Shift Symmetries & AdS/CFT

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"Shift Symmetries and AdS/CFT", arXiv:2211.02055 [hep-th].

Outline

- Review shift symmetric theories in flat spacetime
- Introduce shift symmetric theories in AdS
- Discuss properties of CFTs duals
- Results for AdS/CFT calculations
- Summary/Discussion

Shift Symmetric Fields in Flat Space

- Provide useful classifications of low-energy EFTs
- Appear in spontaneous symmetry breaking
- Lead to non-renormalization theorems
- Have enhanced soft limits in scattering amplitudes
- In exceptional cases, allows scattering amplitudes to be constructed recursively through soft subtracted recursion

Shift Symmetric Fields in Flat Space

❖The simplest example of a shift symmetric theory is a free massless scalar field in flat space.

$$S = \int d^D x \Big(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi \Big)$$

❖It is invariant under an infinite tower of shift symmetries.

$$\delta\phi = c + c_\mu x^\mu + c_{\mu_1\mu_2} x^{\mu_1} x^{\nu_2} + c_{\mu_1\mu_2\mu_3} x^{\mu_1} x^{\nu_2} x^{\nu_3} + \cdots$$

Cartesian spacetime coordinates

$c_{\mu_1 \cdots \mu_k}$ **is a rank-k symmetric traceless constant tensor**

❖We call a symmetry of the form $c_{\mu_1 \cdots \mu_k} x^{\mu_1} \cdots x^{\mu_k}$ a k-level shift.

❖In flat spacetime, they are always massless.

Interacting Scalar Examples in Flat Space

❖ $k = 0$ is a shift by a constant c

Any theory with at least one derivative per field such as $P(X)$ theories where $X = \partial_\mu \phi \partial^\mu \phi$

❖ $k = 1$ $\quad \delta\phi = c + c_\mu x^\mu$

DBI with the action

$$S = -\frac{\Lambda^D}{\alpha}\int d^D x \sqrt{1 + \frac{\alpha}{\Lambda^D}(\partial\phi)^2}$$

Galileons which have actions of the form

$\mathscr{L}_n \sim \phi S_{n-1}(\partial\partial\phi), \quad n = 1,2,\cdots, D+1$

where S_n are symmetric polynomials

❖ $k = 2$

Special Galileons are additionally invariant under

$$\delta\phi = c_{\mu\nu}\left(x^\mu x^\nu + \frac{1}{\Lambda^6}\partial^\mu\phi\partial^\nu\phi\right)$$

$$S = \int d^4x\left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{12\Lambda^6}(\partial\phi)^2\left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2\right]\right)$$

❖ $k \geq 3$

There are no known ghost-free interacting theories with higher level shift symmetries.

Shift Symmetric Fields in AdS

❖Shift symmetric theories in anti-de Sitter spacetime have masses that depend on the shift level k, spacetime dimension D, AdS radius L, and spin s

$$\begin{cases} m_k^2 = \dfrac{k(k+D-1)}{L^2}, & s=0\,, \\ m_k^2 = \dfrac{(k+2)(k+D-3+2s)}{L^2}, & s\geq 1\,, \end{cases} \qquad k = 0,1,2,\ldots$$

❖The equations of motion are the Klein-Gordon equations

$$\begin{cases} \left(\nabla^2 - m^2\right)\Phi = 0\,, & s=0\,, \\ \left(\nabla^2 + \frac{1}{L^2}\left[s+D-2-(s-1)(s+D-4)\right] - m^2\right)\Phi_{\mu_1\cdots\mu_s} = 0\,, & s\geq 1\,, \end{cases}$$

along with transversality and tracelessness in all the indices.

Shift Symmetric Fields in AdS

❖The shift symmetry takes the form

$$\delta\Phi_{\mu_1\cdots\mu_s} = S_{A_1\cdots A_{s+k},B_1\cdots B_s} X^{A_1}\cdots X^{A_{s+k}} \frac{\partial X^{B_1}}{\partial x^{\mu_1}}\cdots\frac{\partial X^{B_s}}{\partial x^{\mu_s}}$$

constant traceless tensor with the symmetries of a two row Young tableau

embedding coordinates of the AdS$_D$ into a $(D+1)$ -dimensional auxiliary flat spacetime

$$S_{A_1\cdots A_{s+k},B_1\cdots B_s} \in \begin{array}{|c|c|}\hline \multicolumn{2}{|c|}{s+k} \\ \hline s & \multicolumn{1}{|c}{} \\ \cline{1-1}\end{array}$$

Some Interacting Scalar Examples in AdS

❖ $k = 0$ is a shift by a constant c

Any scalar with at least one derivative per field such as $P(X)$ theories where $X = \nabla_\mu \phi \nabla^\mu \phi$

