On the IR divergences in dS:

from ***Trees*** *to* ***Loops*** *and* **Back**

with Guilherme Pimentel, Ana Achucarro: 2212.14035
with Sebastian Cespedes, Anne Davis: work in progress

Dong-Gang Wang (王东刚)

DAMTP Cambridge

Correlators in Cortona, Sept. 2023

Plan of the Talk

I. Trees

- Anomalous CFT
- Multi-field non-Gaussianities

II. Loops

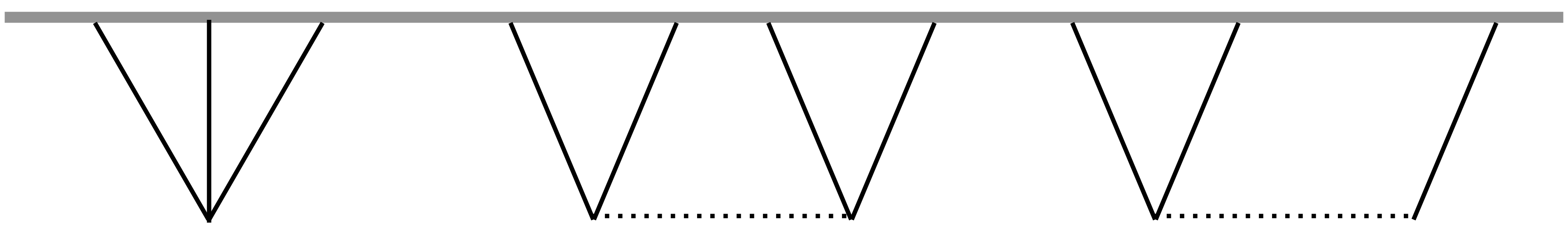
- Wavefunction: Classical v.s. Quantum
- Classical Loops and IR divergences

III. Back to Trees

- Fokker-Planck Equation

I. Trees

- Bootstrap with CFT anomalies
- Multi-field non-Gaussianities

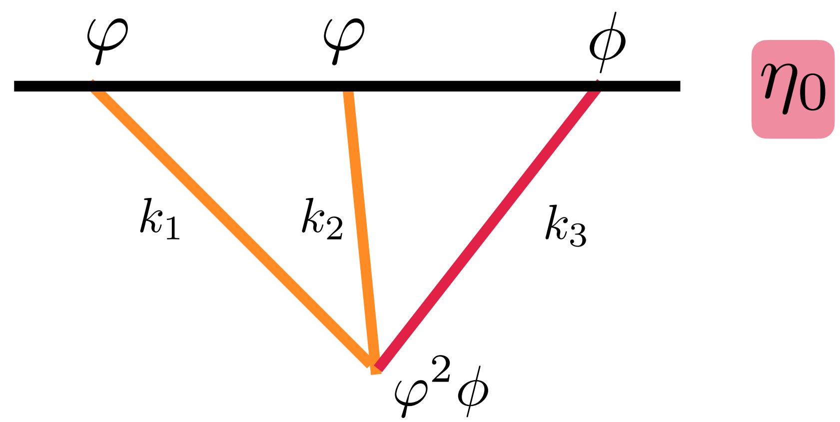


DGW, Pimentel, Achucarro 2212.14035

I. Trees: Contact

Three-Point Contact

$$\langle \varphi_{k_1} \varphi_{k_2} \phi_{k_3} \rangle \sim$$



$$\sim I(u, \eta_0)$$

$$u \equiv \frac{k_3}{k_1 + k_2}$$

- ϕ is massive → IR-finite correlator

Arkani-Hamed, Maldacena 2015

Conformal Ward Identities

$$\left[\Delta_u - 2 + \frac{m^2}{H^2}\right] I(u) = 0$$

with $\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$

- ϕ is massless → IR-divergent correlator

Bzowski, McFadden, Skenderis 2013, 2015, 2018
Pajer, Pimentel, Van Wijck 2016
DGW, Pimentel, Achucarro 2022

Anomalous Conformal Ward Identities

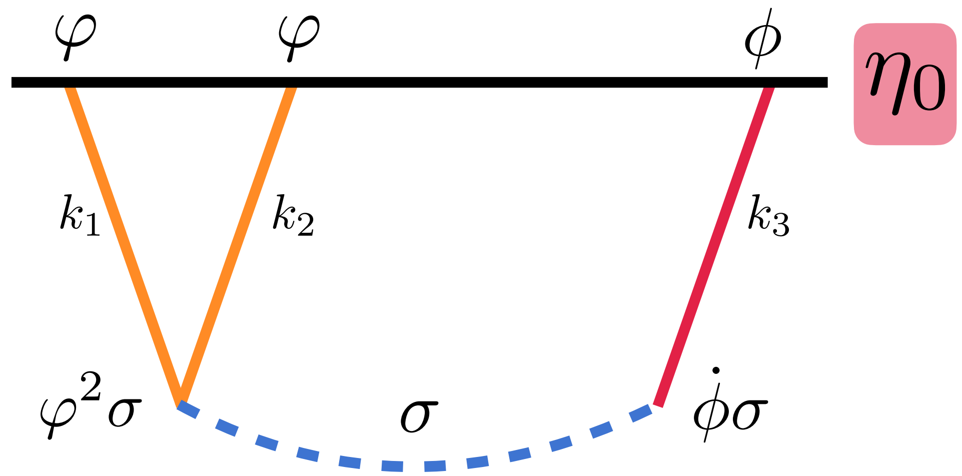
$$\left[\Delta_u - 2\right] I(u, \eta_0) = -\frac{6}{u}$$

