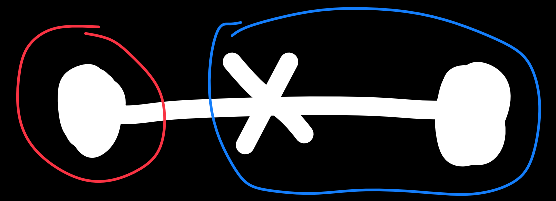
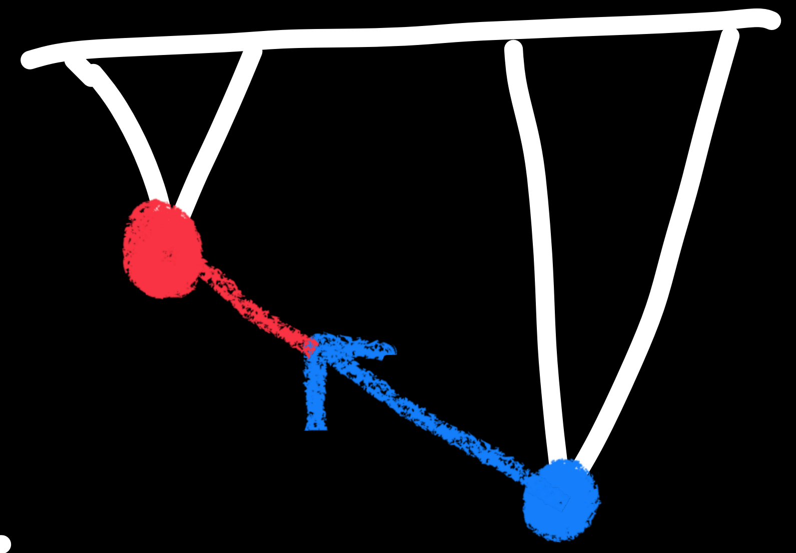
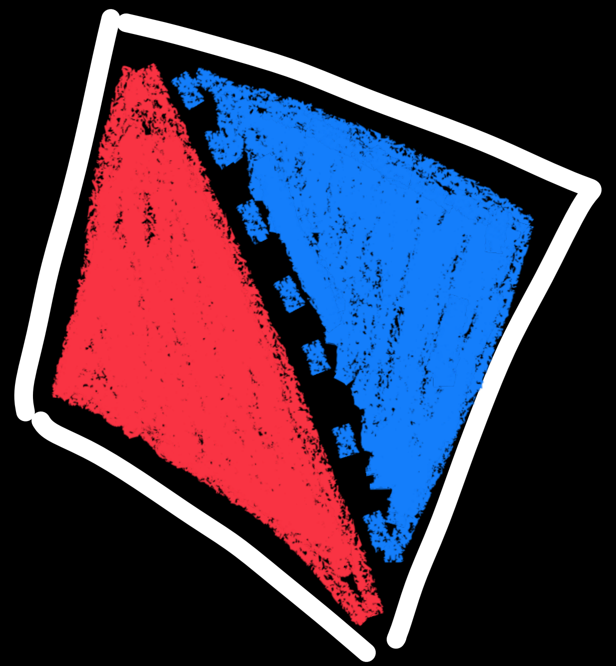
Differential Equations for Cosmological Correlators







Guilherme L. Pimentel

Scuola Normale Superiore & INFN Pisa

WITH

Nima Arkani-Hamed, Daniel Baumann,
Hayden Lee, Aaron Hillman, Austin Joyce.

"Kinematic Flow and the Emergence of Time"

"Differential Equations for Cosmological Correlators"

<u>LOTS</u> OF PEOPLE TO ACKNOWLEDGE

(see the papers)

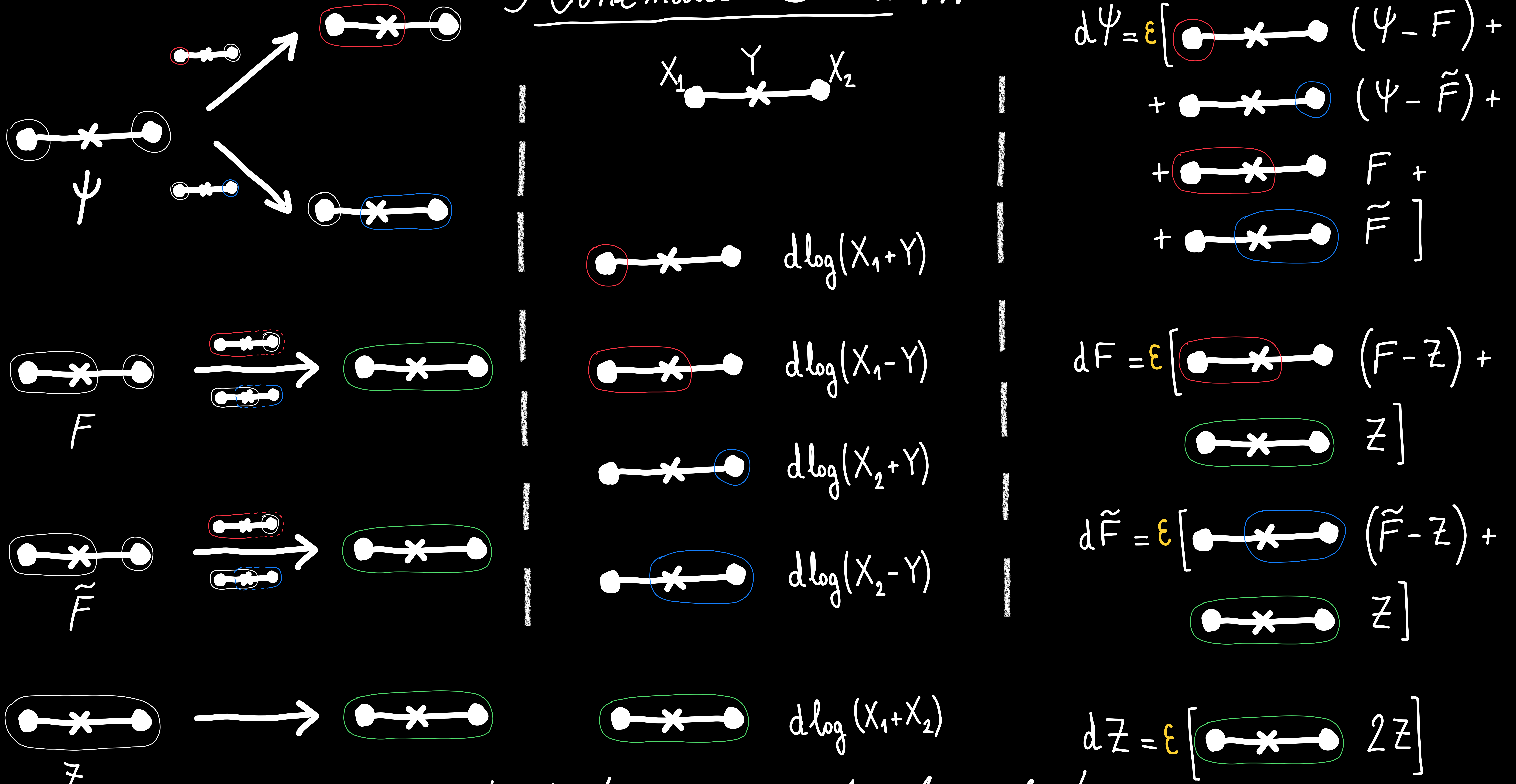
\+ <u>LOTS</u> OF PREVIOUS IMPORTANT WORK

(see the references
in the papers)

The Main Result

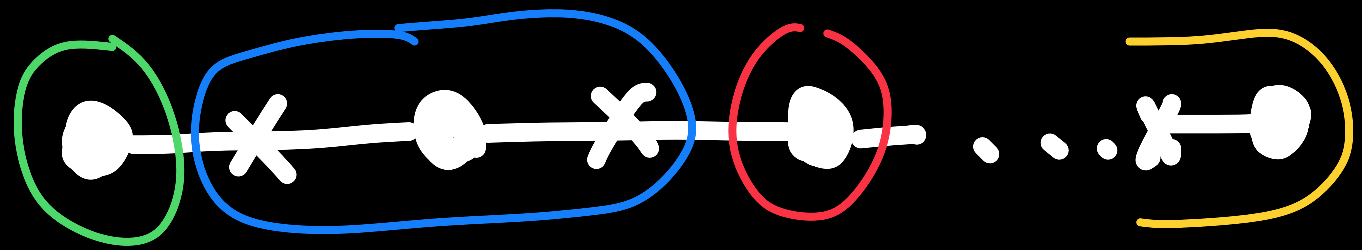
(of short paper)

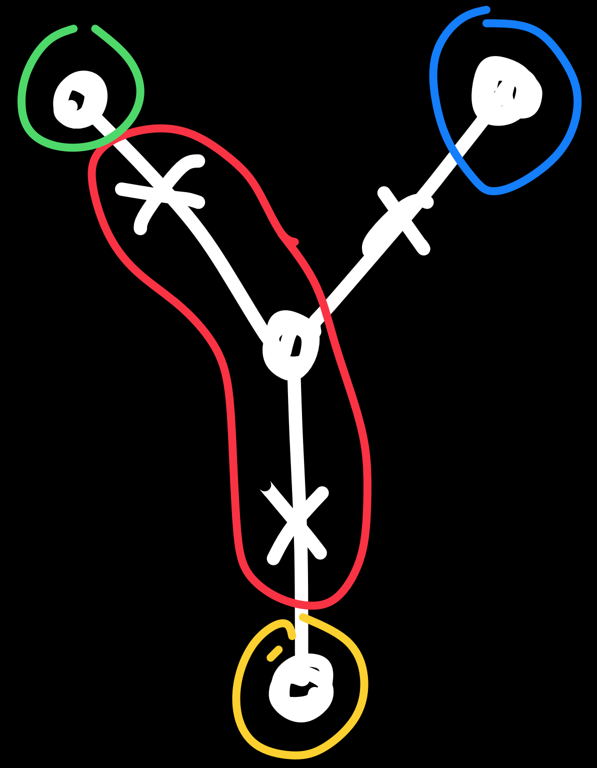
Kinematic Flow...