❖ $k = 1$ $\delta\phi = S_A X^A$ AdS Galileons

$$\mathcal{L}_2(\phi) = \sqrt{-|G|}\left[-\frac{1}{2}(\partial\phi)^2 - \frac{2}{L^2}\phi^2\right],$$

$$\mathcal{L}_3(\phi) = \sqrt{-|G|}\left[-\frac{1}{2}(\partial\phi)^2[\Pi] + \frac{3}{L^2}(\partial\phi)^2\phi + \frac{4}{L^4}\phi^3\right],$$

$$\mathcal{L}_4(\phi) = \sqrt{-|G|}\left[-\frac{1}{2}(\partial\phi)^2\left([\Pi]^2 - [\Pi^2] - \frac{1}{2L^2}(\partial\phi)^2 - \frac{6}{L^2}\phi[\Pi] + \frac{18}{L^4}\phi^2\right) - \frac{6}{L^6}\phi^4\right],$$

$$\mathcal{L}_5(\phi) = \sqrt{-|G|}\left[-\frac{1}{2}\left((\partial\phi)^2 - \frac{1}{5L^2}\phi^2\right)\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)\right.$$
$$\left. + \frac{12}{5L^2}\phi(\partial\phi)^2\left([\Pi]^2 - [\Pi^2] - \frac{27}{12L^2}[\Pi]\phi + \frac{5}{L^4}\phi^2\right) + \frac{24}{5L^8}\phi^5\right]$$
$$\vdots$$

$\Pi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi$ $[\cdots]$ means take the trace

❖ $k = 2$ AdS special Galileon

$$\delta\Phi = S_{AB}\left(X^A X^B - \frac{1}{\Lambda^6}\partial^A\Phi\partial^B\Phi\right)$$

$$\frac{\mathcal{L}_{SG}(\phi)}{|G|} = \sum_{j=1}^{D-1} \frac{\psi^{D-j} + (-1)^j \psi^{*D-j}}{i^j \Lambda^{j(D+2)/2}|\psi|^{D+3} 2\Gamma(j+3)}\left[(j+2)f_l\left(\frac{X}{|\psi|^2}\right)\right.$$
$$\left. -(j+1)f_{j+1}\left(\frac{X}{|\psi|^2}\right)\right]\partial^\mu\phi\,\partial^\nu\phi\, X^{(j)}_{\mu\nu}(\Pi) - \frac{\Lambda^{D+2}L^2}{2(D+1)}\left(1 - \frac{\psi^{*D+1} + \psi^{D+1}}{2|\psi|^{D+1}}\right)$$

$$f_j(x) \equiv {}_2F_1\left(\frac{D+3}{2}, \frac{j+1}{2}; \frac{j+3}{2}; -X\right), \quad \psi \equiv 1 + \frac{2i}{\Lambda^{\frac{D}{2}+1}L^2}\phi, \quad X \equiv -\frac{1}{\Lambda^{D+2}L^2}(\partial\phi)^2$$

❖ $k \geq 3$

There are no known ghost-free interacting theories with higher level shift symmetries.

J. Bonifacio, K. Hinterbichler, A. Joyce, and R. A. Rosen, "Shift Symmetries in (Anti) de Sitter Space," JHEP 02 (2019) 178, arXiv:1812.08167 [hep-th].

Interacting $k = 0$ Vector Examples in AdS

$$\frac{1}{\sqrt{-|G|}}\mathscr{L}(A) = -\frac{1}{2}F_{\mu\nu}^2 - \frac{6}{L^2}A^2 - \Lambda_2^4\left(\frac{\alpha_3}{2}\epsilon^{\mu_1\mu_2\mu_3\lambda}\epsilon^{\nu_1\nu_2\nu_3}{}_{\lambda}B_{\mu_1\nu_1}B_{\mu_2\nu_2}B_{\mu_3\nu_3} + \frac{\alpha_4}{2}\epsilon^{\mu_1\mu_2\mu_3\mu_4}\epsilon^{\nu_1\nu_2\nu_3\nu_4}B_{\mu_1\nu_1}B_{\mu_2\nu_2}B_{\mu_3\nu_3}B_{\mu_4\nu_4}\right)$$

