. Vichi

Can we predict the masses and decay rates of hadrons (mesons and baryons) from first principles?

- Bootstrap: Hadrons coexist in a uniquely determined way
“Nuclear democracy” [Chew 1966]
- QCD: Hadrons emerge from QCD lagrangian
⇒ lattice QCD
- Back to the bootstrap
Meson data $\leftrightarrow$ CFT-data.

$$(m_X^2, g_{\pi\pi X}) \leftrightarrow (\Delta_{\mathcal{O}}, \lambda_{\phi\phi\mathcal{O}})$$



Constraints derive from analyticity, unitarity, and crossing of scattering amplitudes

dispersion relations, positivity, redundancy

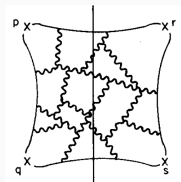
Finite- N

- Unstable mesons $\rightarrow$ resonances
- S-matrix has complicated analytic structure: Branch cuts and threshold effects

Nice work on non-perturbative $2 \rightarrow 2$ pion scattering amplitudes [\[Guerrieri, Penedones, Vieira 2018\]](#) [\[He, Kruczenski 2023\]](#) etc.

Large- N [\[’t Hooft 1974\]](#) [\[Witten 1979\]](#)

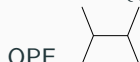
- Hadrons asymptotically stable
- Meson amplitudes form a closed subsector
- Meromorphic amplitudes with poles in s at masses of mesons



Is there a unique theory of large- N QCD with massless quarks?

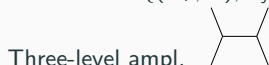
Introduction

CFT-data: $\{(\Delta_i, J_i), \lambda_{ijk}\}$



$\Rightarrow$ Conformal bootstrap

Meson data: $\{(m_i^2, J_i), \lambda_{ijk}\}$



$\Rightarrow$ S-matrix bootstrap

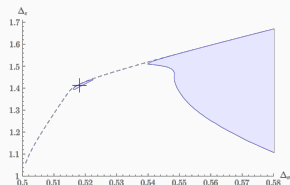
Goal: A “3d Ising island” for large- N QCD

History: single correlator: kink [El-Showk et al 2012], mixed correlators: island [Kos, Poland, Simmons-Duffin 2014]

Method: “EFT bootstrap” developed in [Caron-Huot, Van Duong 2020]

See also [Bellazzini et al 2011], [Tolley, Wang, Zhou 2020], ...

Strategy: Go from single S-matrix to mixed S-matrices

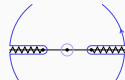


1. EFT bootstrap
2. What we expect from large- N QCD
3. Results from π scattering [\[2312.15013\]](#)
4. Results from ρ scattering [\[WIP\]](#)

1. **EFT bootstrap**
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Dimensional analysis on a firm footing [Caron-Huot, Van Duong 2020]

$$\mathcal{A}_{\text{EFT}} = -\lambda + g_2(s^2 + t^2 + u^2) + g_3 stu + g_4(s^2 + t^2 + u^2)^2 + \dots$$



Sum rules and null constraints

$$g_2 = \left\langle \frac{1}{s^2} \right\rangle$$

$$g_3 = \left\langle \frac{3 - 2J(J+1)}{s^3} \right\rangle$$

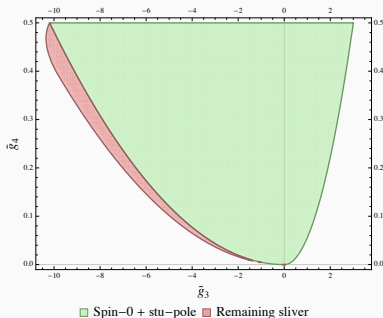
$$g_4 = \left\langle \frac{1}{2s^4} \right\rangle$$

$$0 = \left\langle \frac{J(J+1)(J^2 + J - 8)}{s^4} \right\rangle$$

$$\langle (\dots) \rangle = \sum_J \int_{\Lambda^2}^{\infty} \frac{ds}{s} \rho_J(s) (\dots)$$

