Non-unitary effects in the early universe

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Non-unitarity

- Start with unitary evolution:

$$\rho_{\pi\sigma,\pi'\sigma'}(\eta) \equiv \langle\pi'| \otimes \langle\psi'| \,\widehat{\rho}(\eta)\, |\pi\rangle \otimes |\psi\rangle = \Psi\left[\pi,\sigma\right] \Psi^*\left[\pi',\sigma'\right].$$

- Integrate out σ: cannot write the state of π as a wavefunction

$$\rho_{\pi\pi'}(\eta) \neq \Psi_{\mathrm{red}}\left[\pi\right] \Psi^*_{\mathrm{red}}\left[\pi'\right].$$

- There is an extra piece which does not obey the rules of unitary EFTs.

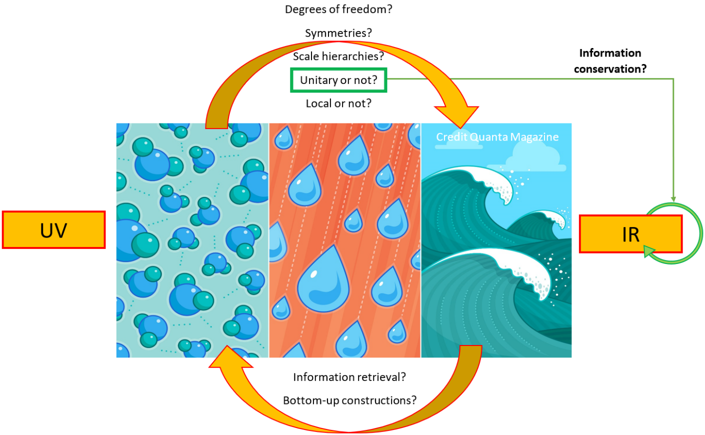
How can we understand it?

Outline

1. Entropy measures

2. Cosmological correlators

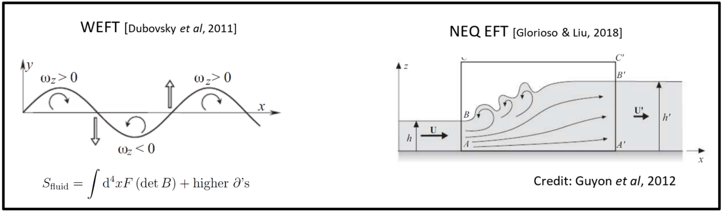
Information conservation



When unitarity matters

Sometimes, $S_{\rm IR}$ local and unitary is not enough:

Energy not conserved



$\Rightarrow$ **dissipative effects** (energy or information losses) can be crucial.

What about cosmology?

- Scales are dynamical: UV/IR mixing;
- Lack of stationarity: out-of-equilibrium system;
- No well segregated energy sectors.

Goal: Extend EFTs to account for dissipative/non-unitary effects in cosmology

The *Holy Trinity* of Open Systems

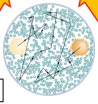
What happens to a ***system*** *immersed into an* ***environment****?* [Brown, 1827]

Effects of environment encoded through noises.

Classical Brownian motion

Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$



Wiener path integral [Wiener, 1923]

$$P = \int \mathcal{D}q\mathcal{D}p e^{iS_C[q,p]} P(t_0)$$

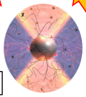
Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

Quantum Brownian motion

Stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right]\mathrm{d}t + \mathrm{d}\widehat{\xi}$$



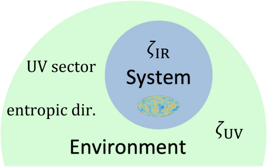
Influence functional [Vernon, 1959]

$$\langle\zeta_1|\,\widehat{\rho}_{\mathrm{red}}\,|\zeta_2\rangle = \int \mathcal{D}\zeta_\pm e^{iS_\zeta[\zeta_+]-iS_\zeta[\zeta_-]} e^{iS_{\mathrm{IF}}[\zeta_+;\zeta_-]}\langle\zeta_1|\,\widehat{\rho}_0\,|\zeta_2\rangle$$

Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{L}[\widehat{\rho}_{\mathrm{red}}]$$

Motivations



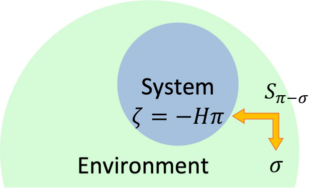
Model impact of:

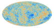
- high-energy extensions;
- multifield hidden sectors;
- short and soft modes;

on adiabatic dof $\zeta_{\rm IR}$ using **Open EFTs**.

