Cosmological Correlators at Finite Coupling

Shota Komatsu

CERN

based on work to appear with Lorenzo Di Pietro, Victor Gorbenko

Punchline

*late-time four-point

We solved* $O(N)$ model @ large N
in dS finite gN

Punch line

*late-time four-point

We solved* O(N) model @ in dS | large N / finite gN

- Confirm & extend results from stochastic approach
  - Starobinski, Yokoyama
  - Gorbenko, Senatore
- Nontrivial check of positivity of spectral density
  - Hogervorst, Penedones, Vaziri
  - Di Pietro, Gorbenko, SK
- Exploration of non-perturbative analyticity

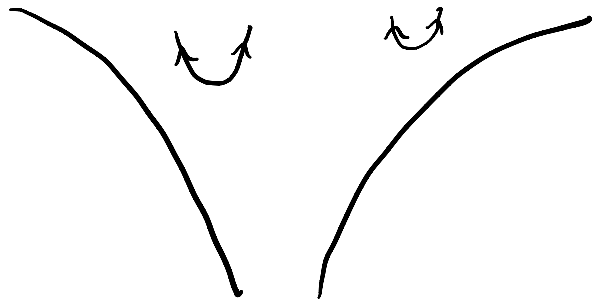
Why non-perturbative QFT

in dS ?

Non-perturbative QFT in dS

- Intuition from perturb theory can be misguiding
- IR problem for light scalar
- For some questions, non-perturbative effects may be crucial.
  - prob. for reheating (Cohen et al)
  - primordial blackhole formation

IR problem

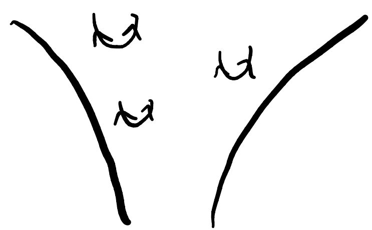


- particle production due to expansion
- secular divergence : 1-loop grows logarithmically @ late time
- Some even argued QFT for light scalar in dS is ill-defined

Of course, we now "know" it is well-defined

- Resummation based on stochastic approach
  $\rightsquigarrow$ sensible answer

- Lesson from $O(N)$

 $\longrightarrow$ Different vacuum

Unitarity & Analyticity

Conformal Partial Wave for de Sitter

- Isometry of dS_{d+1} = $SO(d+1,1)$ ~ conformal group in d-dim

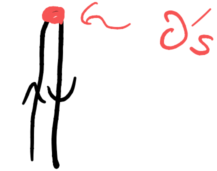
- $\langle \mathcal{O}(x_1)\, \mathcal{O}(x_2)\, \mathcal{O}(x_3)\, \mathcal{O}(x_4) \rangle_{y=0}$

$$= \sum_J \int_{\Delta = \frac{d}{2} + i\mathbb{R}} d\Delta \; \underbrace{\rho_J(\Delta)}_{\text{spectral density}} \; \underbrace{F_{\Delta,J}(x_1, x_2, x_3, x_4)}_{\text{conformal partial wave}} + \cdots$$

