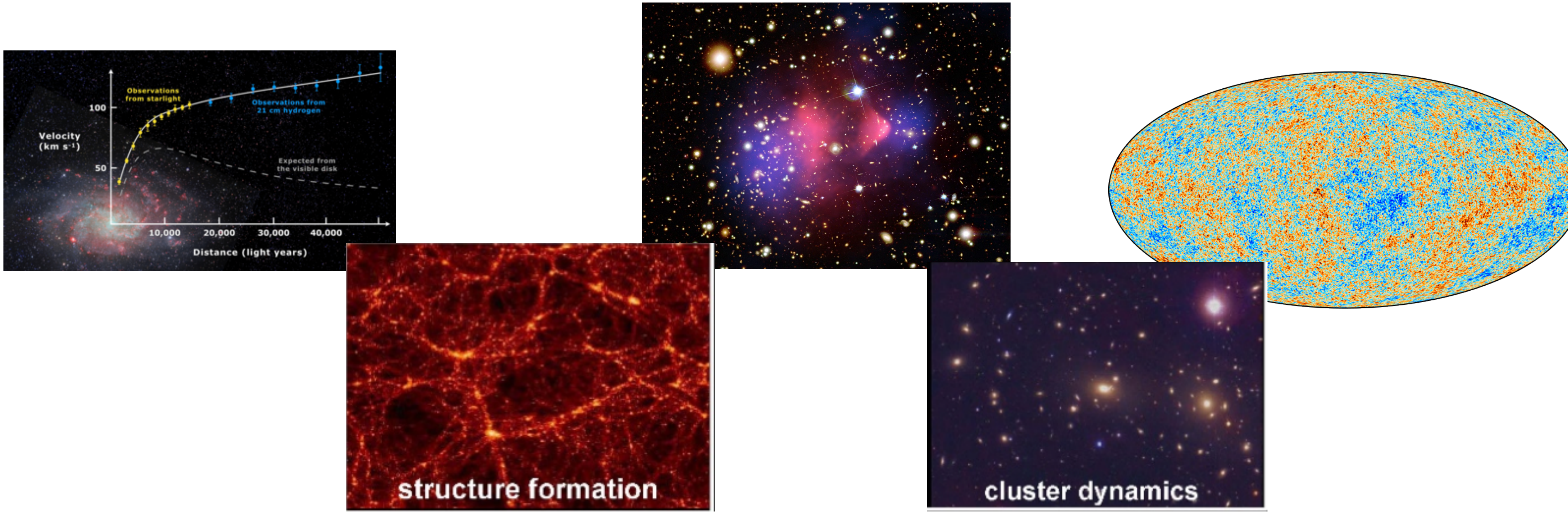
The meso-inflationary axion

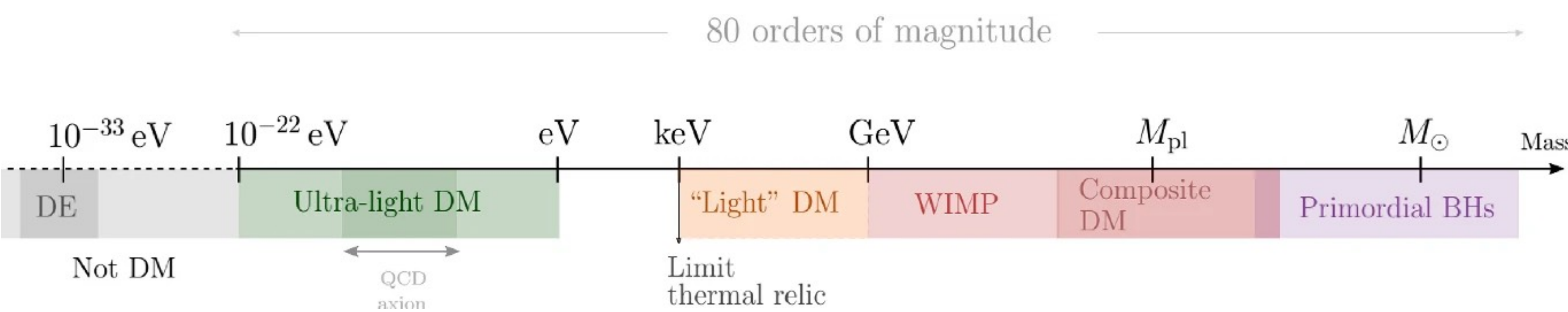
Based on 2211.06421 with Andrea Tesi and 2311.09315 with Gorghetto, Hardy, Nicolaescu, Notari

