Correlators (for all Λs) in Cortona

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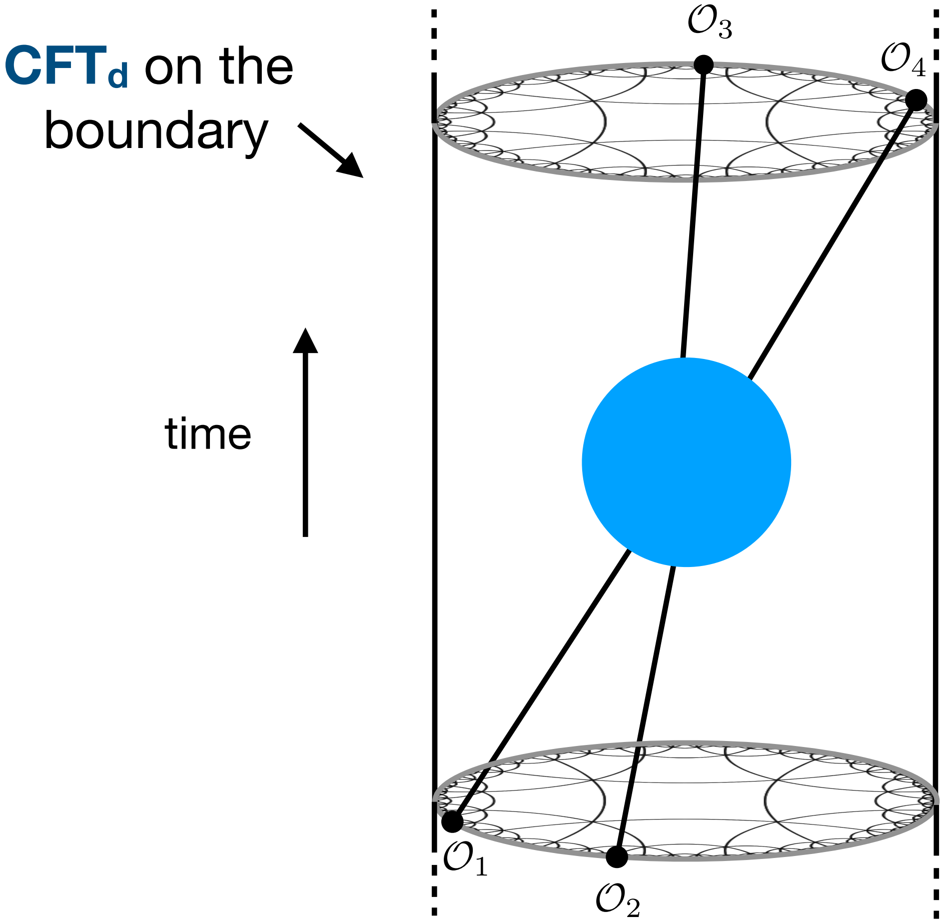
AdS-CFT

Quantum Gravity in AdS$_{d+1}$ = (non-gravitational) CFT in $\mathbb{M}^d$

Observables ?! ↔ **Correlation functions**



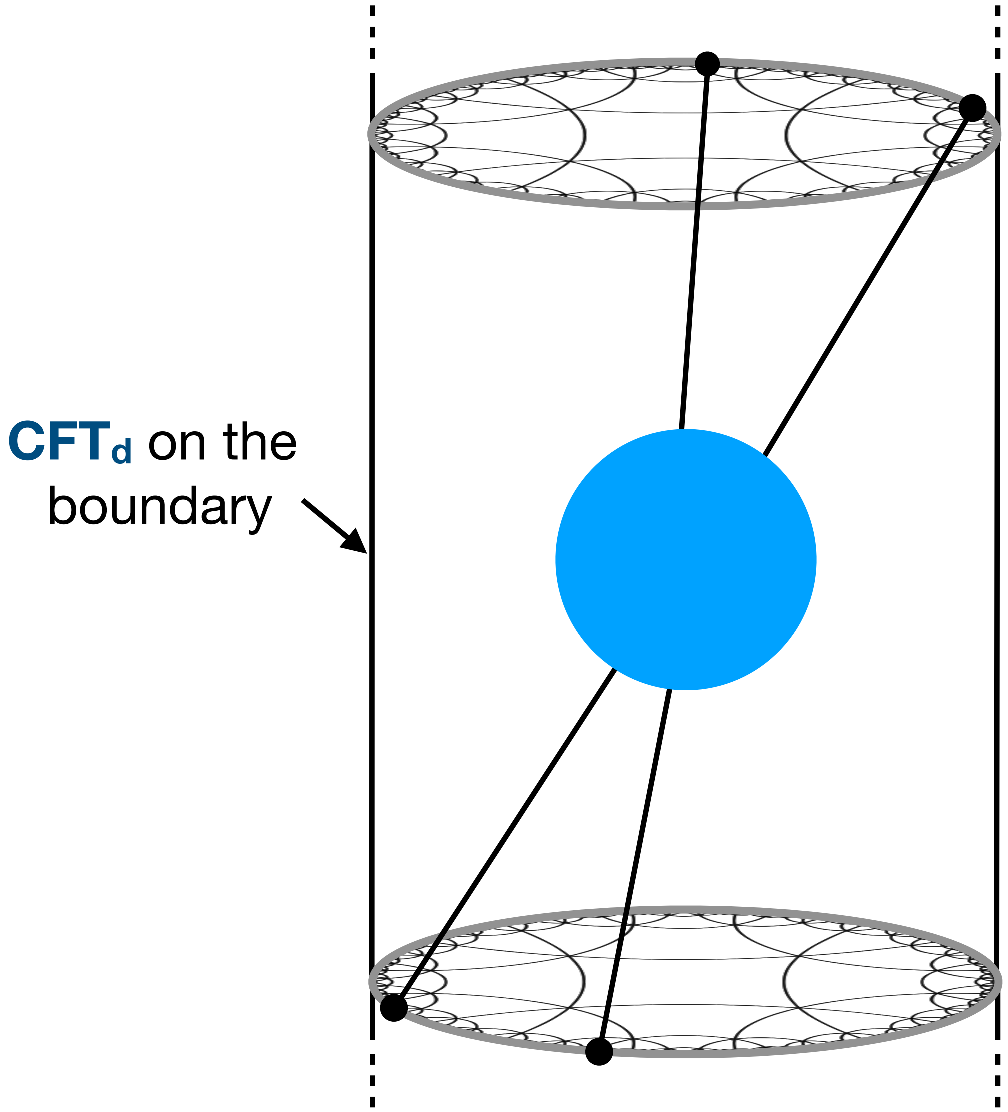
Constrained non-perturbatively by the Conformal Bootstrap:

- Conformal symmetry
- Unitarity
- Associative OPE

$$(\mathcal{O}_1\mathcal{O}_2)\,\mathcal{O}_3 = \mathcal{O}_1\,(\mathcal{O}_2\mathcal{O}_3)$$

[Belavin, Polyakov, Zamolodchikov 1984; Rattazzi, Rychkov, Tonni, Vichi 2008]

AdS-CFT

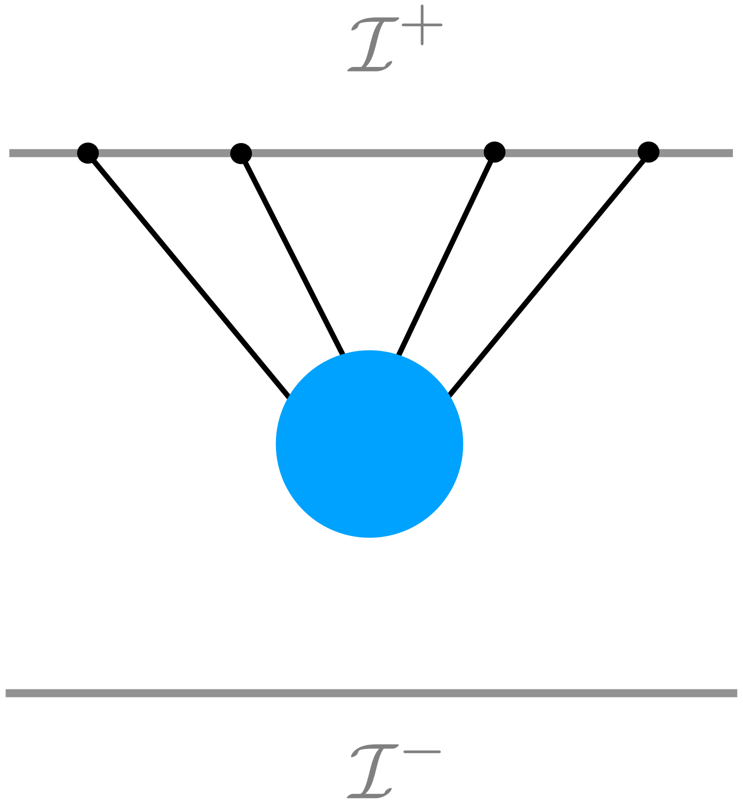


Can we extend this understanding to our own universe?

Holography for all Λs?

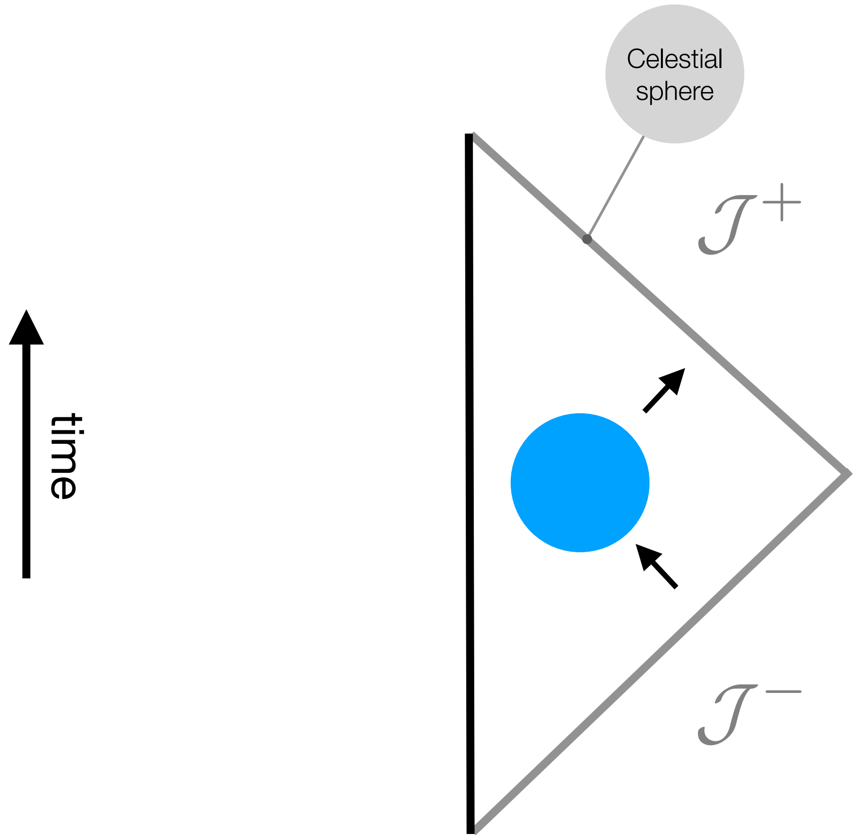
The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter



- Cosmological scales
- Primordial inflation

$\Lambda = 0$ Minkowski

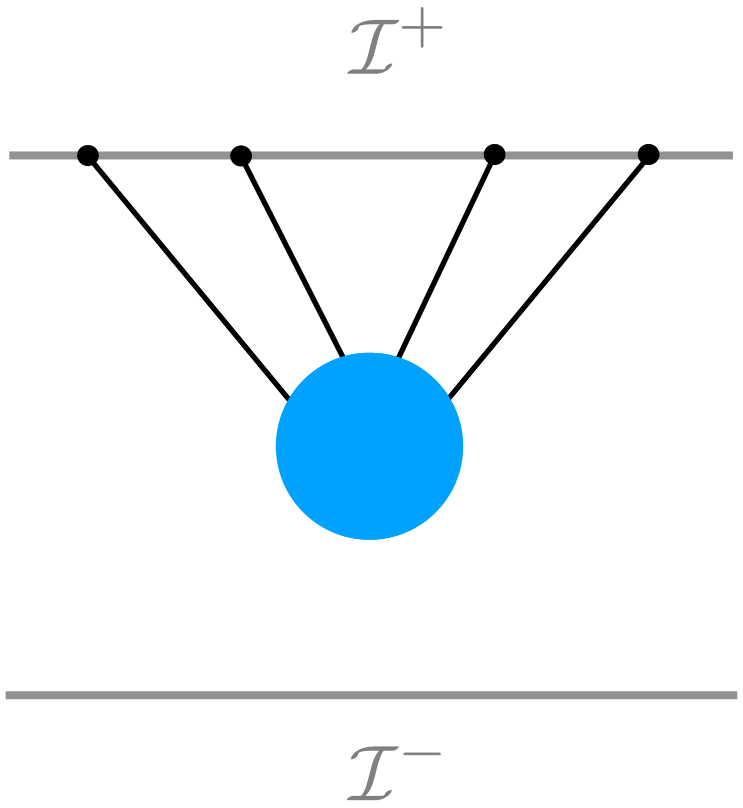


- intermediate scales

Holography for all Λs?

