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Harmonic and polyharmonic maps, isoparametric functions,weak solutions stability, energy minimizing maps

Abstract

The study of higher order energy functionals was first proposed by Eells and Sampson in 1965 and, later, by Eells and Lemaire in 1983. These functionals provide a natural generalization of the classical energy functional which defines harmonic maps. More precisely, Eells and Sampson suggested the investigation of the functionals $E_r^{ES}(\varphi) = (1/2)\int_M |(d^* + d)^r(\varphi)|^2\, dV$, where $\varphi : M \to N$ is a map between two Riemannian manifolds and $r \geq 2$.

In the first part of this talk we shall illustrate some recent progress ([1, 3, 4]) on the study of critical points of this type of functionals when the target manifold is the Euclidean sphere $\mathbb{S}^n$. The involved methods are essentially geometric and provide some interesting connection with the theory of isoparametric functions on spheres.

In the second part of the talk we shall describe a rather different approach which is more suitable to use classical variational methods in order to achieve regularity, stability and minimization properties of critical points. More specifically, let B^n denote the Euclidean n-dimensional unit ball. The *extrinsic k-energy functional* is defined on the Sobolev space $W^{k,2}(B^n, \mathbb{S}^n)$ as follows: $E_k^{\mathrm{ext}}(u) = \int_{B^n} |\Delta^s u|^2\, dx$ when $k = 2s$, and $E_k^{\mathrm{ext}}(u) = \int_{B^n} |\nabla\Delta^s u|^2\, dx$ when $k = 2s + 1$. These energy functionals are a natural higher order version of the classical extrinsic bienergy, also called Hessian energy. The classical equator map $u^* : B^n \to \mathbb{S}^n$, defined by $u^*(x) = (x/|x|, 0)$, is a critical point of $E_k^{\mathrm{ext}}(u)$ provided that $n \geq 2k + 1$. We shall discuss a recent result of [2], where we obtained necessary and sufficient conditions on k and n under which the equator map $u^* : B^n \to \mathbb{S}^n$ is minimizing or unstable for the extrinsic k-energy.

References

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