Firenze - 15 March 2024

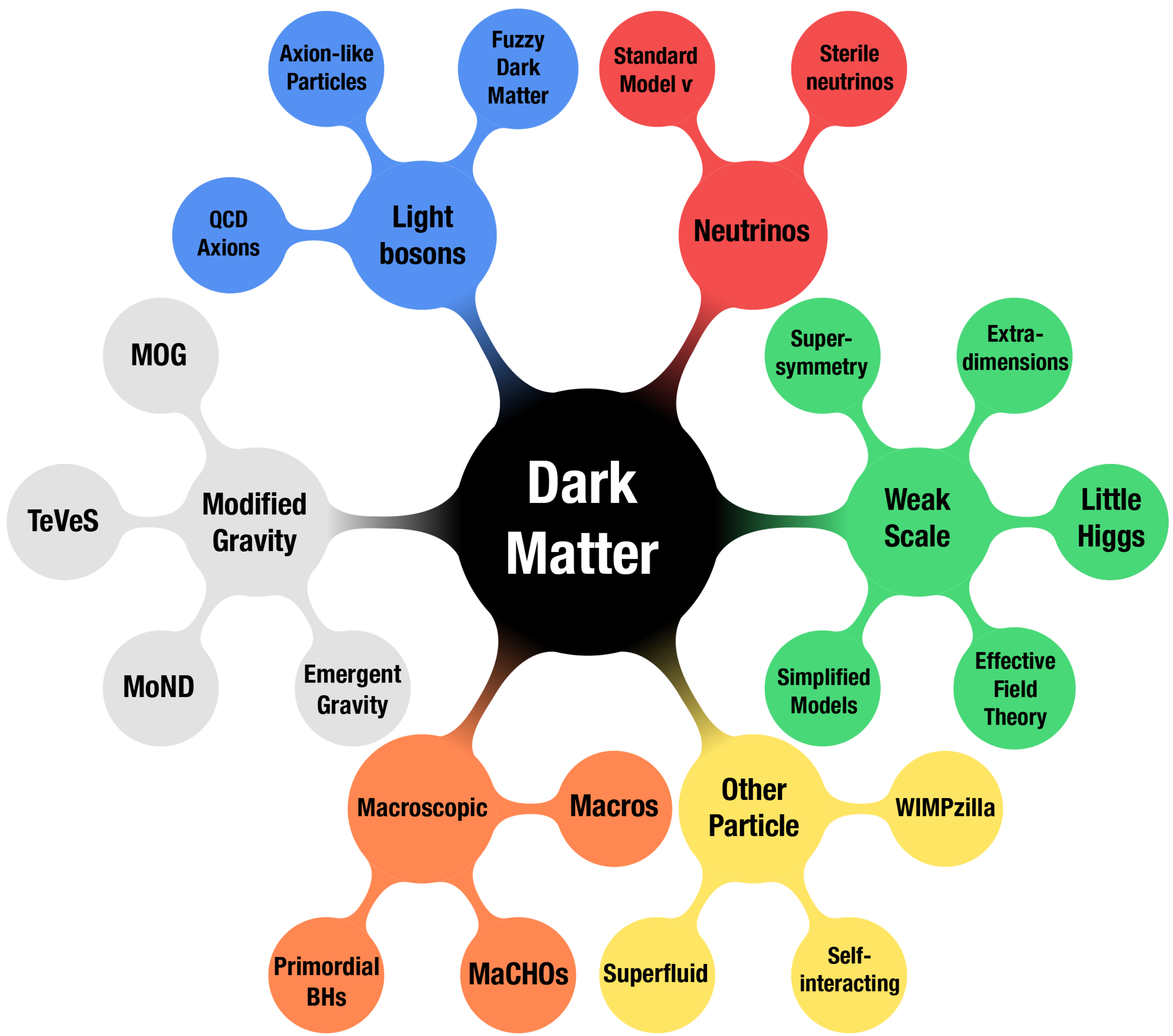


There is extraordinary evidence for the existence of a new non-baryonic cold component of matter — DARK MATTER:

- All evidences are inferred through gravity.
- We don’t know what DM is made of and its mass



DM LANDSCAPE

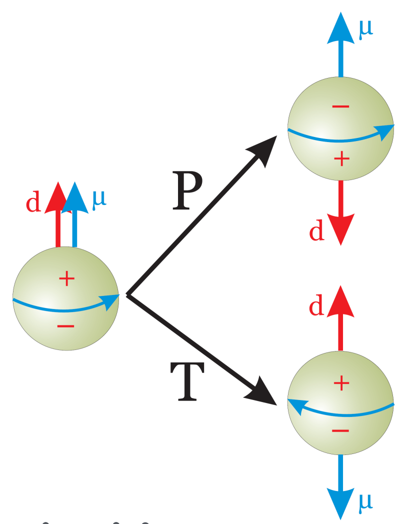


Need some guidance!