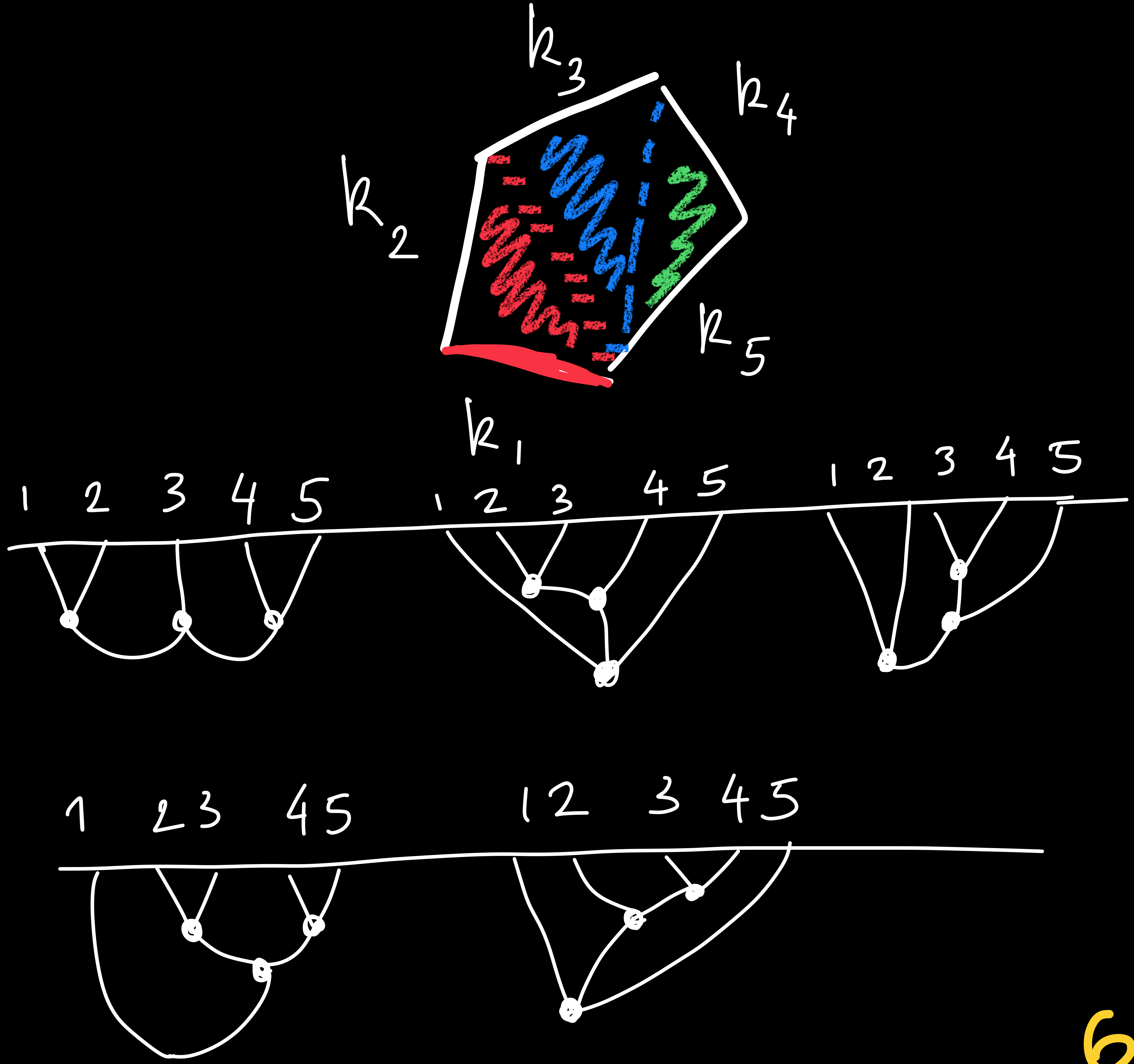


... describes time evolution in power-law cosmology!

Kinematic Flow...



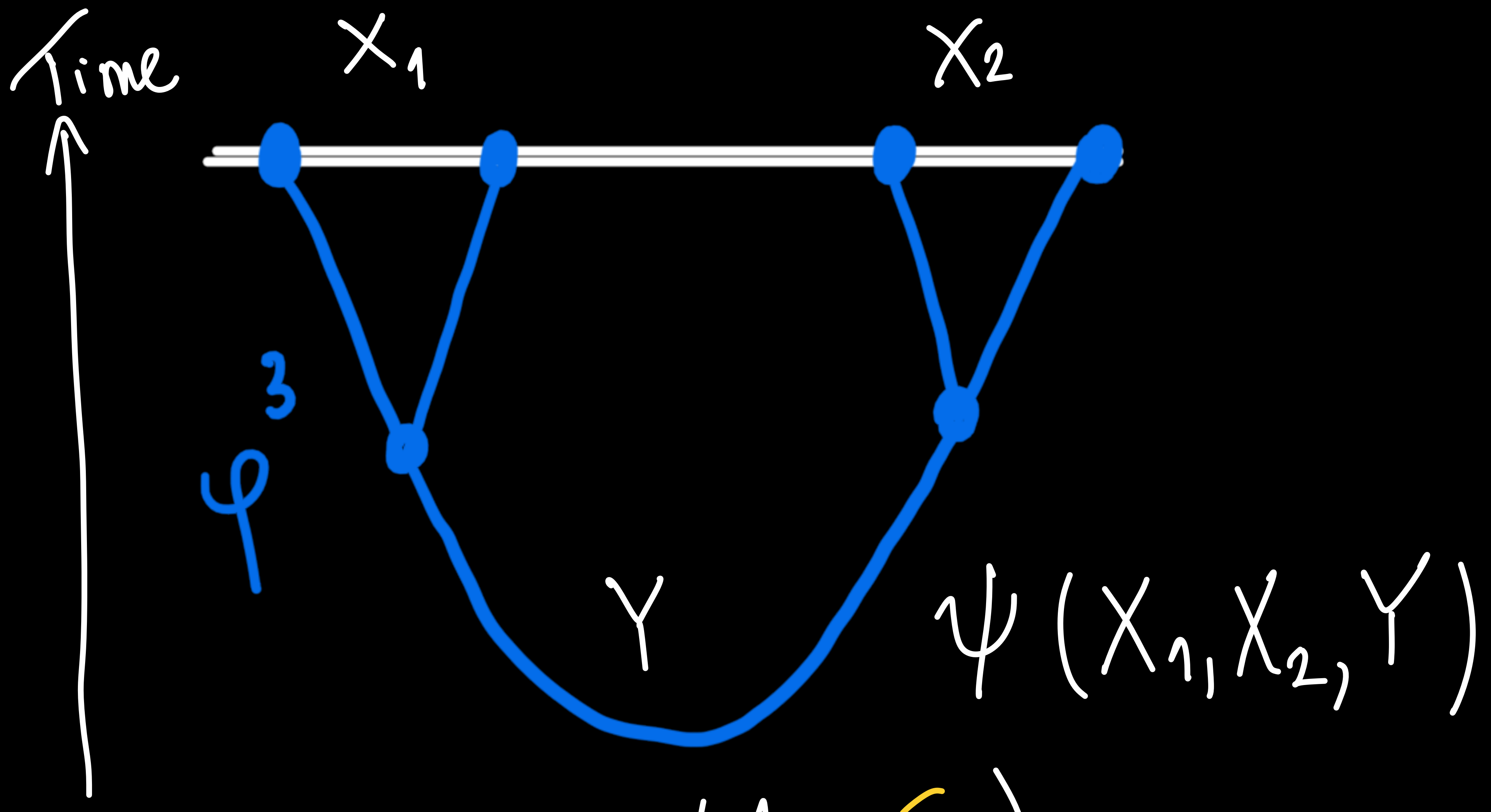




4^{n_v-1} functions

The Main Result

(of long paper)