Solve with SDPB [Simmons-Duffin 2015]

$$\tilde{g}_3 = \frac{g_3 \Lambda^2}{g_2}, \quad \tilde{g}_4 = \frac{g_4 \Lambda^4}{g_2}$$



How to read plots in the EFT bootstrap

A single “primal model” rules in more than a point

$$\text{axis} = \frac{\text{observable}}{\text{normalisation}}$$

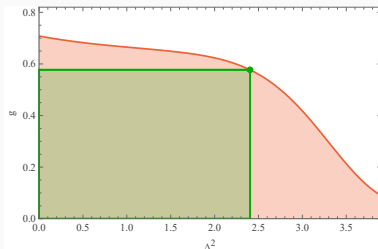
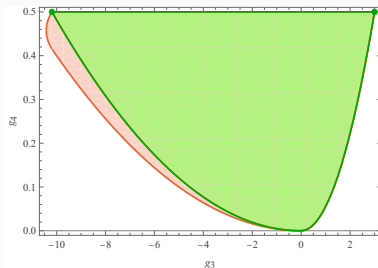
Three mechanisms

- A. Add state that contributes only to the normalization (line to axis/origin)
- B. $\alpha P_1 + (1 - \alpha)P_2$ (straight line between models)
- C. Artificially lowering cutoff: all $\Lambda' \leq \Lambda$ allowed. *Similar to simple CFT bootstrap plots when axis represents a gap*

See [Berman, Elvang, Geiser, Lin 2024] for “ruling in” sliver

infinite-spin-tower

massive scalar



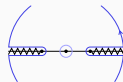
Approximations correct in large- N QCD model

The “EFT bootstrap” setup of [Caron-Huot, Van Duong 2020] can be generalised to

- Spinning states. E.g. photons [JH, McPeak, Russo, Vichi 2021], ...
- Coupling to gravity [Caron-Huot, Mazáč, Rastelli, Simmons-Duffin 2021], ...
- Global symmetry: E.g. massless pions [Albert, Rastelli 2022], ...

A key assumption the analytic structure:

$\mathcal{A}(s, t)$ for fixed $t < 0$ is analytic in the s plane
away from cuts at $s \geq \Lambda^2$ and $s \leq -t - \Lambda^2$.



Generally broken by loop effects, meaning that bounds are valid only in the approximation of **tree-level EFT** for $E \leq \Lambda$.

Pion scattering at large N is precisely compatible with this assumption [Albert, Rastelli 2022]

$$\mathcal{A}(s, t) = \sum_X \lambda_{\pi\pi X}^2 \frac{P_J(1 - 2t/m_X^2)}{m_X^2 - s} + \text{crossed} + \text{regular} \sim \frac{1}{N}, \quad \text{cut} \rightarrow \text{poles}$$

Follow-up work: [Fernandez, Pomarol, Riva, Sciotti 2022] [Albert, Rastelli 2023] [Ma, Pomarol, Sciotti 2023] [Li 2023] [Albert, JH, Rastelli, Vichi 2023] [Albert, Kosva, Rastelli 2026] [Bocchia, Vichi 2026], ...

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Meson spectroscopy

$n^{2s+1}\ell_J$	J^{PC}	$I = 1$ $u\bar{d}, \bar{u}d,$ $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	$I = 0$ f'	$I = 0$ f
1^1S_0	0^{-+}	π	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$f'_2(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$		$\omega(1650)$
1^3D_2	2^{--}			
1^3D_3	3^{--}	$\rho_3(1690)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$		
2^1S_0	0^{-+}	$\pi(1300)$	$\eta(1475)$	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$		
2^3P_2	2^{++}	$a_2(1700)$	$f_2(1950)$	$f_2(1640)$

[PDG]



"If I could remember the names of these particles, I would have been a botanist."