QCD AXION

Strong CP Problem:

$$\frac{\theta}{32\pi^2}\int d^4x\, G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$



$$\longrightarrow \quad \theta + \mathrm{Arg}[\mathrm{Det}(y_u y_d)] < 10^{-10}$$

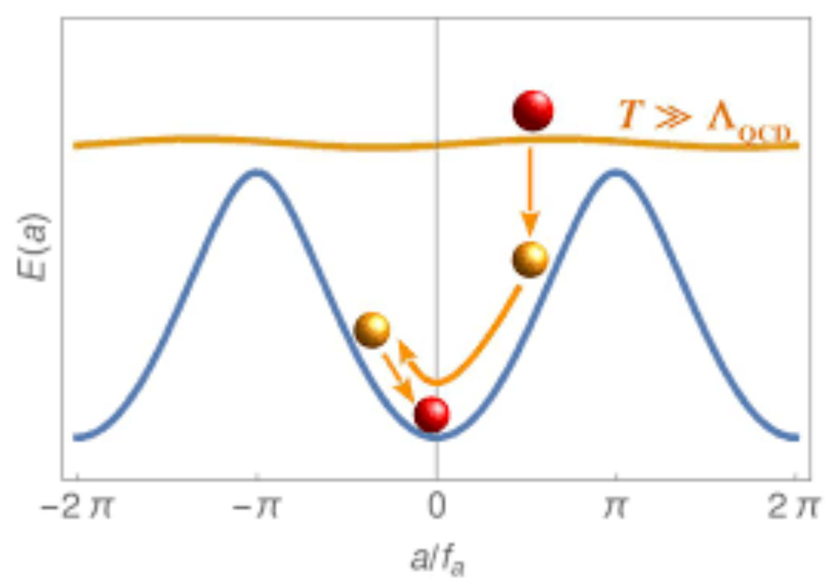
Axion solution:

[Peccei-Quinn '77
Weinberg-Wilczek '78]

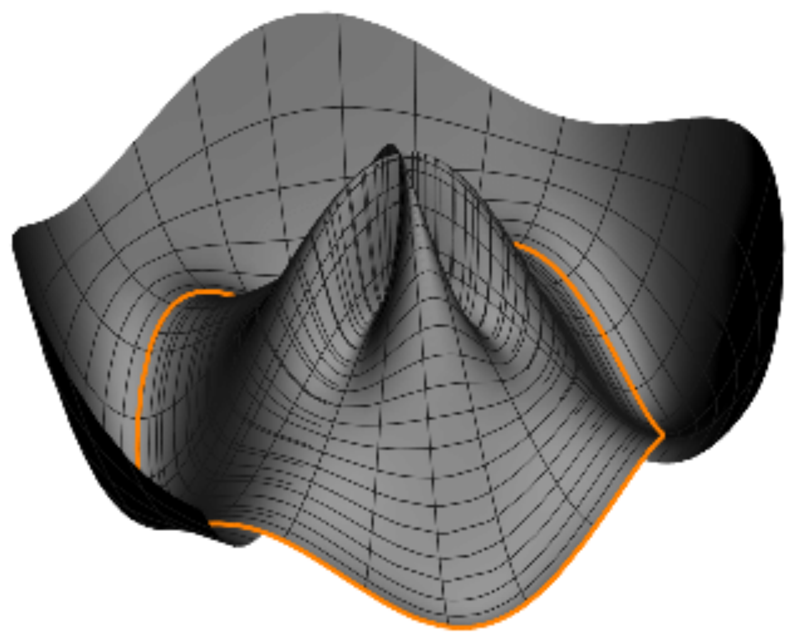
$$\theta \to \frac{a(x)}{f_a}$$

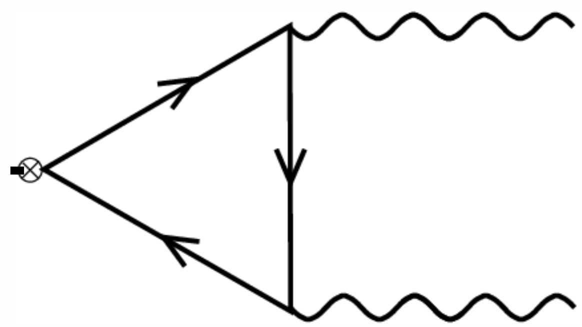
$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}$$

QCD dynamics aligns the vacuum to preserve CP:



The QCD axion is the Nambu-Goldstone boson of a U(1) global symmetry (Peccei-Quinn) broken only by QCD anomalies





$$V(a) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{a}{2 f_a}} + \ldots$$

$$m_a = 5.7 \times 10^{-6}\,\mathrm{eV} \left(\frac{10^{12}\,\mathrm{GeV}}{f_a} \right)$$