The maximally symmetric cousins of AdS

$\Lambda > 0$ de Sitter



Cosmological Bootstrap

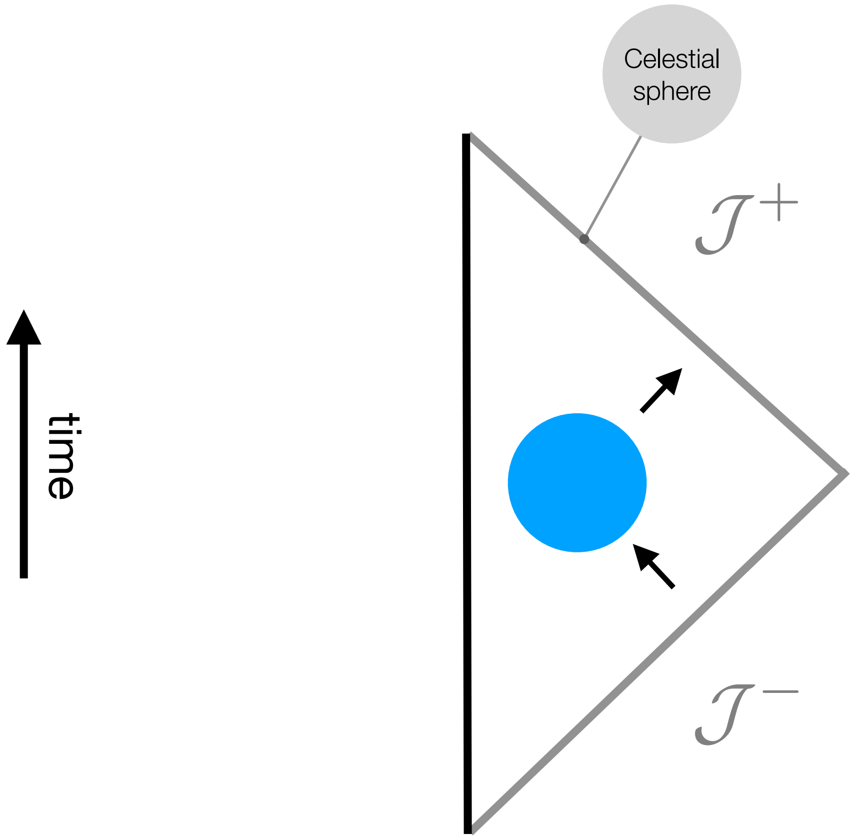
[Arkani-Hamed and Maldacena ’15]

[Arkani-Hamed and Benincasa ’17]

[Arkani-Hamed, Baumann, Lee and Pimentel ’18]

[Sleight and Taronna ’19] [Pajer et al ’20] […]

$\Lambda = 0$ Minkowski



Celestial holography

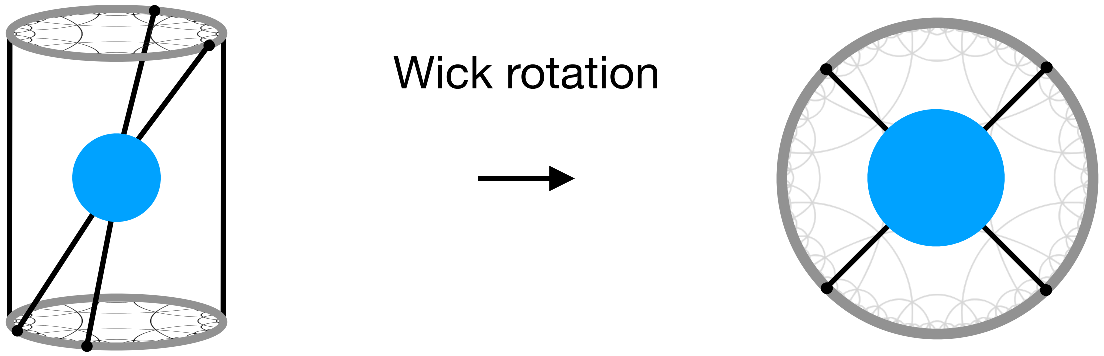
[de Boer and Solodukhin '03]

[Strominger ’17] [Pasterski, Shao, Strominger ’17]

[Pasterski, Shao ’17] […]

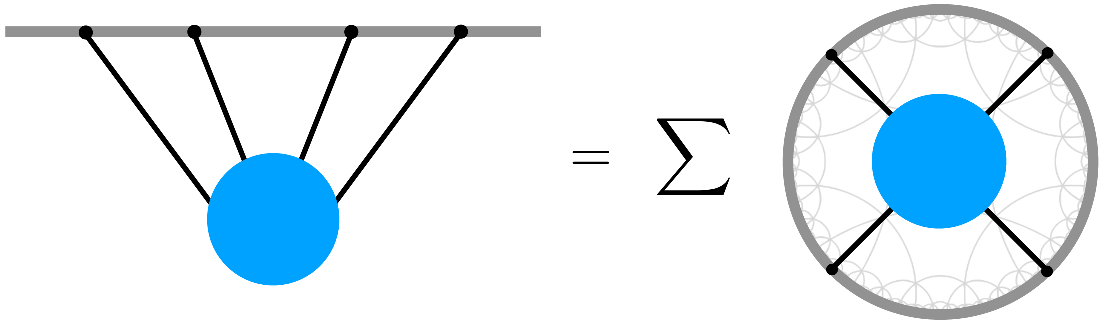
Holography for all Λs?

Boundary correlators in AdS, dS and on the celestial sphere can be reformulated as boundary correlators in Euclidean AdS:



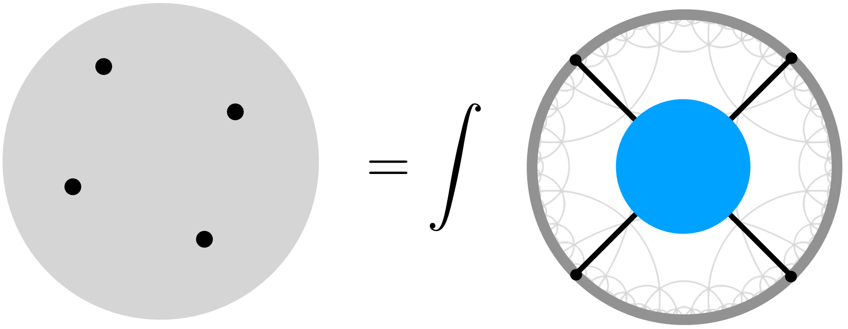
$\Lambda < 0$

Perturbatively:



$\Lambda > 0$

[C.S. & Taronna '19, '20, '21]



$\Lambda = 0$

[Iacobacci, C.S. & Taronna '22]
[C.S. & Taronna '23]

dS and Celestial correlators therefore have a similar analytic structure to their EAdS counterparts! On a practical level, can use such identities to import techniques and understanding from AdS.

Outline

I. $\Lambda < 0$

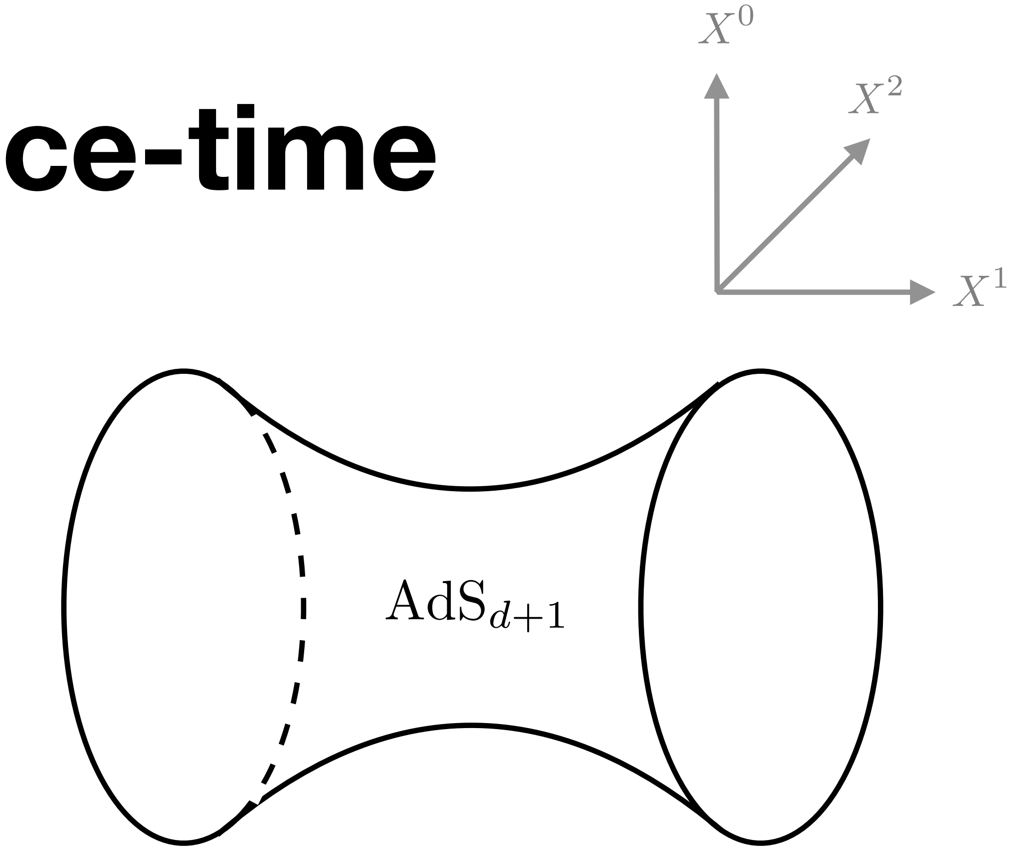
II. $\Lambda > 0$

III. $\Lambda = 0$

IV. Some applications.

$$\Lambda < 0$$

Anti-de Sitter space-time



$\mathrm{AdS}_{d+1} \subset \mathbb{R}^{d,2}$:

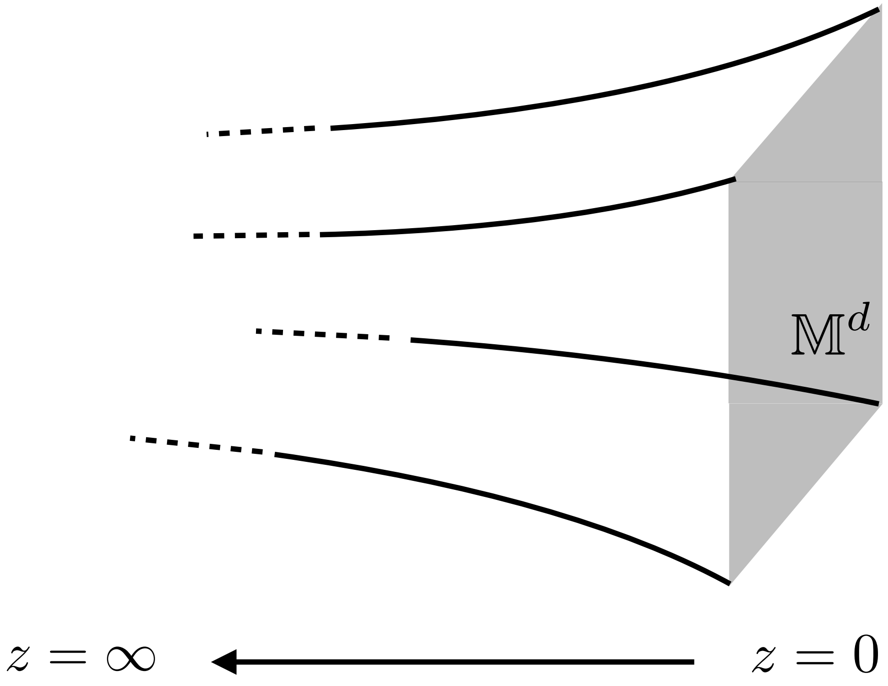
$$-\left(X^0\right)^2 - \left(X^{d+1}\right)^2 + \sum_{i=1}^{d} \left(X^i\right)^2 = -R^2_{\mathrm{AdS}}$$

It is manifest that

Isometry group: $SO\left(d,2\right)$ = conformal group in $\mathbb{M}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R^2_{\mathrm{AdS}} \frac{\mathrm{d}z^2 + \eta_{\mu\nu} dx^\mu dx^\nu}{z^2}$$