$$B_{\mu\nu} = \nabla_\mu A_\nu + \nabla_\nu A_\mu$$

$$\delta A_\mu = -\xi_\mu$$

$$\begin{aligned}\frac{1}{\sqrt{-|G|}}\mathscr{L}(A) = &-\frac{1}{2}F_{\mu\nu}^2 - \frac{6}{L^2}A^2 + \frac{1}{\Lambda_2^2}\left[\frac{\alpha_3}{2}S_3(B) - \frac{1}{2}F^{\mu\alpha}F^{\nu}{}_{\alpha}X^{(1)}_{\mu\nu}(B) - \frac{3}{L^2}A^2B\right]\\
&+\frac{1}{\Lambda_2^4}\left[\frac{1}{8}((F_{\mu\nu}F^{\mu\nu})^2 - F_{\mu\nu}F^{\nu\alpha}F_{\alpha\beta}F^{\beta\mu}) + \frac{\alpha_4}{2}S_4(B) - \frac{3\alpha_3}{4}F^{\mu\alpha}F^{\nu}{}_{\alpha}X^{(2)}_{\mu\nu}(B)\right.\\
&+\frac{1}{4}F^{\mu\alpha}F^{\nu\beta}B_{\mu\nu}B_{\alpha\beta} + \frac{1}{4}F^{\mu\alpha}F^{\nu}{}_{\alpha}B^2_{\mu\nu} - \frac{1}{2}F^{\mu\alpha}F^{\nu\alpha}BB_{\mu\nu}\\
&-\frac{1+6\alpha_3}{L^2}A^2S_2(B) + \frac{1+3\alpha_3}{L^2}A^\mu A^\nu X^{(2)}_{\mu\nu}(B)\\
&\left.+\frac{2}{L^2}(A^2F_{\mu\nu}F^{\mu\nu} - A^\mu A^\nu B_{\mu\alpha}F_\nu{}^\alpha + \frac{1}{2}A^\mu A^\nu F_{\mu\alpha}F_\nu{}^\alpha) + \frac{12}{L^4}A^4\right] + \cdots\end{aligned}$$

$$S_n(M) = n!\, M^{[\mu_1}_{\mu_1}M^{\mu_2}_{\mu_2}\cdots M^{\mu_n]}_{\mu_n}$$

$$X^{(n)\mu}{}_{\nu}(M) = (n+1)!\,\delta^{[\mu}_{\nu}M^{\mu_2}_{\mu_2}\cdots M^{\mu_n]}_{\mu_n}$$

$$\delta A_\mu = -\frac{2}{\Lambda_2^2}\nabla_\mu\xi^\nu A_\nu - \xi_\mu\sqrt{1+\frac{4A^2}{(\Lambda_2^2 L)^2}}$$

C. De Rham, K. Hinterbichler, and L. A. Johnson, "On the (A)dS Decoupling Limits of Massive Gravity," JHEP 09 (2018) 154, arXiv:1807.08754 [hep-th]

J. Bonifacio, K. Hinterbichler, L. A. Johnson, and A. Joyce, "Shift-Symmetric Spin-1 Theories," JHEP 09 (2019) 029, arXiv:1906.10692 [hep-th].

Parent Fields

- Shift symmetric theories can be constructed as the decoupled longitudinal mode of a massive field as it approaches a partially massless value.

- Partially massless values occur for spins ≥ 1 at the mass values

$$\bar{m}_t^2 = -\frac{1}{L^2}(s-t-1)(s+t+D-4)\,, \quad t = 0,1,\ldots,s-1$$

where t is called the field depth, with a massless field occurring at $t = s-1$ for D > 2.

Spin-1 Parent Fields Example

❖ Starting with a massive vector

$$S = \int d^4x \sqrt{|G|} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A^2 \right]$$

❖ Taking the depth $t = 0$ massless limit ($m \to 0$) using the Stückelberg trick to preserve the number of degrees of freedom leads to

$$S = \int d^4x \sqrt{|G|} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \nabla_\mu \phi \, \nabla^\mu \phi \right]$$

parent field

shift symmetric
longitudinal mode

- The scalar field is shift symmetric under a $k = 0$ shift.

$$\delta\phi = c$$

Spin-2 Parent Fields Examples

❖Starting with a massive graviton

$$S = \int d^4x \left[\mathscr{L}_{m=0}(h) - \frac{1}{2} m^2 \sqrt{|G|} \left(h_{\mu\nu} h^{\mu\nu} - (h^\mu_\mu)^2 \right) \right]$$

❖Taking the depth $t = 1$ massless limit ($m \to 0$)

$$S = \int d^4x \left[\mathscr{L}_{m=0}(h) + \sqrt{|G|} \left(-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{6}{L^2} A^2 \right) \right]$$

- The vector field is shift symmetric under a $k = 0$ shift.

$$\delta A_\mu = -\,\xi_\mu, \qquad \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$

Spin-2 Parent Fields Examples

❖ Starting with a massive graviton

$$S = \int d^4x \left[\mathcal{L}_{m=0}(h) - \frac{1}{2} m^2 \sqrt{|G|} \left(h_{\mu\nu} h^{\mu\nu} - (h^\mu_\mu)^2 \right) \right]$$

❖ Taking the depth $t = 0$ partially massless limit $\left(m^2 \to -\frac{2}{L^2} \right)$

$$S = \int d^4x \left[\mathcal{L}_{m=0}(h) + \frac{2}{L^2} \sqrt{|G|} \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) + 3\sqrt{|G|} \left(-\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{2}{L^2} \phi^2 \right) \right]$$

- The scalar field is shift symmetric under a $k = 1$ shift.

$$\delta\phi = S_A X^A$$

Our Question

Does the shift symmetry in AdS manifest itself in the dual CFT, and if so how?