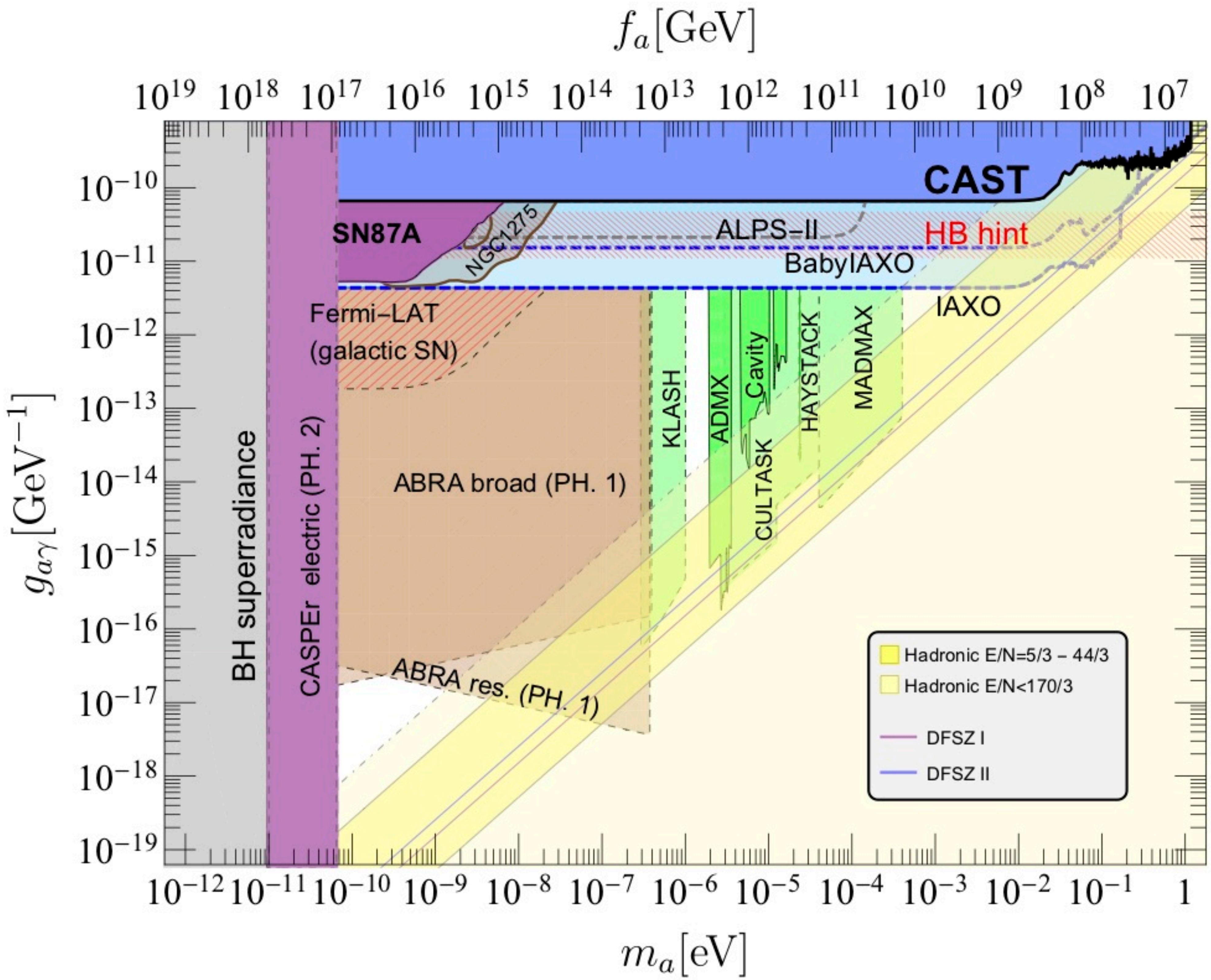
[Grilli di Cortona et al.'15]

Couplings to SM fields are predicted:

$$\frac{g_{a\gamma\gamma}}{4} a F \tilde{F}$$

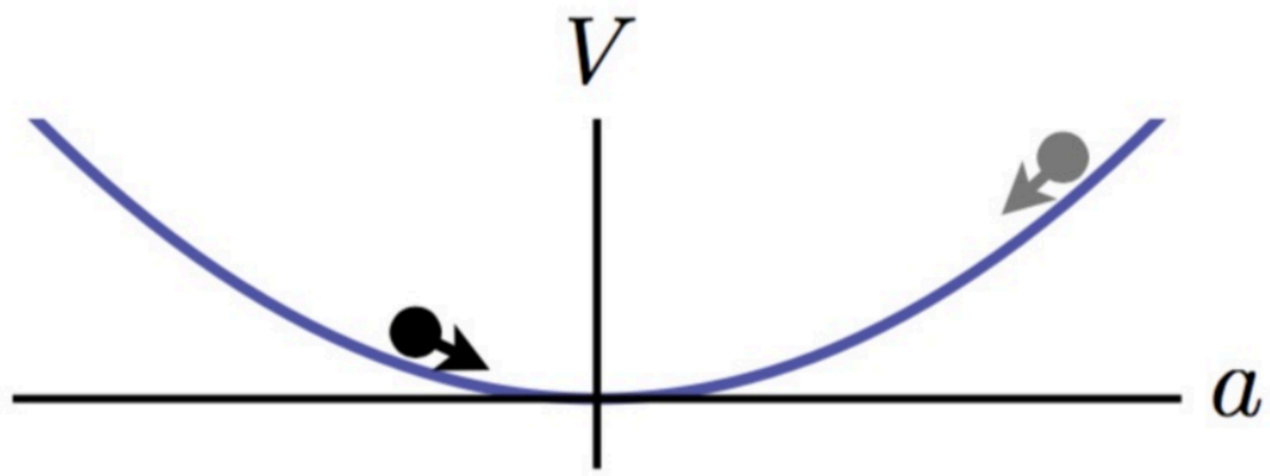
$$g_{a\gamma\gamma} = \frac{\alpha_{\mathrm{em}}}{2\pi f_a} \left[\frac{E}{N} - 1.92 \right]$$

Experiments:



If axion is DM it will be likely found!

Axions are excellent DM candidate for $f_a \gtrsim 10^8\,\mathrm{GeV}$ ($m_a \lesssim 10^{-2}\,\mathrm{eV}$)
They are naturally produced through misalignment mechanism.



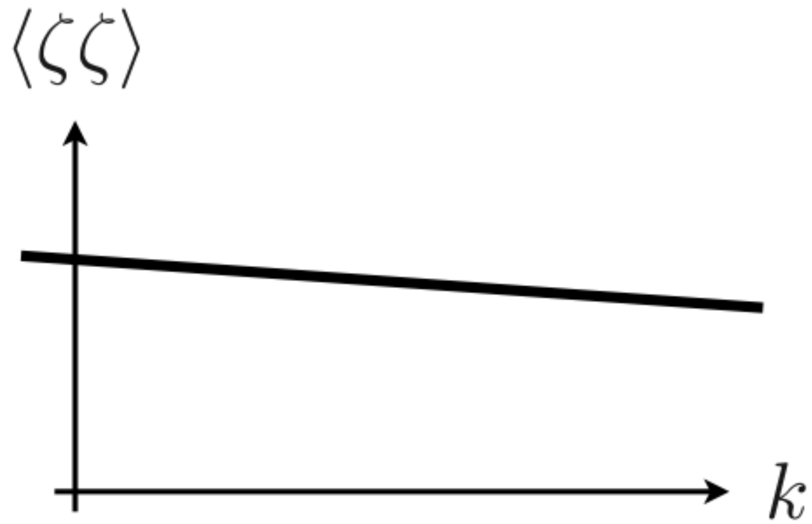
- $H \sim m_a(T)$

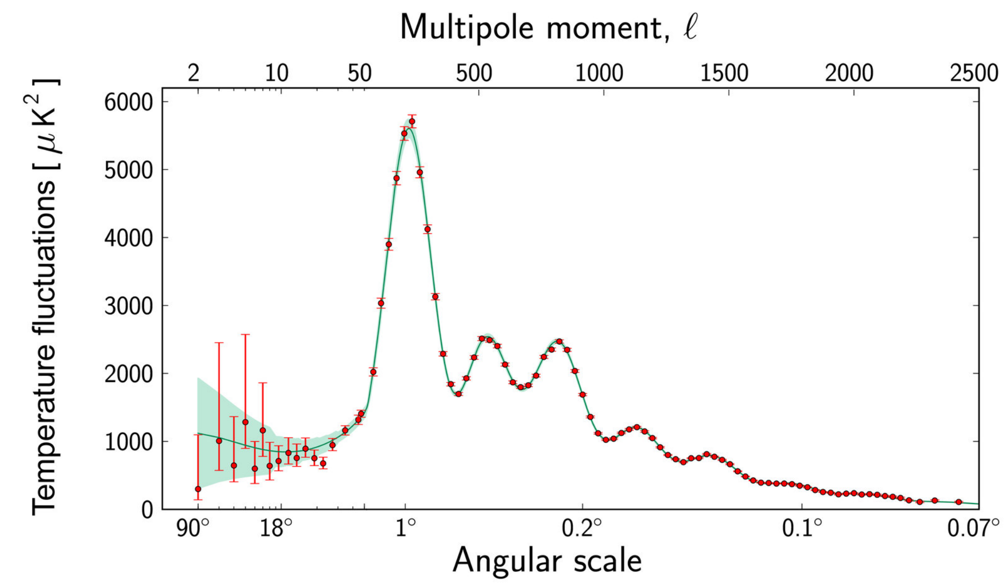
Axion condensate starts to oscillate around minimum.
Coherent oscillations behave as cold DM.