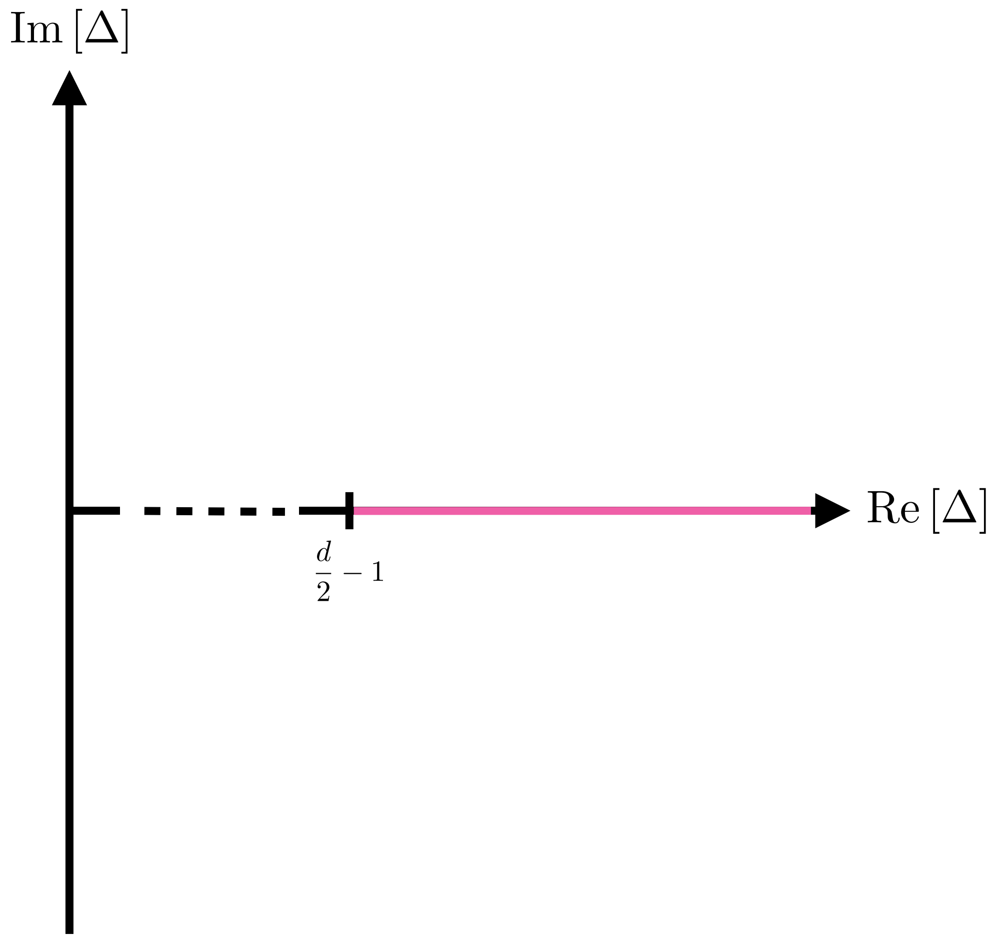


Particles in AdS

Particles in AdS$_{d+1}$ $\longleftrightarrow$ unitary irreducible representations of $SO\left(d,2\right)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

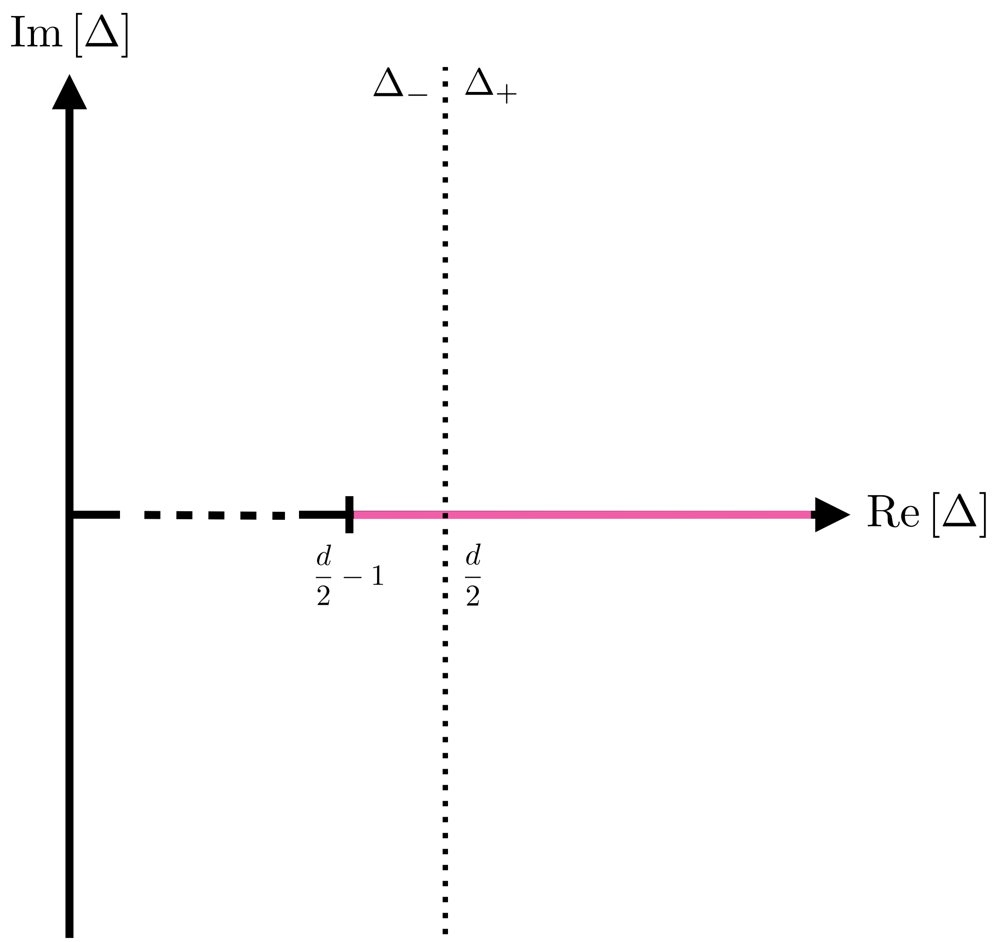
- $\Delta \in \mathbb{R}$
- Bounded from below $\Delta \geq \dfrac{d}{2} - 1$

Particles in AdS

Particles in AdS$_{d+1}$ ⟷ unitary irreducible representations of $SO\left(d,2\right)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in AdS$_{d+1}$:

E.g. Spin J=0 representations



Quadric Casimir equation

$$\overbrace{\left(\mathcal{C}_2 - \langle\mathcal{C}_2\rangle\right)}^{\langle\mathcal{C}_2\rangle = \Delta\left(\Delta - d\right)}$$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle\mathcal{C}_2\rangle\right)\varphi = 0$$

$$m^2 R^2_{\mathrm{AdS}} = \Delta\left(\Delta - d\right)$$

Boundary behaviour ($\Delta_- = d - \Delta_+$):

$$\lim_{z\to 0}\varphi\left(z,x\right) = \underbrace{O_{\Delta_+}\left(x\right)}_{\text{Dirichlet boundary condition}} z^{\Delta_+} + \underbrace{O_{\Delta_-}\left(x\right)}_{\text{Neuman boundary condition}} z^{\Delta_-}$$

N.B. Δ_- may be ruled out by unitarity

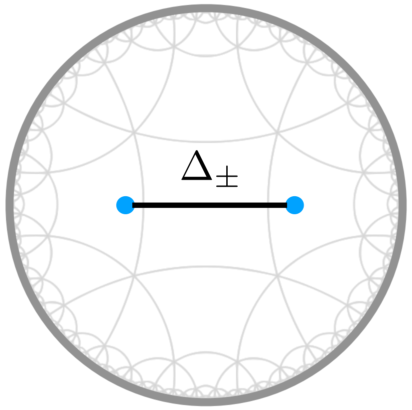
$O_{\Delta_\pm}\left(x\right)$ transform as primary fields with scaling dimension $\Delta_\pm$ in Minkowski CFT$_d$

AdS boundary correlators

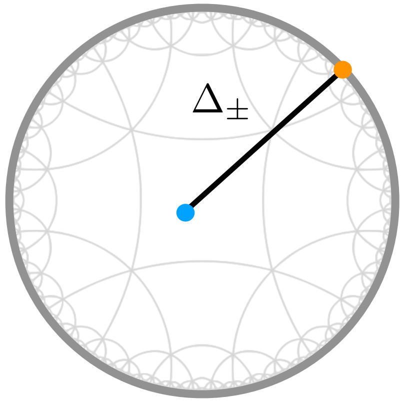
$$\lim_{z\to 0} z^{-(\Delta_1+\ldots+\Delta_n)}\langle\varphi_1\left(x_1,z\right)\ldots\varphi_n\left(x_n,z\right)\rangle \overset{!}{=} \langle\mathcal{O}_{\Delta_1}\left(x_1\right)\ldots\mathcal{O}_{\Delta_n}\left(x_n\right)\rangle$$

Feynman rules:

Bulk-to-bulk propagator, $\Delta_\pm$ boundary condition:



Bulk-to-boundary propagator, $\Delta_\pm$ boundary condition:

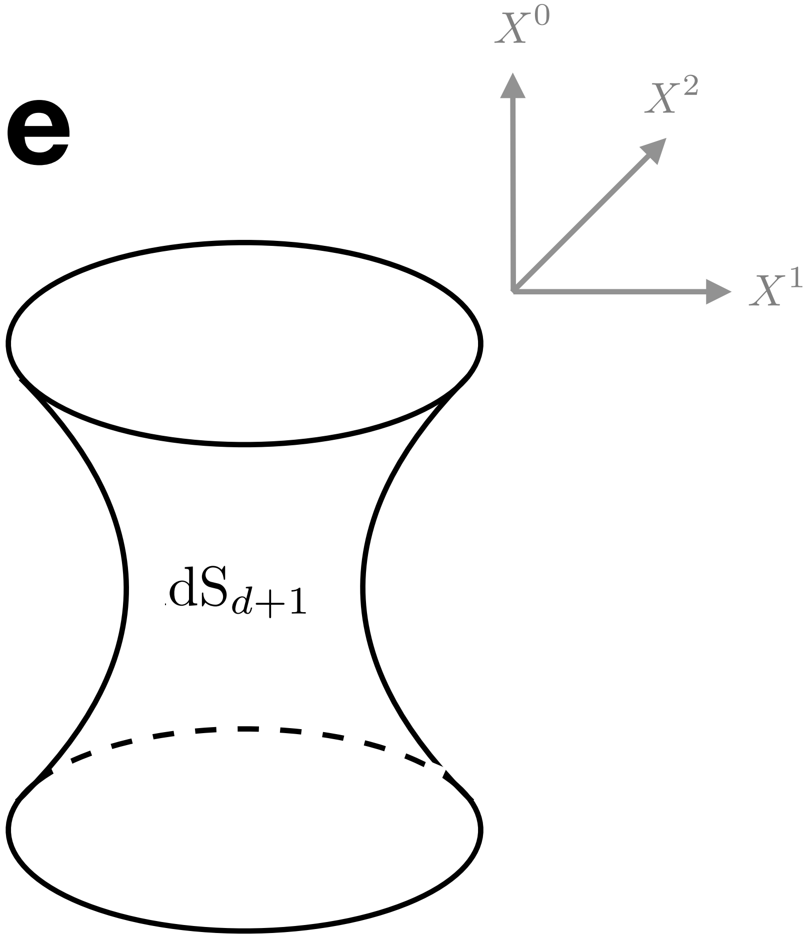


$$\Lambda > 0$$

de Sitter space-time

$\mathrm{dS}_{d+1} \subset \mathbb{M}^{d+2}$:

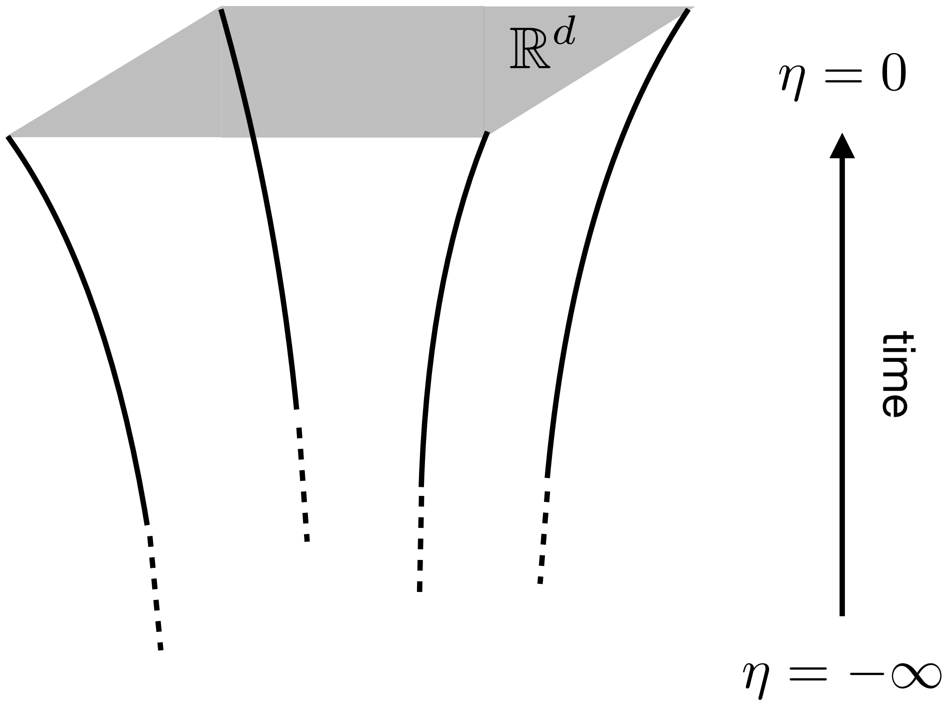
$$-\left(X^0\right)^2 + \sum_{i=1}^{d+1} \left(X^i\right)^2 = R_{\mathrm{dS}}^2$$



