

# **Turbulence on the Banks of the Arno**

**Wednesday, 31 January 2024 - Friday, 2 February 2024**

**Palazzo del Castelletto**

## **Scientific Programme**

**Michele Coti Zelati***Deterministic and stochastic dynamics in incompressible fluid mixing*

**Abstract:** We describe the concepts of mixing and enhanced/anomalous dissipation in incompressible fluids. Fluid mixing is perhaps the most fundamental mechanism for stability and instability in incompressible fluids: it serves as a cornerstone for understanding both stability and instability in incompressible fluids, contributing to the emergence of fine-scale structures. Throughout this course, we will explore various facets of fluid mixing, including the Lagrangian perspective, which involves the folding and twisting of the flow-map generated by the velocity field, as well as the Eulerian viewpoint, which unveils the convergence of macroscopic quantities such as velocity and vorticity towards asymptotic states.

**Jérémie Bec***Anomalies and Spontaneous Symmetry Breaking in Infinite-Reynolds-Number Turbulence*

**Abstract:** The dynamics of viscous, incompressible fluid flows are governed by the Navier–Stokes equations. As the injection of kinetic energy surpasses viscous damping—occurring at sufficiently large Reynolds numbers ( $Re = UL/\nu$ , where  $U$  and  $L$  are typical velocity and length scales, and  $\nu$  is the fluid kinematic viscosity)—these flows transition into unsteady, turbulent states, with significant implications for various practical applications. In the limit as  $Re$  approaches infinity, three distinct anomalies characterize the behavior: the dissipative anomaly, signifying the persistence of finite dissipation of kinetic energy even in the absence of viscosity; multifractal scaling, manifesting as systematic deviations from Kolmogorov self-similarity; and super-diffusive separation of fluid particles, occurring both backward and forward in time.

These anomalies are respectively associated with the spontaneous breaking of three symmetries: time reversal, scale invariance, and the uniqueness of the Lagrangian flow leading to what is commonly referred to as spontaneous stochasticity. While the intricate interplay between time irreversibility, intermittency, and spontaneous stochasticity has been partially elucidated in specific turbulent models such as the Burgers equation, advection by random flows from the Kraichnan ensemble, and surface quasi-geostrophic dynamics, it remains a challenge within the context of Navier–Stokes hydrodynamical turbulence.

The aim of these lectures is to provide an overview of these different concepts.