$$a(\eta) = \eta^{-(1+\varepsilon)}$$

Gravity

Time integrals

Differential Equation Method

Loop Integrals

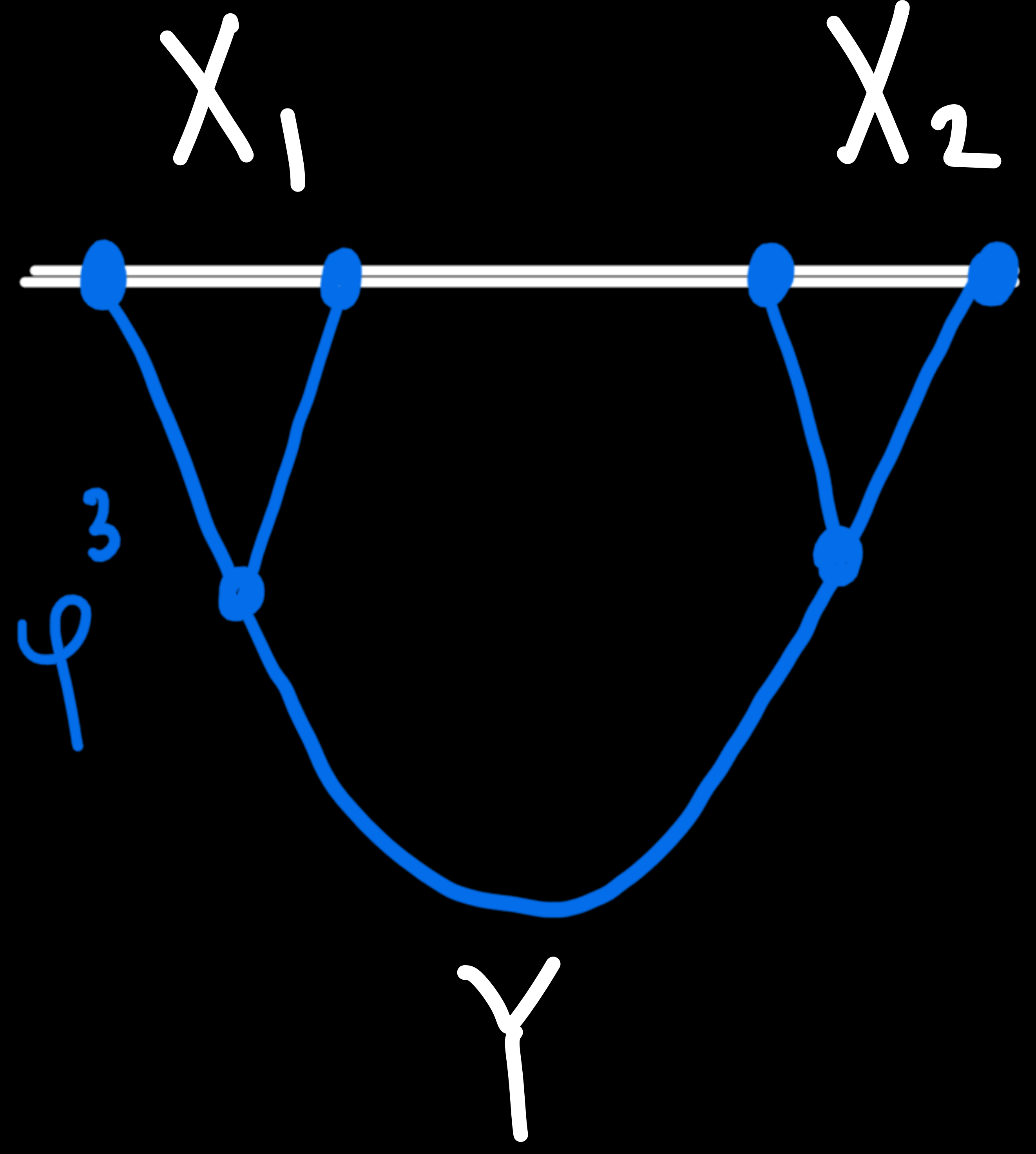
Twisted Cohomology

Master Integrals

Canonical forms

QCD

$$d\mathcal{I} = \varepsilon A \mathcal{I}$$



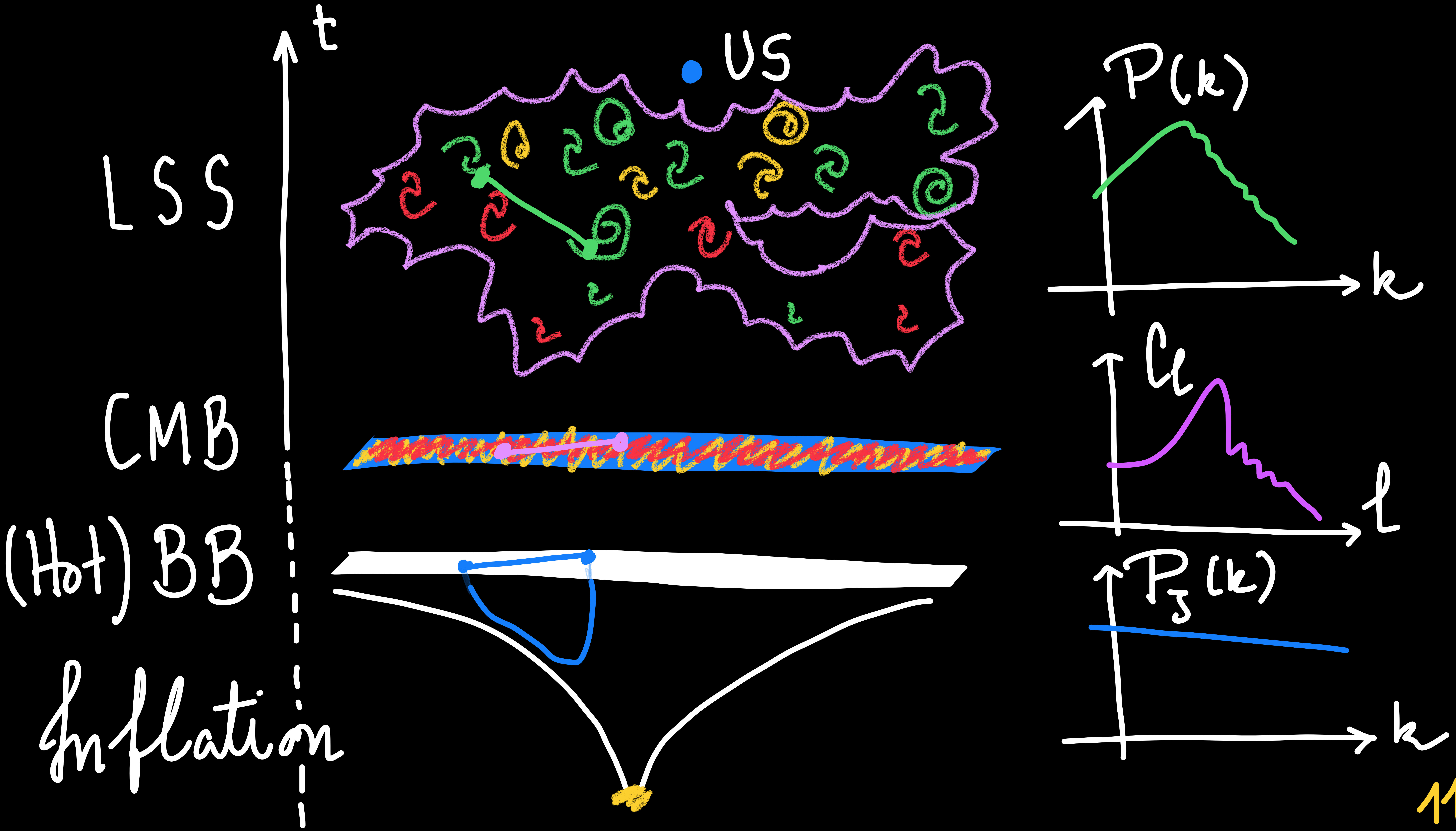
$$= c_1 (Y+X_1)^{\varepsilon}(Y+X_2)^{\varepsilon} + c_2 \, (X_1+X_2)^{2\varepsilon}$$

$$+ c_3 \; {}_2F_1\left[\begin{matrix} 1,\ \varepsilon \\ 1-\varepsilon \end{matrix} \middle| \frac{Y-X_2}{Y+X_1}\right](X_1+X_2)^{2\varepsilon}$$

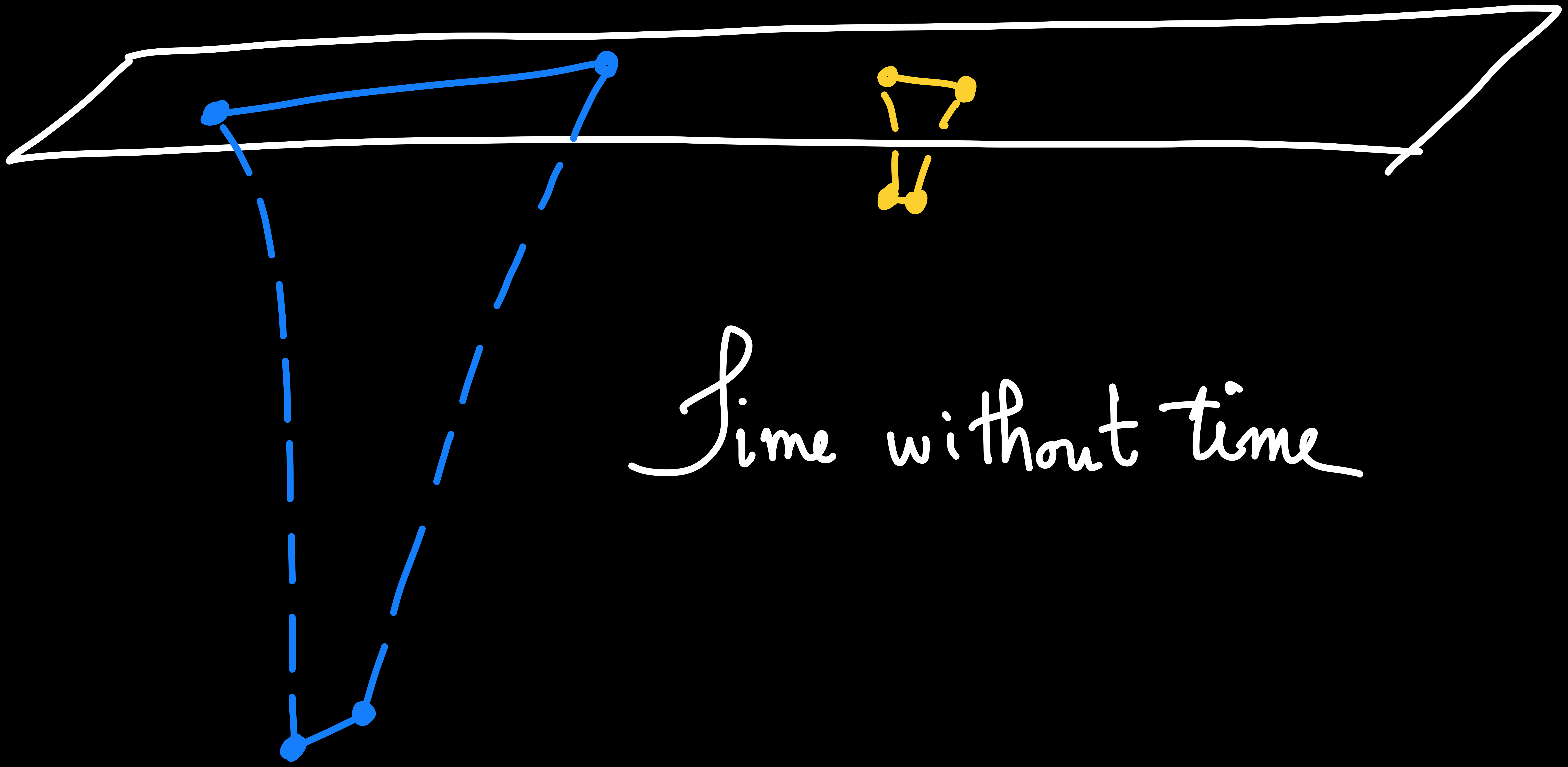
$$+ c_4 \; {}_2F_1\left[\begin{matrix} 1,\ \varepsilon \\ 1-\varepsilon \end{matrix} \middle| \frac{Y-X_1}{Y+X_2}\right](X_1+X_2)^{2\varepsilon}$$