Isometry group: $SO\left(d+1,1\right)$ = conformal group in $\mathbb{R}^d$

Poincaré coordinates:

$$\mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

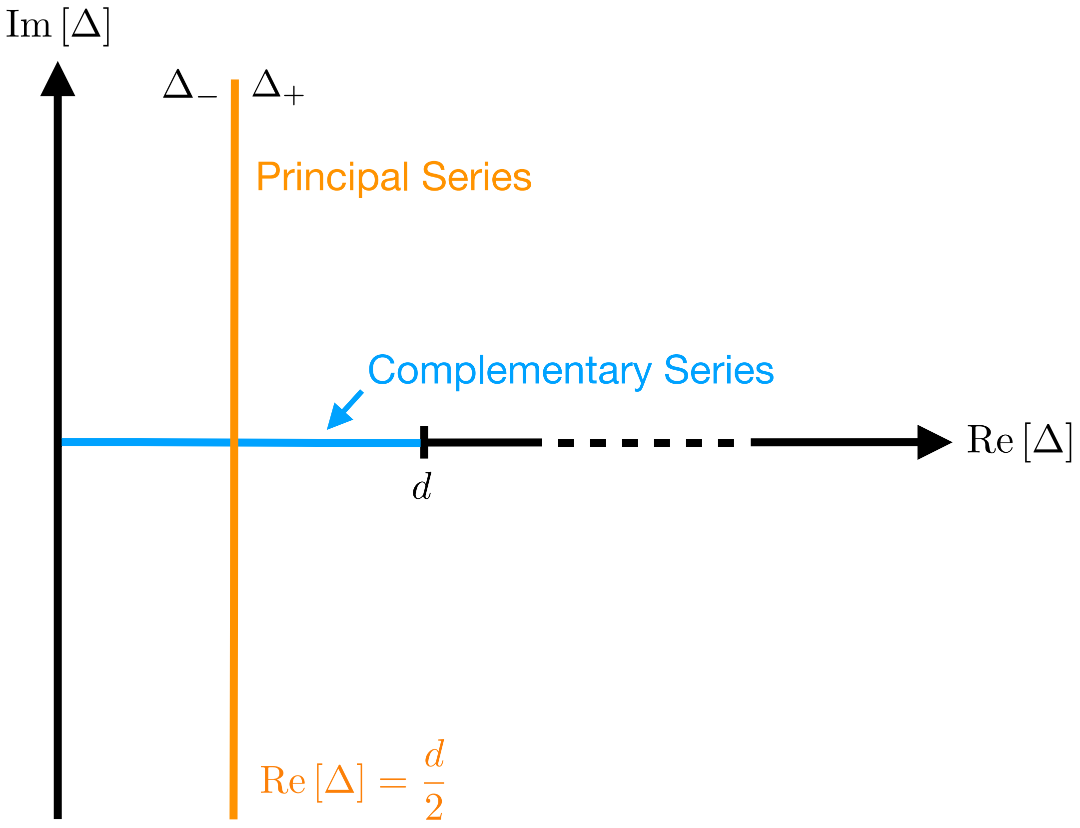


Particles in dS

Particles in dS_{d+1} ⟷ unitary irreducible representations of $SO\left(d+1,1\right)$

Labelled by a scaling dimension Δ and spin J. Unitarity constrains Δ :

E.g. Spin J=0 representations



Notes:

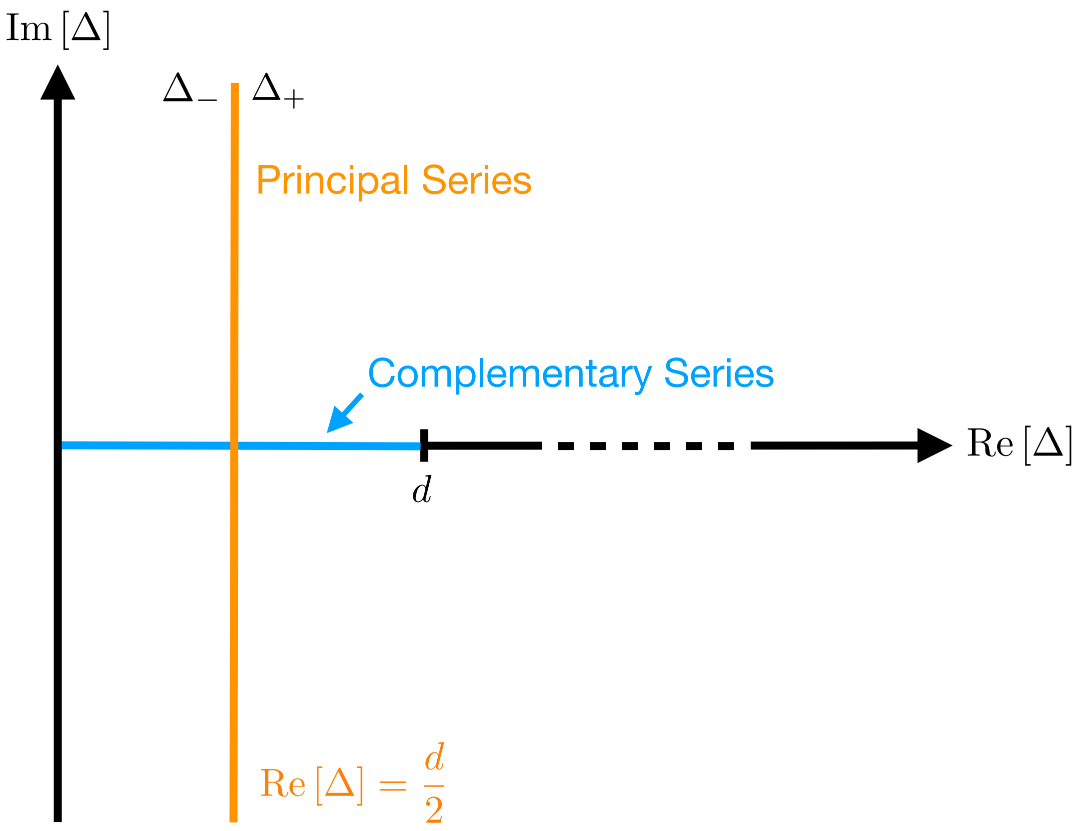
- Both Δ_+ and Δ_- are unitary
- Δ can be complex - Principal Series

Particles in dS

Particles in dS$_{d+1}$ ⟷ unitary irreducible representations of $SO\left(d+1,1\right)$

Labelled by a scaling dimension Δ and spin J. Can be realised by fields in dS$_{d+1}$.

E.g. Spin J=0 representations



Quadric Casimir equation

$$\langle \mathcal{C}_2 \rangle = \Delta\left(d-\Delta\right)$$

$$\left(\nabla^2 - m^2\right)\varphi = 0 \quad \leftrightarrow \quad \left(\mathcal{C}_2 - \langle \mathcal{C}_2 \rangle\right)\varphi = 0$$

$$m^2 R_{\mathrm{dS}}^2 = \Delta\left(d-\Delta\right)$$

Boundary behaviour:

$$\lim_{\eta\to 0}\varphi\left(\eta, x\right) = O_{\Delta_+}\left(\mathbf{x}\right)\eta^{\Delta_+} + O_{\Delta_-}\left(\mathbf{x}\right)\eta^{\Delta_-}$$

Determined by
the initial state

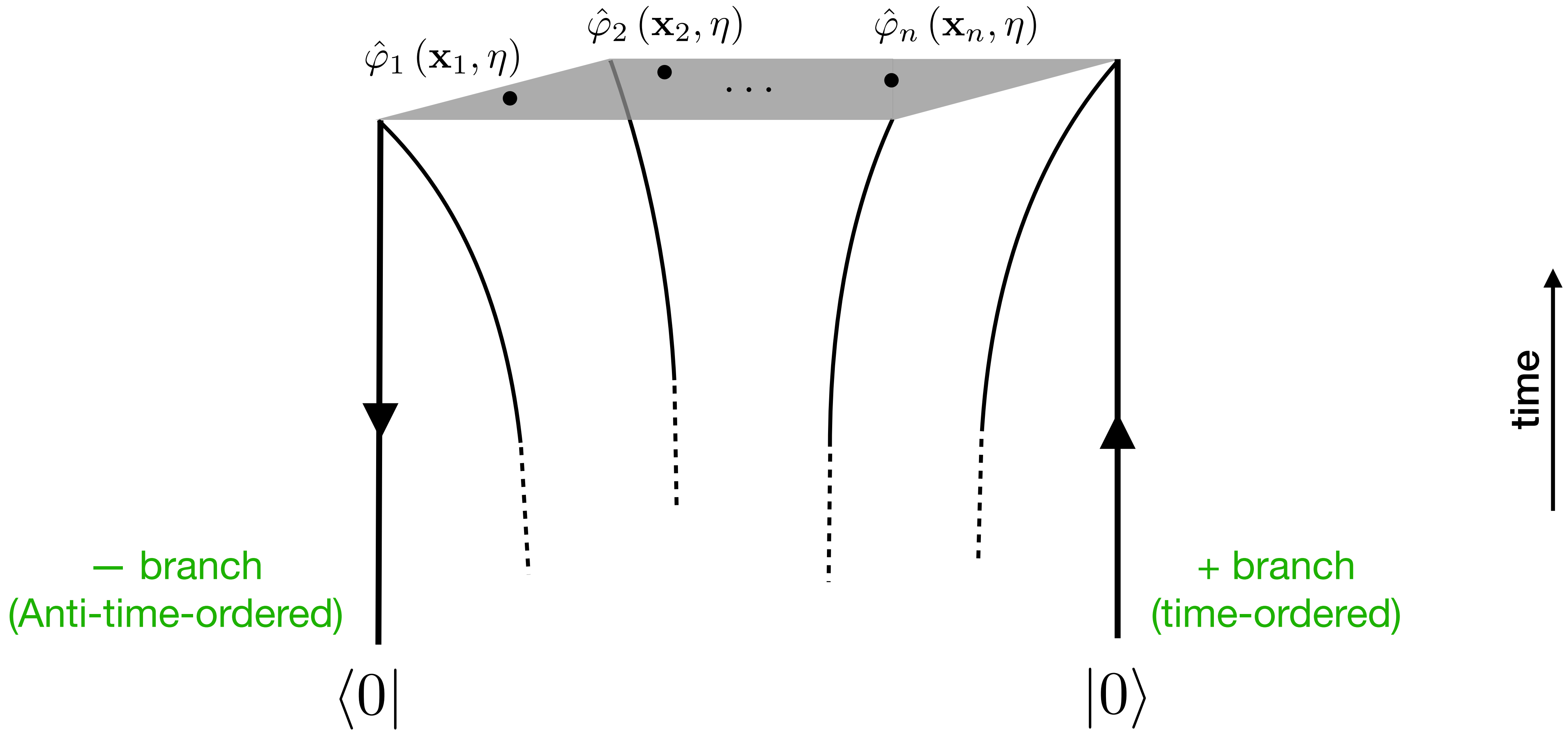
$O_{\Delta_\pm}\left(\mathbf{x}\right)$ transform as primary fields with scaling dimension $\Delta_\pm$ in Euclidean CFT$_d$

dS Boundary Correlators

in-in formalism

[Maldacena ’02, Weinberg '05]

$$\lim_{\eta\to 0}\langle 0|\hat{\varphi}_1\left(\mathbf{x}_1,\eta\right)\ldots\hat{\varphi}_n\left(\mathbf{x}_n,\eta\right)|0\rangle$$



We take $|0\rangle$ to be the Bunch-Davies vacuum.

[some work to appear with Alistair Chopping and Massimo Taronna on other vacua]

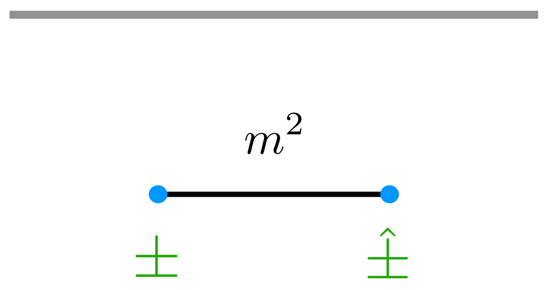
dS Boundary Correlators

in-in formalism [Maldacena ’02, Weinberg '05]

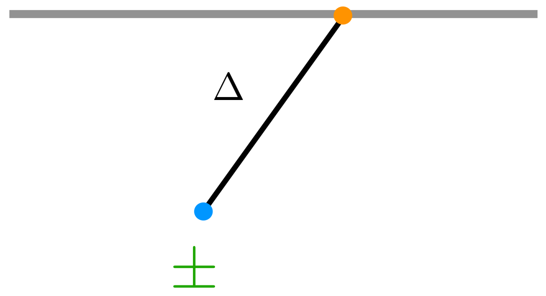
$$\lim_{\eta\to 0}\langle 0|\hat{\varphi}_1\left(\mathbf{x}_1,\eta\right)\ldots\hat{\varphi}_n\left(\mathbf{x}_n,\eta\right)|0\rangle$$

Feynman rules:

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:



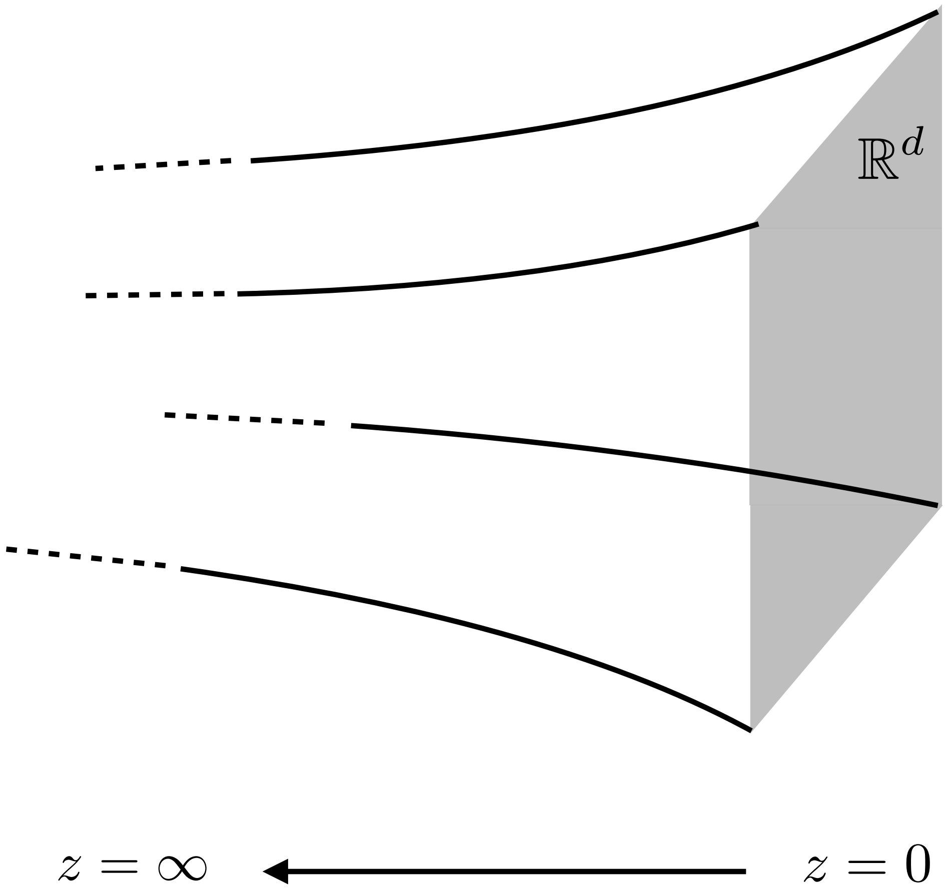
$\pm$ bulk-to-boundary propagator:



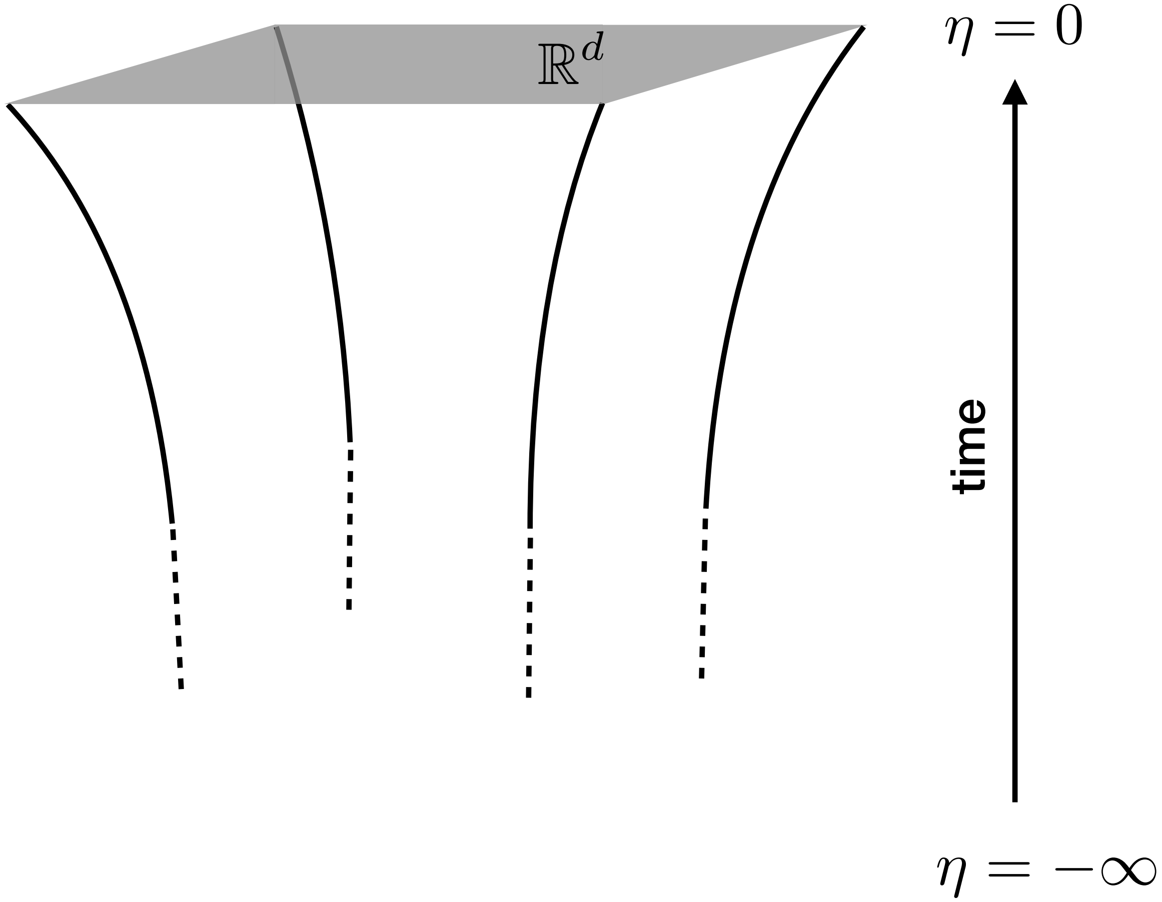
Sum contributions from each branch (±) of the time (in-in) contour!

From dS to Euclidean AdS

Euclidean AdS



dS



$$\mathrm{d}s^2 = R_{\mathrm{AdS}}^2 \frac{\mathrm{d}z^2 + \mathrm{d}\mathbf{x}^2}{z^2} \longleftrightarrow \mathrm{d}s^2 = R_{\mathrm{dS}}^2 \frac{-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2}{\eta^2}$$

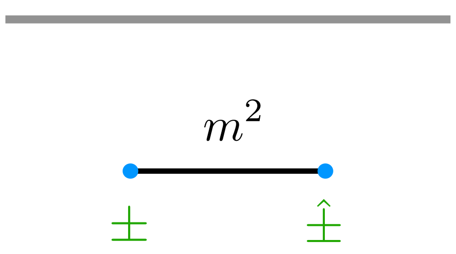
EAdS and dS are identified under:

$$R_{\mathrm{AdS}} = \pm i R_{\mathrm{dS}} \qquad z = \pm i\,(-\eta)$$

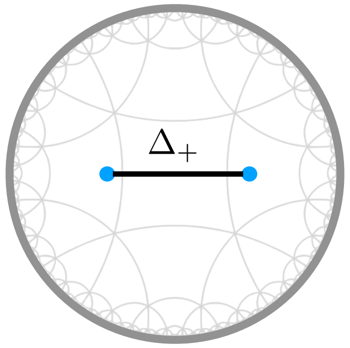
From dS to Euclidean AdS

$\pm$ bulk-to- $\hat{\pm}$ bulk propagator:

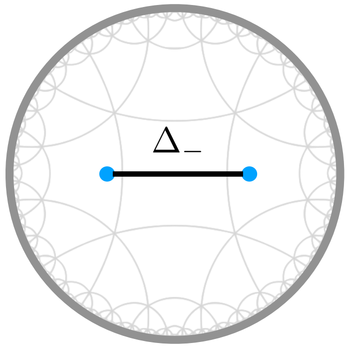
[C.S. and M. Taronna ’19, ’20, ’21]

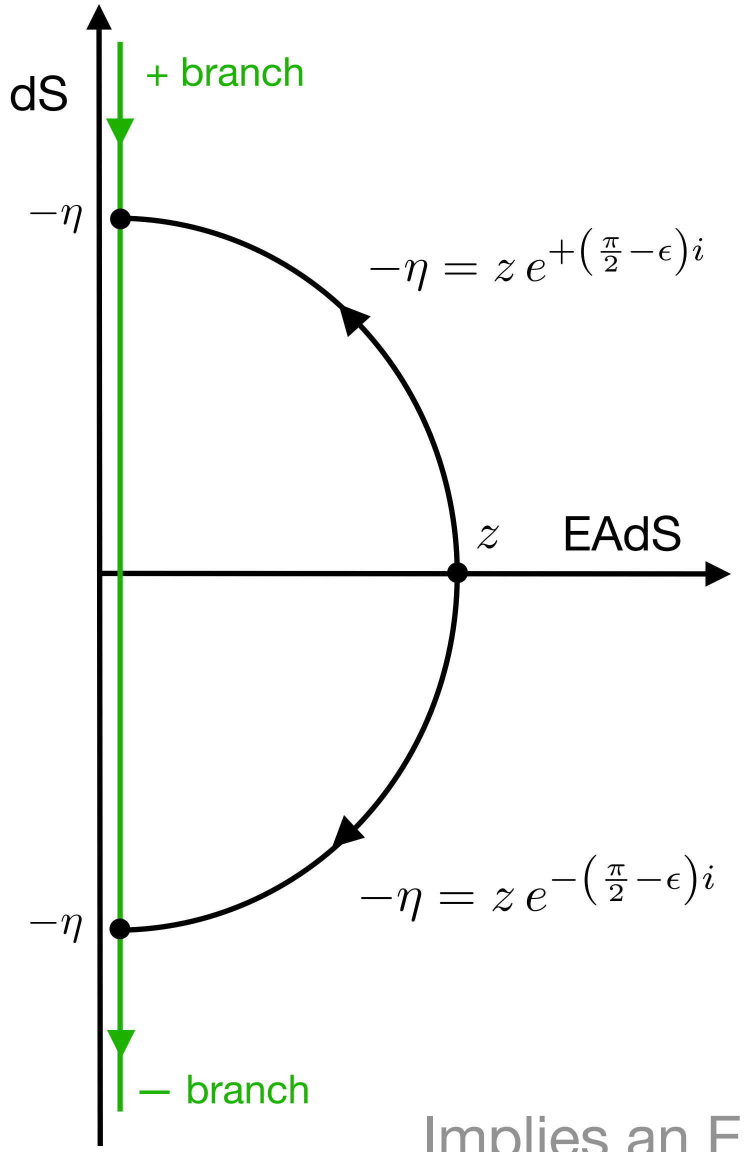


$= e^{\mp\frac{i\pi}{2}\Delta_+} e^{\hat{\mp}\frac{i\pi}{2}\Delta_+}$

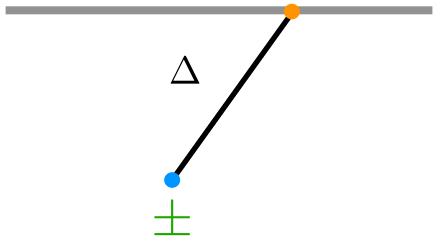


$+ e^{\mp\frac{i\pi}{2}\Delta_-} e^{\hat{\mp}\frac{i\pi}{2}\Delta_-}$

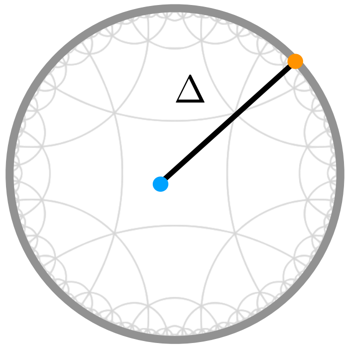




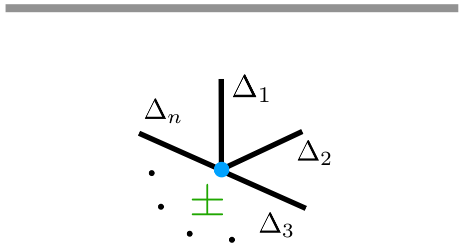
$\pm$ bulk-to-boundary propagator:



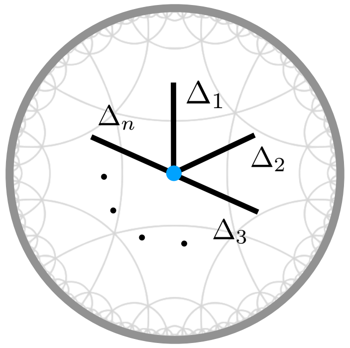
$= e^{\mp\frac{i\pi}{2}\Delta}$



$\pm$ bulk integrals:



$= e^{\pm(d-1)\frac{\pi i}{2}}$



Implies an EAdS Lagrangian for dS correlators [di Pietro, Gorbenko and Komatsu ’21]

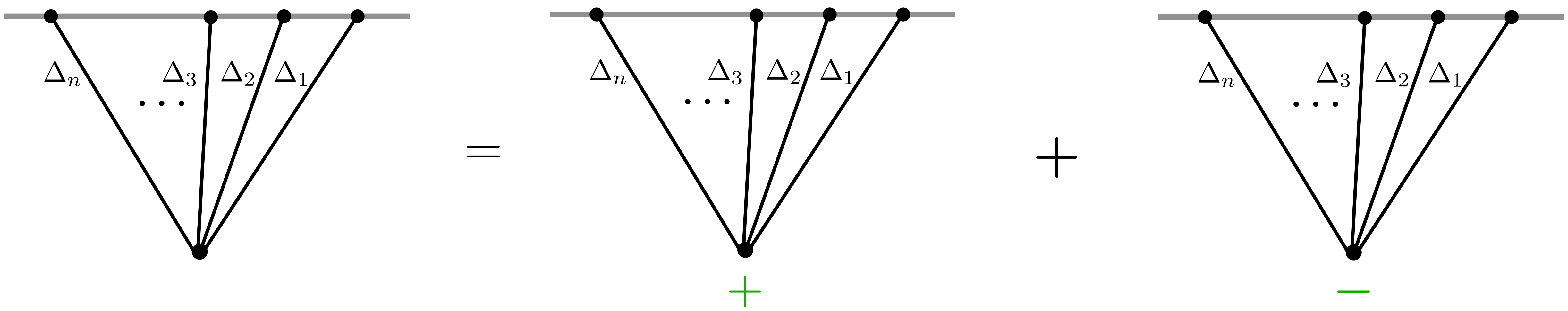
From dS to Euclidean AdS

Examples.