1. *Technical:* Open EFTs may go **beyond Standard Perturbation Theory** (SPT) by implementing late-time resummations [Boyanovsky, 2015], [Burgess *et al.*, 2015], [Burgess, Holman & Tasinato, 2015].
2. *Conceptual:* By assessing departure from unitarity, Open EFTs improve our understanding of the **emergence of unitary descriptions** [Kaplanek & Burgess, 2022], [TC, Grain & Vennin, 2212.09486].
3. *Phenomenological:* Non-unitary effects leave **signatures on cosmological observables** $\Rightarrow$ If neglected, there is a risk to misinterpret the physics [Lopez Nacir *et al.*, 2011], [Creminelli *et al.*, 2023].

Early universe phenomenology



Quantum fluctuations of $\langle\widehat{\zeta}^2\rangle = H^2\langle\widehat{\pi}_c^2\rangle$
leads to $\langle(\delta T/T)^2\rangle \sim$ 

Minimal framework: EFT of Inflation [Cheung *et al.*, 2007]: $\delta g^{00} = -2\dot{\pi} - \dot{\pi}^2 + (\partial_i\pi)^2/a^2$

$$S_\pi = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\left(\partial_\mu\pi_c\right)^2 - \frac{1}{2}M_2^4(t)(\delta g^{00})^2 - \frac{1}{3!}M_3^4(t)(\delta g^{00})^3 + \cdots\right]$$

Extensions: Scalar hidden sector [Assassi *et al.*, 2013], [Jazayeri, *et al.*, 2023]

$$S_\sigma = -\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\left(\partial_\mu\sigma\right)^2 + \frac{1}{2}m^2\sigma^2 + \mu\sigma^3 + \cdots\right]$$

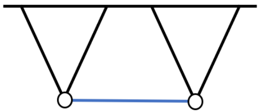
Interactions: Fixed by symmetries

$$S_{\pi-\sigma} = \int \mathrm{d}^4x\sqrt{-g}\left[\widetilde{M}_1^3\delta g^{00}\sigma + \widetilde{M}_3^3(\delta g^{00})^2\sigma + \widetilde{M}_2^2\delta g^{00}\sigma^2 + \cdots\right]$$

Outline

Purity and entropy measures

Corrections to the observables leads to corrections to $\widehat{\rho}_{\rm red} \equiv {\rm Tr}_{\mathcal{E}}\,\widehat{\rho}$



$$\left\langle \widehat{O}(\eta) \right\rangle^{(n)} = i^n g^n \left\langle \int_{\eta_0}^{\eta} {\rm d}\eta_1 \int_{\eta_0}^{\eta_1} {\rm d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} {\rm d}\eta_n \left[\widetilde{\mathcal{H}}_{\rm int}(\eta_n), \left[\cdots \left[\widetilde{\mathcal{H}}_{\rm int}(\eta_1), \widetilde{O}(\eta) \right] \right] \right] \right\rangle_{\widehat{\rho}_0}$$

leads to

$$\widetilde{\rho}_{\rm red}^{(n)}(\eta) = (-i)^n g^n {\rm Tr}_{\mathcal{E}} \int_{\eta_0}^{\eta} {\rm d}\eta_1 \int_{\eta_0}^{\eta_1} {\rm d}\eta_2 \cdots \int_{\eta_0}^{\eta_{n-1}} {\rm d}\eta_n \left[\widetilde{\mathcal{H}}_{\rm int}(\eta_1), \left[\cdots \left[\widetilde{\mathcal{H}}_{\rm int}(\eta_n), \widehat{\rho}_0 \right] \right] \right]$$