For the axion the potential is temperature dependent and oscillations begin at $T \sim \mathrm{GeV}$.

Axion DM depends on the dynamics during inflation.

Inflation:





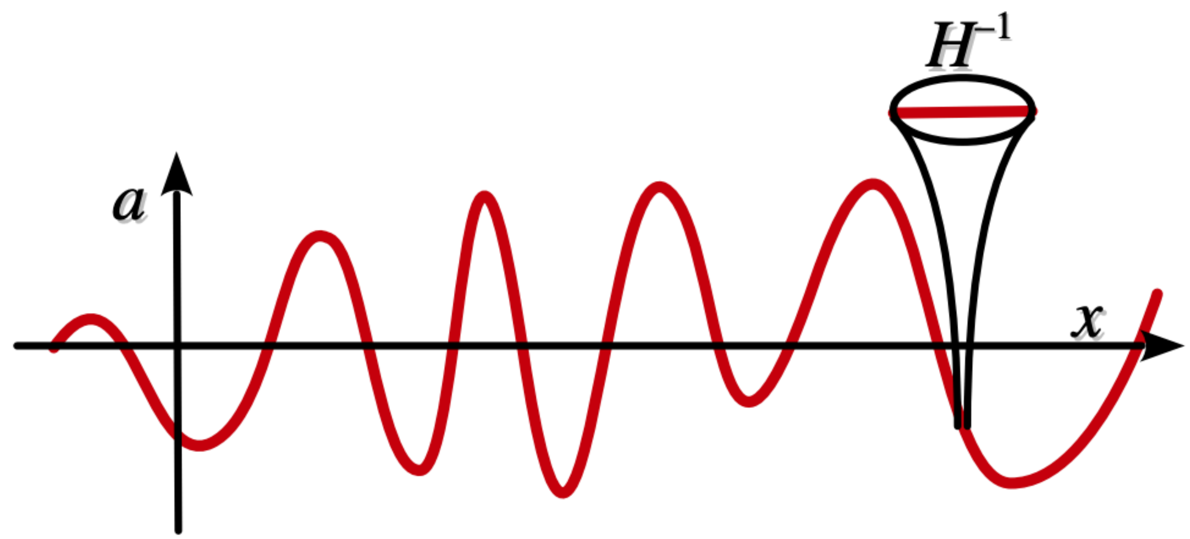
A phase of accelerated expansion — inflation — appears necessary to explain the initial conditions of our universe.

$$a(t) \approx a_i e^{H_I t} \qquad\qquad N \equiv Ht$$

- Classically inflation makes the universe flat and homogeneous by exponentially stretching space.
- Small quantum fluctuations generate the seeds of structures in the universe.

Axions DM:

- Pre-inflationary $f_a > \mathrm{Max}[H_I, T_{\mathrm{Max}}]$



PQ symmetry always broken.
Misalignment is constant over the visible universe.