CFT duals to Shift Symmetric Theories

❖A field in $\mathrm{AdS_D}$ of mass m and spin s has a dual CFT operator on the $d = D - 1$ dimensional boundary with a scaling dimension related to the AdS mass by

$$\begin{cases} m^2L^2 = \Delta(\Delta - d)\ , & s = 0\,, \\ m^2L^2 = (\Delta + s - 2)(\Delta - s - d + 2)\ , & s \geq 1\,. \end{cases}$$

❖Or solving for the conformal dimensions,

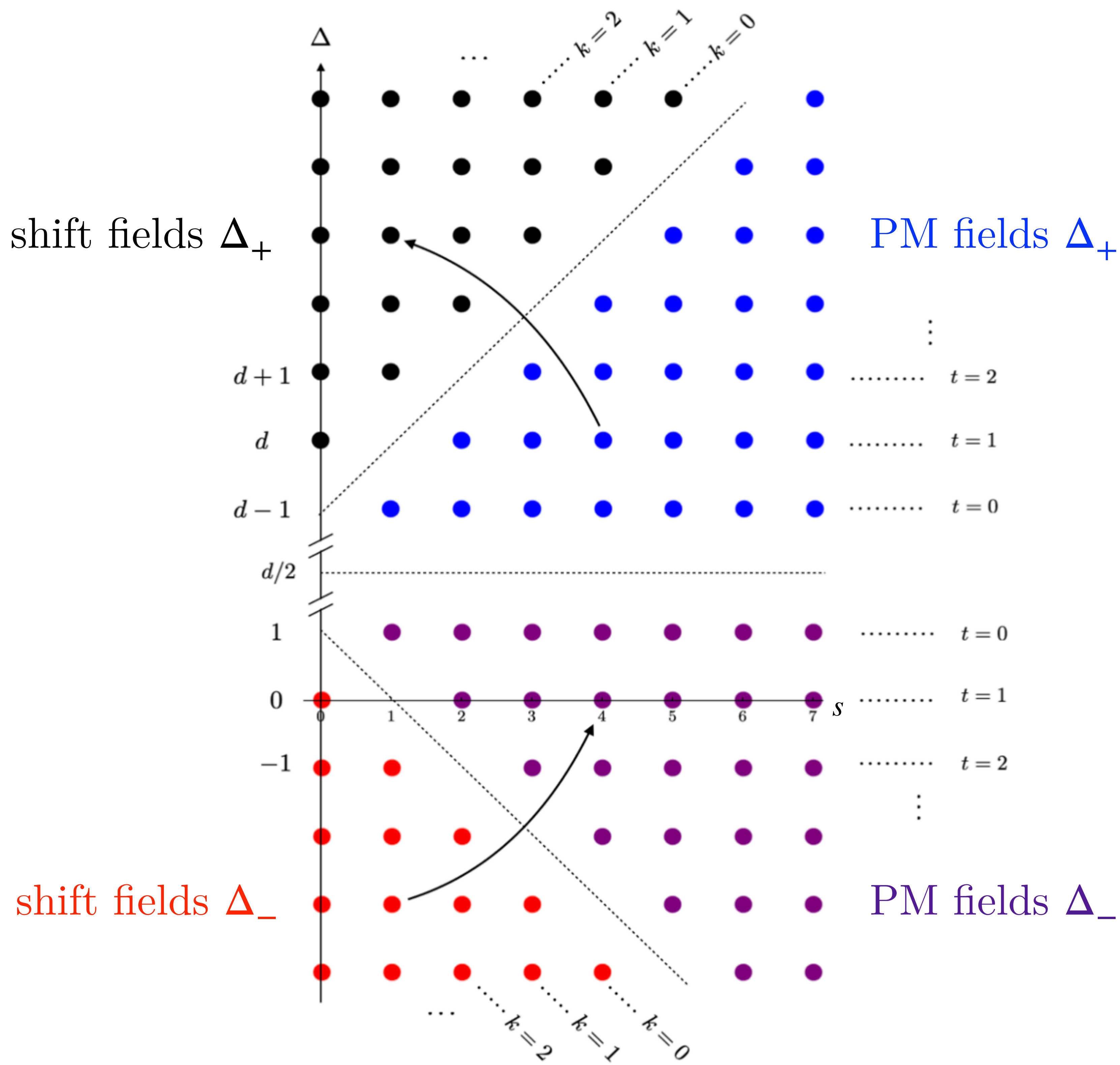
$$\begin{cases} \Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2L^2}\ , & s = 0\,, \\ \Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{\left(d + 2(s-2)\right)^2}{4} + m^2L^2}\ , & s \geq 1\,. \end{cases}$$

❖There are two different scaling dimensions for the dual CFT, Δ_+ and Δ_-.

