Recent results on Z production at $\mathcal{O}(\alpha\alpha_s)$

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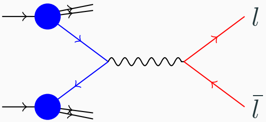
Drell-Yan



✓ **One of the standard candle processes**

- Large cross section and clean experimental signature - important for detector calibration and constraining parton distribution functions

✓ **Precise predictions for electroweak parameter**

- W boson mass (m_W), Weak mixing angle ($\sin^2\theta_{\text{eff}}$) ... ($\delta m_W < 5$ MeV and $\delta\sin^2\theta_{\text{eff}} < 0.0001$ would provide very stringent test of the SM likelihood.)

✓ **New physics potential**

- Many BSM scenarios with same final states - W', Z', KK modes *etc.*

Chronicles of the inclusive Drell-Yan

NLO

Politzer (1977)

Sachrajda (1978)

Altarelli, Ellis, Martinelli (1979)

Humpert, van Neerven (1979)

Baur, Brein, Hollik, Schappacher, Wackeroth (2002)

Carloni Calame, Montagna, Nicrosini, Vicini (2007)

Arbuzov, Bardin, Bondarenko, Christova, Kalinovskaya, Nanava, Sadykov (2008)

Dittmaier, Huber (2010)

NNLO

Altarelli, Ellis, Martinelli (1979)

Hamberg, Matsuura, van Neerven (1991)

Harlander, Kilgore (2002)

N^{3}LO

Ahmed, Mahakhud, Rana, Ravindran (2014) (in threshold limit)

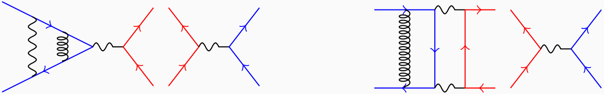
Duhr, Dulat, Mistlberger (2020)

$\alpha_s(m_Z) \simeq 0.118$	$\alpha(m_Z) \simeq 0.0078$	$\frac{\alpha_s(m_Z)}{\alpha(m_Z)} \simeq 15.1$	$\frac{\alpha_s^2(m_Z)}{\alpha(m_Z)} \simeq 1.8$

1. From naive argument of coupling strength, N^{3}LO QCD $\sim$ mixed NNLO QCD$\otimes$EW.
2. However, in specific phase-space points, fixed order EW corrections can become very large because of logarithmic (Sudakov type) enhancement. They need to be resummed to all orders, if possible.
3. NLO EW effects are large for W mass measurements. Hence, one needs to include mixed QCD$\otimes$EW corrections while aiming for $<$10 MeV precision.
4. The factorized contributions (NLO EW x NLO QCD) or in a more general argument a re-scaling of NLO QCD may catch partial effects in the soft and collinear limit due to parton shower. However it can not compensate for the non-factorized corrections. Hence, computing the non-factorized contributions exactly is needed and very important for matching.
5. Because of initial-final interaction and photon FSR, the EW corrections become more important in the tail of the distribution.
6. The appearance of photon induced processes $\Rightarrow$ photon PDFs.

NNLO contributions to full Drell-Yan

Pure Virtual



Real-Virtual

Double Real

Our goal

Here, we consider the production of a on-shell Z boson, specially the quark initiated channel.

[Bonciani, Buccioni, Rana, Triscari, Vicini]

Notation

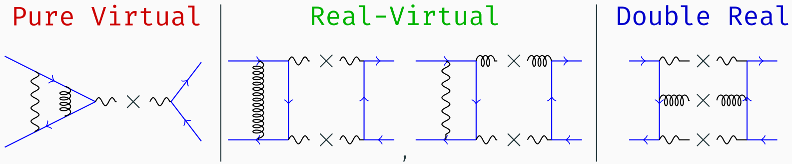
$$\sigma_{tot}(z) = \sum_{i,j\in q,\bar{q},g,\gamma} \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1,\mu_F) f_j(x_2,\mu_F) \sigma_{ij}(z,\varepsilon,\mu_F)$$

