New constraints on inflation in the Hartle-Hawking state?

based on work [2305.15440] with T. Hertog & J. Karlsson

very speculative, very incomplete, very fun

$$\Psi[h_{ab}, \chi] = \int_{?}^{g=h,\ \phi=\chi} Dg\, D\phi \; e^{-S/\hbar}$$

fluctuating

$$\approx \sum_{i \in A} e^{-S_i/\hbar}$$

optimistically

Witten [2111.06514], inspired by Kontsevich & Segal [2105.10161]

+ Halliwell-Hartle '90 + Louko-Sorkin '95

$$A = \{\text{allowable complex geometries}\}$$

$= \{$ those g_i on which the Euclidean path integral for perturbative free p-form matter converges (all p) $\}$

exciting : new ingredient for QC so we were eager to investigate

goal : explore whether A makes sense, what kind (if any) predictions does it make (WIP), not final story

motivation

1) complex metrics seem to be a must in gravity

$$S \supset \left(-\frac{1}{2}\right)\int \sqrt{g}\, R \quad \text{unbounded below}$$

2) Ψ ~ superposition of classical spacetimes at large h

$$\sim \sum e^{-S^{Re}} e^{iS^{Im}} \,, \qquad |\nabla S^{Im}| \gg |\nabla S^{Re}|$$

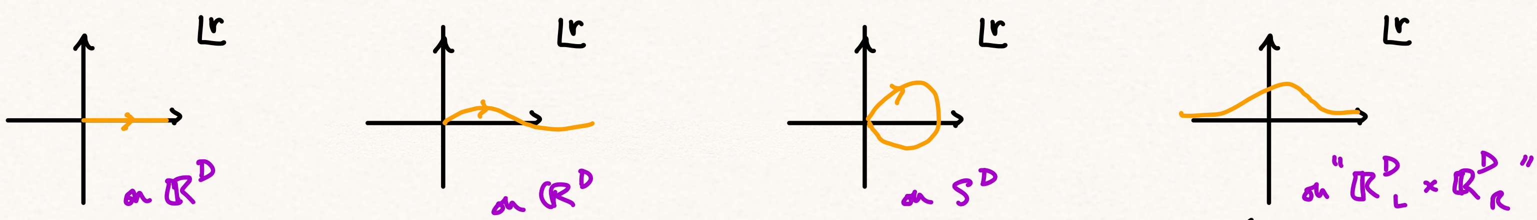
3) • some complex metrics are good

(double cone to study spectral form factor $\langle Z(-\beta) Z(\beta) \rangle$ in holographic theory  ,

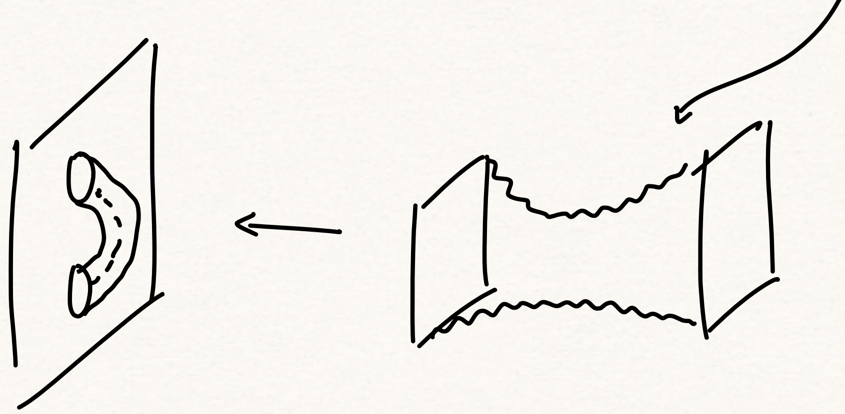
"quasi - Euclidean" Kerr)

• some are bad

$ds^2 = dr^2 + r^2 d\Omega^2_{D-1}$ and complexify $r = r(u) \in \mathbb{C}$



$ds^2 = r'(u)^2 du^2 + r(u)^2 d\Omega^2_{D-1}$ all metrics are flat, have vanishing action