At large N much less is known about spectrum of mesons

[Bali et al 2013], [Bonanno et al 2025]

Large- N pion amplitude expected to exchange

Even J : $J^{++} \ni a_J, f_J$

Odd J : $J^{--} \ni \rho_J, \omega_J$

Want to see

1. Exclusion plots with interesting structure (kinks/corners)
2. Kinks saturated by non-trivial amplitudes (not just “single scalar at $m^2 = \Lambda^2$ ”)
3. Kinks saturated by large- N QCD
4. Precision island for large- N QCD

End-goal: Novel $1/N$ expansion towards the spectrum of real-world QCD



A salient feature of confining gauge theories is the presence of higher-spin mesons forming asymptotically linear Regge trajectories

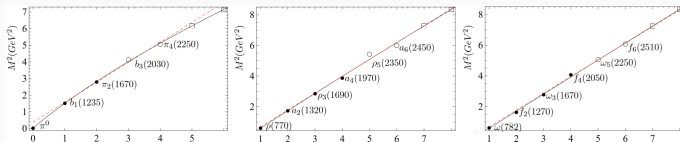


fig: [Chen 2021]

How can we force the presence of higher-spin particles and ultimately Regge trajectories?

Will be focus of study 1: *Bootstrapping mesons at large N : Regge trajectory from spin-two maximization* [Albert, JH, Rastelli, Vichi 2023]

Dispersion relations and Regge trajectories

Pedagogical example from Simon Caron-Huot [\[Caron-Huot 2017\]](#)

$$f(s) = -\log(1 - s^2) = \sum_{J=0}^{\infty} C_J s^J$$

Properties: $f(s)/s \rightarrow 0$ at $|s| \rightarrow \infty$, cuts at $s > 1$, $s < -1$

Dispersion relation

$$\begin{aligned} C_J &= \frac{1}{2\pi i} \oint_0 \frac{ds}{s} s^{-J} f(s) \\ &= \frac{1}{2\pi} \int_1^{\infty} \frac{ds}{s} s^{-J} \left(\text{Disc } f(s) + (-1)^J \text{Disc } f(-s) \right) \\ &= (1 + (-1)^J) \frac{1}{J}, \quad J \geq 1 \end{aligned}$$

If we change a single C_J , the whole argument breaks down.

Physical case:

- Particles on Regge trajectories: $\mathcal{A} \sim s^{\alpha(t)}$
- Isolated spin- J particles: $\mathcal{A} \sim s^J$

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Study 1. Pion kink

[Albert, JH, Rastelli, Vichi 2023]

Spin $J > 1$ cannot appear alone

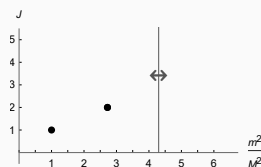
Assumptions:

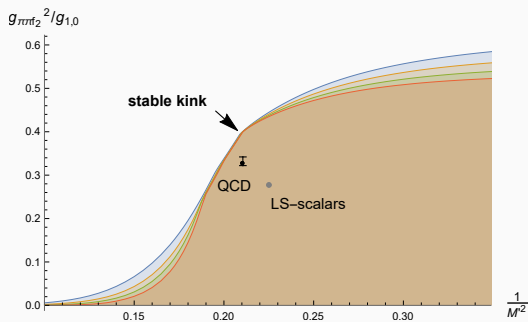
- Spin-1 state at $m = 1$ ($m_{\rho}^{\text{phys}} = 775 \text{ MeV}/c^2$)
- Spin-2 state at $m = 1.65$ ($m_{f_2}^{\text{phys}} = 1275 \text{ MeV}/c^2$)
- All other states above Λ (free parameter)

Force presence by maximising $\frac{g_{\pi\pi f_2}^2}{g_{1,0}^2}$

Expect

- $\Lambda \gg 1$ — nothing allowed
- $\Lambda \sim m_{\rho_3}$ — extremal solution with all spins ($m_{\rho_3}^{\text{phys}} = 1688 \text{ MeV}/c^2$)