From $\widehat{\rho}_{\rm red}$, quantify **amount of information shared** between system and environment

$$S_{\rm ent} \equiv -{\rm Tr}\left[\widehat{\rho}_{\rm red} \log \widehat{\rho}_{\rm red}\right] = \lim_{q\to 1} S_q \qquad \text{with} \qquad S_q \equiv \frac{1}{q-1}{\rm Tr}\left[\widehat{\rho} - \widehat{\rho}^q\right]$$

Focus on purity $\gamma \equiv 1 - S_2$ to assess non-unitarity: if $\gamma \ll 1$, system has decohered.

Linear interactions [TC, Grain & Vennin, 2212.09486]

At **linear order**: entropic and adiabatic linear mixing of multifield inflation.

System Environment Interactions

$$S^{(2)} = \int \mathrm{d}^4 \boldsymbol{x} \frac{a^2}{2} \left[\pi_c'^2 - c_s^2 \left(\partial_i \pi_c\right)^2 + \sigma'^2 - \left(\partial_i \sigma\right)^2 - m^2 a^2 \sigma^2 + \rho a \pi_c' \sigma \right]$$

Does an entropic sector lead to quantum decoherence ($\gamma \ll 1$) of the curvature perturbations?

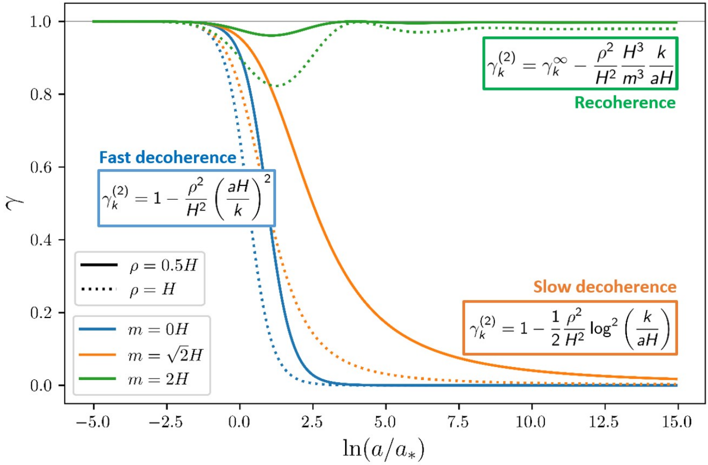
Gaussian systems: purity γ **fully determined** by det $\boldsymbol{\Sigma}$ [Serafini *et al.*, 2003]

$$\boldsymbol{\Sigma} \equiv \begin{pmatrix} \left\langle \widehat{\pi}_c^2 \right\rangle_\Omega & \left\langle \left\{ \widehat{\pi}_c, \widehat{p}_{\pi_c} \right\} \right\rangle_\Omega \\ \left\langle \left\{ \widehat{\pi}_c, \widehat{p}_{\pi_c} \right\} \right\rangle_\Omega & \left\langle \widehat{p}_{\pi_c}^2 \right\rangle_\Omega \end{pmatrix}$$

with $\widehat{p}_{\pi_c}$ conjugate momentum and Ω Bunch-Davies vacuum.

Solve **numerically** and **analytically** in asymptotic regimes (exact).

De(re)coherence phenomenology



$$\sigma_l \propto \frac{1}{a^{3/2-\nu}}, \quad \nu \equiv \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \qquad \text{and} \qquad \sigma_h \propto \frac{\cos\left(\mu \ln \frac{a}{a_*}\right)}{a^{3/2}}, \quad \mu \equiv \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

Purity as unequal time correlators [with G. Kaplanek, in prep.]

Is the recoherence mechanism robust to the inclusion of non-linear interactions?

