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**PhD
Nanoscience**

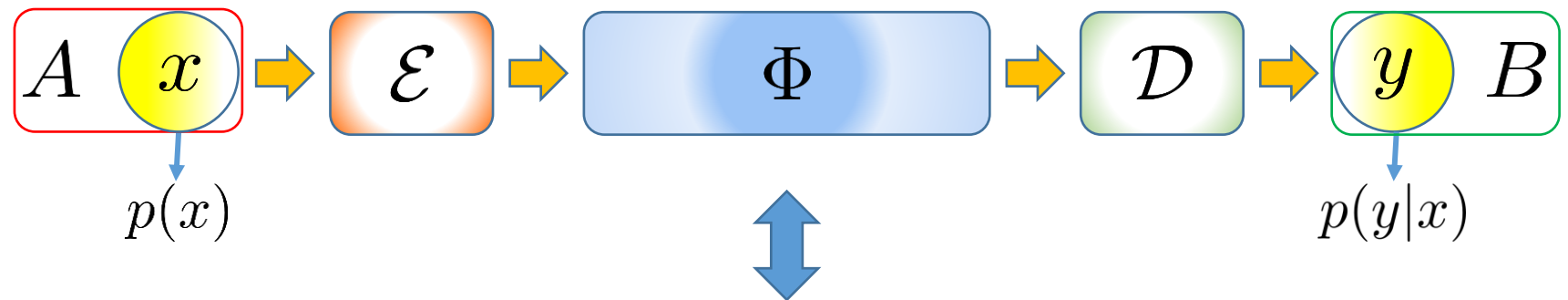
Year II

Research Project

- Research project

Quantum Information & Communication

- Communication channels:



Characterization

- Conferences

Research Project

Last year:


- Research project

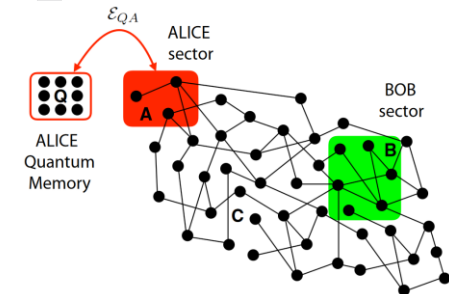
- Quantum channel: spin networks

- Conferences

PHYSICAL REVIEW A **100**, 032311 (2019)


Quantum-capacity bounds in spin-network communication channels

Stefano Chessa , Marco Fanizza, and Vittorio Giovannetti
NEST, Scuola Normale Superiore and Istituto Nanoscienze-CNR, I-56126 Pisa, Italy



PHYSICAL REVIEW A **100**, 052309 (2019)

Time-polynomial Lieb-Robinson bounds for finite-range spin-network models

Stefano Chessa  and Vittorio Giovannetti

Research Project

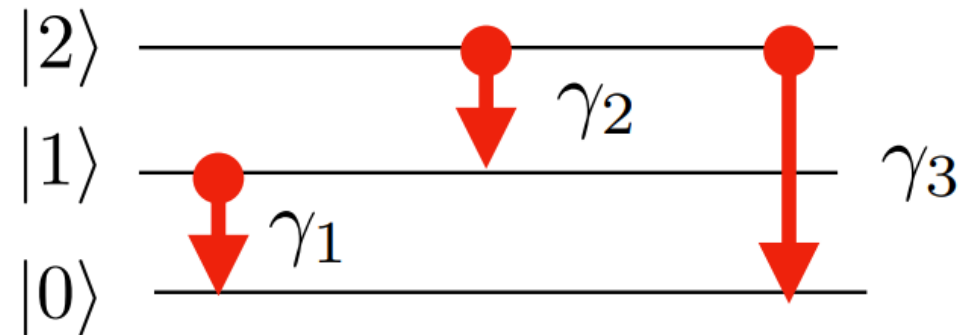
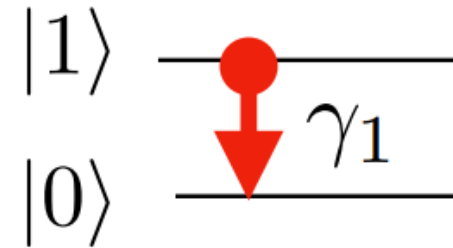
This year:

- Research project

- Quantum channel:
Amplitude damping



**MORE
LEVELS**



Research Project

This year:

- Research project

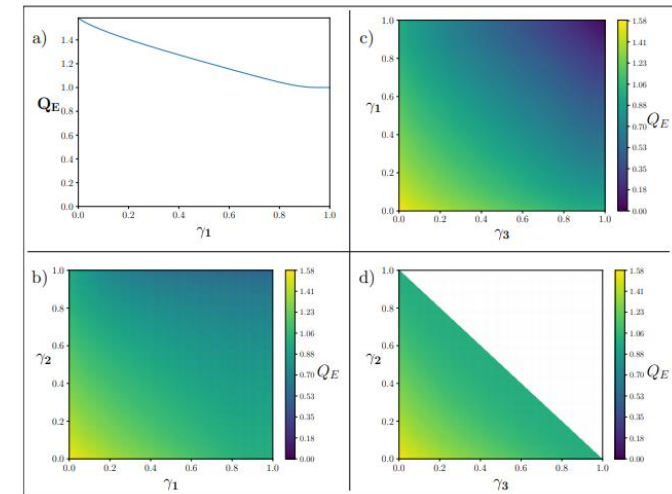
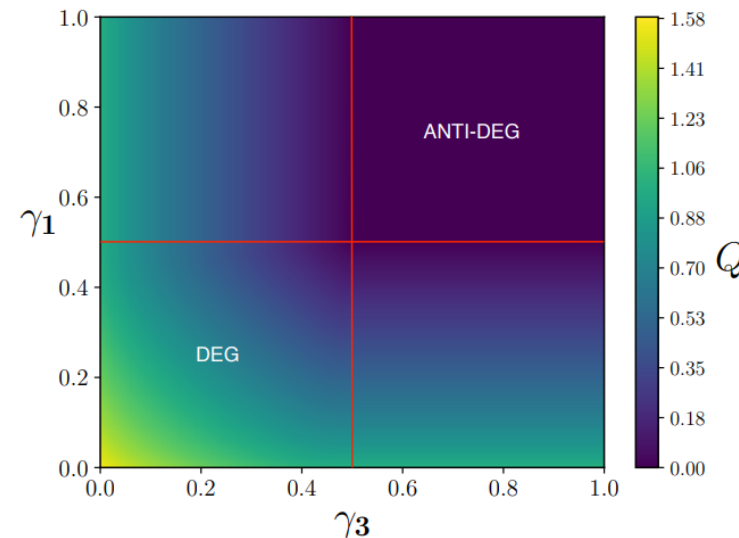
Multi-level Amplitude Damping channels: quantum capacity analysis

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(Dated: August 5, 2020)

- Conferences



Research Project

This year:

- Research project

- Quantum channel:

Partially Coherent Direct Sum channels

- Conferences

$$\Phi_{CC} \left[\begin{array}{c|c} \hat{\Theta}_{AA} & \hat{\Theta}_{AB} \\ \hline \hat{\Theta}_{BA} & \hat{\Theta}_{BB} \end{array} \right] = \left[\begin{array}{c|c} \Phi_{AA}[\hat{\Theta}_{AA}] & \Phi_{AB}^{(\text{off})}[\hat{\Theta}_{AB}] \\ \hline \Phi_{BA}^{(\text{off})}[\hat{\Theta}_{BA}] & \Phi_{BB}[\hat{\Theta}_{BB}] \end{array} \right]$$

Research Project

This year:

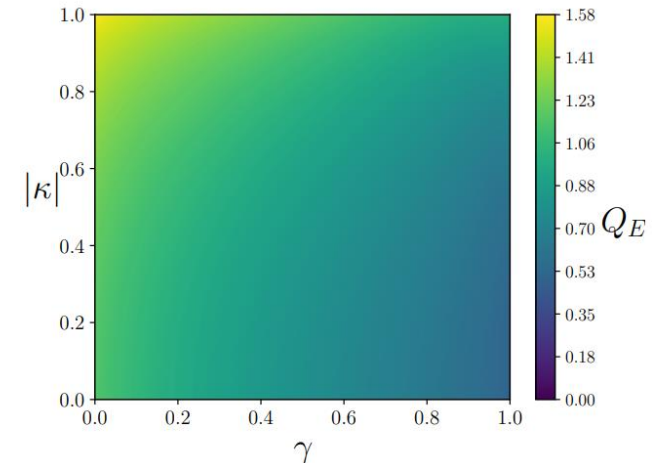
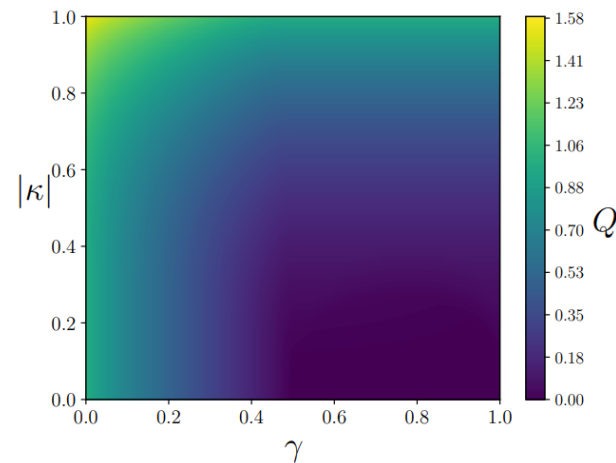
Partially Coherent Direct Sum Channels

Stefano Chessa^{1,*} and Vittorio Giovannetti¹

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(Dated: August 5, 2020)

$$\Omega_{\text{CC}}^{[\gamma](\kappa)}[\hat{\rho}_{\text{CC}}] = \begin{pmatrix} \rho_{00} + \gamma\rho_{11} & \sqrt{1-\gamma}\rho_{01} & \kappa\rho_{02} \\ \sqrt{1-\gamma}\rho_{01}^* & (1-\gamma)\rho_{11} & \kappa\sqrt{1-\gamma}\rho_{12} \\ \kappa^*\rho_{02}^* & \kappa^*\sqrt{1-\gamma}\rho_{12}^* & \rho_{22} \end{pmatrix}$$