- $\rho_J(\Delta) \geq 0$ ⟸ Unitarity

Positivity : A rough sketch

- $\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$: In-in correlators



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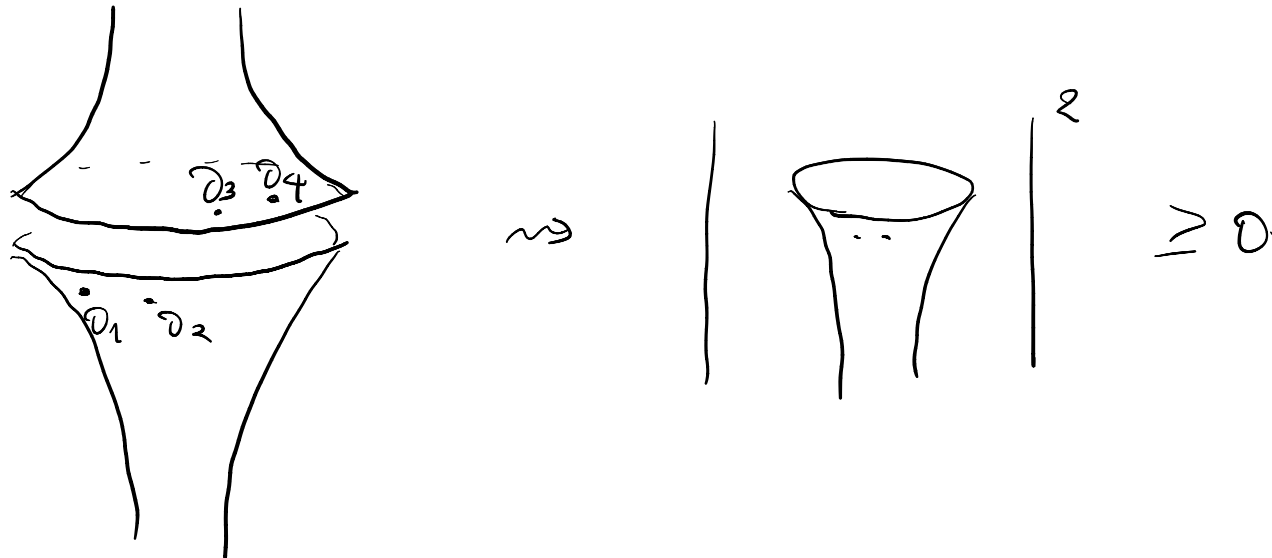
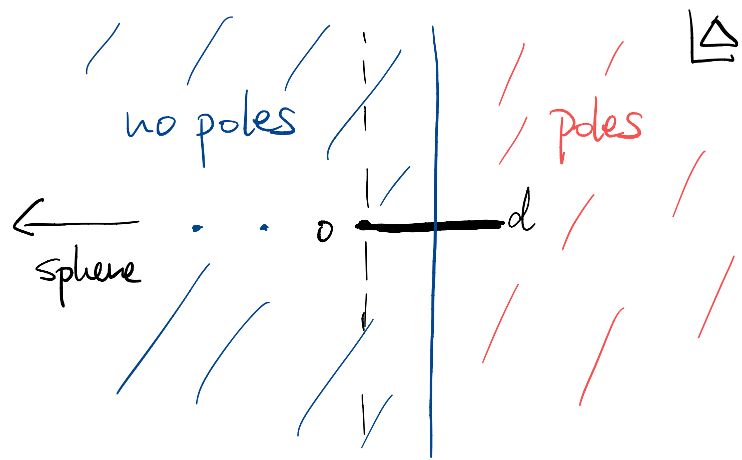


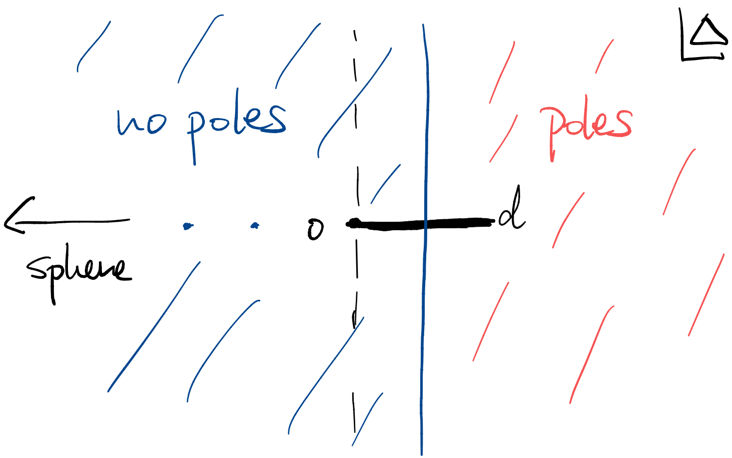
Analyticity

- In all the examples we know, $\rho_J(\Delta)$ only has poles (NO branch cuts)

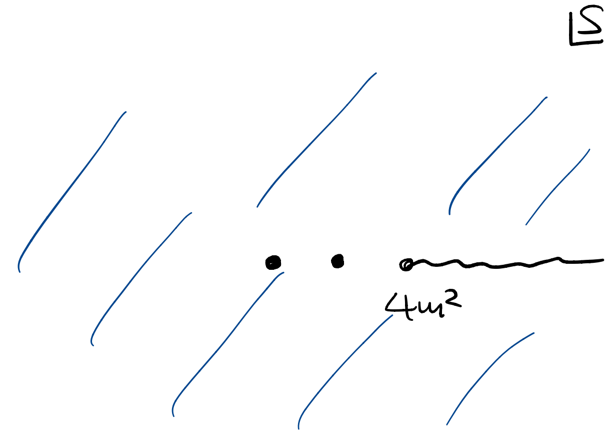
- Spectral Amplitude : $f_J(\Delta)$ $\left(\rho_J(\Delta) = \frac{1}{2}\left(f_J(\Delta) + f_J(d-\Delta)\right)\right)$