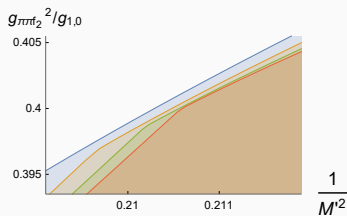




[Albert, JH, Rastelli, Vichi 2023]

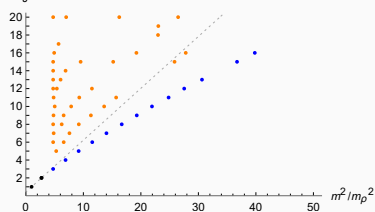
$$M_{\text{LS-scalars}} = -\frac{\Gamma(1/2 - s/2)\Gamma(1/2 - t/2)}{\Gamma(-s/2 - t/2)} - \text{scalars}$$

[Fernandez, Pomarol, Riva, Sciotti 2022]

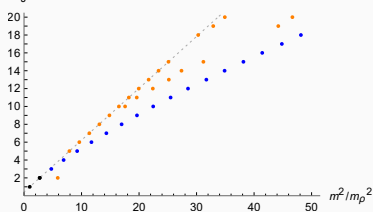


Extremal functional method [El-Showk & Paulos 2012]

$$m^2 \geq \Lambda^2$$



$$m^2 \geq a + bJ$$



J	Dominant state		Other states					
	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$	m^2	$\tilde{g}_X^2$
3	4.747774	0.33527						
4	6.902792	0.28933						
5	9.181336	0.25334	5.278811	1.515×10^{-4}				
6	11.54579	0.22174	4.747774	3.297×10^{-6}	6.582414	1.773×10^{-4}		
7	14.01378	0.19857	4.835758	7.862×10^{-7}	7.581251	1.237×10^{-4}		
8	16.67318	0.18599	4.747774	8.771×10^{-8}	6.235041	1.265×10^{-6}	9.207674	1.352×10^{-4}
9	19.28674	0.16358	4.808180	1.895×10^{-8}	6.571938	4.367×10^{-7}	11.31167	2.537×10^{-4}
10	21.93016	0.14912	5.019793	7.308×10^{-9}	7.879411	3.754×10^{-7}	13.61458	4.242×10^{-4}
11	24.82063	0.11649	4.825621	6.643×10^{-10}	9.289181	1.875×10^{-6}	15.69828	1.554×10^{-4}
12	27.53345	0.10811	4.747774	8.380×10^{-11}	5.390215	7.235×10^{-11}	11.48907	7.067×10^{-6}

[Albert, JH, Rastelli, Vichi 2023]

Is this large- N QCD?

What we have achieved:

- Spin 2 maximisation forces presence of Regge trajectory
- Interesting kink gives extremal amplitude
- Many observables are close to real-world QCD ($N = 3$)

Problems:

- No daughter trajectories
- Curved trajectory?

⇒ Expect real large- N QCD to sit somewhat below the kink

Open problem: Exhibit the curved-trajectory amplitude explicitly!

1. EFT bootstrap
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Next: mixed scattering amplitudes

Physical (helicity) amplitudes

$$\mathcal{A}(s, t) = \mathcal{A}_{\mathcal{R}}^{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(s, t)$$

Partial waves: Wigner d functions

$$d_{\lambda_1 - \lambda_2, \lambda_4 - \lambda_3}^{(J)}(\theta)$$

	$m = 0$	$m > 0$
$J = 0$		
$J > 0$		

Complications for massive spinning

All helicity amplitudes mix under crossing!