❖For the shift symmetric fields, the scaling dimensions dictated by the masses are

$$\Delta_+ = d + s + k\,, \quad \Delta_- = -\,s - k$$

Operator Dimensions For Shift Symmetric and PM Fields



Review of CFT Properties

❖There is a unitarity bound for conformal primary operators.

$$\Delta \geq \begin{cases} \frac{d}{2} - 1, & s = 0 \\ s + d - 2, & s \geq 1 \end{cases}$$

❖For shift-symmetric theories, Δ_+ values lie above the unitarity bound, and the Δ_- values lie below the unitarity bound.

❖The two-point functions are completely fixed (up to an overall constant)

$$\langle \phi_{i_1 \cdots i_s}(x) \phi^{j_1 \cdots j_s}(0) \rangle = \frac{1}{x^{2\Delta}} I^{(j_1}_{(i_1} \cdots I^{j_s)_T}_{i_s)_T}, \quad I_{ij} \equiv \eta_{ij} - 2\frac{x_i x_j}{x^2}$$

where $(\cdots)_T$ denotes the symmetric traceless part.

Near Boundary Expansion

- The near boundary Fefferman-Graham expansion of a spin-s field of generic mass contains two leading fall-off behaviours. In Poincaré coordinates $ds^2 = \frac{L^2}{z^2}\left(dz^2 + \delta_{ij}dx^i dx^j\right)$, it is given by:

$$\Phi_{i_1\cdots i_s}(x,z) = z^{\Delta_- - s}\left[\phi_{(0)i_1\cdots i_s}(x) + \cdots\right] + z^{\Delta_+ - s}\left[\psi_{(0)i_1\cdots i_s}(x) + \cdots\right]$$

- In standard quantization, $\phi_{(0)}$ is proportional to the source in the CFT's generating functional, while in alternate quantization, $\psi_{(0)}$ is proportional to the source.

In standard quantization
$\phi_{(0)} \propto J$

In alternate quantization
$\psi_{(0)} \propto J$

Near Boundary Expansion of Shift-Symmetric Fields

- For shift-symmetric spinning fields, $\Delta_+ = d + k + s\,, \quad \Delta_- = -k - s$, the near boundary Fefferman-Graham expansion is:

$$\Phi(z,x)_{i_1\cdots i_s} = \frac{1}{z^{k+2s}}\left[\phi_{(0)i_1\cdots i_s}(x) + z^2\phi_{(2)i_1\cdots i_s}(x) + \cdots\right]$$
$$+z^{d+k}\left[\psi_{(0)i_1\cdots i_s}(x) + z^2\psi_{(2)i_1\cdots i_s}(x) + z^4\psi_{(4)i_1\cdots i_s}(x) + \cdots\right]$$

- The z-directed field components can be related to this one by the EOM and the transversality and tracelessness constraints.

Shift Symmetries and the Fefferman-Graham Expansion

❖ Applying a shift symmetry $\delta\Phi = S_{A_1\cdots A_k} X^{A_1}\cdots X^{A_k}$ to the near boundary expansion,

$$\Phi_{i_1\cdots i_s}(x,z) = \frac{1}{z^{k+2s}}\left[\phi_{(0)i_1\cdots i_s}(x) + \cdots\right] + z^{d+k}\left[\psi_{(0)i_1\cdots i_s}(x) + \cdots\right]$$

we find that $\phi_{(0)}$ is shifted by the transformation, while $\psi_{(0)}$ is unaffected.

$$\delta\phi_{(0)i_1\cdots i_s} = L^{k+2s} S_{A_1\cdots A_{s+k},B_1\cdots B_s} \tilde{X}^{A_1}\cdots\tilde{X}^{A_{s+k}} \frac{\partial \tilde{X}^{B_1}}{\partial x^{i_1}}\cdots\frac{\partial \tilde{X}^{B_s}}{\partial x^{i_s}}$$

$$\delta\psi_{(0)i_1\cdots i_s} = 0$$

Holographic Renormalization Procedure

1. Asymptotic Solution: Use the EOM to find the asymptotic solution in terms of two pieces of boundary data $\phi_{(0)}$ and $\psi_{(0)}$, which are then determined by the choice of boundary conditions and requiring the solution to be regular in the bulk.

2. Regularization: Regularize the action by cutting off the integral near the boundary at $z = \epsilon$. In general there will be terms that diverge in the limit $z \to 0$.

3. Counterterms: Construct the counterterm action from all possible local functions of Φ and the induced metric $\gamma_{ij} = \frac{L^2}{\epsilon^2}\eta_{ij}$ with arbitrary coefficients, evaluated on the $z = \epsilon$ boundary.

4. Renormalized On-shell Action: Add the counterterm action to the regularized action and tune the coefficients of the counterterm action to remove divergences giving the finite renormalized action.