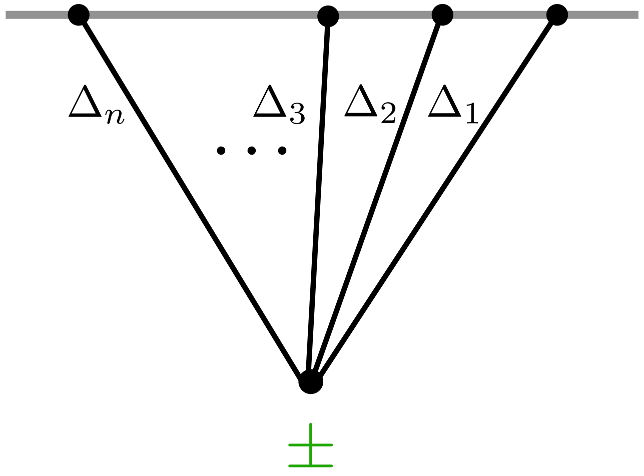
[C.S. and M. Taronna ’19]

Non-derivative vertex of scalars fields $\mathcal{V}(X) = g\phi_1(X)\ldots\phi_n(X)$

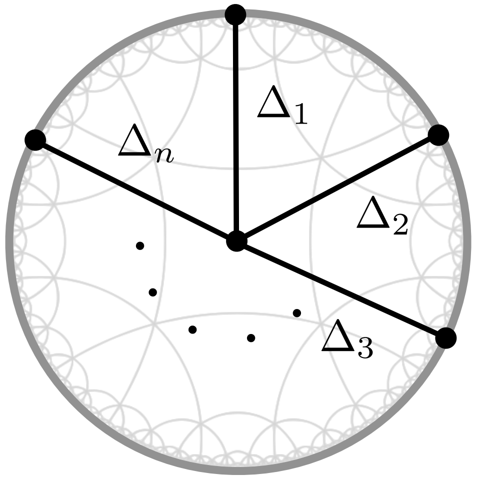
Contact diagram:



Where



$$= e^{\pm\frac{i\pi}{2}(d-1)}\prod_{j=1}^{n} e^{\mp\frac{i\pi}{2}\Delta_j}$$



Same contact diagram in EAdS

From dS to Euclidean AdS

Examples.

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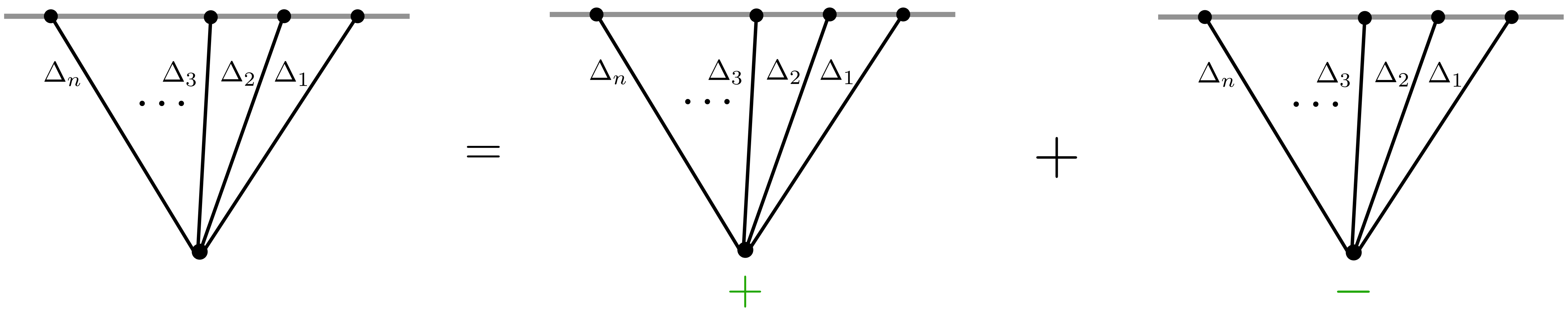
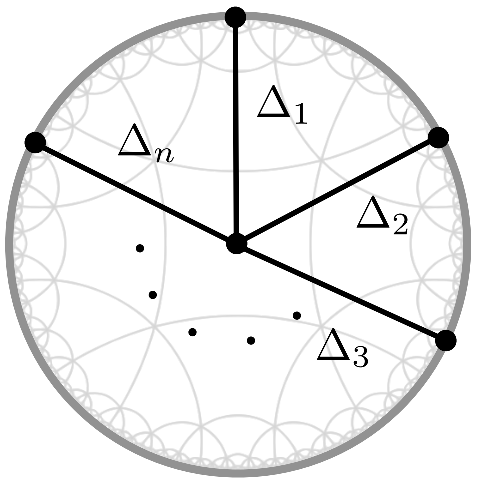


$$= \underbrace{\sin\left(-\frac{d}{2}+\frac{1}{2}\sum_{i=1}^{n}\Delta_i\right)\pi}_{\text{Encodes unitary time evolution}}$$

Same contact diagram in EAdS

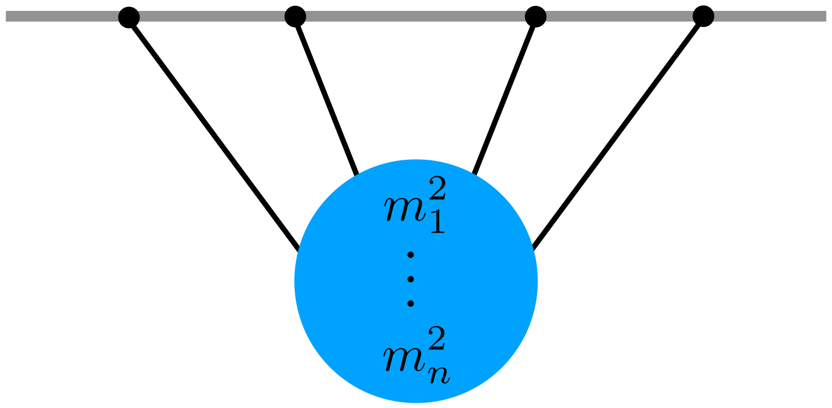


c.f. *“Cosmological Optical Theorem”* [Goodhew et al, 2020]

From dS to EAdS, and back

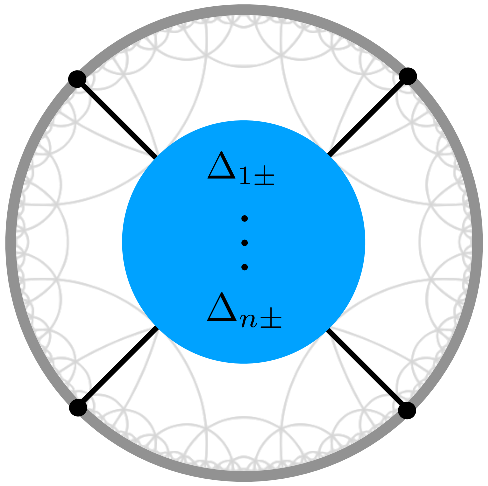
dS boundry correlators are perturbatively recast as Witten diagrams in EAdS:

e.g. four-points



Process with M vertices

$$= \sum_{\underbrace{\Delta_{1\pm}\dots\Delta_{n\pm}}_{\text{Sum over boundary conditions for exchanged particles}}} \overset{\text{Combines contributions from each branch of the in-in contour}}{\underbrace{C_{\Delta_{1\pm}\dots\Delta_{n\pm}}}_{\text{factorises into: } C_1^{\text{contact}} \times \dots \times C_M^{\text{contact}}}}$$



Notes:

- Contributions from both $\Delta_\pm$ modes
- $\Delta_{i\,\pm} \in$ Unitary Irreducible Representation of **dS** isometry

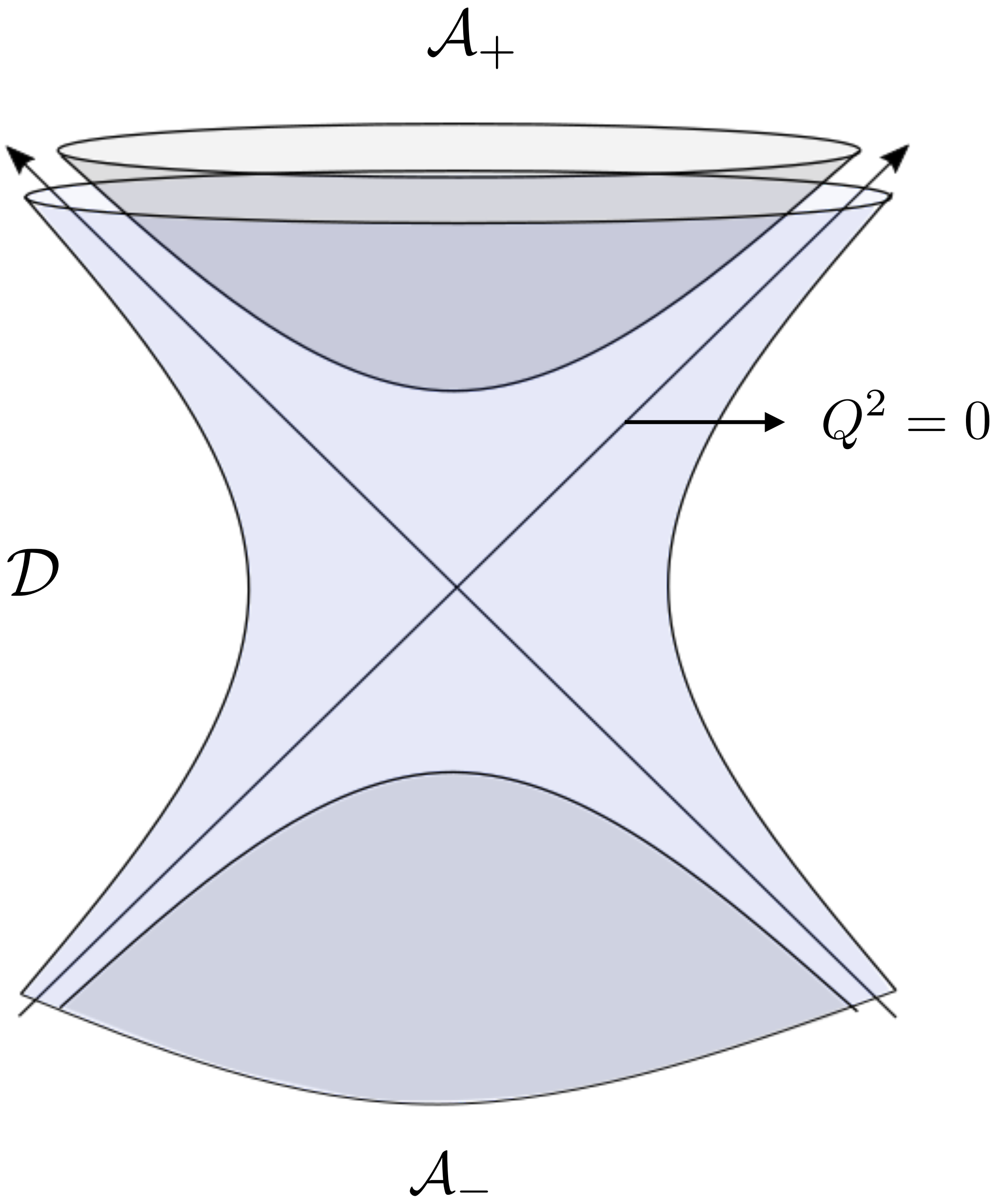
Can use to import techniques, results and understanding from AdS to dS!

$$\Lambda = 0$$

Hyperbolic slicing of Minkowski space

[de Boer and Solodukhin '03]

(d+2)-dimensional Minkowski space $\mathbb{M}^{d+2}$, coordinates $X^A, \quad A = 0, \dots d+1$



$\mathcal{A}_\pm : \quad X^2 = -t^2$ (EAdS$_{\text{d+1}}$, radius t)

$\mathcal{D} : \quad X^2 = R^2$ (dS$_{\text{d+1}}$, radius R)

Conformal boundary:

$$Q^2 = 0, \quad Q \equiv \lambda Q, \quad \lambda \in \mathbb{R}^+$$

Introduce projective coordinates:

$$\xi_i = Q^i / Q^0, \quad i = 1, \dots, d+1$$

$$\xi_1^2 + \dots + \xi_{d+1}^2 = 1$$

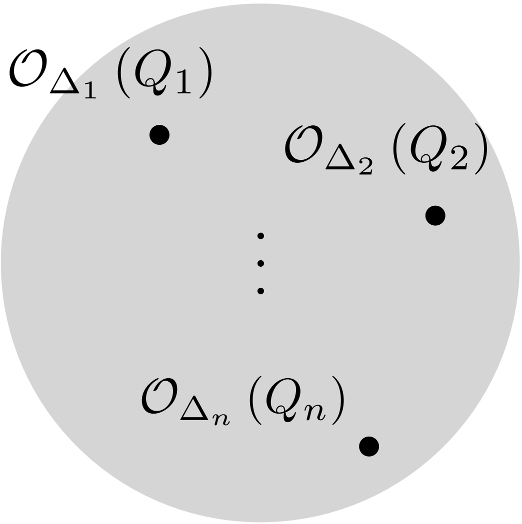
d-dimensional unit sphere (Celestial sphere)

$SO\left(d+1,1\right)$ acts on the celestial sphere as the Euclidean conformal group!