an extra source term

I. Trees: Exchange

<u>Three-Point Scalar Seed</u>

$$\langle \varphi_{k_1}\varphi_{k_2}\phi_{k_3}\rangle \sim$$



$$\sim \hat{\mathcal{I}}(u,\eta_0)$$

$$u \equiv \frac{k_3}{k_1+k_2}$$

- σ is massive → IR-finite correlator

Arkani-Hamed, Baumann, Lee, Pimentel 2018
Pimentel, DGW 2022

Conformal Ward Identities

$$\left(\Delta_u - 2 + \frac{m^2}{H^2}\right)\hat{\mathcal{I}} = \frac{u}{1+u}$$

with $\Delta_u \equiv u^2(1-u^2)\partial_u^2 - 2u^3\partial_u$

- σ is massless → IR-divergent correlator

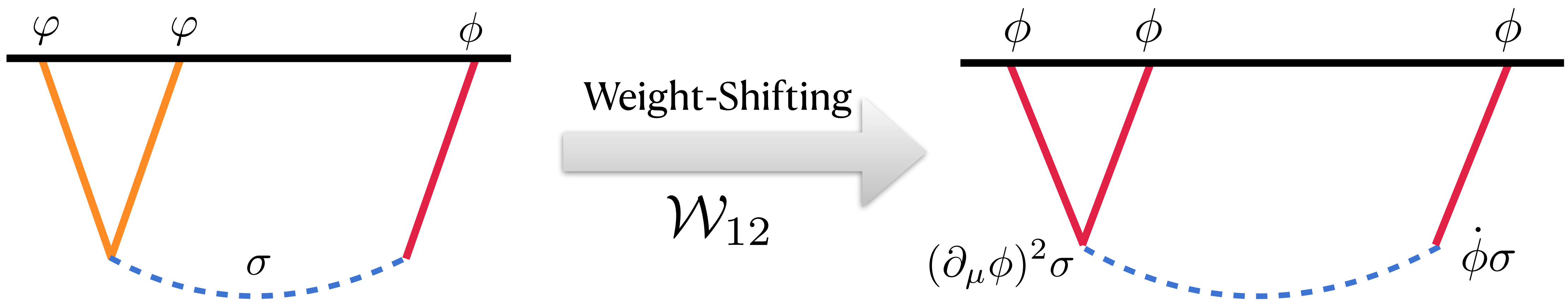
DGW, Pimentel, Achucarro 2022

Anomalous Conformal Ward Identities

$$(\Delta_u - 2)\hat{\mathcal{I}}(u,\eta_0) = \frac{u}{1+u} + \frac{6}{u}\hat{\mathcal{K}}(k\eta_0)$$

an extra source term caused by the IR cutoff

Application: Bootstrapping multi-field inflation



massless exchange:

the minimal setup for multi-field inflation

Here is the full shape:

logarithmic k_t-pole: from the cubic vertex

$$S(k_1,k_2,k_3) \propto \frac{1}{k_1^3k_2^3k_3^3}\Big[\left(\gamma_E - 3 - \log(-k_t\eta_0)\right)(k_1^3+k_2^3+k_3^3) + k_t e_2 - 4e_3$$
$$+ (k_2^3+k_3^3)\log(-2k_1\eta_0) + (k_1^3+k_3^3)\log(-2k_2\eta_0) + (k_1^3+k_2^3)\log(-2k_3\eta_0)\Big]$$

logarithmic k_n-pole: from the linear mixing

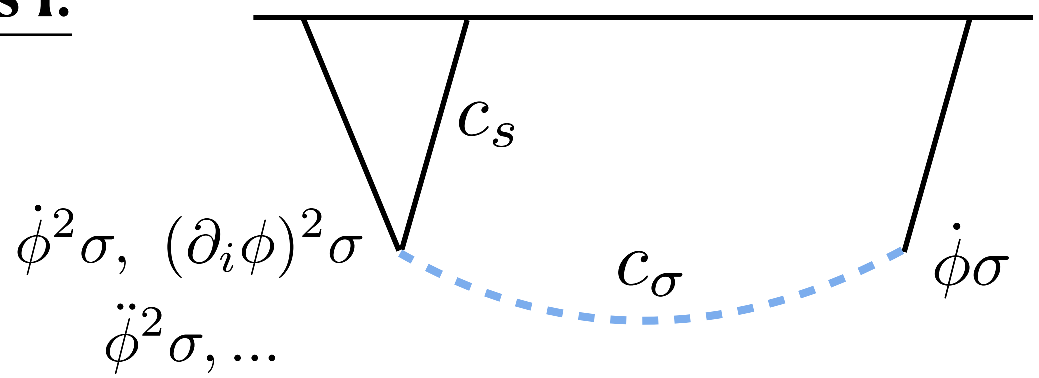
For comparison:

$$S^{\rm local} = \frac{k_1^3+k_2^3+k_3^3}{k_1^3k_2^3k_3^3}$$

these poles were missed in the previous δN formalism

Massless Exchange in Boost-Breaking Scenarios

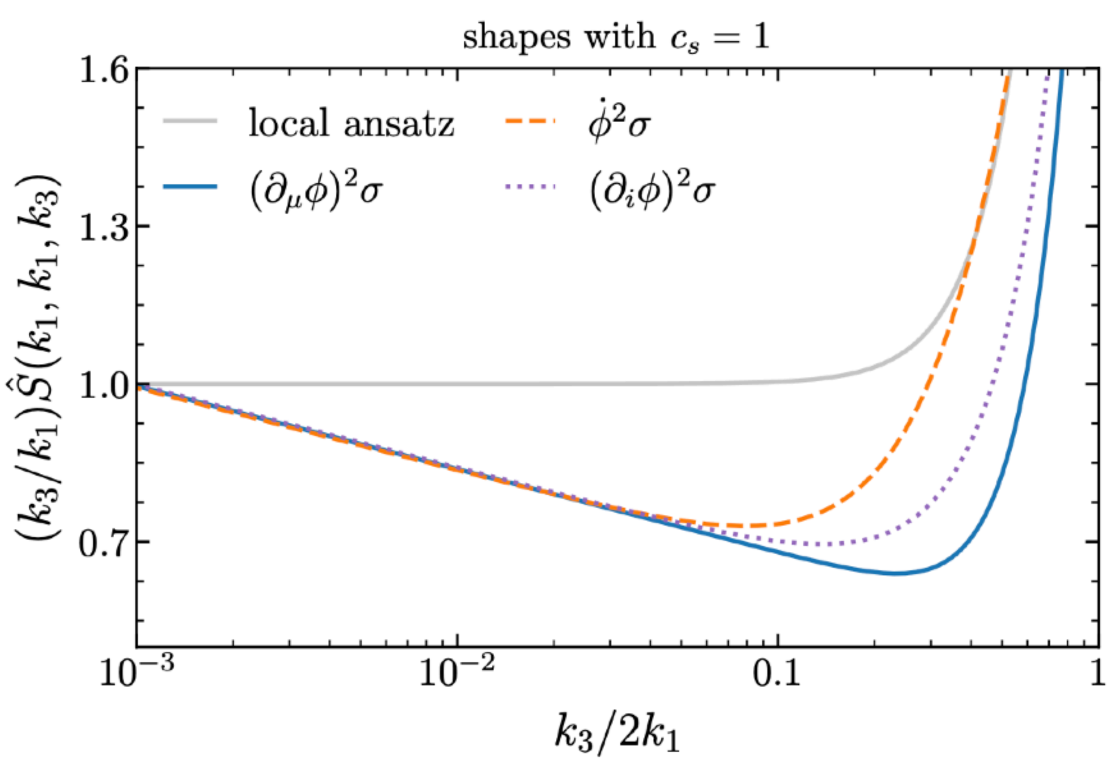
Class I:

