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Hilbert Modules and Singular Vector Bundles on Bounded Symmetric Domains

Abstract

A Hilbert space H of holomorphic functions on a bounded domain D in several complex variables is called a Hilbert module if the coordinate functions, and hence all polynomials, act on H by (bounded) multiplication operators. In this case the adjoint operators have a joint eigenspace at every point of D, and the collection of all joint eigenspaces forms a "singular vector bundle" (linearly fibered complex analytic space), in duality with a coherent analytic sheaf. In the talk we consider bounded symmetric domains $D = G/K$ and study K-invariant Hilbert modules of functions vanishing on determinantal varieties. It is shown that the fibres of the associated singular vector bundle can be described in terms of Kähler geometry, namely as spaces of holomorphic line bundle sections over compact flag manifolds. The main technique is the Jordan theoretic description of bounded symmetric domains, and the Peter-Weyl decomposition of polynomials under the natural K-action, related to the theory of partitions and Young diagrams.