For $\mathcal{L}_{\rm int} = \lambda(t) f(\pi_c) g(\sigma)$, second-order perturbative purity:

Coupling — System — Environment

$$\frac{\mathrm{d}\gamma^{(2)}}{\mathrm{d}t} = -4\lambda(t)\int_{-\infty(1-i\epsilon)}^{t} \mathrm{d}s\lambda(s)\Re\mathrm{e}\left[\Big\langle f(\pi_c)(t) f(\pi_c)(s)\Big\rangle_\Omega \Big\langle g(\sigma)(t) g(\sigma)(s)\Big\rangle_\Omega\right]$$

NL extensions: $\mathcal{L}_{\rm int} \supset \rho \delta g^{00}\sigma$ with $\delta g^{00} \to -2\dot{\pi} - \dot{\pi}^2 + (\partial_i \pi)^2/a^2$

$$S_{\pi_c \sigma} = \int \mathrm{d}^3 x \mathrm{d}\eta a^2 \left[\rho a \pi_c' \sigma + \frac{1}{\Lambda_1}\pi_c'^2 \sigma + \frac{c_s^2}{\Lambda_2}\left(\partial_i \pi_c\right)^2 \sigma\right]$$

Robustness of recoherence

- Heavy environment *(recoherence)*:

$$\gamma_k^{(2)} = \gamma_k^{\infty} - \frac{\rho^2}{H^2}\frac{H^3}{m^3}\frac{k}{aH}\left[1 - \frac{1}{(2\pi)^8}\frac{3}{16\epsilon}\frac{1}{c_s^4}\frac{H^4}{M_{\mathrm{Pl}}^2 m^2}\right] + \mathcal{O}\left[\left(\frac{k}{aH}\right)^2\right].$$

- Massless environment *(fast decoherence)*:

$$\gamma_k^{(2)} = 1 - \frac{\rho^2}{H^2}\left(\frac{aH}{k}\right)^2\left[1 - \frac{1}{(2\pi)^8}\frac{3}{8\epsilon}\frac{1}{c_s^4}\frac{H^2}{M_{\mathrm{Pl}}^2}\right] + \mathcal{O}\left[\left(\frac{aH}{k}\right)\right]$$

- Conformal environment *(slow decoherence)*:

$$\gamma_k^{(2)} = 1 - \frac{1}{2}\frac{\rho^2}{H^2}\log^2\left(\frac{k}{aH}\right)\left[1 - \frac{1}{(2\pi)^8}\frac{3}{8\epsilon}\frac{1}{c_s^4}\frac{H^2}{M_{\mathrm{Pl}}^2}\right] + \mathcal{O}\left[\log\left(\frac{k}{aH}\right)\right]$$

Open questions: environmental non-linearities, higher-order interactions, $\cdots$

Outline

Non-unitary effects on correlators [with H. Goodhew, in prep.]

Goal:

- Assess the impact of non-unitary effects on cosmological observables;
- Clarify the regime of validity of different integration scheme.

Observables:

- Power spectrum with heavy mediator;
- Trispectrum with heavy mediator.

Techniques:

1. **In-in computation**: standard perturbation theory [Chen *et al.*, 2017], $\cdots$;
2. Wavefunction: cosmological bootstrap [Arkani-Hamed *et al.*, 2018], [Pajer *et al.*, 2020], $\cdots$;
3. **Open EFTs**: unitary and non-unitary effects [TC, Grain & Vennin, 2209.01929], $\cdots$;
4. Non-local EFT: integrate heavy mediator from eom [Jazayeri *et al.*, 2023], $\cdots$.

From in-in to Open EFTs

In-in computation recasted in terms of a **master equation** [TC, Grain & Vennin, 2209.01929]

$$\frac{\mathrm{d}\widetilde{\rho}_{\text{red}}}{\mathrm{d}\eta} = -g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathrm{Tr}_{\mathcal{E}} \left[\widetilde{\mathcal{H}}_{\text{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\text{int}}(\eta'), \widetilde{\rho}_{\text{red}}(\eta) \otimes \rho_{0,\mathcal{E}}\right]\right]$$