$c_i \begin{cases} \underline{\text{No}} \text{ folded singularity} \\ \text{factorization} \end{cases}$

Motivation

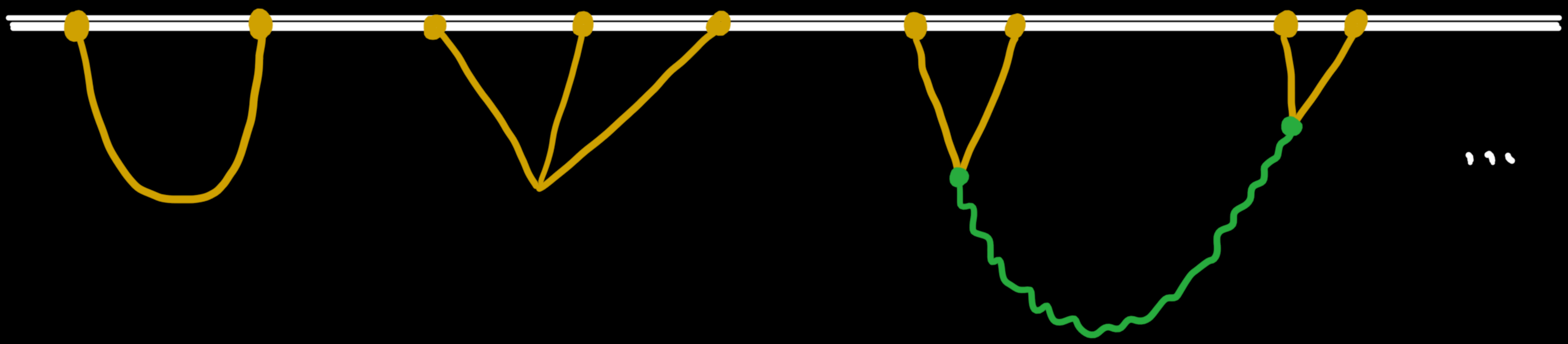

t
US
LSS
CMB
(Hot) BB
Inflation
$P(k)$
k
C_ℓ
ℓ
$P_\zeta(k)$
k

Time Evolution ⇒ Boundary Differential Equation



Time without time

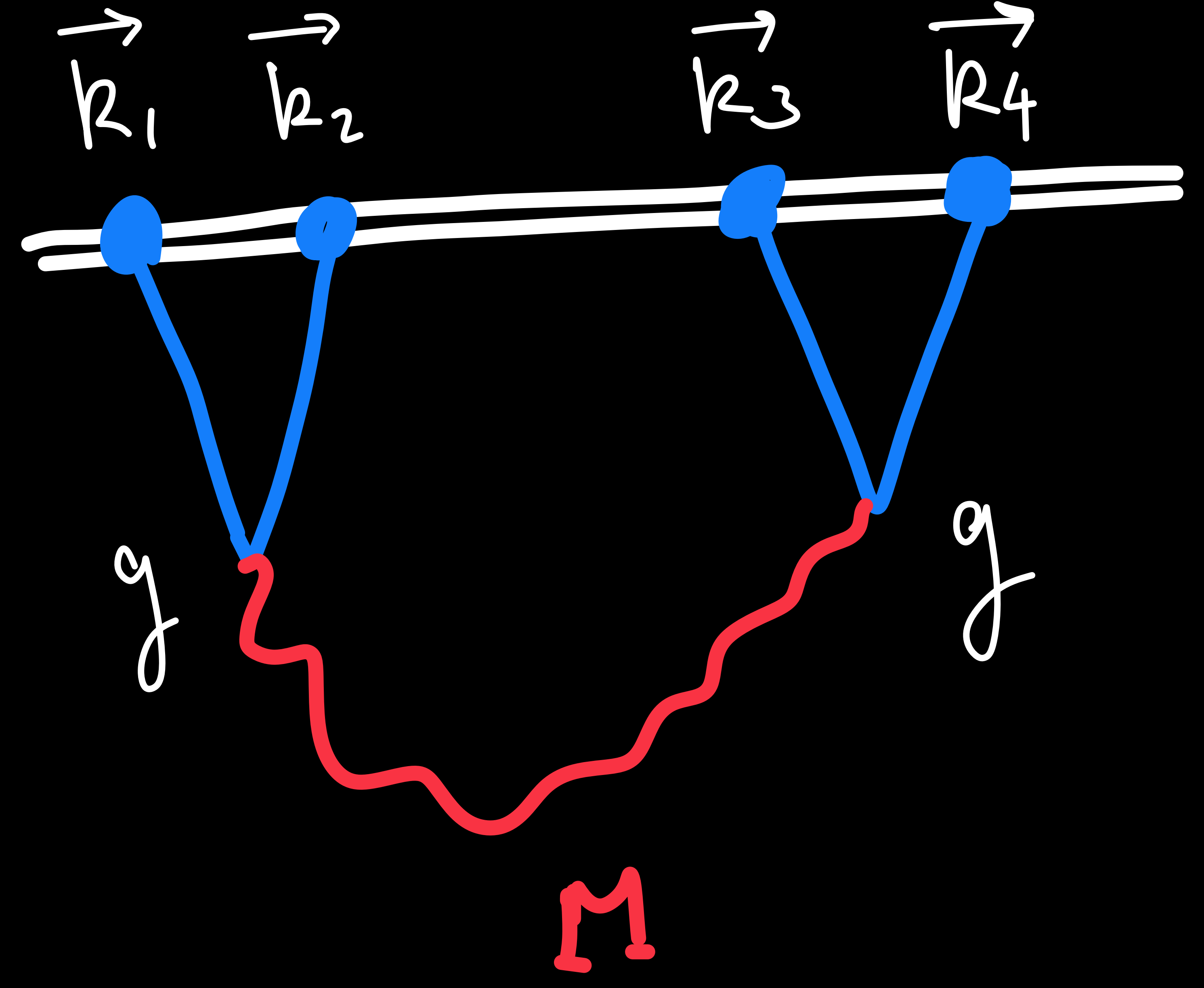
Cosmological Bootstrap



In dS_4 (inflation), with & without boost symmetry

Locality, Unitarity, perturbative & non-perturbative

...

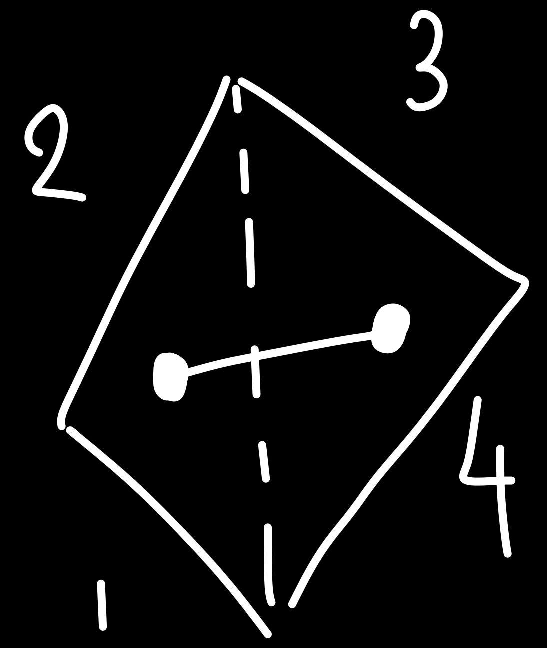


$$\psi(X_1, X_2, Y)$$

$$\left[\Delta_{X_1} + \left(\frac{M}{H}\right)^2 - 2\right]\psi = \frac{g^2}{X_1 + X_2}$$

$$\left[\Delta_{X_2} + \left(\frac{M}{H}\right)^2 - 2\right]\psi = \frac{g^2}{X_1 + X_2}$$