M. Bianchi, D. Z. Freedman and K. Skenderis, "Holographic renormalization," Nucl. Phys. B 631 (2002) 159 [arXiv:hep-th/0112119].

Renormalized Action

❖ The renormalization procedure leads to the renormalized action

$$S_{\text{ren}} = \lim_{\epsilon \to 0} (S_\epsilon + S_{\text{c.t.}}) = \int \frac{d^d p}{(2\pi)^d} \left[-\left(k + \frac{d}{2} \right) \phi_{(0)}(p) \psi_{(0)}(-p) \right]$$

❖ For standard quantization, we identify $\phi_{(0)} = J$ and for alternate quantization, we identify $\psi_{(0)} = J$.

Result for Standard Quantization

❖ For standard quantization of shift symmetric scalars

$$S_{\mathrm{ren}} = \int \frac{d^d p}{(2\pi)^d} \left[-\frac{\left(k+\frac{d}{2}\right)}{4^{\left(k+\frac{d}{2}\right)}} \frac{\Gamma\left(-k-\frac{d}{2}\right)}{\Gamma\left(k+\frac{d}{2}\right)} p^{2k+d} |J(p)|^2 \right]$$

leading to the position space correlation function for the CFT dual to take the form

$$\langle \mathcal{O}(0)\mathcal{O}(x)\rangle \propto \frac{1}{x^{2(d+k)}}$$

which has the canonical structure of a 2-point correlation function of a primary field.

Ward Identities

❖ The Noether procedure leads to an integral constraints*

$$S_{\text{ren}} = \int d^d x \sqrt{-\gamma} \langle \mathcal{O} \rangle_s S_{A_1 \cdots A_k} \tilde{X}^{A_1} \cdots \tilde{X}^{A_k} = 0$$

which can be differentiated to give a constraint for any n-point correlation function.

For 2-point functions, the Ward identity is

$$\int d^d x \langle \mathcal{O}(x) \mathcal{O}(0) \rangle S_{A_1 \cdots A_k} \tilde{X}^{A_1} \cdots \tilde{X}^{A_k} = 0.$$

* M. M. Caldarelli, A. Christodoulou, I. Papadimitriou, and K. Skenderis, "Phases of planar AdS black holes with axionic charge," JHEP 04 (2017) 001, arXiv:1612.07214 [hep-th].

Ward Identities

- For example, considering the scalar with $k = 0$, the Ward identity becomes

$$\int d^d x \langle \mathcal{O}(x)\mathcal{O}(0)\rangle = 0$$

- This becomes a soft limit in momentum space, which is satisfied by the momentum space correlator.

$$\lim_{p\to 0} \langle \mathcal{O}(p)\mathcal{O}(-p)\rangle \sim \lim_{p\to 0} p^d = 0$$

Result for Alternate Quantization

❖ For alternate quantization of shift symmetric theories,

$$S_{ren} = \int \frac{d^d p}{(2\pi)^d} \left[-\left(k + \frac{d}{2}\right) 4^{\left(k+\frac{d}{2}\right)} \frac{\Gamma\left(k + \frac{d}{2}\right)}{\Gamma\left(-k - \frac{d}{2}\right)} \frac{1}{p^{2k+d}} |J(p)|^2 \right]$$

leading to the position space correlation function for the CFT dual to take the form

$$\langle \mathcal{O}(0)\mathcal{O}(x) \rangle \propto x^{2k} \log(x^2)$$

which violates the canonical structure of a 2-point correlation function for a primary field.

Analogous to the Case of a Free Massless Scalar in 2d

❖ For this familiar case (which is shift-symmetric)

$$S_{CFT} = \int d^2x \left(-\frac{1}{2}\partial_\mu \phi \partial^\mu \phi \right)$$

where the two-point function is given in complex coordinates z, w by

$$\langle \phi(z,\bar{z})\phi(w,\bar{w})\rangle \propto \log(z-w) + \log(\bar{z}-\bar{w})$$

❖ This indicates that ϕ is not a primary operator. However, $\partial_z \phi$ is a primary operator of spin 1 and conformal dimension 1, the two-point function of which is given by

$$\langle \partial_z \phi(z,\bar{z})\partial_w \phi(w,\bar{w})\rangle \propto \frac{1}{(z-w)^2}$$

Result for Alternate Quantization

❖We find an analogous result for the alternately quantized CFTs from shift symmetric theories.