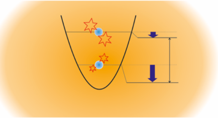
which can be decomposed into [Breuer & Petruccione, 2002]

$$\frac{\mathrm{d}\widehat{\rho}_{\text{red}}}{\mathrm{d}\eta} = \underbrace{-i\left[\widehat{H}_0(\eta) + \widehat{H}^{(\text{LS})}(\eta), \widehat{\rho}_{\text{red}}(\eta)\right]}_{\text{Unitary evolution}} + \underbrace{\sum_{i,j} \mathcal{D}_{ij}(\eta)\left[\widehat{z}_i \widehat{\rho}_{\text{red}}(\eta) \widehat{z}_j - \frac{1}{2}\left\{\widehat{z}_j \widehat{z}_i, \widehat{\rho}_{\text{red}}(\eta)\right\}\right]}_{\text{Non-unitary evolution}}$$

● Unitary
✸ Non-unitary

Lamb shift

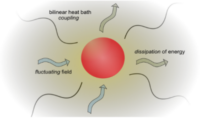
Energy levels of the system shifted by interactions with environment: dressing of the system Hamiltonian



CREDIT : [RENTROP ET AL., 2016]

Dissipation

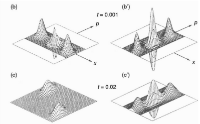
Energy can be lost onto the environment: time-translation symmetry breaking.



CREDIT : ULM UNIVERSITY

Diffusion

Noise-induced spreading: information loss and quantum decoherence of the system.



CREDIT: [ZUREK, 2005]

Effective cosmological flow [Werth, Pinol & Renaux-Petel, 2023]

Power spectra from $\mathcal{L}_{\mathrm{int}} \supset \rho \pi_c' \sigma$: consider $\boldsymbol{z} \equiv (\widehat{\pi}_c, \widehat{p}_{\pi_c})^{\mathrm{T}}$ and $\boldsymbol{\Sigma} \equiv \left\langle \{\boldsymbol{z}^\dagger, \boldsymbol{z}\} \right\rangle_\Omega$

$$\frac{\mathrm{d}\boldsymbol{\Sigma}^{(2)}}{\mathrm{d}\eta} = \mathbf{Hom} + \boldsymbol{S}_{\mathrm{LS}} + \boldsymbol{S}_{\mathrm{dissip}} + \boldsymbol{S}_{\mathrm{diff}}$$

with homogeneous evolution from $\boldsymbol{H}_0$ the *free Hamiltonian*

$$\mathbf{Hom} \equiv \boldsymbol{\omega} \boldsymbol{H}_0 \boldsymbol{\Sigma}^{(2)} - \boldsymbol{\Sigma}^{(2)} \boldsymbol{H}_0 \boldsymbol{\omega}, \qquad \boldsymbol{\omega} \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

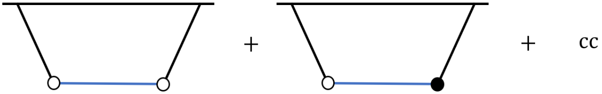
and source terms with $\boldsymbol{D}$ and $\boldsymbol{\Delta}$ the *noise* and *dissipation* kernels

$$\boldsymbol{S}_{\mathrm{LS}} \equiv \boldsymbol{\omega} \boldsymbol{\Delta} \boldsymbol{\Sigma}^{(0)} - \boldsymbol{\Sigma}^{(0)} \boldsymbol{\Delta} \boldsymbol{\omega}$$

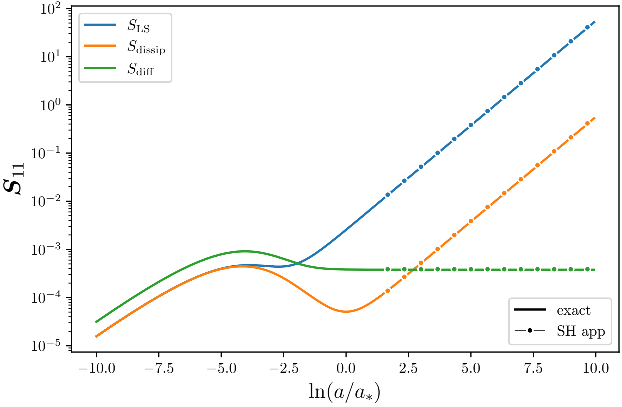
$$\boldsymbol{S}_{\mathrm{dissip}} \equiv 2 \boldsymbol{\Delta}_{12} \boldsymbol{\Sigma}^{(0)}$$

$$\boldsymbol{S}_{\mathrm{diff}} \equiv -\boldsymbol{\omega} \boldsymbol{D} \boldsymbol{\omega}$$