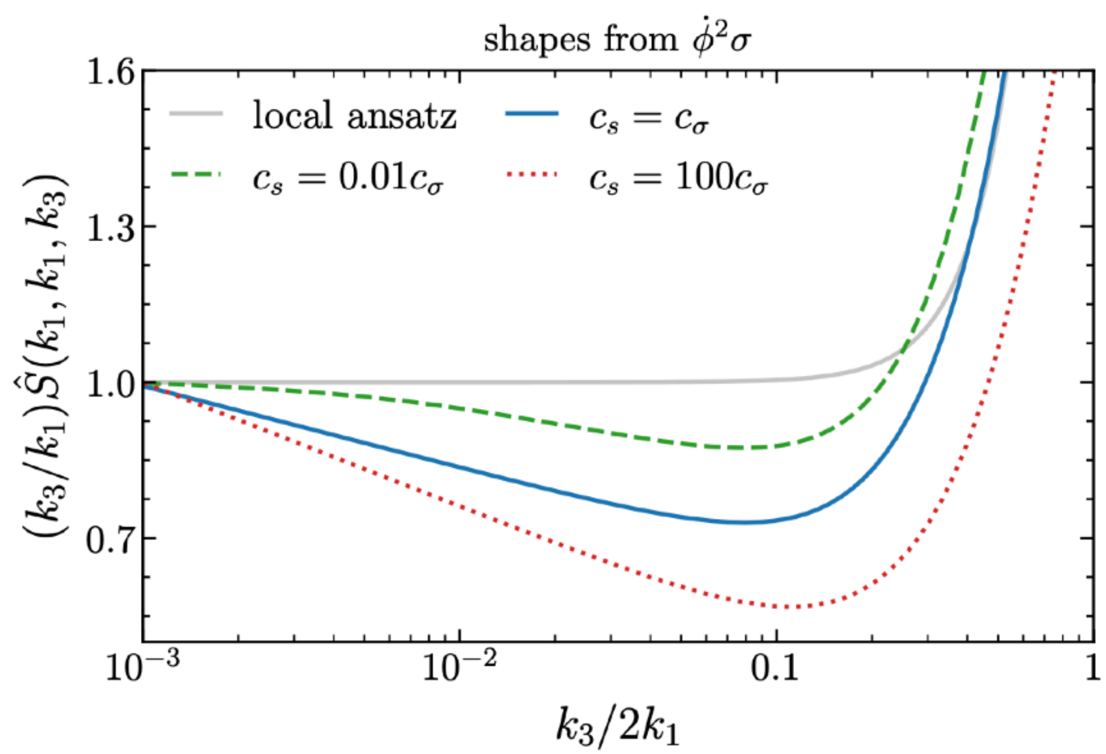


IR divergence

local-type non-Gaussianity

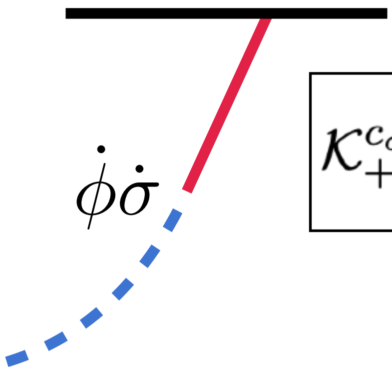
with more complicated analytical structures





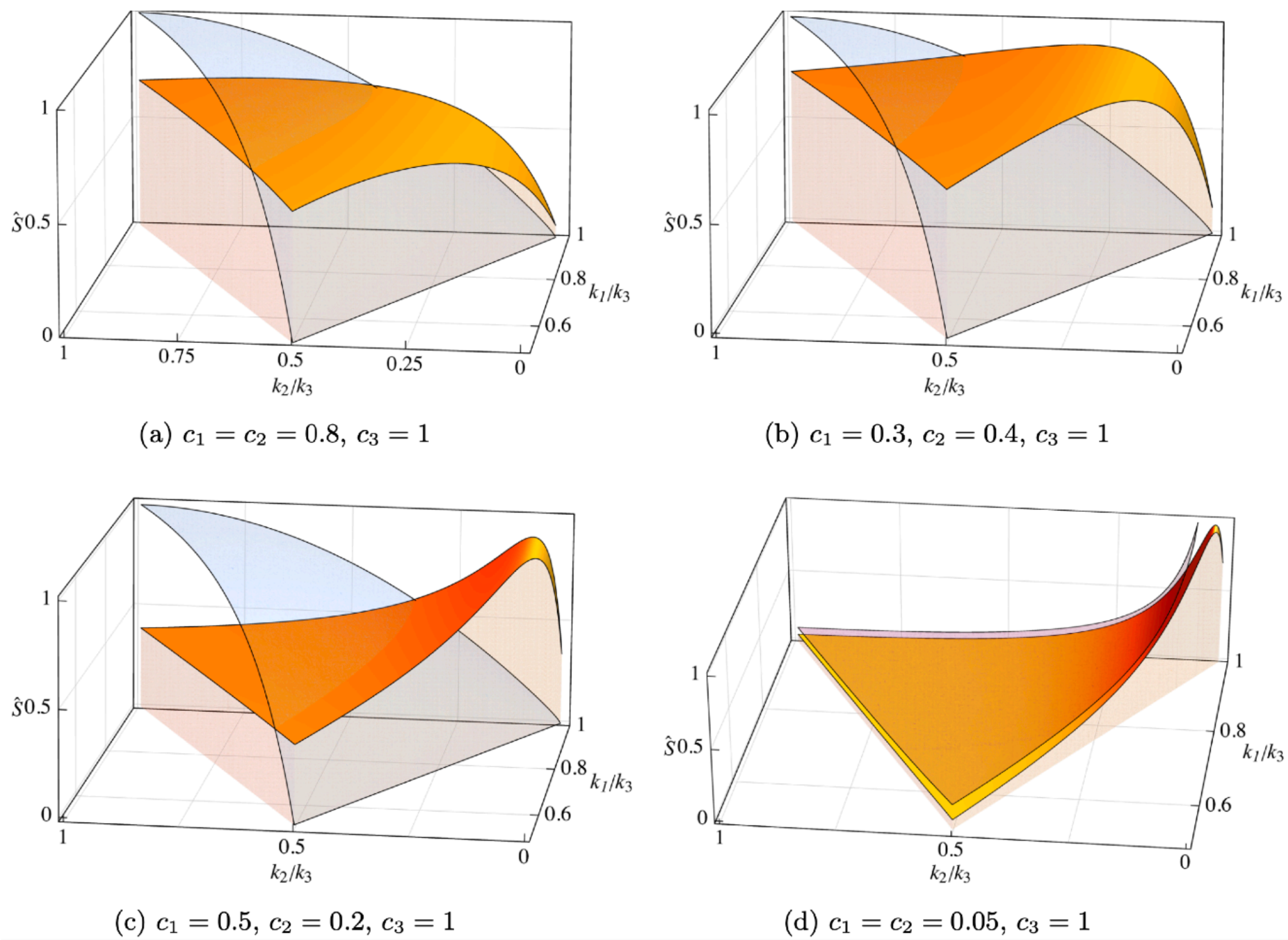
Class II: Linear mixing with higher derivatives $\ddot{\phi}\sigma,\ \dot{\phi}\partial_i^2\sigma$, etc. → **IR-finite** correlators