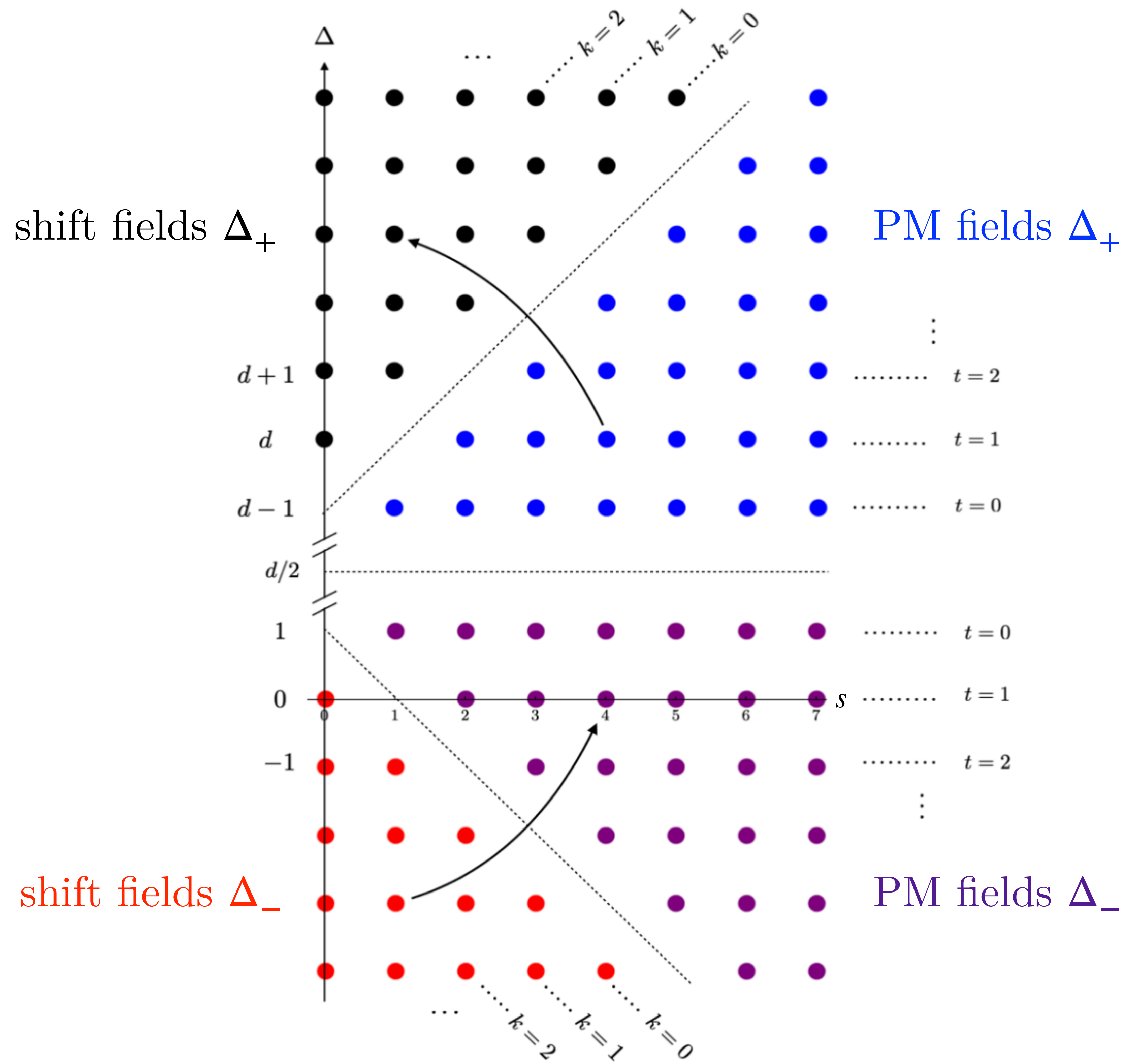
❖The logarithmic structure implies that $\mathcal{O}$ is not a primary field. However, we can construct a primary field by taking

$$F_{i_1\cdots i_{k+1}} = \partial_{(i_1}\cdots\partial_{i_{k+1})_T}\mathcal{O}$$

❖This leads to a correlation function corresponding to a primary field with spin $k+1$ and conformal dimension $\Delta = 1$.

$$\langle F_{i_1\cdots i_{k+1}}(x)F^{j_1\cdots j_{k+1}}(0)\rangle = \frac{1}{x^2}I_{(i_1}^{(j_1}\cdots I_{i_{k+1})_T}^{j_{k+1})_T} \qquad I_{ij} \equiv \delta_{ij} - 2\frac{x_i x_j}{x^2}$$

Operator Dimensions For Shift Symmetric and PM Fields



Near Boundary Expansion

- For shift-symmetric scalars, $\Delta_+ = d + k\,, \quad \Delta_- = -k$, the near boundary Fefferman-Graham expansion splits into two cases:

$$\Delta_+ - \frac{d}{2} = k + \frac{d}{2} \neq \mathbb{Z}, \text{ corresponding to odd dimensions}$$

$$\Phi(z,x) = \frac{1}{z^k}\left[\phi_{(0)}(x) + z^2\phi_{(2)}(x) + \cdots\right] + z^{d+k}\left[\psi_{(0)}(x) + z^2\psi_{(2)}(x) + \cdots\right]$$

$$\Delta_+ - \frac{d}{2} = k + \frac{d}{2} \in \mathbb{Z}, \text{ corresponding to even dimensions}$$

$$\begin{aligned}\Phi(z,x) = {} & \frac{1}{z^k}\left[\phi_{(0)}(x) + z^2\phi_{(2)}(x) + \cdots + z^{d+2k-2}\phi_{(d+2k-2)}(x)\right] \\ & + z^{d+k}\left[\left(\psi_{(0)}(x) + \phi_{(d+2k)}(x)\log(\mu z)\right) + z^2\left(\psi_{(2)}(x) + \phi_{(d+2k+2)}(x)\log(\mu z)\right) + \cdots\right]\end{aligned}$$