Result is **equivalent to in-in treatment** [TC, Grain & Vennin, 2209.01929]



Heavy environment $m \gg H$



Heavy environment $m \gg H$

In the super-Hubble $k \ll aH$ regime:

- **Lamb shift** dominates:

$$\boldsymbol{S}_{\mathrm{LS},11} = \frac{\rho^2}{H^2}\frac{H^2}{m^2}\left(1+\frac{H^2}{m^2}\right)\frac{aH}{k} + \mathcal{O}\left[\left(\frac{k}{aH}\right)^0, \frac{H^6}{m^6}\right]$$

- **Dissipation** is $\mathcal{O}(H^2/m^2)$ suppressed:

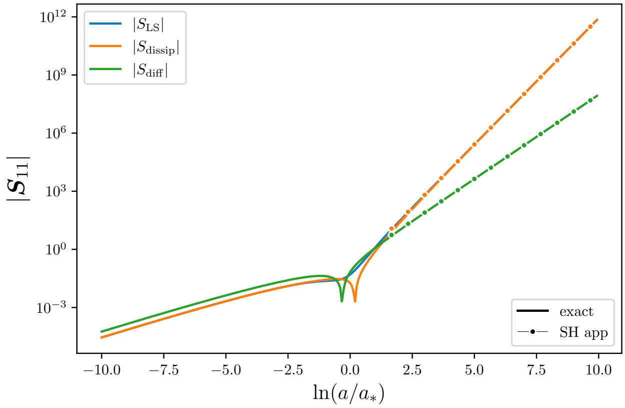
$$\boldsymbol{S}_{\mathrm{dissip},11} = \frac{\rho^2}{H^2}\frac{H^4}{m^4}\frac{aH}{k} + \mathcal{O}\left[\left(\frac{k}{aH}\right)^0, \frac{H^6}{m^6}\right]$$

- **Diffusion** is $\mathcal{O}(k/aH)$ and $\mathcal{O}(H/m)$ suppressed:

$$\boldsymbol{S}_{\mathrm{diff},11} = \frac{3}{2}\frac{\rho^2}{H^2}\frac{H^3}{m^3} + \mathcal{O}\left[\left(\frac{k}{aH}\right), \frac{H^5}{m^5}\right]$$

$\Rightarrow$ Non-unitary effects are $\mathcal{O}(H^2/m^2)$ suppressed.

Conformal environment $m = \sqrt{2}H$



Conformal environment $m = \sqrt{2}H$

In the super-Hubble $k \ll aH$ regime:

- **Lamb shift** cancels with **dissipation**:

$$\boldsymbol{S}_{\mathrm{LS},11} = -\frac{1}{4}\frac{\rho^2}{H^2}\left(\gamma_{\mathrm{E}} + \log 2\right)\left(\frac{aH}{k}\right)^3 + \mathcal{O}\left[\left(\frac{aH}{k}\right)\right]$$

- **Dissipation** also has subleading contributions:

$$\boldsymbol{S}_{\mathrm{dissip},11} = \frac{1}{4}\frac{\rho^2}{H^2}\left(\gamma_{\mathrm{E}} + \log 2\right)\left(\frac{aH}{k}\right)^3 - \frac{\pi}{4}\frac{\rho^2}{H^2}\left(\frac{aH}{k}\right)^2 + \mathcal{O}\left[\left(\frac{aH}{k}\right)\right]$$

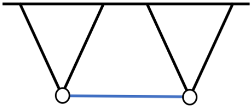
- **Diffusion** dominates with **dissipation**:

$$\boldsymbol{S}_{\mathrm{diff},11} = -\frac{\pi}{4}\frac{\rho^2}{H^2}\left(\frac{aH}{k}\right)^2 + \mathcal{O}\left[\left(\frac{aH}{k}\right)\right]$$

$\Rightarrow$ Non-unitary effects are not suppressed!