Multi-Speed Non-Gaussianity



$$\mathcal{K}_{+}^{c_\sigma}(c_\sigma k, \eta) = \frac{H^2\eta}{2c_\sigma k} e^{ic_\sigma k\eta}$$

$$\hat{S}^{\text{multi}-c_s}(k_1, k_2, k_3) = \frac{k_1 k_2 k_3}{(c_1 k_1 + c_2 k_2 + c_3 k_3)^3} + 5 \text{ perms}$$



(a) $c_1 = c_2 = 0.8,\ c_3 = 1$

(b) $c_1 = 0.3,\ c_2 = 0.4,\ c_3 = 1$

(c) $c_1 = 0.5,\ c_2 = 0.2,\ c_3 = 1$

(d) $c_1 = c_2 = 0.05,\ c_3 = 1$

I. Trees: Summary

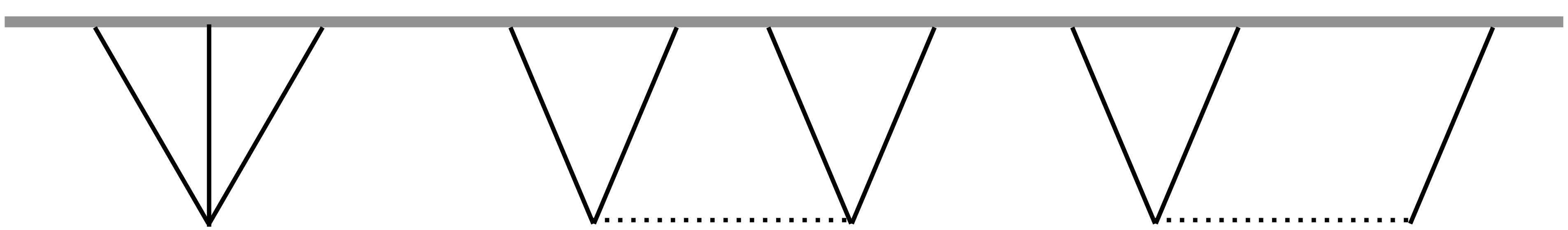
- **IR divergent correlator**: logarithmic secular growth

$$\lim_{\eta_0 \to 0} \langle \phi^n \rangle \propto \log(-k\eta_0)^v$$

with local-type shape function

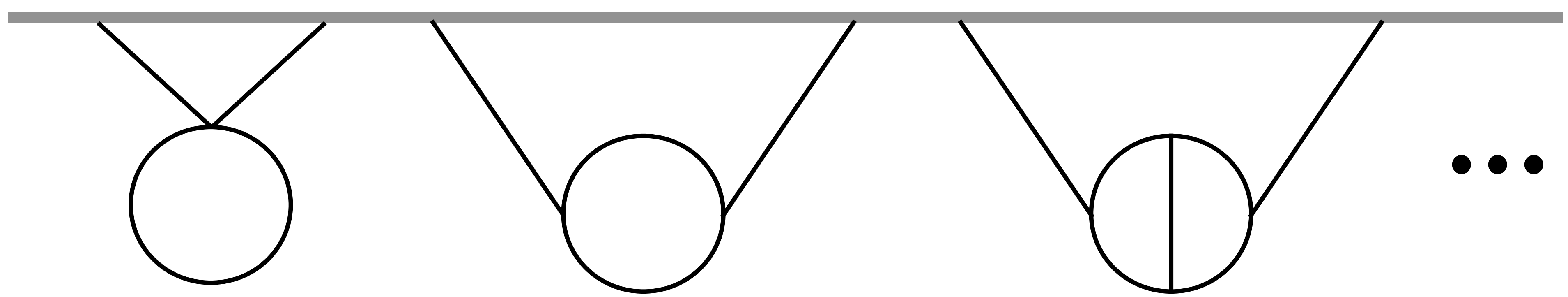
Are they really divergent? Do we need to worry about loops?

- **IR finite correlator**: multi-speed non-G



II. Loops

- Wavefunction: Classical v.s. Quantum
- Classical Loops and IR divergences



Cespedes, Davis, DGW
work in progress
comments are welcome!

The Wavefunction of the Universe

$$P = |\Psi|^2$$

Fokker-Planck Equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H}P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

- stochastic effects
- non-perturbative
- equilibrium behaviour

Cosmological Bootstrap

$$\Psi[\phi] = \exp\left[\sum_{n=2}^{\infty} \frac{1}{n!} \int_{\mathbf{k}_1,\ldots,\mathbf{k}_n} \psi_n(\mathbf{k}_1, \ldots, \mathbf{k}_n)\ \phi_{\mathbf{k}_1} \cdots \phi_{\mathbf{k}_n}\right]$$

- related to correlators
- perturbative
- secular growth

?