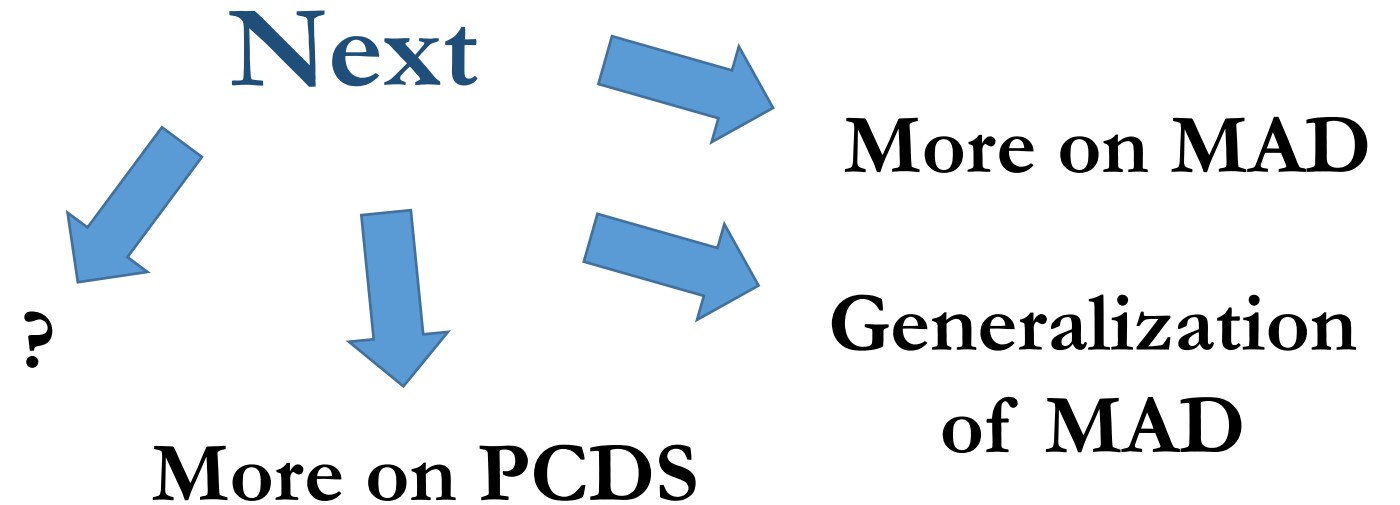
- Research project

- Conferences

Research Project

- Research project

- Conferences



Research Project

• Research project

QIP 2020

TQC 2020

YIQIS 2020

• Conferences

NEST

SCUOLA NORMALE SUPERIORE

Lieb-Robinson bound constrains information capacities for quantum spin network channels

S. Chessa, V. Giovannetti, Time-Dependent Lieb-Robinson bounds for finite-range spin-network models, arXiv preprint 1905.1171 (2019).

S. Chessa, M. Fannes, V. Giovannetti, Quantum capacity bounds in spin-network communication channels, arXiv preprint 1905.1109 (2019).

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SCNRNANO

PISA

INTRO: LIEB-ROBINSON (LR) BOUND

• $d(x, y)$, a metric which measures the shortest path between vertices x and y ;

IMPROVING THE BOUND: POLYNOMIAL IN TIME

Being $|A|, |B|$ the number of spins in A, B and $\lambda > 0$ a free parameter, a more recent expression for the LR bound [2] is cast in the following form

$$\| \hat{A}(t), \hat{B} \| \leq 2|A||B| \| \hat{A} \| \| \hat{B} \| (e^{2\lambda \| \hat{B} \| t} - 1) e^{-\lambda d(A, B)}, \quad (3)$$

with $\| \hat{B} \|_\lambda := \sup_{x \in V} \sum_{Y \ni x} |X| M_X^{2\lambda(X)} \| B_X \| < \infty$, being $M_X := \max_{i \in X} \| H_i \|$.

Assuming a short range interaction s.t. $D(X) < D$, it can be proved that there exists a $\zeta > 0$ depending on the topology and interaction strengths s.t. $\forall \lambda \geq 0$ we have $\| \hat{B} \|_\lambda \leq \zeta e^{\lambda D}$. Now we perform an optimization w.r.t. λ obtaining a λ_{opt} : inserting it in (3) we get the optimal bound

$$\| \hat{A}(t), \hat{B} \| \leq 2|A||B| \| \hat{A} \| \| \hat{B} \| \left(\frac{2\zeta D \| \hat{B} \|}{d(A, B)} \right)^{\frac{d(A, B)}{D}}. \quad (4)$$

• This bound is polynomial in t , is 0 at $t = 0$ and gets smaller at larger distances.

1D PERTURBATIVE METHOD AND A SIMULATION

Consider now a 1-D next-neighbour Hamiltonian

$$\hat{H} := \sum_{i=1}^{L-1} h_{i,i+1},$$

s.t. $\| h_{i,i+1} \| = \max \| h_{i,i+1} \|$. Via Campbell-Baker-Hausdorff formula we can rewrite $\hat{A}(t)$ in $\| \hat{A}(t), \hat{B} \|$. Bounding the effective number of non zero terms in the series expansion we get

$$\| \hat{A}(t), \hat{B} \| \leq \frac{2|A||B|}{d(A, B)} \left(\frac{2\zeta D \| \hat{B} \|}{d(A, B)} \right)^d e^{2\lambda \| \hat{B} \| t}. \quad (7)$$

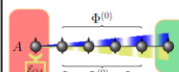
• At small t behaves polynomially like (4).

We test our predictions (Fig. 4) simulating $\| \hat{A}(t), \hat{B} \|$ for an Heisenberg next-neighbour chain at different lengths L (Fig. 3) and Hamiltonian

$$\hat{H} = J \sum_{i=1}^{L-1} \sigma_{i,i+1}^x + \sigma_{i,i+1}^y. \quad (8)$$

MESSAGE DISCRIMINATION

For the communication to succeed, Bob must be able to discern the natural network unitary evolution $\Phi^{(0)}$ from Alice's action $\Phi := \Phi^{(0)} \circ \mathcal{E}_{Q,A}$.