In the full QCD-EW SM, we have a double expansion of the partonic cross sections in the electromagnetic and strong coupling constants, α and α_s, respectively:

$$\begin{aligned}
\sigma_{ij}(z) &= \sigma_{ij}^{(0)} \sum_{m,n=0}^{\infty} \alpha_s^m \alpha^n \; \sigma_{ij}^{(m,n)}(z) \\
&= \sigma_{ij}^{(0)} \Big[\sigma_{ij}^{(0,0)}(z) \\
&\quad + \alpha_s \sigma_{ij}^{(1,0)}(z) + \alpha \sigma_{ij}^{(0,1)}(z) \\
&\quad + \alpha_s^2 \sigma_{ij}^{(2,0)}(z) + \alpha\alpha_s \sigma_{ij}^{(1,1)}(z) + \alpha^2 \sigma_{ij}^{(0,2)}(z) \\
&\quad + \alpha_s^3 \sigma_{ij}^{(3,0)}(z) + \alpha\alpha_s^2 \sigma_{ij}^{(2,1)}(z) + \alpha^2\alpha_s \sigma_{ij}^{(1,2)}(z) + \cdots \Big]
\end{aligned}$$

We compute $\sigma_{q\bar{q}}^{(1,1)}(z)$.

Anatomy of NNLO contributions for Z production



For different vector bosons, the contribution can be organized into four types

- QCD⊗QED : γ propagator in the loop / emission of γ
- EW1 : single Z propagator in the loop
- EW2 : single W propagator in the loop
- EW3 : Contributions with WWZ vertex

Emission of massive boson is infrared finite, hence, is treated as separate process.

gauge invariant and finite : QCD⊗QED, EW1, EW2+EW3

Challenges (in perturbative calculations)

- Large number of Feynman diagrams

  Complicated SU(3) color flow and Dirac-γ trace $\Rightarrow$ complicated numerator

- Divergences : Needs regularization (Dim. Reg.)

  Ultraviolet : UV Renormalization

  Infrared : Cancels after sum over all degenerate states and mass factorization

- Multi-loop integrals

  Computing them in d dimension is itself the biggest challenge

  Presence of many mass scales makes it harder

- Phase-space integrals

  Computing them analytically in d dimension is almost impossible

  Numerical computation is 10-20 dimensional

The generic procedure

$\boxed{d = 4 - 2\epsilon}$

- Diagrammatic approach -> QGRAF [Nogueira] to generate diagrams $(10^2 - 10^3)$
- In-house FORM routines [Vermaseren] for algebraic manipulation :
  Lorentz, Dirac and Color [Ritbergen, Schellekens, Vermaseren] *algebra*
- Reverse unitarity : phase-space integrals to loop integrals

$$\delta(k^2 - m^2) \sim \frac{1}{2\pi i}\left(\frac{1}{k^2 - m^2 + i0} - \frac{1}{k^2 - m^2 - i0}\right)$$

- Decomposition of the dot products to obtain scalar integrals $(10^5 - 10^6)$

$$\frac{2l.p}{l^2(l-p)^2} = \frac{l^2 - (l-p)^2 + p^2}{l^2(l-p)^2} = \frac{1}{(l-p)^2} - \frac{1}{l^2} + \frac{p^2}{l^2(l-p)^2}$$

- Identity relations $(10^5 - 10^6)$ among scalar integrals : *IBPs, LIs & SRs*
- Algebraic linear system of equations relating the integrals

$$\Downarrow \text{LiteRed}$$

Master integrals (MIs) $(10 - 10^2)$

- Computation of MIs : *Differential eqns.*

Iterated integrals and Harmonic polylogarithms (HPLs)