Results for even dimensions

❖ For standard quantization of shift symmetric theories

$$S_{\text{ren}} = \int \frac{d^d p}{(2\pi)^d} \frac{(-1)^{\left(k+\frac{d}{2}\right)}}{2^{2k+d-1}\Gamma\left(k+\frac{d}{2}\right)^2} p^{2k+d} \log(p/\mu)\,|J(p)|^2, \quad \nu \in \mathbb{Z}$$

leading to the position space correlation function for the CFT dual to take the form

$$\langle \mathcal{O}(0)\mathcal{O}(x)\rangle \propto \frac{1}{x^{2(d+k)}}$$

which has the canonical structure of a 2-point correlation function for a primary field.

Results even dimensions

- For alternate quantization of shift symmetric theories

$$S_{\text{ren}} = \int \frac{d^d p}{(2\pi)^d} \frac{(-1)^{\left(k+\frac{d}{2}\right)} 2^{2k+d-1} \Gamma\left(k+\frac{d}{2}\right)^2}{p^{2k+d}(\log(p/\mu)+c)} |J(p)|^2$$

which is not conformally invariant and neither are any of its derivatives.

- We suspect this is due to needing an alternate regularization procedure for these cases.

Vector Case

- We also looked at holographic renormalization for shift-symmetric vectors. The procedure is the same, but with more complicated expressions.

Standard in Odd Dimensions

$$\langle \mathcal{O}_i(0)\mathcal{O}_j(x)\rangle \propto \frac{1}{x^{2(d+k+1)}}\left(\eta_{ij} - 2\frac{x_i x_j}{x^2}\right)$$

Standard in Even Dimensions

$$\langle \mathcal{O}_i(0)\mathcal{O}_j(x)\rangle \propto \frac{1}{x^{2(d+k+1)}}\left(\eta_{ij} - 2\frac{x_i x_j}{x^2}\right)$$

Alternate in Odd Dimension

$$\langle \mathcal{O}_i(0)\mathcal{O}_j(x)\rangle \propto x^{2(k+1)}\left(\eta_{ij} - 2\frac{x_i x_j}{x^2}\right)\log(x^2)$$

Create primary operator of spin $k+2$ and $\Delta = 0$

$$F_{i_1\cdots i_{k+2}} = \partial_{(i_i}\cdots\partial_{i_{k+1}}\mathcal{O}_{i_{k+2})_T}$$

$$\langle F_{i_1\cdots i_{k+2}}F^{j_1\cdots j_{k+2}}\rangle \sim I^{(j_1}_{(i_1}\cdots I^{j_{k+2})_T}_{i_{k+2})_T}$$

Alternate in Even Dimension

Same issue with $\dfrac{1}{\log(p/\mu)+c}$

- We expect higher spin theories to follow the same pattern.

Summary

❖ The CFT dual obtained by standard quantization is affected by the shift symmetry.

- It has 2-point correlation functions in position space with the canonical form for primary operators.

- We find Ward identities that take the form of integral constraints.

❖ The CFT dual obtained by alternate quantization preserves the shift symmetry.

- In odd dimensions, this leads to two-point correlation functions in position space with logarithmic behavior violating the canonical form for primary operators.

- The shift invariant field strength can be constructed by taking $k+1$ traceless symmetrized derivatives of the operator giving a primary operator of spin $s+k+1$ and conformal dimension $1-s$.

Discussion

❖ There are some AdS flux vacua where the dual operators have integer conformal dimensions.

- F. Apers, J. P. Conlon, S. Ning, and F. Revello, "Integer conformal dimensions for type IIa flux vacua," Phys. Rev. D 105 no. 10, (2022) 106029, arXiv:2202.09330 [hep-th].
- E. Plauschinn, "Mass spectrum of type IIB flux compactifications - comments on AdS vacua and conformal dimensions," arXiv:2210.04528 [hep-th].
- F. Apers, "Aspects of AdS flux vacua with integer conformal dimensions," arXiv:2211.04187 [hep-th].

❖ Some interesting things to look at in the future would be

- Consider interacting shift symmetric theories, which we expect to exhibit interesting behavior.
- Understand what is happening with alternate quantization of shift symmetric theories in even dimensions.