Define stripped amplitudes $M_I(s, t)$ with nice crossing properties

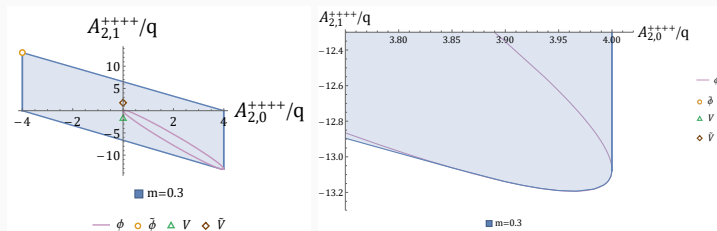
$$\mathcal{A}_I(s, t) = E_{IJ}(s, t) M_I(s, t)$$

$$M_1(s, t) = M^{(1)}(s, t), M_2(s, t) = M^{(1)}(t, s) \text{ etc.}$$

Matrix E_{IJ} non-sparse matrix

$$\text{Crossing: } \mathcal{A}_I(s, t) = E_{IJ}(s, t) C_{JK}^{st} E_{KL}^{-1}(t, s) \mathcal{A}_L(t, s)$$

First full EFT bootstrap for massive spinning particles [Bertucci, JH, McPeak, Ricossa, Riva, Vichi 2024]



Some counting

System	Physical $\mathcal{A}(s, t)$	Independent $M(s, t)$
Pions	3	1
Massive vectors	17	5
ρ mesons	51	10

- Take-home: tedious but systematic method to derive null constraints

Compared to our work on neutral massive spin-1, the ρ scattering amplitude has **additional technical obstacles**

- ρ self-couplings and exchange of massless pion

$$-\frac{\lambda_{\rho\rho\pi}}{2}d^{abc}\epsilon^{\mu\nu\sigma\tau}F_{\mu\nu}^aF_{\sigma\tau}^b\pi^c - \frac{\lambda_{\rho\rho\rho}^{(1)}}{2}f^{abc}\rho_\mu^a\rho_\nu^bF^{c,\mu\nu} - \frac{\lambda_{\rho\rho\rho}^{(2)}}{3}f^{abc}F_{\mu}{}^\nu{}_aF_{\nu}{}^\rho{}_bF_{\rho}{}^\mu{}_c$$

lead to low-lying poles and issues with normalisation.

- Usual mnemonics such as
 - Spin- $J \leftrightarrow \mathcal{A} \sim s^J$
 - Higher-derivative couplings (e.g. $\lambda_{\rho\rho\rho}^{(2)}$) worse than lower derivative-couplings

hold only up to various modifications

- Threshold at $s = 4m_\rho^2$:
 - Kinematic singularities $\sim \sqrt{stu}$
 $\rightarrow$ factor out
 - Residues of P-odd states change sign below $s = 4m_\rho^2$
 $\rightarrow$ modify normalisation



Explicit exchange amplitudes

notation: J^{PC}

- Scalars: $\phi \in 0^{++}$, $\alpha \in 0^{-+}$

$$M^{(1)}(s, t)|_{\phi} = -\frac{(c_{\phi}^{(1)} + c_{\phi}^{(2)}(2m^2 - s))^2}{s - M^2}, \quad M^{(1)}(s, t)|_{\alpha} = -c_{\alpha}^2 \frac{t(s-u)}{t - M_{\alpha}^2}, \dots$$

Special case $\pi = \alpha|_{M_{\alpha}=m_{\pi}=0}$

- Vectors: $A \in 1^{++}$, $B \in 1^{+-}$, $X \in 1^{-+}$, $V \in 1^{--}$

$$M^{(10)}(s, t)|_A = -\frac{c_A^2}{M_A^2} \frac{1}{s - M_A^2}, \quad \dots$$

Special case $\rho = \alpha|_{M_V=m_{\rho}=1}$

All exchanges break Regge assumption on their own

Marginally allowed: $\lambda_{\rho\rho\rho}^{(1)} + \text{contact (Higgs model)}$

Aside: $X \in J_{\text{odd}}^{-+}$ represents “exotic mesons”. Our system rules out everything if we don’t allow for exotic mesons!