Gorbenko, Senatore 2019

The Wavefunction Method: perturbation theory

Classical Way:

- Saddle-point approximation

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

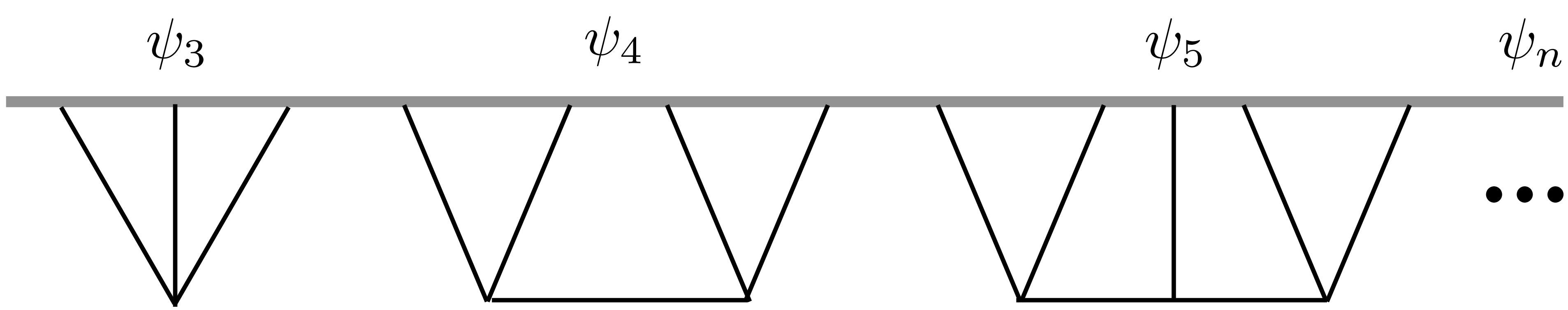
On-shell condition

bulk-to-boundary

$$(\Box - m^2)\Phi = -\frac{1}{\sqrt{-g}}\frac{\delta S_{\rm int}}{\delta \Phi} \quad \Rightarrow \quad \Phi_{\rm cl}(\eta, \mathbf{k}) = \phi_{\mathbf{k}} K(k,\eta) + \frac{i}{\hbar}\int {\rm d}\eta' G(k;\eta,\eta') \frac{\delta S_{\rm int}}{\delta \Phi_{\mathbf{k}}(\eta')}\Big|_{\Phi=\Phi_{\rm cl}}$$

bulk-to-bulk

All the tree-level wavefunction coefficients:



$\hbar^{-1}$

(semi-)classical wavefunction coefficients

The Wavefunction Method: perturbation theory

Quantum Way:

- Functional Quantization

$$\Psi[\phi(\mathbf{x})] = \int_{\substack{\Phi(t_0)=\phi \\ \Phi(-\infty)=0}} \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S[\Phi]\right)$$

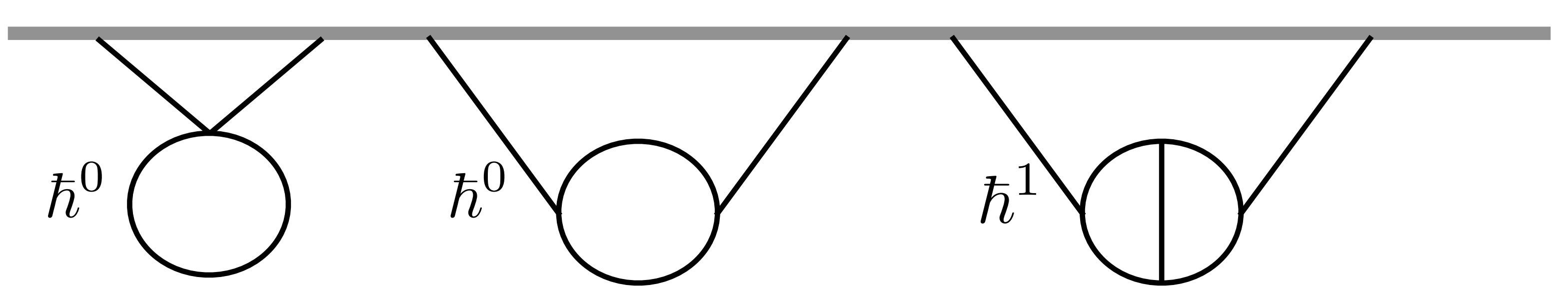
- Propagators

$$\langle \Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta')\rangle \equiv \frac{\int \mathcal{D}\Phi\Phi_{\mathbf{k}}(\eta)\Phi_{\mathbf{k}'}(\eta')\exp\left(\frac{i}{\hbar}S_0[\Phi]\right)}{\int \mathcal{D}\Phi \exp\left(\frac{i}{\hbar}S_0[\Phi]\right)} = G(k,\eta,\eta')(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

$$\langle \Pi_{\mathbf{k}}(\eta_0)\Phi_{\mathbf{k}'}(\eta)\rangle = 2i\hbar K(k,\eta)(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$$