Minkowski boundary correlators

[C.S. and M. Taronna '23]

Radial Mellin transform of Minkowski correlators implements a radial reduction onto the hyperbolic slicing:



$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i}\, t_i^{\Delta_i} \left\langle \phi_1(\underset{\underset{\text{radial coordinate}}{\uparrow}}{t_1}\overset{\overset{\text{Hyperbolic coordinate}}{\downarrow}}{\hat{X}_1}) \dots \phi_n(t_n \hat{X}_n) \right\rangle$$

Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

Mellin transform

$$\int_0^\infty \frac{dt}{t} t^{\Delta} (\dots)$$

Inverse Mellin transform

$$\underbrace{\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}}_{\text{Unitary Principal Series representations of SO(d+1,1)}} \frac{d\Delta}{2\pi i}\, t^{-\Delta} (\dots)$$

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[C.S. and M. Taronna '23]

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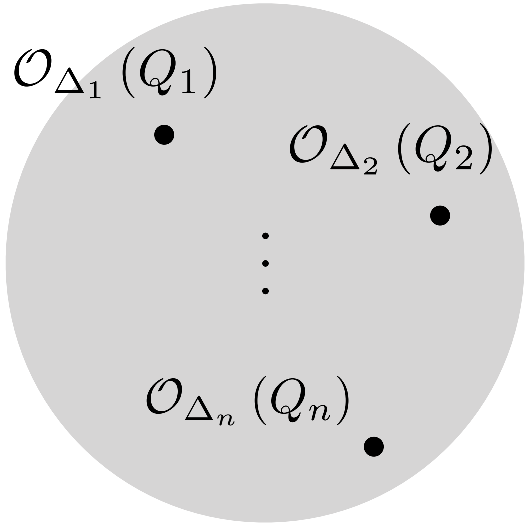


Hyperbolic coordinate

$$= \prod_i \lim_{\hat{X}_i \to Q_i} \int_0^\infty \frac{dt_i}{t_i} t_i^{\Delta_i} \left\langle \phi_1(t_1 \hat{X}_1) \dots \phi_n(t_n \hat{X}_n) \right\rangle$$

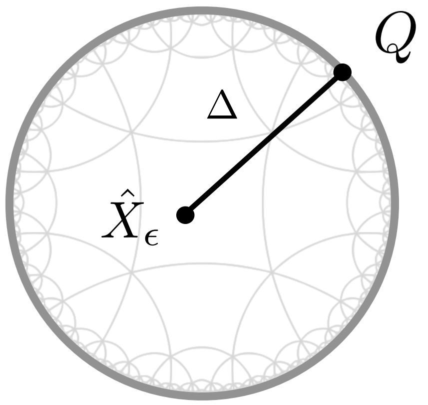
radial coordinate

Celestial correlators then arise in the boundary limit $\hat{X}_i \to Q_i$!

"Celestial" bulk-to-boundary propagator:

bulk-to-boundary propagator in EAdS

$$G^{\text{flat}}_{\Delta}(X,Q) = \lim_{\hat{Y} \to Q} \int_0^\infty \frac{dt}{t} t^{\Delta} G_F\left(X, t\hat{Y}\right) = \overbrace{\mathcal{K}^{(m)}_{i\left(\frac{d}{2}-\Delta\right)}\left(\sqrt{X^2+i\epsilon}\right)}^{\text{Kernel of the radial reduction (Bessel-K function)}} \times$$



From the Celestial Sphere to EAdS

[C.S. and M. Taronna ’23]

Examples.

Free theory Celestial two point function:

$$\langle \mathcal{O}_{\Delta_1}\left(Q_1\right)\mathcal{O}_{\Delta_2}\left(Q_2\right)\rangle = \lim_{\hat{X}\to Q_2}\int_0^\infty \frac{dt}{t}\, t^{\Delta_2} G^{\text{flat}}_{\Delta_1}(t\hat{X}, Q_1)$$

Q_i can be null separated

$$= \underbrace{\frac{C^{\text{flat}}_{\Delta_1}\left(m\right)}{(-2Q_1\cdot Q_2+i\epsilon)^{\Delta_1}}}_{\text{Form required by Conformal Symmetry}} \underbrace{(2\pi)\delta(i(\Delta_1-\Delta_2))}_{\text{Consequence of continuous spectrum}}$$

From the Celestial Sphere to EAdS

[C.S. and M. Taronna '23]

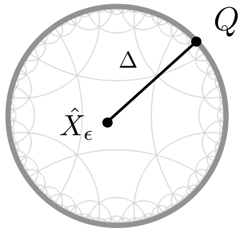
Examples.

Non-derivative vertex of scalars fields $\mathcal{V}\left(X\right) = g\phi_1\left(X\right)\ldots\phi_n\left(X\right)$

Contact diagram:

$$\langle \mathcal{O}_{\Delta_1}(Q_1)\ldots\mathcal{O}_{\Delta_n}(Q_n)\rangle = -ig\int d^{d+2}X\, G^{\text{flat}}_{\Delta_1}(X,Q_1)\cdots G^{\text{flat}}_{\Delta_n}(X,Q_n)\,.$$

$$G^{\text{flat}}_{\Delta}\left(X,Q\right) \quad = \quad \mathcal{K}^{(m)}_{i\left(\frac{d}{2}-\Delta\right)}\left(\sqrt{X^2+i\epsilon}\right) \quad \times$$

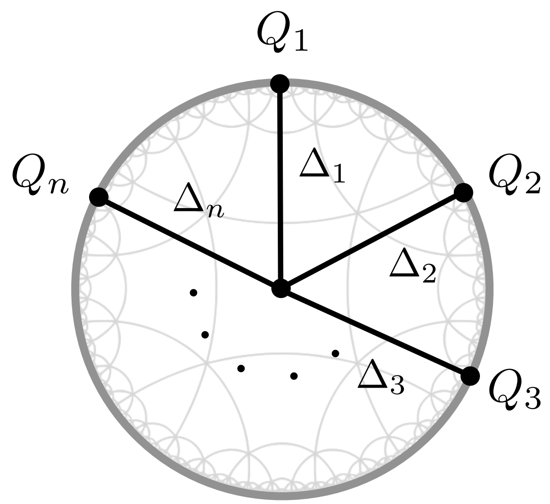


$$= \quad \underbrace{R_{\Delta_1\ldots\Delta_n}\left(m_1,\ldots,m_n\right)} \quad \times \quad \underbrace{\sin\left(-\frac{d}{2}+\frac{1}{2}\sum_{i=1}^{n}\Delta_i\right)\pi}$$

Contribution from radial integral.
Encodes all mass dependence.
(Generalised hypergeometric)

Same factor as for dS contact diagrams.
Comes from combining contributions
from regions inside and outside lightcone

(Analytically continued)
EAdS contact diagram

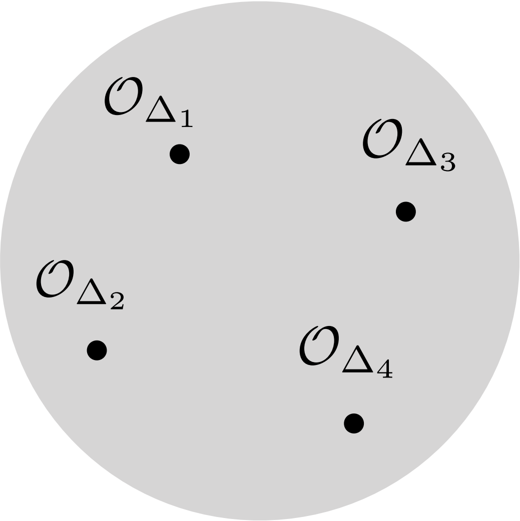


Like in dS, Celestial contact diagrams are proportional to their EAdS counterparts

From the Celestial Sphere to EAdS

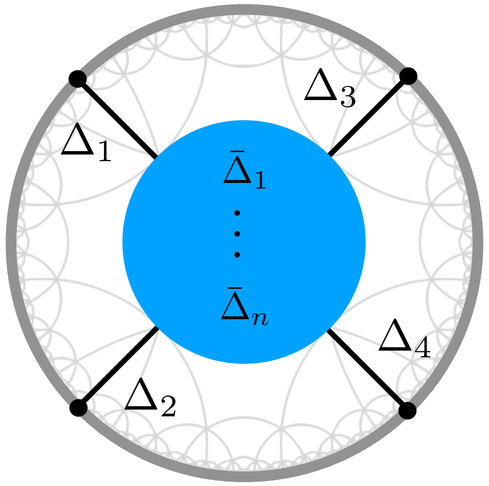
[C.S. and M. Taronna '23]

In general, for exchanges of particles of mass $m_i, \quad i = 1, \dots, n$



Process with M vertices

$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \dots \frac{d\bar{\Delta}_n}{2\pi i} \, C_{\bar{\Delta}_1 \dots \bar{\Delta}_n}(m_1, \dots, m_n)$$



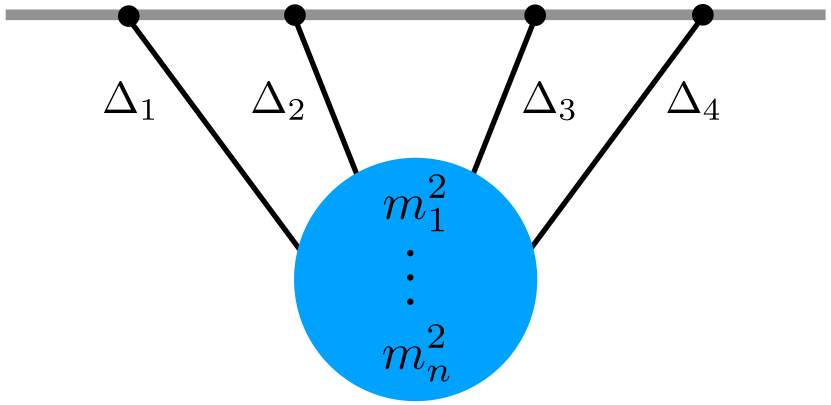
Minkowski exchanges are a *continuum* of EAdS exchanges

factorises into:

$$C_1^{\text{contact}} \times \dots \times C_M^{\text{contact}}$$

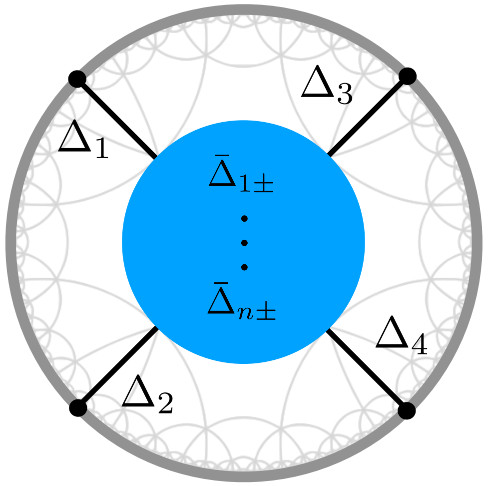
Makes manifest conformal symmetry

Compare with de Sitter:



Process with M vertices

$$= \sum_{\bar{\Delta}_{1\pm} \dots \bar{\Delta}_{n\pm}} C_{\bar{\Delta}_{1\pm} \dots \bar{\Delta}_{n\pm}}$$

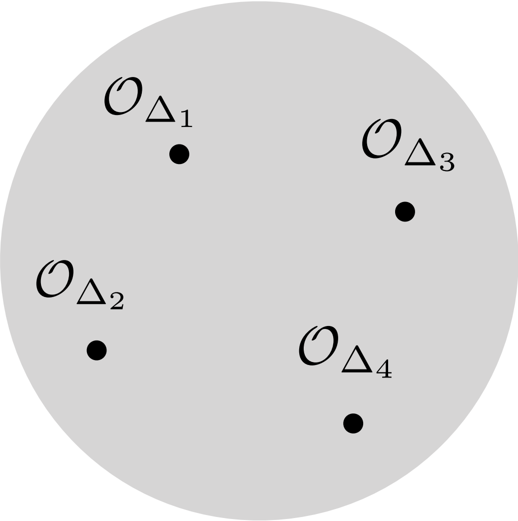


dS exchanges are a *discrete sum* of EAdS exchanges

From the Celestial Sphere to EAdS

[C.S. and M. Taronna '23]

In general, for exchanges of particles of mass $m_i, \quad i = 1, \ldots, n$



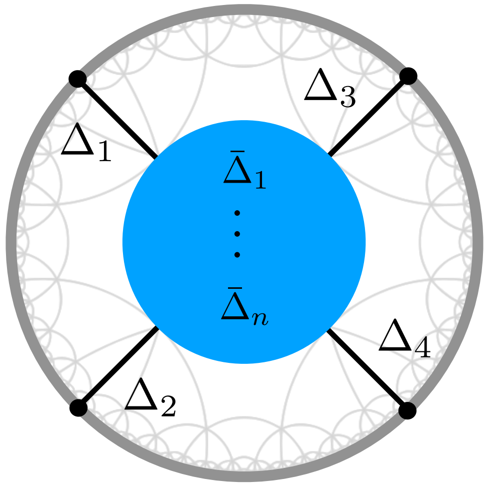
Process with M vertices

$$= \int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty} \frac{d\bar{\Delta}_1}{2\pi i} \cdots \frac{d\bar{\Delta}_n}{2\pi i}\, \underbrace{C_{\bar{\Delta}_1 \ldots \bar{\Delta}_n}(m_1, \ldots, m_n)}$$

Minkowski exchanges are a *continuum* of EAdS exchanges

factorises into:

$$C_1^{\text{contact}} \times \ldots \times C_M^{\text{contact}}$$



Makes manifest conformal symmetry

Comments:

- Relation to definition [Pasterski, Shao, Strominger '17] of celestial correlators as scattering amplitudes in a conformal basis?