Hypergeometric (2^{nd} order) op.



$$X_1 \equiv k_1 + k_2$$

$$X_2 \equiv k_3 + k_4$$

$$Y \equiv |\vec{k}_1 + \vec{k}_2|$$

<u>Singular</u> at $\begin{bmatrix} X_1 + Y = 0 \\ X_2 + Y = 0 \\ X_1 + X_2 = 0 \end{bmatrix}$. <u>Shouldn't</u> be at $\begin{bmatrix} X_1 - Y = 0 \\ X_2 - Y = 0 \end{bmatrix}$

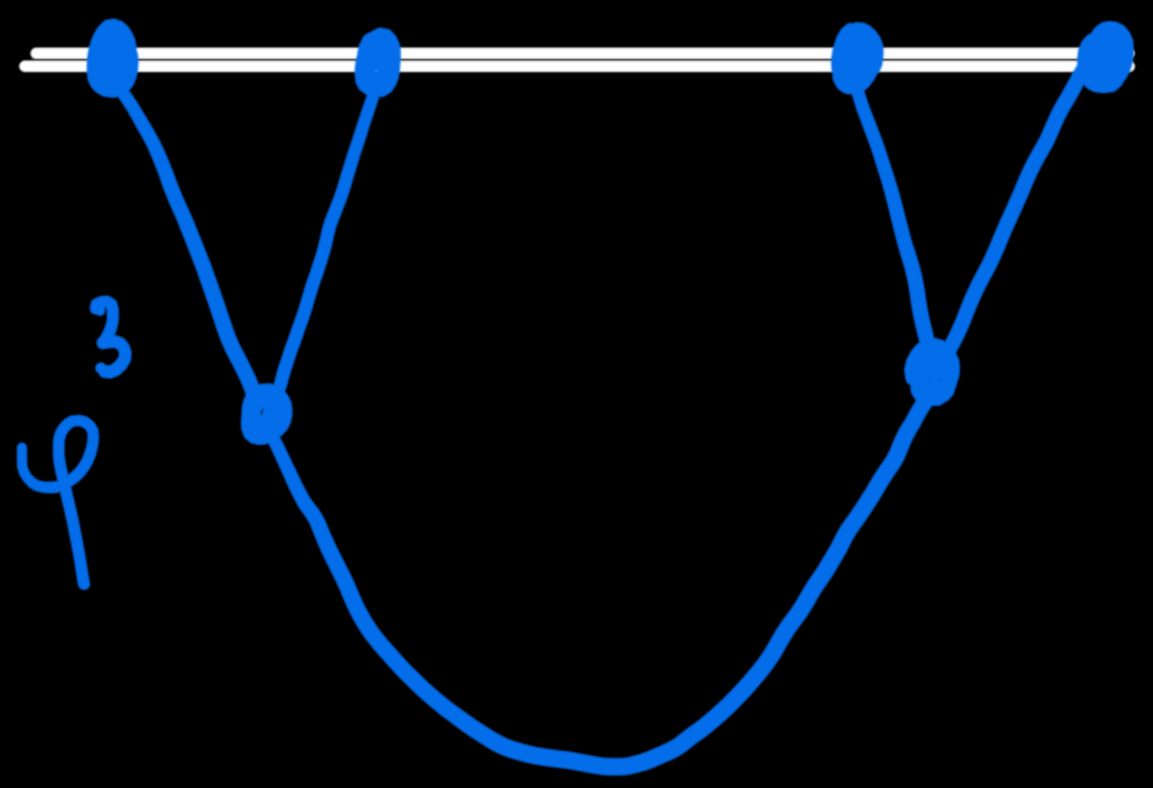
Where do the differential equations come from?

Can we derive them (+ solve them) systematically?

Model

Prototype: Conformal Scalars (φ) In Power Law (ε) Cosmologies

φ^3



$$\Psi(X_1, X_2, Y) = \int^{\infty} \frac{(x_1 x_2)^{\varepsilon}\, dx_1\, dx_2}{(x_1 + X_1 + Y)(x_2 + X_2 + Y)(x_1 + x_2 + X_1 + X_2)}$$

$$ds^2 = \eta^{-2\varepsilon}\left[\frac{-d\eta^2 + d\vec{x}^2}{\eta^2}\right] \begin{cases} \varepsilon = 0 & \text{dS } 0 \\ \varepsilon = -1 & \text{Flat} \\ \varepsilon = -2 & \text{Radiation} \\ \varepsilon = -3 & \text{Matter} \end{cases}$$

How to find D.E. for Ψ?

Key Point

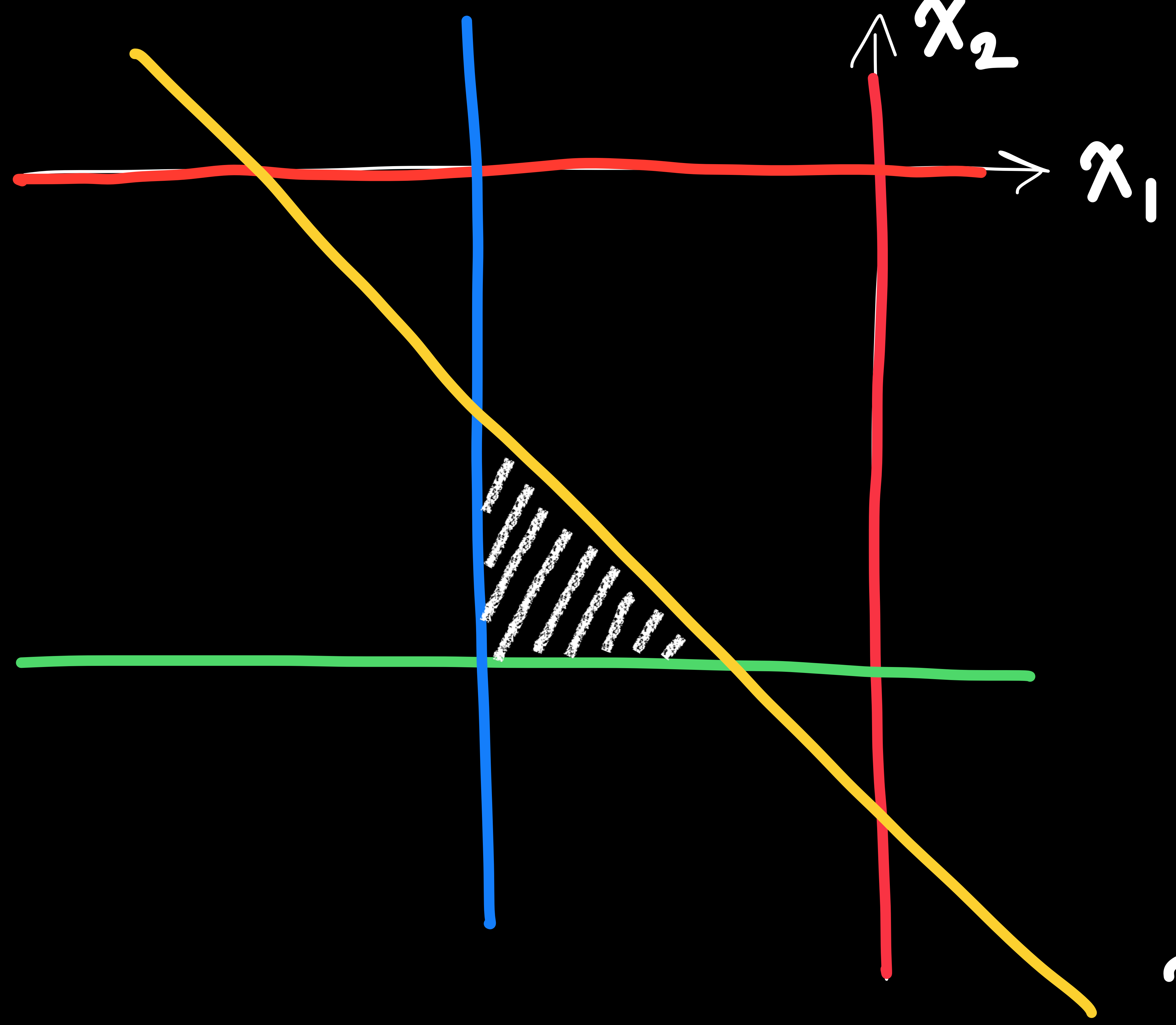
"twist"

$$F = \int (x_1 x_2)^{\varepsilon}\, \Omega$$

Canonical Form for bounded region in (x_1, x_2) plane (Cosmological Polytope)

Strategy: Exploit (twisted) cohomology of integrands

Consider $$F_{n_1,\dots,n_5} = c_{n_1\dots n_5}\int_0^\infty \frac{dx_1\,dx_2\,(x_1x_2)^{\varepsilon}}{T_1^{n_1}\,T_2^{n_2}\,L_1^{n_3}\,L_2^{n_4}\,L_3^{n_5}}$$