Both tree-level & loop-level wavefunction coefficients:

$$\psi_n(\mathbf{k}_1,...,\mathbf{k}_n) \equiv \left.\frac{\delta^n\Psi[\phi]}{\delta\phi_{\mathbf{k}_1}...\delta\phi_{\mathbf{k}_n}}\right|_{\phi=0} = \frac{(i/2\hbar)^n}{\Psi[0]}\int \mathcal{D}\Phi\Pi_{\mathbf{k}_1}(\eta_0)...\Pi_{\mathbf{k}_n}(\eta_0)\exp\left(\frac{i}{\hbar}S[\Phi]\right)$$

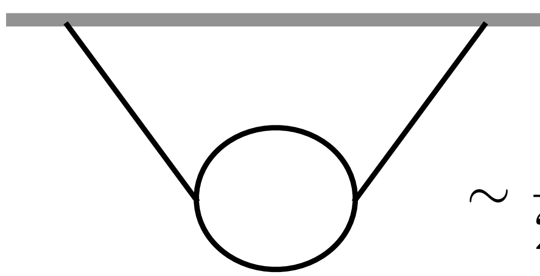


quantum wavefunction coefficients

II. Loops: Back to IR divergences

From Wavefunction to Correlators: One-Loop Example for $\frac{g}{3!}\Phi^3$

$\psi_2^{1-\mathrm{loop}}$



$\sim \frac{\quad}{2k^3}$

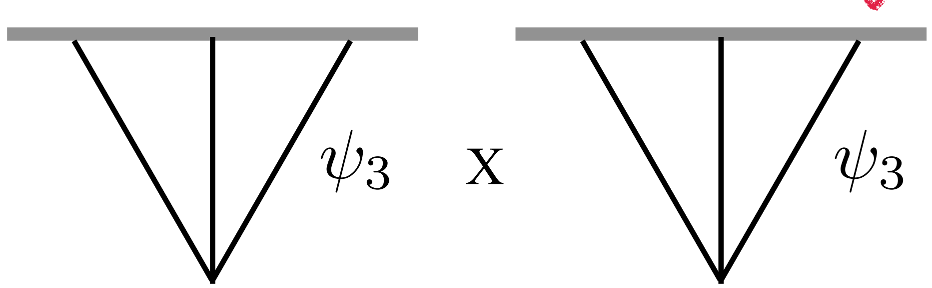
ψ_4

$$\langle \Sigma(\mathbf{x}_1)\cdots\Sigma(\mathbf{x}_n)\rangle = \frac{\int \mathcal{D}\Sigma\ \Sigma(\mathbf{x}_1)\cdots\Sigma(\mathbf{x}_n)\,|\Psi[\Sigma,\eta_0]|^2}{\int \mathcal{D}\Sigma\ |\Psi[\Sigma,\eta_0]|^2}$$

$$= \frac{g^2}{8k^3}\frac{-5}{18\pi^2}\log(kL)\log(-2k\eta_0)^2$$

$$\langle\phi_{\mathbf{k}_1}\phi_{\mathbf{k}_2}\rangle'_{1-\mathrm{loop}} = \frac{\mathrm{Re}\,\psi'^{1-\mathrm{loop}}_{\mathbf{k}_1\mathbf{k}_2}}{2\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)} - \frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\frac{\mathrm{Re}\,\psi'_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)}$$
$$+\frac{1}{8\mathrm{Re}\,\psi'_2(k_1)\mathrm{Re}\,\psi'_2(k_2)}\int_{\mathbf{p}}\Big[\frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{p},-\mathbf{p}-\mathbf{k}_1)\mathrm{Re}\,\psi'_3(\mathbf{k}_2,-\mathbf{p},\mathbf{p}+\mathbf{k}_1)}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{p}+\mathbf{k}_1|)}$$
$$+\frac{\mathrm{Re}\,\psi'_3(\mathbf{k}_1,\mathbf{k}_2,-\mathbf{k}_1-\mathbf{k}_2)\mathrm{Re}\,\psi'_3(\mathbf{k}_1+\mathbf{k}_2,\mathbf{p},-\mathbf{p})}{\mathrm{Re}\,\psi'_2(p)\mathrm{Re}\,\psi'_2(|\mathbf{k}_1+\mathbf{k}_2|)}\Big]$$

Classical Loops



$$= \frac{g^2}{8k^3}\frac{1}{3\pi^2}\log(kL)\log(-2k\eta_0)^2$$

II. Loops: Wavefunction is IR safe

Gorbenko, Senatore 2019

L-loop *n*-point wavefunction coefficient:

$$\psi_n^{L-\text{loop}} \sim \int d\eta_1 ... d\eta_m a(\eta_1)^4 ... a(\eta_m)^4 K(k_1, \eta_1) ... K(k_n, \eta_m)$$
$$\int_{\mathbf{p_1}, ..., \mathbf{p_L}} G(p_1, \eta_a, \eta_b) ... G(p_L, \eta_c, \eta_d) G(|\mathbf{p_x} + \mathbf{k_y}|, \eta_e, \eta_f) ...$$