Trispectrum

$$\mathcal{L} = \frac{1}{2}a^2\varphi'^2 - \frac{1}{2}a^2(\partial_i\varphi)^2 + \frac{1}{2}a^2\sigma'^2 - \frac{1}{2}a^2(\partial_i\sigma)^2 - \frac{1}{2}a^4m^2\sigma^2 + \lambda a^4\varphi^2\sigma$$



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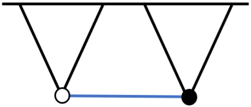

$+ \quad \mathrm{cc}$

$$\mathcal{C}^s_{\mathrm{CIE}} = 2\Re\mathrm{e}\left[-\lambda^2 \int^{\eta_0} \mathrm{d}\eta a^4(\eta) \int^{\eta_0} \mathrm{d}\eta' a^4(\eta') \right.$$
$$\left. G^{\varphi}_{+}(k_1,\eta) G^{\varphi}_{+}(k_2,\eta) G^{\sigma}_{++}(p_s,\eta,\eta') G^{\varphi}_{+}(k_3,\eta') G^{\varphi}_{+}(k_4,\eta') \right]$$

$$\mathcal{C}^s_{\mathrm{CIM}} = 2\Re\mathrm{e}\left[\lambda^2 \int^{\eta_0} \mathrm{d}\eta a^4(\eta) \int^{\eta_0} \mathrm{d}\eta' a^4(\eta') \right.$$
$$\left. G^{\varphi}_{+}(k_1,\eta) G^{\varphi}_{+}(k_2,\eta) G^{\sigma}_{+-}(p_s,\eta,\eta') G^{\varphi}_{-}(k_3,\eta') G^{\varphi}_{-}(k_4,\eta') \right]$$

Flat space limit

$$\mathcal{C}^{s}_{\mathrm{CIE}} + \mathcal{C}^{s}_{\mathrm{CIM}} = 2\frac{1}{E_T}\frac{1}{\prod_{i=1}^{4} 2k_i}\frac{\lambda^2}{E_s^{\sigma 2}}\left(1 - \frac{k_{12}k_{34}}{E_s^{\sigma 2}} + \frac{k_{12}^2 k_{34} + k_{12}k_{34}^2}{E_s^{\sigma 3}} + \cdots\right)$$

with $E_s^\sigma \equiv \sqrt{p_s^2 + m^2} \gg k_{ij} \equiv k_i + k_j$, whereas

$$iS_{\mathrm{EFT}}\left[\varphi\right] = -\lambda^2 \int \mathrm{d}^4 x \int \mathrm{d}^4 y \varphi^2(x) \Box_\sigma^{-1}(x-y)\varphi^2(y)$$

only captures **even powers** of the mass through

$$\Box_\sigma^{-1}(x-y) = i\left[\frac{1}{m^2} - \frac{\partial^2}{m^4} + \mathcal{O}\left(\frac{\partial^4}{m^6}\right)\right]\delta(x-y).$$

Odd powers come from the noise kernel, $\mathrm{Re}[G^{\sigma}_{-+}(p_s, \eta, \eta')]$: **non-unitary effect**.

Summary and prospects

In the presence of *heavy environments* ($m \geq 3H/2$):

1. $S_{\rm ent}$ freezes: recoherence and decoupling;
2. Non-unitary effects on power spectrum are $\mathcal{O}(H^2/m^2)$ suppressed.

In the presence of *light environments* ($m < 3H/2$):

1. $S_{\rm ent}$ grows: late-time decoherence;
2. Non-unitary effects on power spectrum are not suppressed.

Now, need to:

- Extend trispectrum computation in de Sitter;
- Generalize to other interactions and environments.

Short-term: Wavefunction approach and bootstrap [with H. Goodhew, in prep.];

Future direction: Bottom-up Open EFTs [Lopez Nacir *et al.*, 2011], [Hongo *et al.*, 2018].