$T_1 : x_1$

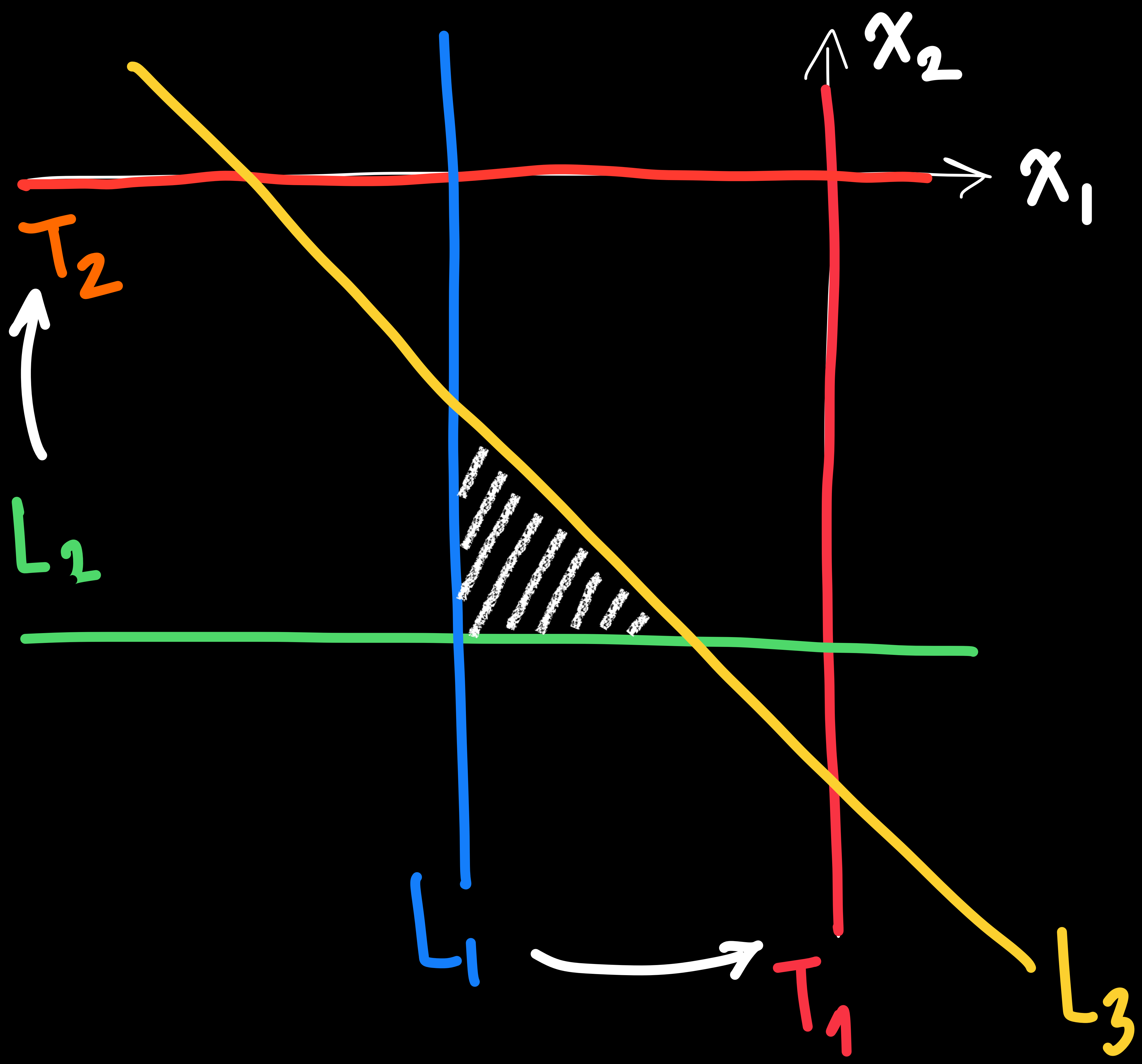
$T_2 : x_2$

$L_1 : x_1 + X_1 + Y$

$L_2 : x_2 + X_2 + Y$

$L_3 : x_1 + x_2 + X_1 + X_2$

Basis size: # of bounded regions



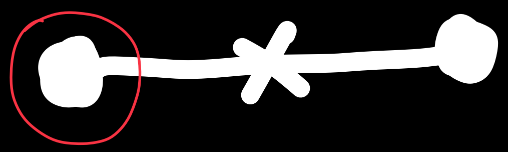
$$
\begin{tikzcd}
 & F_{T_1L_2L_3} \arrow[dr, "L_2 \to T_2"] & \\
\psi_{L_1L_2L_3} \arrow[ur, "L_1 \to T_1"] \arrow[dr, "L_2 \to T_2"'] & & Z_{T_1T_2L_3} \\
 & \tilde{F}_{L_1T_2L_3} \arrow[ur, "L_1 \to T_1"'] &
\end{tikzcd}
$$

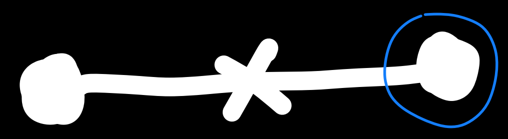
$$
d\begin{bmatrix} \psi \\ F \\ \tilde{F} \\ Z \end{bmatrix} = \varepsilon A \begin{bmatrix} \psi \\ F \\ \tilde{F} \\ Z \end{bmatrix}
$$

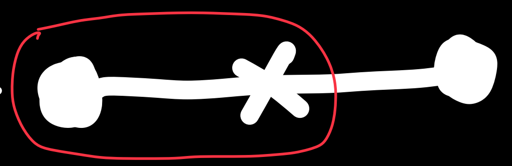
$$
\begin{tikzcd}
 & F_{T_1L_2L_3} \arrow[dr, "L_2 \to T_2"] & \\
\Psi_{L_1L_2L_3} \arrow[ur, "L_1 \to T_1"] \arrow[dr, "L_2 \to T_2"'] & & Z_{T_1T_2L_3} \\
 & \tilde{F}_{L_1T_2L_3} \arrow[ur, "L_1 \to T_1"'] &
\end{tikzcd}
$$

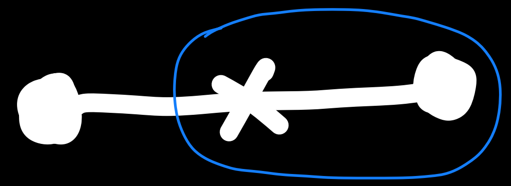
$$
d\begin{bmatrix} \Psi \\ F \\ \tilde{F} \\ Z \end{bmatrix} = \varepsilon A \begin{bmatrix} \Psi \\ F \\ \tilde{F} \\ Z \end{bmatrix}
$$

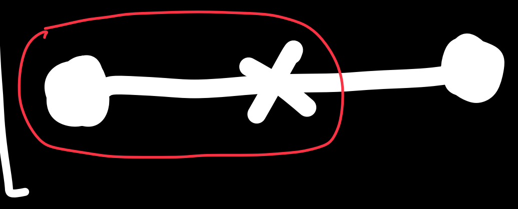
A : follows Kinematic flow!

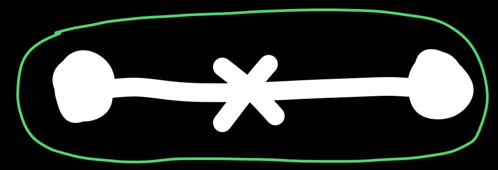
$d\Psi = \varepsilon\Big[$  $(\Psi - F) +$

