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Contributed Talks 5: Daniel Boutros, Daniel Goodair & Victor Navarro Fernandez

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Daniel Boutros. We consider the analogue of Onsager's conjecture for the hydrodynamic helicity, which is a topological conserved quantity of the incompressible Euler equations. We establish a local helicity balance with a defect term capturing the local helicity flux at small scales. We find a sufficient criterion for helicity conservation which is more general than the previous criteria in the literature. Using these results we can establish a Kolmogorov-type scaling law for the helicity. We also make some remarks on the non-barotropic compressible Euler equations, where the helicity is no longer conserved but bounds on the evolution of a related topological quantity can still be deduced. We also prove new results for the conservation of magnetic helicity for the ideal MHD equations. We observe that the conservation laws of the MHD system only hold under the assumption that the magnetic field remains divergence-free, which is not guaranteed at the level of weak solutions. We therefore show that this property is preserved for Leray-Hopf weak solutions of the viscous MHD equations and their vanishing viscosity limits. Finally, if time allows we will show new results on convex integration for a wide class of geophysical models. These results are joint works with John D. Gibbon, Simon Markfelder and Edriss S. Titi.

Daniel Goodair. Fluids under Transport Noise and Boundary Conditions: Energy Estimates, Well- Posedness and Inviscid Limits. The seemingly random nature of turbulence has long suggested the use of noise in fluid equations, inviting the study from numerous perspectives of physically motivated forms of noise in such models. Backed by variational principles, model reduction and regularisation phenomena, there is a strong trend towards the use of transport noise. Introducing a stochastic term dependent on the gradient of the solution necessitates precise cancellation in the energy estimates at the core of our solution theory. I will explore the muddied waters of boundary conditions and their interplay with transport noise and energy estimates, discussing what does or does not play out like the torus, with implications for well-posedness and inviscid limit results concerning the Stochastic Navier-Stokes and Euler Equations.

Victor Navarro Fernandez. Exponential mixing and enhanced dissipation with random cellular flows via hypocoercivity. In this work we study the evolution of a passive scalar advected by a cellular flow on a two-dimensional periodic box, where the center of the flow undergoes a random walk. We prove exponential decay of correlations for mean-free H^s functions, uniformly in diffusivity, leading to almost sure exponential mixing and optimal enhanced dissipation rates. Despite the stochastic nature of the flow, our approach is entirely analytical. We introduce a hypoelliptic PDE for the expectation of the two-point process, and establish its hypocoercivity in a weighted H^1 norm that degenerates at the diagonal. Hypocoercivity of the two-point PDE can be understood as an L^2-analogue of geometric ergodicity of the two-point chain in the random dynamical systems framework. This is a joint work with Christian Seis, Univeristat Munster.