Recap: Abelian Higgs model and restoration of Regge

$$F_{\mu\nu a} = \partial_\mu \rho_{\nu a} - \partial_\nu \rho_{\mu a} + g f_a^{bc} \rho_{\mu b} \rho_{\nu c}$$

$$\mathcal{L} \supset -\frac{\lambda_{\rho\rho\rho}^{(1)}}{2} f^{abc} F_{\mu\nu a} \rho_b^\mu \rho_c^\nu + B f^{abe} f^{cde} \rho_{\mu a} \rho_{\nu b} \rho_c^\mu \rho_d^\nu + c_\phi^{(1)} f^{abc} \rho_{\mu a} \rho_b^\mu \phi_c$$

Longitudinal amplitude

$$\mathcal{A}_5 = \mathcal{A}_{AdjS}^{0000} \sim \frac{\lambda_{\rho\rho\rho}^{(1)2} - 4B}{6m^4} s^2 + O(s^1)$$

Leading growth cancels if $B = \lambda_{\rho\rho\rho}^{(1)2}/4$. This corresponds to gauge-invariant mass term $F_{\mu\nu a}^2 = (\partial_\mu \rho_{\nu a} - \partial_\nu \rho_{\mu a} - \lambda f_{abc} \rho_\mu^b \rho_\nu^c)^2$

$$\rightarrow \mathcal{A}_{AdjS}^{0000} \sim \left(-\frac{c_\phi^{(1)2}}{6m^4} + \frac{3m^2 - t}{6m^2(m^2 - t)} \lambda_{\rho\rho\rho}^{(1)2} \right) s^1 + O(s^0)$$

No further cancellation in Regge limit (fixed- t). Fixed-angle expansion

$$\mathcal{A}_{AdjS}^{0000} \sim \frac{\lambda_{\rho\rho\rho}^{(1)2} - 4B}{12m^4} (3 - \cos^2 \theta) s^2 + \left(16B - 3\lambda_{\rho\rho\rho}^{(1)2} - \frac{c_\phi^{(1)2}}{m^2} \right) \frac{s}{6m^2} + O(s^2)$$

We can now cancel s^1 by $B = \lambda_{\rho\rho\rho}^{(1)2}/4$, $c_\phi^{(1)} = m^2 \lambda_{\rho\rho\rho}^{(1)}$.

Fully mixed system of 51 scattering amplitudes $\mathcal{A}_{\mathcal{R}}^{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$

Eight “sectors” for J^{PC} plus special cases at low J . Multiple couplings

Null constraint orders:

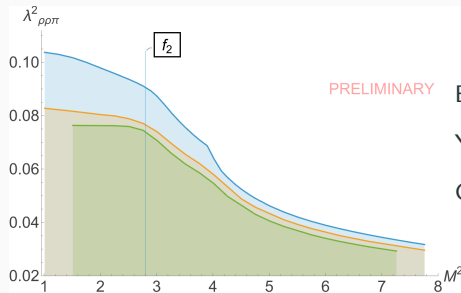
- $-2, \dots, 1$: 169 null constraints
- $-2, \dots, 2$: 271 null constraints
- $-2, \dots, 3$: 398 null constraints
- $-2, \dots, 4$: 551 null constraints

Assumptions:

- Exchange of π at $m = 0$, and ρ at $m = 1$
- All other states at $m^2 \geq M^2$.
- Maximise coupling $\lambda_{\rho\rho\pi}$ or $\lambda_{\rho\rho\rho}^{(i)}$
- Normalise by positive quantity with support on all sectors

Assumptions:

- Exchange of π at $m = 0$, and ρ at $m = 1$
- All other states at $m^2 \geq M^2$.