The class A

Those $g_{\mu\nu}$ on which QFT could be defined:

$$\int_{A_p \text{ real}} DA_p \, e^{-S[A_p; g]} < \infty$$

$$S = \int F_{p+1} \wedge * F_{p+1} \quad , \quad F = dA$$

$$= \int d^D x \sqrt{\det g} \; g^{\alpha_1 \beta_1} \cdots g^{\alpha_{p+1} \beta_{p+1}} F_{\alpha_1 \cdots \alpha_{p+1}} F_{\beta_1 \cdots \beta_{p+1}}$$

$$\rightsquigarrow \; \mathrm{Re}\left(\sqrt{\det g} \; g^{\alpha_1 \beta_1} \cdots g^{\alpha_{p+1} \beta_{p+1}} F_{\alpha_1 \cdots \alpha_{p+1}} F_{\beta_1 \cdots \beta_{p+1}} \right) > 0$$

$$\forall x \in M$$

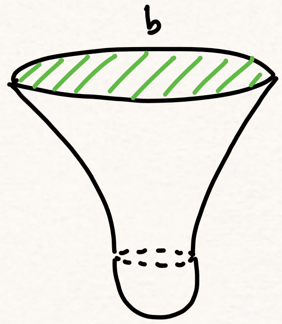
$$\forall p \in \{-1, \dots, D-1\}$$

KS showed that these conditions are equivalent to

$$\boxed{\sum_{i=1}^{D} |\arg \lambda_i| < \pi}$$

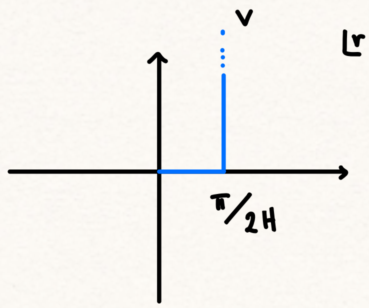
Lorentzian ✗ Euclidean ✓✓ Lorentzian $+ i\varepsilon$ ✓

The Hartle-Hawking state

$$\Psi_{HH}(b) \approx \exp\Big(- $$  $$\Big)$$

metric

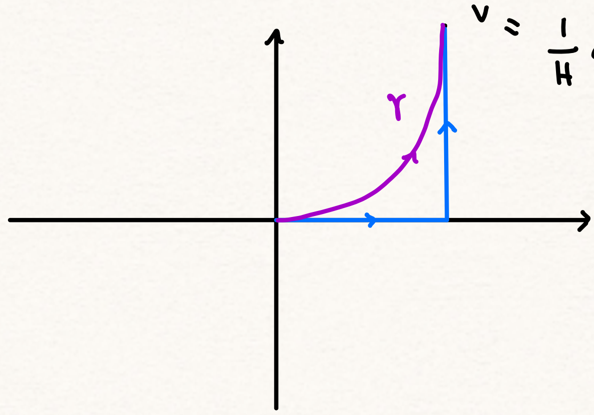
$h_{ab} = b^2 \Omega_{ab}$

$$= \exp\Big(- $$  $$\Big)$$

$$= \exp\left(\frac{1}{H^2}\right) \times \exp\left(i\, b^3 H\right)$$

$$ds^2 = dr^2 + a(r)^2\, d\Omega_3^2 \quad , \quad a(r) = \frac{1}{H}\sin(Hr)$$

blue contour is not allowed, but $\exists$ deformations that are allowed



$v = \frac{1}{H}\cosh^{-1}(bH)$

$$ds^2 = \dot{\gamma}(\ell)^2\, d\ell^2 + a(\gamma(\ell))^2\, d\Omega_3^2$$

$$\text{KS}: \quad \sum_i |\arg \lambda_i| = |\arg(\dot{\gamma}^2)| + 3\,|\arg a^2|$$

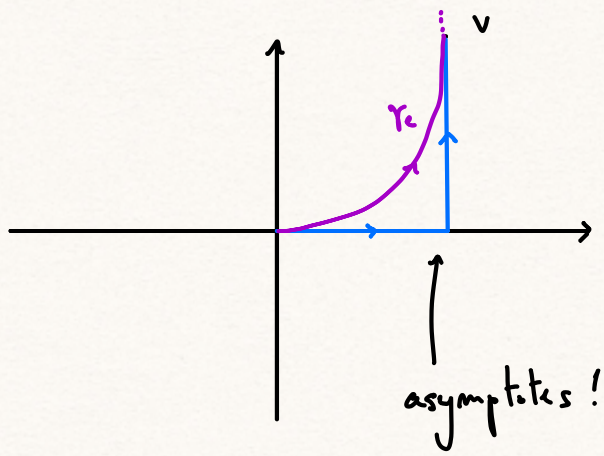
$$< \pi \quad \text{along all } \gamma \quad \checkmark$$

- How to go about showing $\exists \gamma$ in a systematic way ?