Bulk-to-bulk propagator at IR:

$$\lim_{p \to 0} G(p, \eta, \eta') = -\frac{i}{6} H^2 (\eta^3 + \eta'^3) + \mathcal{O}(p)$$

Momentum integration from loops:

$$\int_{\mathbf{p}} \frac{1}{p^n} = \frac{1}{(2\pi)^3} \int_{1/L}^{\Lambda} 4\pi p^{2-n} dp \xrightarrow{L \to \infty} \begin{cases} \text{IR-finite} \ , & n < 3 \\ \frac{1}{2\pi^2} \log(kL) \ , & n = 3 \ . \end{cases}$$

<u>**only secular divergence from time integrals,**</u>

<u>**no IR divergences from loop integrals**</u>

II. Loops: leading IR logs in correlators

L-loop *n*-point correlator (with *V* vertices):

$$\langle \phi^n \rangle_{L-\mathrm{loop}} \sim \frac{\mathrm{Re}\ \psi_n^{L-\mathrm{loop}}}{(\mathrm{Re}\psi_2)^n} + \int_{\mathbf{p_1,\dots,p_a}} \frac{(\text{lower-order loop } \psi)}{\mathrm{Re}\ \psi_2 \dots \mathrm{Re}\ \psi_2} + \dots$$

$$+ \int_{\mathbf{p_1,\dots,p_b}} \frac{(\text{exchange } \psi)}{\mathrm{Re}\ \psi_2 \dots \mathrm{Re}\ \psi_2} + \dots + \int_{\mathbf{p_1,\dots,p_L}} \frac{(\text{contact Re } \psi_3)^V}{(\mathrm{Re}\psi_2)^{(3V+n}/2}$$

$$\propto \lambda^V \log(k L_{\mathrm{IR}})^L \log(-k\eta_0)^V$$

in agreement with
Baumgart, Sundrum 2019

IR-divergent correlators are always dominated by Classical loops

IR divergence ⟷ **Superhorizon physics** ⟷ **Semi-classical effect**

III. Back To Trees

- What does the stochastic formalism resum?

A tempting thought

saddle-point approx.

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar} S[\Phi_{\rm cl}]\right)$$

$\Longleftrightarrow$

Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H} P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

First, screen out the short wavelength modes:

$$P[\phi_l] = \int \mathcal{D}\phi \;\; \delta\left(\phi_l - \int \frac{d^3k}{(2\pi)^3} \Omega_k \phi_k\right) |\Psi(\phi_k)|^2$$

Then, let's check the perturbative regime of $\lambda\phi^4$, where the equilibrium has not been reached.

$$\text{coupling} \times \log^2 < 1$$

- controllable playground to explicitly match two computations
- could be interesting for pheno

Correlators from stochastic formalism

$$\langle \phi^n \rangle = \int d\phi \phi^n P(\phi) \qquad \frac{d}{dt}\langle \phi^n \rangle = \int d\phi \phi^n \frac{P(\phi,t)}{dt} = \frac{n}{3H}\langle \phi^{n-1} V_\phi \rangle + \frac{n(n-1)H^3}{8\pi^2}\langle \phi^{n-2} \rangle$$

a set of diff. eqs.
that can be solved
perturbatively

$$\begin{cases} \frac{d}{dt}\langle \phi^2 \rangle = \frac{1\lambda}{9H}\langle \phi^4 \rangle + \frac{H^3}{4\pi^2} \\ \frac{d}{dt}\langle \phi^4 \rangle = \frac{2\lambda}{9H}\langle \phi^6 \rangle + \frac{3H^3}{4\pi^2}\langle \phi^2 \rangle \\ \vdots \\ \frac{d}{dt}\langle \phi^n \rangle = \frac{n\lambda}{18H}\langle \phi^{n+2} \rangle + \frac{n(n-1)H^3}{8\pi^2}\langle \phi^{n-2} \rangle \end{cases}$$

$$\langle \phi^2 \rangle = \frac{H^2}{4\pi^2}\log a + \frac{\lambda H^4}{144\pi^4}(\log a)^3 + \frac{\lambda^2 H^6}{2880\pi^6}(\log a)^5 + \mathcal{O}(\lambda^3 (\log a)^7)$$

Free theory — one-loop — two-loop — ......

Gaussian variance

classical loops from tree-level wavefunction coefficients

quantum loops are absent

Beyond perturbation theory?

saddle-point approx.

$$\Psi[\phi] \simeq \exp\left(\frac{i}{\hbar}S[\Phi_{\rm cl}]\right)$$

Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V_\phi}{3H}P\right) + \frac{H^3}{8\pi^2}\left(\frac{\partial^2 P}{\partial \phi^2}\right)$$

Now we believe we can do it! Stay tuned.

Summary

I. Trees

- Cosmological Bootstrap with anomalous Ward identities
- a classification of multi-field non-Gaussianities: local & multi-speed shapes

II. Loops

- Wavefunction: Classical v.s. Quantum
- Classical Loops and IR divergences

III. Back to Trees

- Saddle-point approx. <=> Fokker-Planck Equation