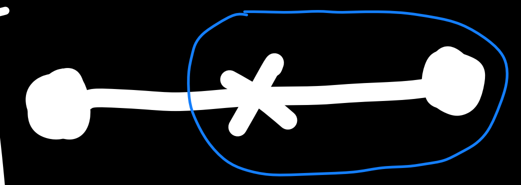
$+$  $(\Psi - \tilde{F}) +$

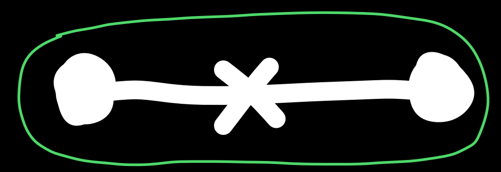
$+$  $F +$

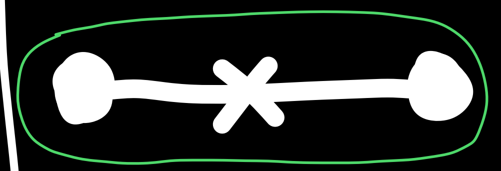
$+$  $\tilde{F} \Big]$

$dF = \varepsilon\Big[$  $(F - Z) +$

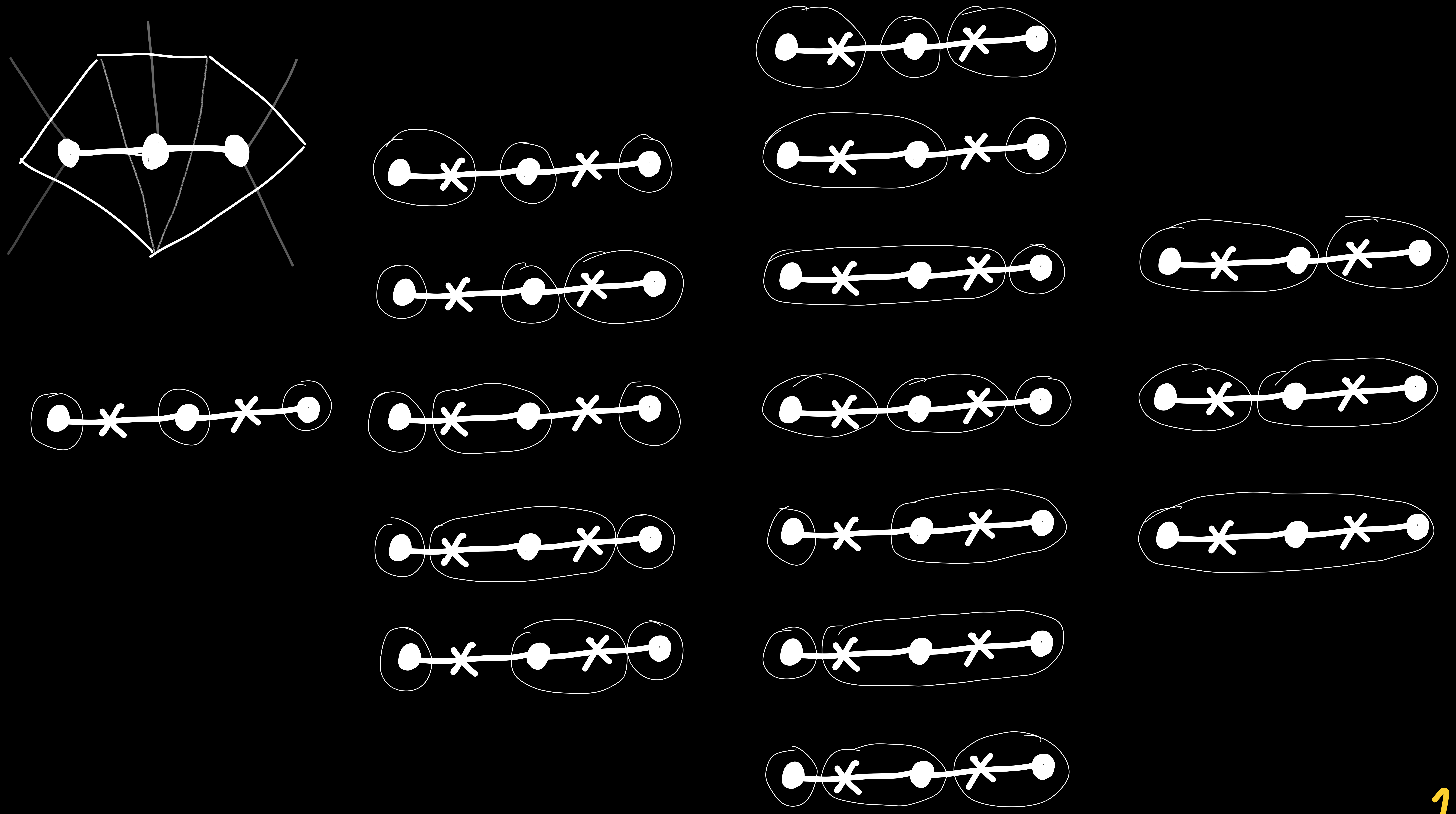
 $Z \Big]$

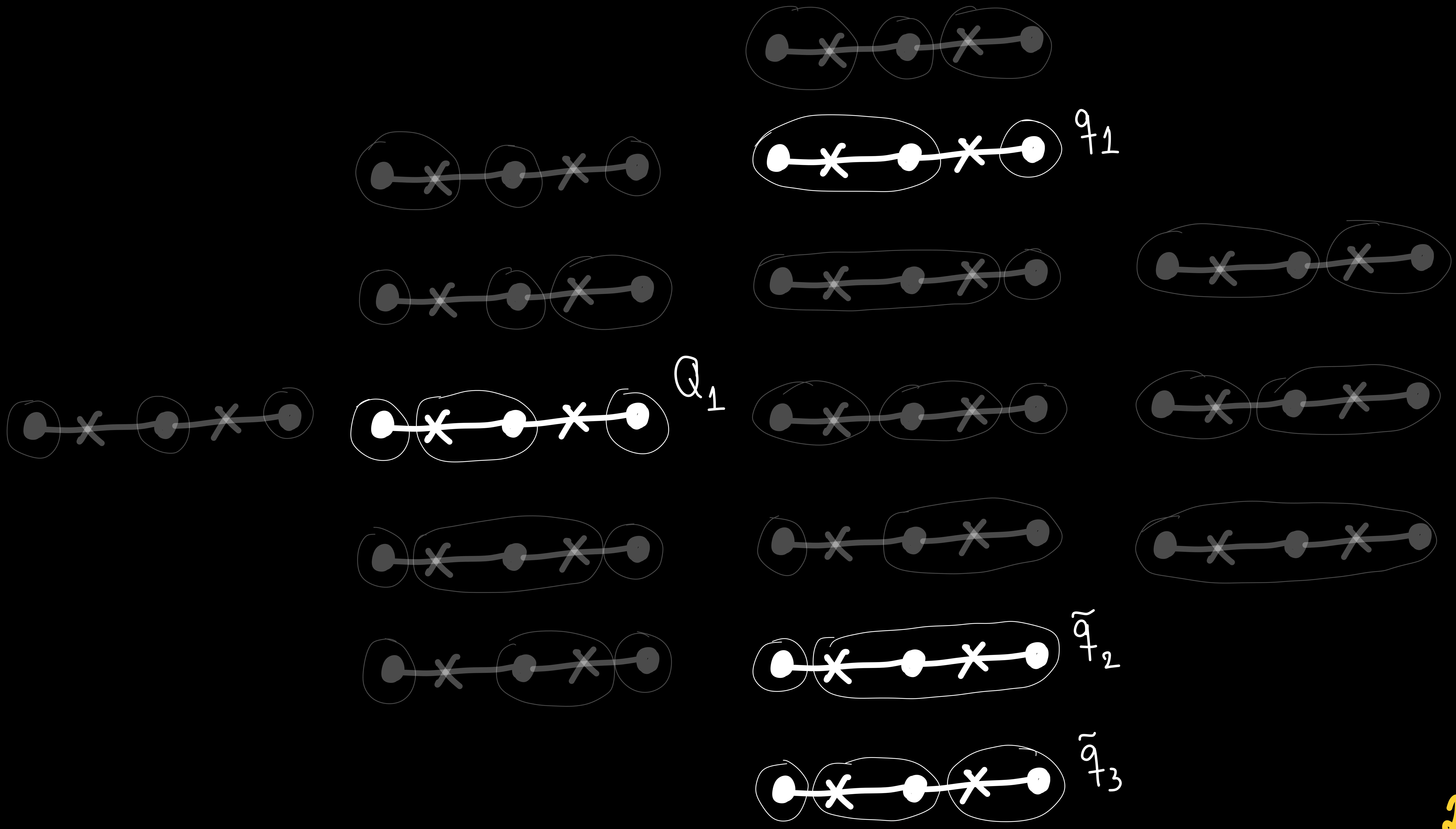
$d\tilde{F} = \varepsilon\Big[$  $(\tilde{F} - Z) +$

 $Z \Big]$

$dZ = \varepsilon\Big[$  $2Z \Big]$

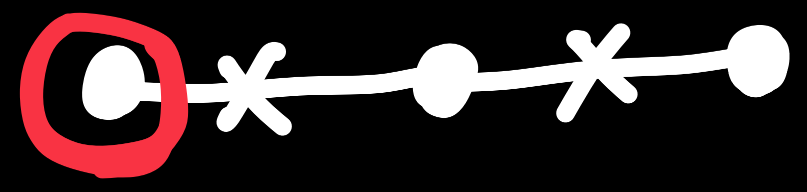
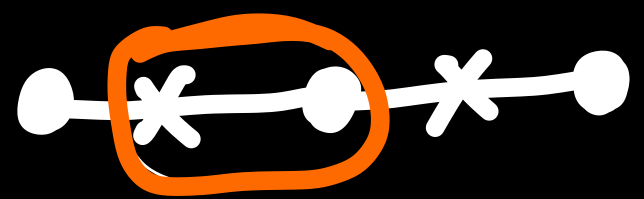
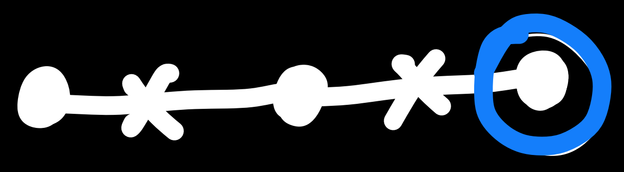
The Rules of the Flow . . .

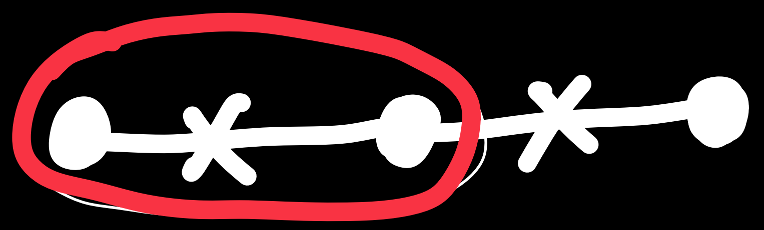
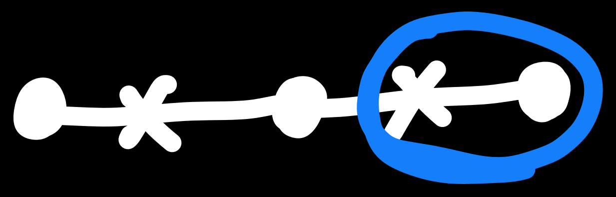


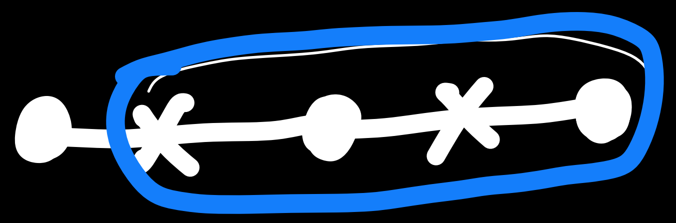


q_1
Q_1
$\tilde{q}_2$
$\tilde{q}_3$


growth
merge
q_1
Q_1
growth
Absorption
$\tilde{q}_3$
$-\tilde{q}_2$
Activation

$$d Q_1 = \varepsilon \Big[(Q_1 - q_1)$$  $$+\, Q_1$$  $$+\, (Q_1 - \tilde{q}_3)$$ 

$$+\, q_1$$  $$+\, (\tilde{q}_3 + \tilde{q}_2)$$ 

$$-\, \tilde{q}_2$$  $$\Big]$$

Works for sums of graphs!