Given a set of integration kernels $K_i(t)$, one can define

$$\mathcal{I}(i_n, \dots i_1, x) = \int_{x_0}^{x} K_{i_n}(t)\mathcal{I}(i_{n-1}, \dots i_1, t)dt$$

$\{i_n\}$ are called the indices. $\{K_{i_n}\}$ is called the alphabet. Classic example is $\mathrm{Li}_n(x)$.

$$\mathrm{Li}_n(x) = \int_0^x \frac{1}{t}\,\mathrm{Li}_{n-1}(t)\,dt$$

Now, instead of $1/t$, we can have many kernels K_m

$$\Big\{-1, 0, 1, \{6,0\}, \{6,1\}, \{4,1\}, \dots\Big\} \equiv \Big\{\frac{1}{1+x}, \frac{1}{x}, \frac{1}{1-x}, \frac{1}{1-x+x^2}, \frac{x}{1-x+x^2}, \frac{x}{1+x^2}, \dots\Big\}$$

and correspondingly we define the HPLs as

$$H(m_n, \dots, m_1, x) = \int_0^x K_{m_n}(t)H(m_{n-1}, \dots, m_1, t)dt$$

Some important properties :

Shuffle algebra, Scaling invariance and integration-by-parts identities

Ultraviolet renormalization

⊛ The Born contribution is zeroth order in α_s, hence no α_s renormalization is needed for $\sigma_{ij}^{(1,1)}$.

⊛ Renormalization of quark wave function receives one-loop EW and two-loop mixed QCD⊗EW contributions in the on-shell scheme.

⊛ The neutral current vertex is renormalized using background field gauge, with the advantage that the vertex and propagator contributions are separately UV finite.

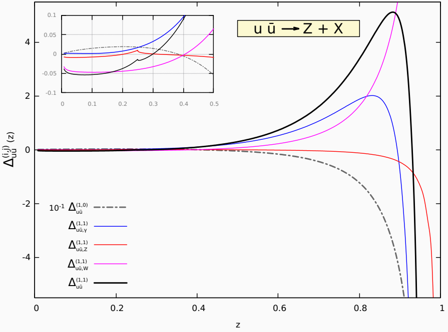
⊛ The UV counter terms get contributions from two-point functions with massive propagator insertion which we have obtained up to required accuracy.

Treatment of γ_5 : *As all the* γ_5 *appear in a single quark line, we can safely use the naive anti-commutation rule.*

Results!

Mixed QCD⊗EW corrections For the process $u\bar{u} \to Z$

- ✓ QCD⊗QED
- ✓ EW1 (Contributions from Z interchange)
- ✓ EW2+EW3 (Contributions from W interchange and WWZ vertex)



Summarizing

- The mixed NNLO QCD-EW contributions to Drell-Yan production are much sought for. We make an advancement by obtaining analytic results for on-shell Z boson production in quark initiated channel.
- We have computed analytically 12 different matrix element squared purely at two-loop level. Additionally there are one-loop contributions. Combining them to obtain a finite partonic cross section is non-trivial and acts as a strong check on our calculation.
- The method of reverse unitarity allows us to use the techniques (IBP, DE) of loop calculation for the phase-space integrals.
- The solutions are obtained mostly in terms of harmonic poly-logarithms (HPL) and special constants (MZV and cyclotomic HPL at 1). The contributions from elliptic functions are obtained as expansion near threshold.
- We also perform a parallel independent computation to cross check.

Future direction

1. Towards obtaining the complete $\mathcal{O}(\alpha\alpha_s)$ corrections to full Drell-Yan, the next step is to compute the double virtual (two-loop four-point with up to two massive lines) contributions. After performing the algebra and IBP reduction, the most challenging part is compute the master integrals.
2. For the real emission cases, we then try to use the method of reverse unitarity to solve the phase-space master integrals.
3. In parallel, we wish to establish a subtraction technique in this case. With proper subtraction terms, we will be able to obtain both inclusive and differential distributions for full Drell-Yan.

Thank you for your attention!