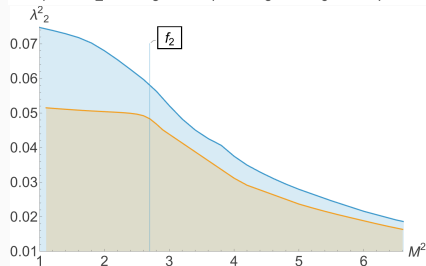
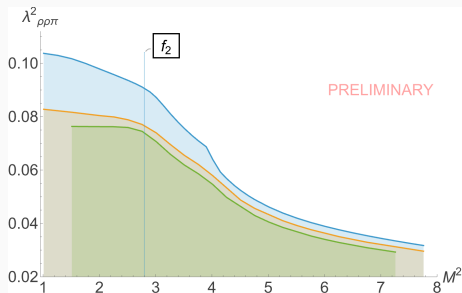
Blue: 169 null constraints

Yellow: 271 null constraints

Green: 396 null constraints

Mass of the f_2 for comparison only

Kinks at the same place in two variables



Blue: 169 null constraints

Yellow: 396 null constraints

What lives at the kink?

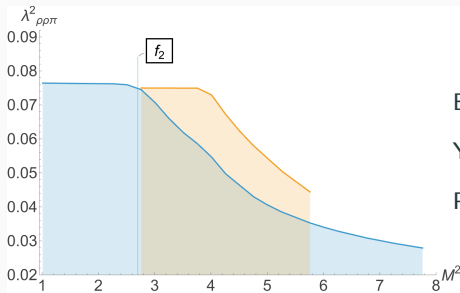
- Extremal spectrum not as clear as before: many sectors, different couplings in each sector
- More tedious but more rigorous approach:
 1. Kink near mass of meson of type X .
 2. Conjecture that the kink is saturated by a spectrum of the type

$$m_\pi = 0, \quad m_\rho = 1, \quad m_X = m_{\text{kink}}, \quad m_{\text{else}} \geq M > m_X$$

3. Make new plots under the above assumptions (allowing π, ρ, X): *Expect the plateau to extend until we reach the mass of the next state Y*
 4. Iterate
- If this pattern consists: evidence that kink is saturated by a non-trivial amplitude

In our case, we can expect $X = f_2(1270) \in 2^{++}$

Extending the plateau

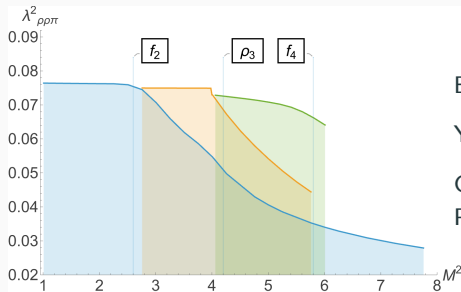


Blue: $\pi, \rho, \text{ else } \geq M^2$

Yellow $\pi, \rho, f_2, \text{ else } \geq M^2$

Plot at 396 null constraints

Extending the plateau further



Blue: $\pi, \rho, \text{ else } \geq M^2$

Yellow: $\pi, \rho, f_2, \text{ else } \geq M^2$

Green: $\pi, \rho, f_2, (\rho, \rho_3)$

Plot at 396 null constraints

We are working on a further analysis

- We developed the setup for bounding scattering of ρ mesons at large N
- We discovered a series of kinks in the allowed region
- We know almost no simple amplitudes that can populate our bound, so ...
- are we closing in on large- N QCD?

Next steps

- Kink from pion-system showed no daughter trajectories
- Kink from ρ system has weak indications of daughter trajectories
- Can we get stronger constraints from the fully mixed system?

$$\mathcal{A}_{\pi\pi\pi\pi}, \quad \mathcal{A}_{\rho\rho\rho\rho}, \quad \mathcal{A}_{\pi\pi\rho\rho}, \quad \mathcal{A}_{\pi\rho\rho\rho}, \quad \mathcal{A}_{\pi\rho\rho\pi}, \dots$$

[Albert, JH, Rastelli, Vichi: WIP]

Thank you for your attention!