$g \, \phi^a{}_b \, \phi^b{}_c \, \phi^c{}_a$

# pts	$\mathbb{A}$ naive	$\mathbb{A}$ real
4	8	7
5	80	56
6	896	502

Is there some structure that contains A in it???

F.A.Q.

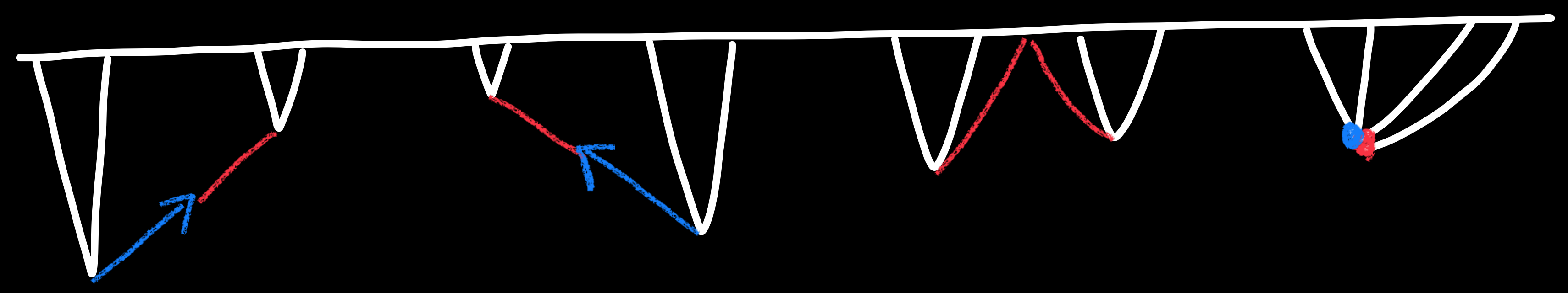
& OUTLOOK

Q: Boundary Conditions?

A: No folded singularity + Single factorization limit

Q: What's the physical interpretation of $\mathcal{I}$?

A: Flow: Bubblings → ? Bulk: Breaking propagator in 4 pieces



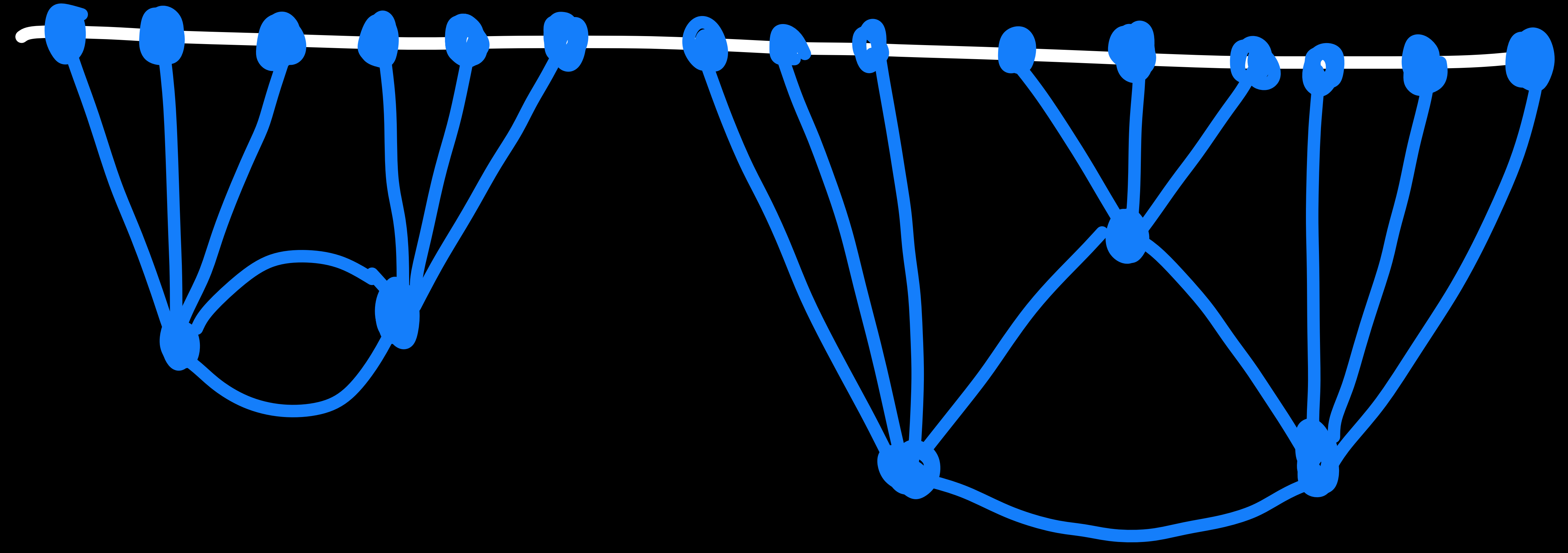
Q: Can you solve them?

A: To some extent. Beyond my wildest dreams for sure.

Q: What about (mass, spin, loops)?

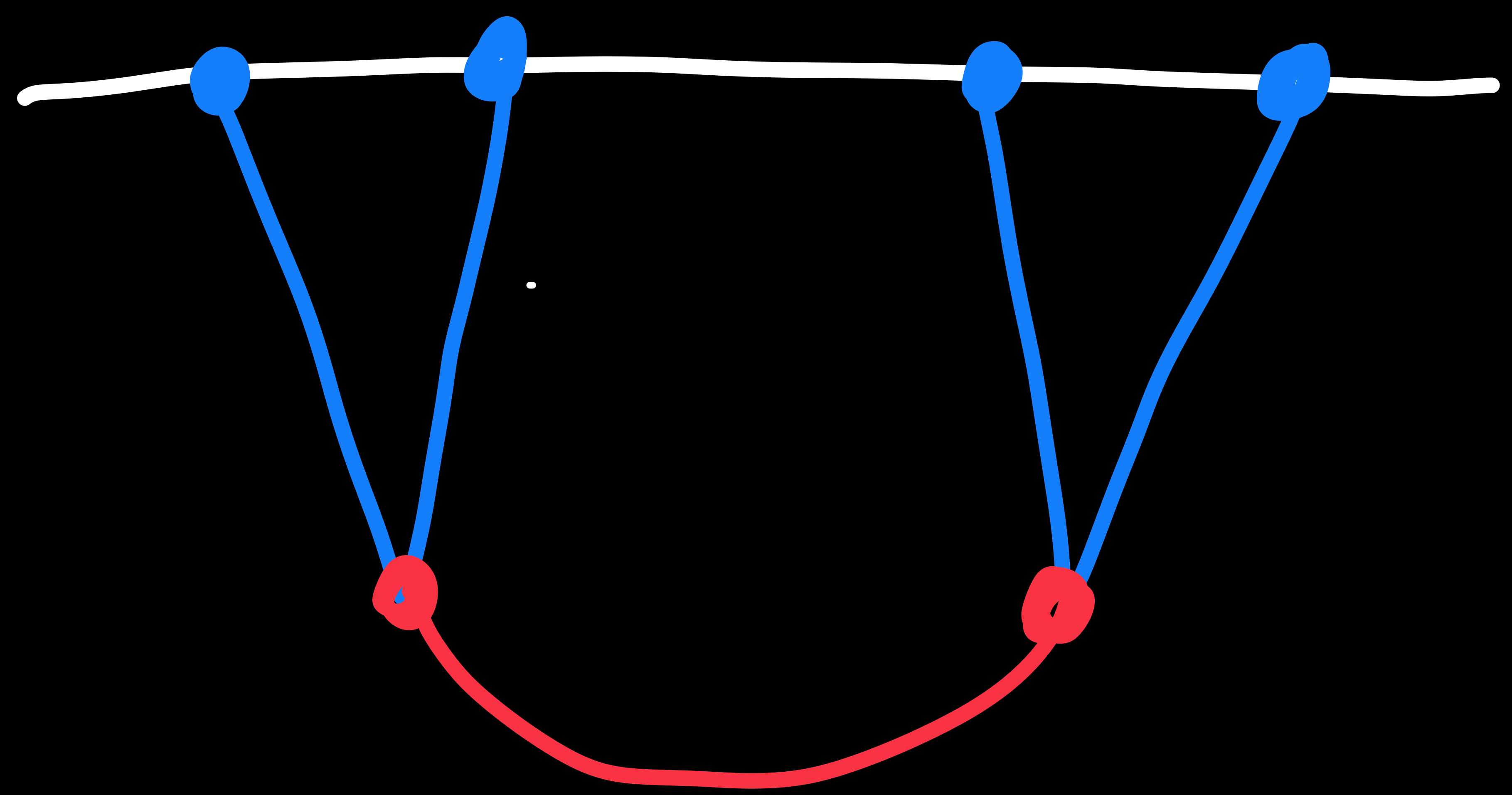
A: Working on it (with brilliant students)

New Directions



$$\int \Omega \, d\mathrm{loop} \, d\mathrm{cosmo}$$

spoils canonical form



$M_g \ll H$: Unparticle

$M_g \sim H$: (Holographic) RG

$M_g \gg H$ flow geometry

Differential Equations May Be Entryway To New Framework For Cosmology!

Masses
Loops
Emergence of Time
Stochasticity
Unitarity
Spins
Strong Coupling