  [Pasterski, Shao, Strominger '17] = LSZ ([Sleight, Taronna '23]) ?

- Celestial correlators defined as an extrapolation of bulk Minkowski correlators give a definition of celestial correlators for theories without an S-matrix.

  What lessons can we draw from Minkowski CFT?

Some applications.

Perturbative OPE data

Perturbative OPE data on the boundary of dS and Minkowski space from EAdS

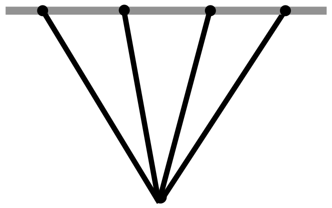
E.g. Composite operators on the boundary

[C.S. and M. Taronna ’20]

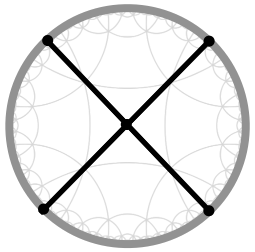
$$[\mathcal{O}\mathcal{O}]_{n,\ell} \sim \mathcal{O}\left(\partial^2\right)^n \partial_{i_1}\ldots\partial_{i_\ell}\mathcal{O}+\ldots$$

scaling dimension: $\Delta_{n,\ell} = \underbrace{2\Delta+2n+\ell}_{\text{Free theory}} + \underbrace{\gamma_{n,\ell}}_{\text{anomalous dimension}}$

- $\gamma_{n,\ell}$ induced by bulk ϕ^4 contact diagram in dS:

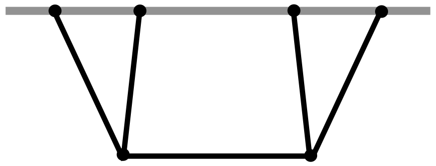


$$= \sin\left(-\frac{d}{2}+2\Delta\right)\pi$$

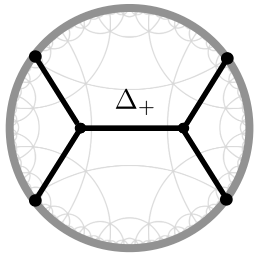


$$\longrightarrow \quad \gamma^{\phi^4}_{n,\ell} = \sin\left(-\frac{d}{2}+2\Delta\right)\pi \times {}^{(\text{EAdS})}\gamma^{\phi^4}_{n,\ell}$$

- $\gamma_{n,\ell}$ induced by an exchange diagram in dS:



$$= \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi$$



$$+ \quad (\Delta_+ \to \Delta_-)$$

$$\longrightarrow \quad \gamma^{\phi^3\,\text{exch}}_{n,\ell} = \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \sin\left(\frac{-d+2\Delta+\Delta_+}{2}\right)\pi \times {}^{(\text{EAdS})}\gamma^{\phi^3\,\text{exch}\,\Delta_+}_{n,\ell} + (\Delta_+ \to \Delta_-)$$

Perturbative OPE data

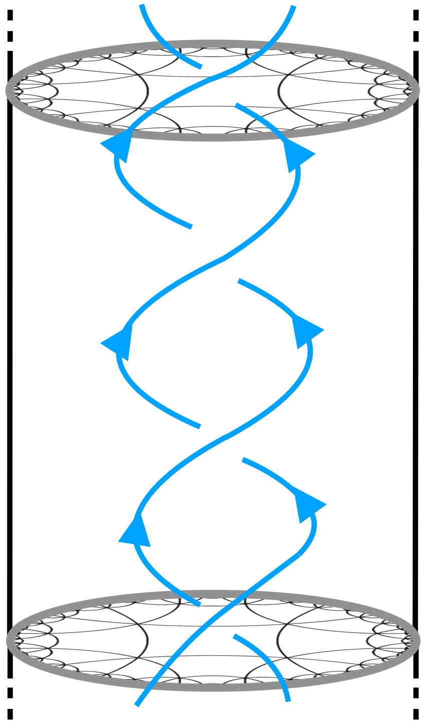
Perturbative OPE data on the boundary of dS and Minkowski space from EAdS

E.g. Composite operators on the boundary

$[\mathcal{O}\mathcal{O}]_{n,\ell} \sim \mathcal{O}\left(\partial^2\right)^n \partial_{i_1}\ldots\partial_{i_\ell}\mathcal{O} + \ldots$

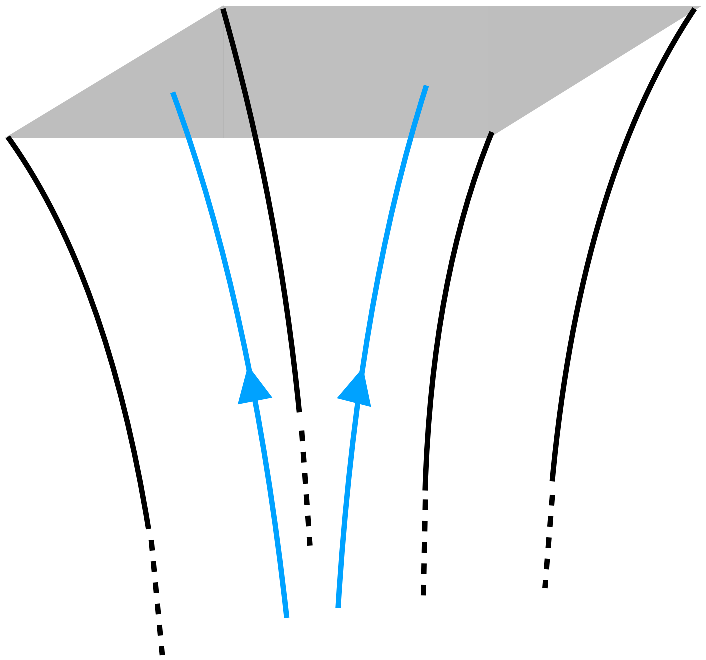
scaling dimension: $\Delta_{n,\ell} = \underbrace{2\Delta + 2n + \ell}_{\text{Free theory}} + \underbrace{\gamma_{n,\ell}}_{\text{anomalous dimension}}$

AdS



vs.

dS



$\Delta_{n,\ell}$ is unitary

⟶ stable particle (bound state)

$\Delta_{n,\ell}$ is (generally) non-unitary

⟶ resonance

[Bros et al ’06-’09, Marolf and Morrison ’10, Krotov and Polyakov ’10, Arkani-Hamed and Maldacena ’15]

Conformal Partial Wave Expansion

[Sleight, Taronna ’20] [Hogervorst, Penedones, Vaziri ’21] [di Pietro, Komatsu, Gorbenko '21]

Perturbative dS and celestial correlators have a similar analytic structure to those in AdS.

→ Like in AdS they admit a conformal partial wave expansion

$$\left\langle \mathcal{O}\left(\mathbf{x}_1\right)\mathcal{O}\left(\mathbf{x}_2\right)\mathcal{O}\left(\mathbf{x}_3\right)\mathcal{O}\left(\mathbf{x}_4\right)\right\rangle = \sum_{J=0}^{\infty}\int_{\frac{d}{2}-i\infty}^{\frac{d}{2}+i\infty}\frac{d\Delta}{2\pi i}\overset{\overset{\text{Spectral density, meromorphic in }\Delta}{\downarrow}}{\rho_J}\left(\Delta\right)\,\underbrace{\mathcal{F}_{\Delta,J}\left(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4\right)}_{\text{Conformal Partial Wave}}$$

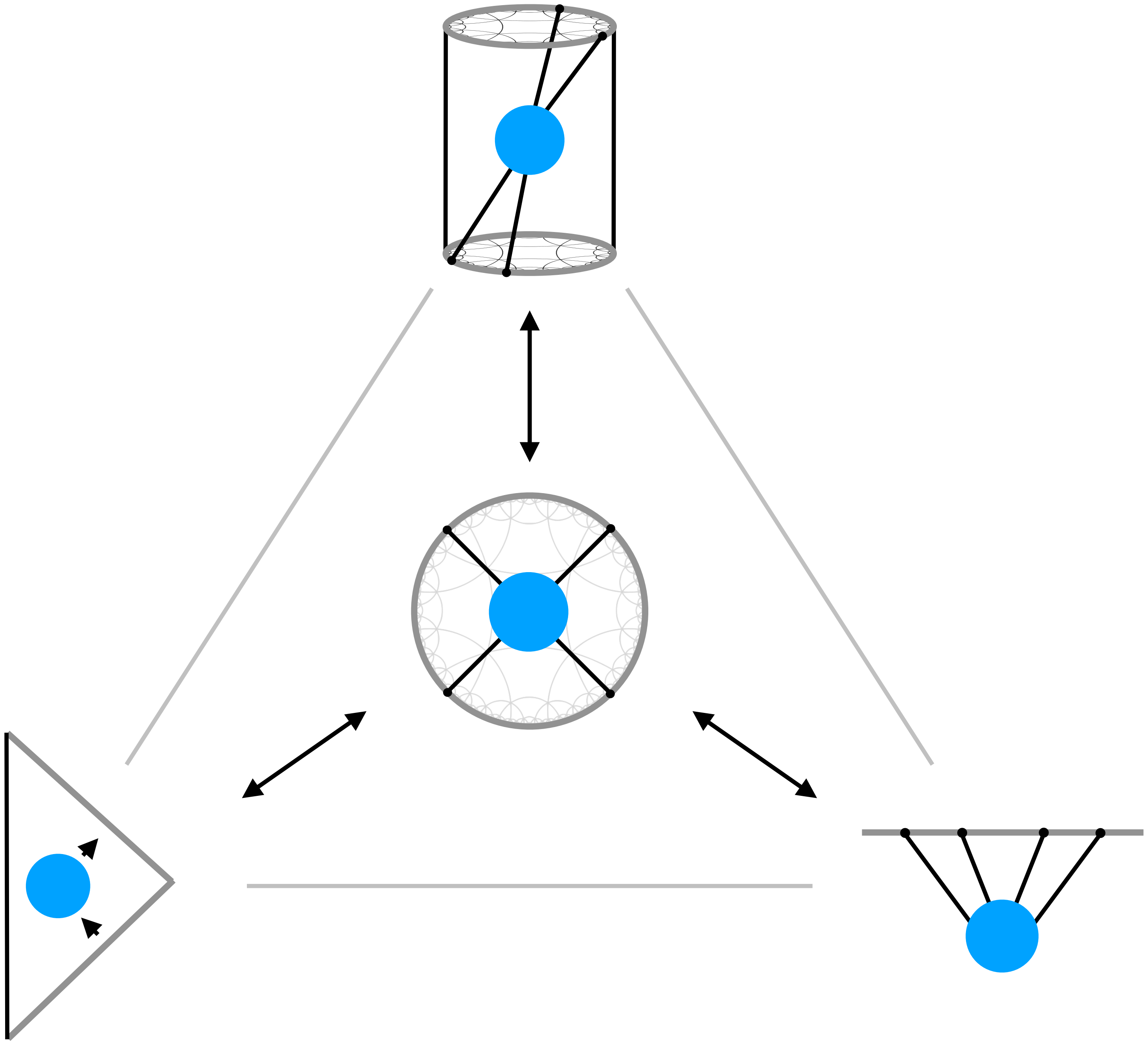
This has been argued to hold non-perturbatively as well [Hogervorst, Penedones, Vaziri ’21, di Pietro, Komatsu, Gorbenko '21]

Unitarity: $\rho_J\left(\Delta\right)\geq 0$ + crossing → Bootstrap for Euclidean CFTs?

Cf. Lorentzian CFT:

$$\left\langle \mathcal{O}\left(\mathbf{x}_1\right)\mathcal{O}\left(\mathbf{x}_2\right)\mathcal{O}\left(\mathbf{x}_3\right)\mathcal{O}\left(\mathbf{x}_4\right)\right\rangle = \sum_{\Delta,J}^{\infty} C^2_{\Delta,J}\,\underbrace{G_{\Delta,J}\left(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4\right)}_{\text{Conformal Block}}$$

Unitarity: $C^2_{\Delta,J}\geq 0$ + crossing → Conformal Bootstrap



Thank you.