- Complementary ~ bound state
- Poles ~ resonance



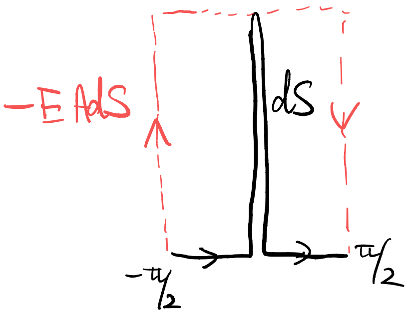
dS



flat

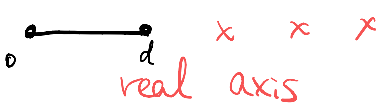
Argument for analyticity

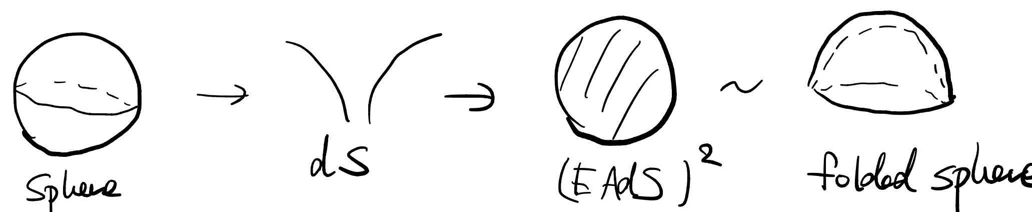
- Many examples
- Rotation to ("minus" EAdS)^2
  - Hartle, Hertog
  - Stanford, Harlow



  - Taronna, Sleight
  - Di Pietro, Gorbenko, SK

- CFT in dS
  - Hogervorst et al.





$O(N)$ model : sample of results

Sketch of Derivation

- $S \sim \int \sum_i (\partial\phi^i)^2 + m^2 \sum_i \phi_i^2 + \frac{\lambda}{2N}\left(\sum_i \phi_i^2\right)^2$

- Hubbard-Stratonovich

  $S \sim \int \sum_i (\partial\phi^i)^2 + m^2 \sum_i \phi_i^2 - \frac{N}{2\lambda}\sigma^2 + \sigma \sum_i \phi_i^2$

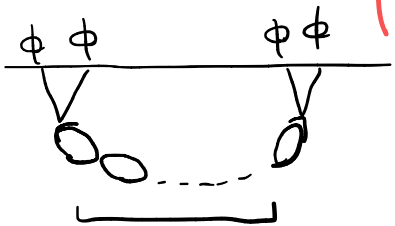
- Quadratic in ϕ^i $\rightsquigarrow$ integrate out ϕ^i

- $S_{eff}(\sigma) = N\left[-\frac{\sigma^2}{2\lambda} - \log\det(\partial^2 + m^2 + \sigma)\right]$

  $\rightsquigarrow$ saddle point σ_*

- Correlator from fluctuations around $S_{eff}(\sigma^*)$

Sample of results

- $\sum_n$  n $\sim \int d\nu \, \dfrac{1}{\frac{1}{\lambda} + B(\nu)} \, \mathcal{F}_{\text{partial wave}}$

$$\hat{B}(\nu')|_{d=2} = \frac{i}{8\pi\nu'}\left[\pi - i\coth(\pi\nu)\left(\psi\left(-i\nu + \frac{i\nu'}{2} + \frac{1}{2}\right) - \psi\left(i\nu + \frac{i\nu'}{2} + \frac{1}{2}\right)\right)\right],$$

- ρ  $+ \ \rho$  $\gtrsim 0$

  - Both are $\mathcal{O}(1)$
  - ρ  oscillates

- $m^2_{\text{bare}} \ll \sqrt{\lambda}\, m^2_{\text{Hubble}}$ $\rightarrow$ $M^2_{\text{phys}} = \sqrt{\lambda}\, m^2_{\text{Hubble}} + \cdots$

  perturbation is "IR divergent"

  $(\sim m^2 + \sigma_*)$

  Agrees with stochastic approach.

Future Direction

Conclusion

- Large N theory in dS is useful for checks of positivity, analyticity, stochastic

Future

- Large N theories as solvable toy models for nonperturbative physics in cosmology?
  - PBH, reheating
- CFT + perturb in dS → EAdS ?
- Bounding $(\partial\phi)^4$ using positivity ... ?
  - Sum rule [Bonifacio, Mazac, Pal]