• $\Phi^{(0)}[\cdot] = e^{iH_0[\cdot]t}$, for fixed t and input state it's a depolarizing channel.

• $\mathcal{E}_{Q,A}[\cdot] = \sum_{k=1}^K \hat{U}_k[\cdot] \hat{U}_k^\dagger$, with $K \leq (M_A M_Q)^2$.

Fig. 3: Unitary evolution $\Phi^{(0)}$ vs Alice's encoding Φ .

We will evaluate $D(\hat{\rho}_B(t), \hat{\rho}_B^{(0)}(t)) := \frac{1}{2} \| \hat{\rho}_B(t) - \hat{\rho}_B^{(0)}(t) \|_1 = \max_{\theta_B} \left[\text{Tr}_B \left[\hat{\theta}_B (\hat{\rho}_B(t) - \hat{\rho}_B^{(0)}(t)) \right] \right]$, with $\| X \|_1 := \text{Tr} \sqrt{X^\dagger X}$ and $\hat{\theta}_B \geq 0$ and $\text{Tr} \hat{\theta}_B = 1$. From Kraus decomposition of $\mathcal{E}_{Q,A}$, trace cyclicity, positivity of $\hat{\theta}_B$ and triangular inequality follows that

$$D(\hat{\rho}_B(t), \hat{\rho}_B^{(0)}(t)) \leq \max_{\theta_B} \sum_k \| \hat{M}_k^\dagger \hat{\theta}_B(t) \| \| \hat{M}_k \|.$$

Since \hat{M}_k are local on A we adopt LR machinery in (5) and, being $\epsilon_{AB}(t)$ the r.h.s. in (4), get:

$$D(\hat{\rho}_B(t), \hat{\rho}_B^{(0)}(t)) \leq (K/2) \epsilon_{AB}(t). \quad (6)$$

BOUNDING CAPACITIES

Being $|\psi\rangle_{QQ'}$ the purification of ρ_Q , the diamond distance of channels Φ, Φ' from Q to B is $\| \Phi - \Phi' \|_\diamond = \max_{|\psi\rangle_{QQ'}} \| \Phi \otimes \mathbb{I} - \Phi' \otimes \mathbb{I} \|_1(|\psi\rangle_{QQ'}\langle\psi|)$. From (6), assuming $M_A \leq M_B M_Q$ we can show:

$$\| \Phi - \Phi^{(0)} \|_\diamond \leq 2M_A^2 \epsilon_{AB}(t). \quad (9)$$

By $C_E(\Phi)$ the entanglement assisted capacity for Φ . Defining $g(x) := (1+x)H_2(x)/(1+x)$, being $H_2(x)$ the Shannon entropy, via Fannes inequality [3] in [4] it is shown to be

$$|C_E(\Phi) - C_E(\Phi^{(0)})| \leq \| \Phi - \Phi^{(0)} \|_\diamond \log M_A + \frac{1}{2} \| \Phi - \Phi^{(0)} \|_\diamond.$$

Since $\Phi^{(0)}$ acts as a depolarizing channel, $C_E(\Phi^{(0)}) = 0$. Information capacities of a channel Φ satisfy $C_P(\Phi), C(\Phi), Q(\Phi) \leq C_E(\Phi)$. Then from (9) all capacities are bounded by

$$C_E(\Phi) \leq 2M_A^2 \epsilon_{AB}(t) \log M_A + \frac{1}{2} M_A^2 \epsilon_{AB}(t). \quad (11)$$

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[1] E. Lieb and D. Robinson, Commun. Math. Phys. **28**, 251-257, (1972).

[2] B. Nachtergaele and R. Sims, Commun. Math. Phys. **265**, 119 (2006).

[3] M. Fannes, Commun. Math. Phys. **31**, 291 (1973).

[4] M. E. Shirokov, J. Math. Phys. **58**, 102302 (2017).