  Construct curve γ_e that <u>saturates</u> KS :

$$|\arg(\dot{\gamma}_e^2)| + 3\,|\arg a(\gamma_e)^2| = \pi$$

has analytic solution in this case



any curve "below" γ_e allowed

so take $\pi \to \pi - \varepsilon$: every endpoint can be reached by allowable γ

- can show $\mathrm{Re}\left(\frac{\pi}{2H} - \gamma_e\right) = \mathcal{O}\left(\frac{1}{H} e^{-3H \,\mathrm{Im}\, \gamma_e}\right)$

  illustrates numerical difficulty !

  if $b = \frac{1}{H} e^{N_e}$ then $v \sim \frac{1}{H} N_e \sim \mathrm{Im}\,\gamma_e$

  so $\mathrm{Re}\left(\frac{\Delta}{H}\right) = \mathcal{O}\left(\frac{1}{H} e^{-3N_e}\right)$. for $N_e = 60$

  $\mathrm{Re}(\Delta) = \mathcal{O}(10^{-80})$

Inflation

include scalar, potential V

$$\Psi_{HH}[h_{ab}, \chi] \sim \exp\left(- \right.$$


$$\left. \right)$$

here : $\Psi_{HH}(b, \chi)$

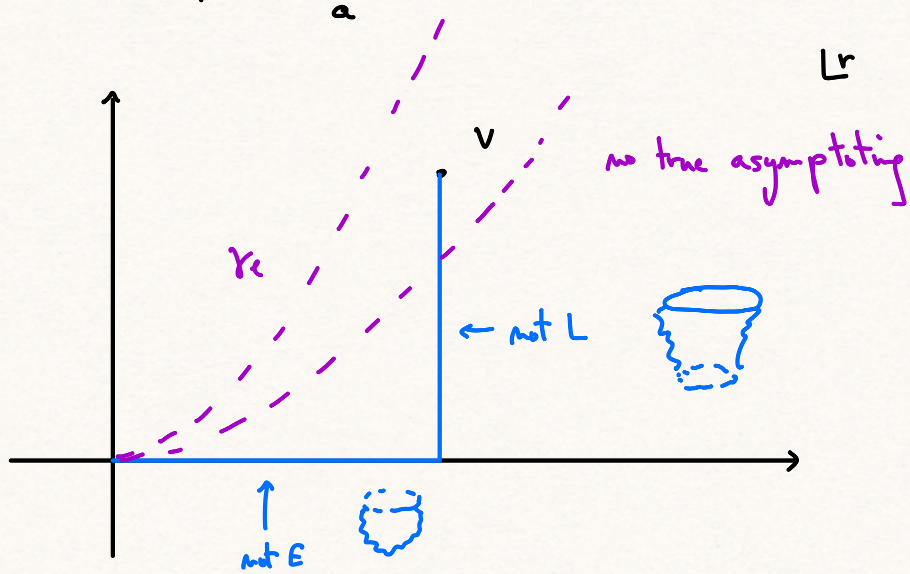
round S^3 boundary "radius" b, homogeneous χ

assume saddle (g, ϕ) maximal symmetry on B^4

Ansatz :
$$ds^2 = dr^2 + a(r)^2 d\Omega_3^2$$
$$\phi = \phi(r)$$

EoM :
$$\left(\frac{a'}{a}\right)^2 = \frac{1}{a^2} + \frac{1}{3}\left(\frac{\phi'^2}{2} - V\right)$$
$$\phi'' + 3\frac{a'}{a}\phi' - V' = 0$$

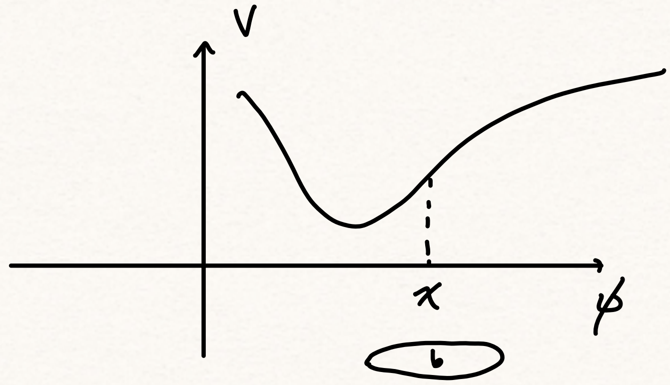
same idea, but:



- it is now possible for some configurations (b, χ) to be "not allowed": there is no $O(4)$-symmetric instanton on B^4 that induces (b, χ) on its S^3 boundary and is everywhere allowed. they would be (at least) exponentially suppressed in Ψ_{HH}

- we examined large universes at the end of inflation,

$$b = \frac{1}{H} e^{N_e}, \quad \chi = \phi_{end}$$



<u>1-line summary</u>: allowability likes concave slow-roll regions

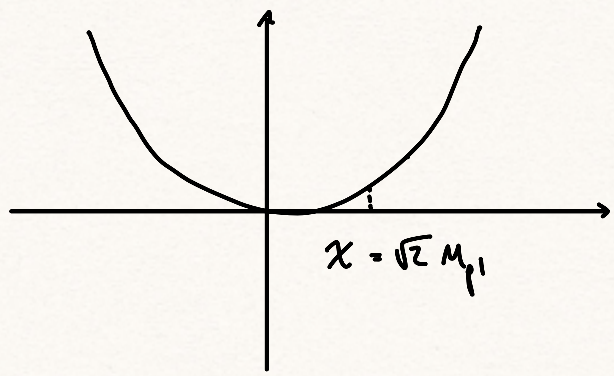
concave : not allowed $\rightarrow$ allowed as $N_e \nearrow$

convex : allowed $\rightarrow$! allowed

can be absent

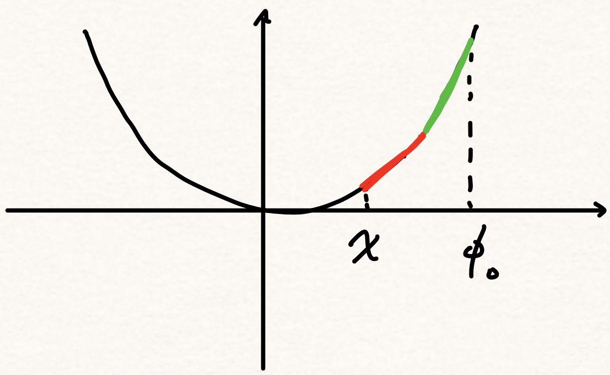
- allowability is exponentially subtle: whether or not a late-time configuration is allowed depends on details of how instanton was set up in very early phase of inflation! memory of initial state is not washed away

example: $V = m^2 \phi^2$

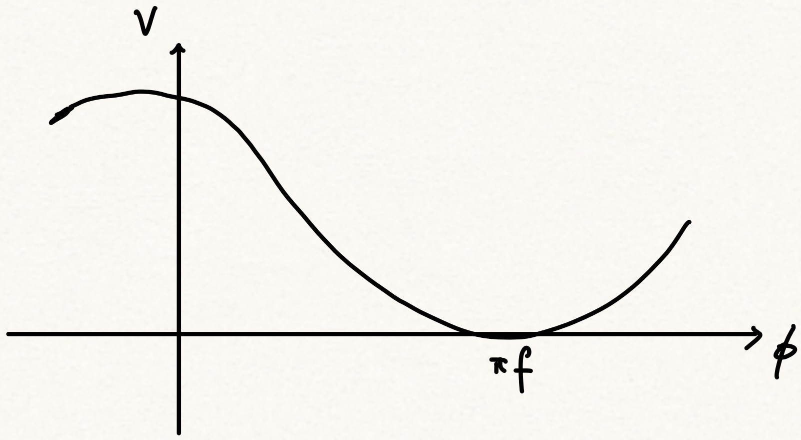


nothing is allowed

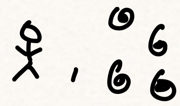
what happens:



BUT consider $V = \Lambda\left(1 + \cos(\phi/f)\right)$



large $f \gg M_{pl}$, inflation $\sim m^2 \phi^2$. so $N_e < N_e^{max}$ efolds not allowed. BUT many $N_e \gg N_e^{max}$ allowed.

- what does this mean?
  - in a given landscape, a set of inflationary trajectories will be ruled out (hard to make general prediction, but small r seems to be preferred)
  - what are <u>predictions</u>? (future work)
    - ↳ include $\Psi_{HH} \sim e^{-S}$
    - ↳ include $P(\theta \mid D)$, $D = \{$  $, \ldots \}$