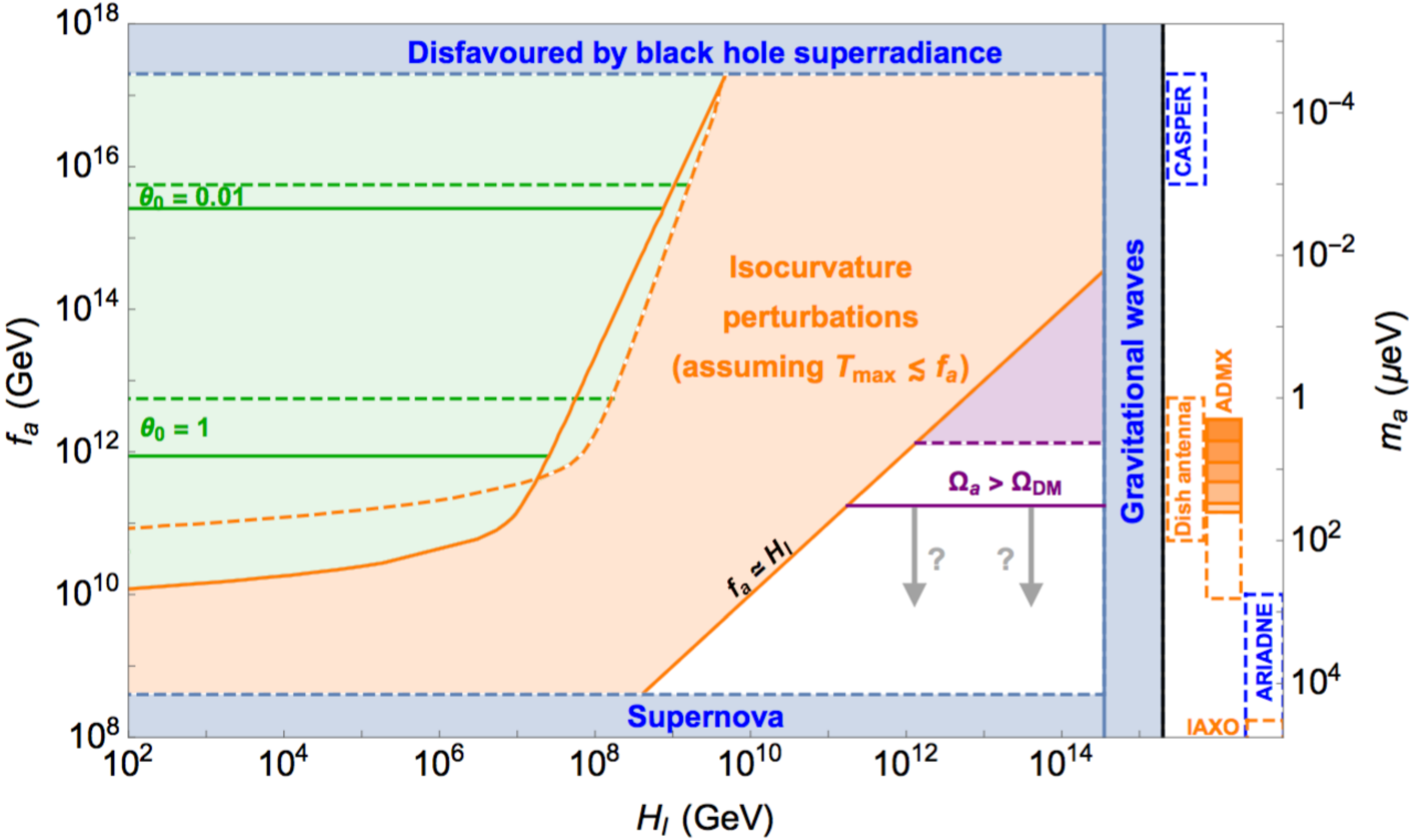
$$\Omega_a^{\mathrm{mis}} \sim \theta^2 \left[\frac{f_a}{10^{12}\mathrm{GeV}}\right]^{7/6}$$

Quantum fluctuations of the axion generate isocurvature.

- Post-inflationary $f_a < \mathrm{Max}[H_I, T_{\mathrm{Max}}]$

PQ symmetry is initially restored.
String network forms. θ scans over all values so that abundance is in principle predicted. Constrained by domain walls.

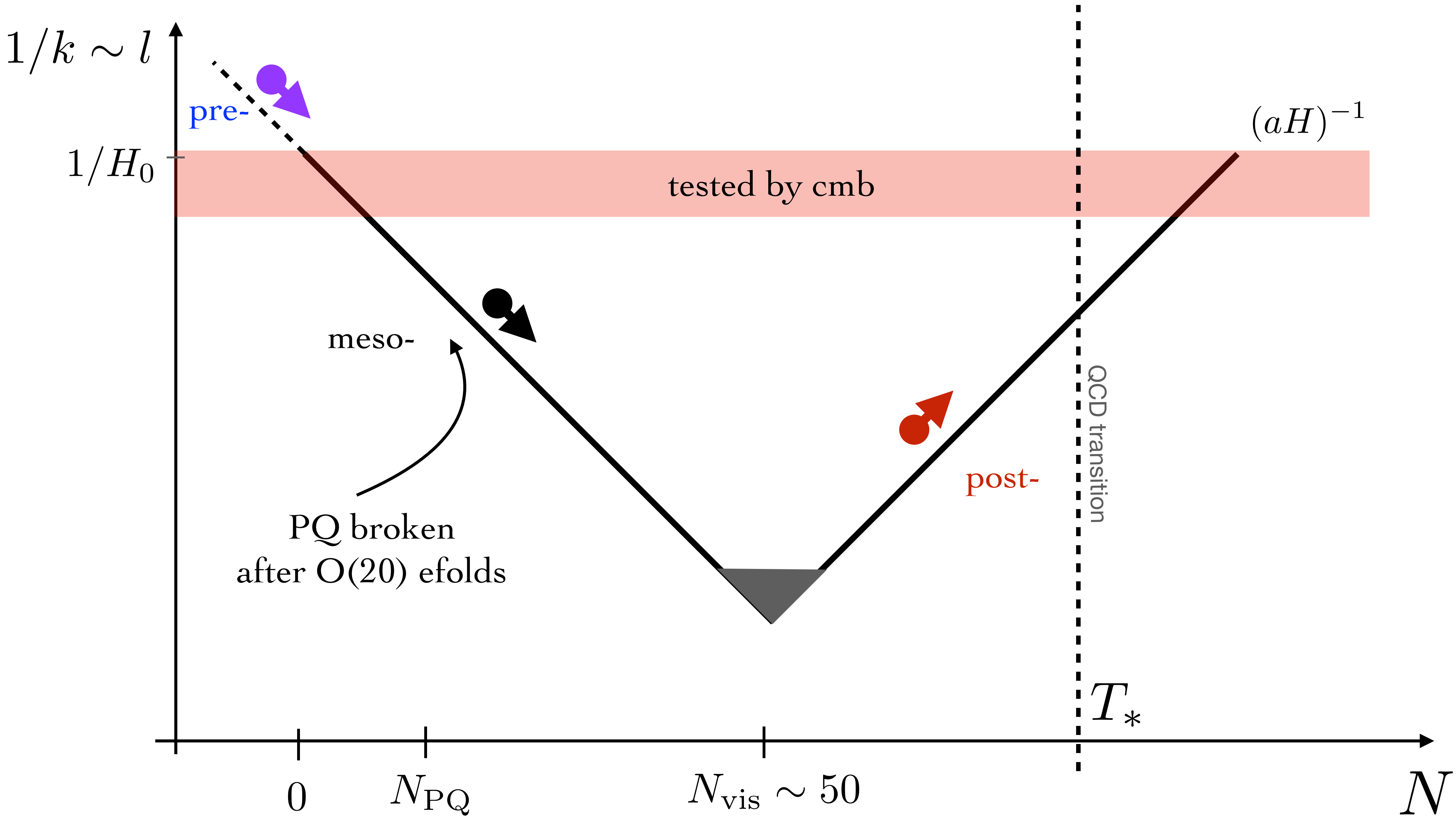
Abundance:



[Grilli Di Cortona et. al.'15]

- Meso-inflationary axion:

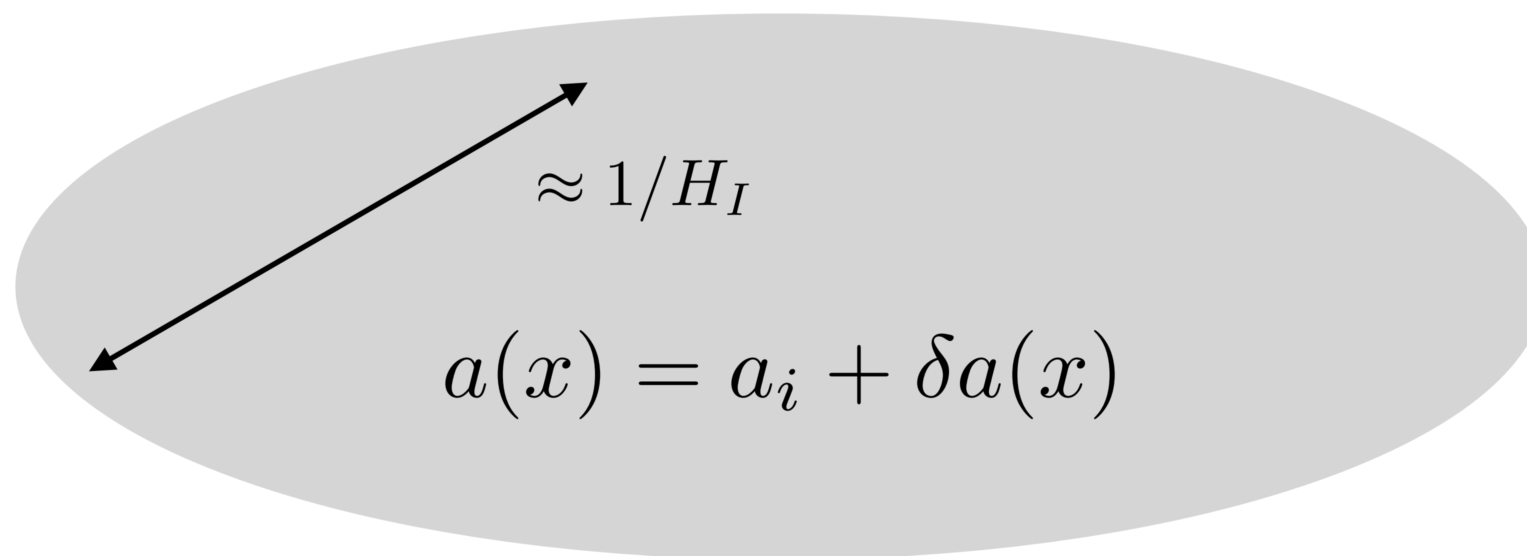
[Redi, Tesi '22]



PQ symmetry is broken after N_{PQ} e-foldings of visible inflation.

If PQ symmetry is broken after $N_{\rm PQ}$ e-foldings the axion does not exist at the beginning of inflation.

$$a > a_{\rm PQ}$$



Due to the inflationary expansion the axion becomes homogeneous over macroscopic scales,

$$d \sim \frac{1}{k_{\rm PQ}} \equiv \frac{1}{H_0} \exp[-N_{\rm PQ}]$$

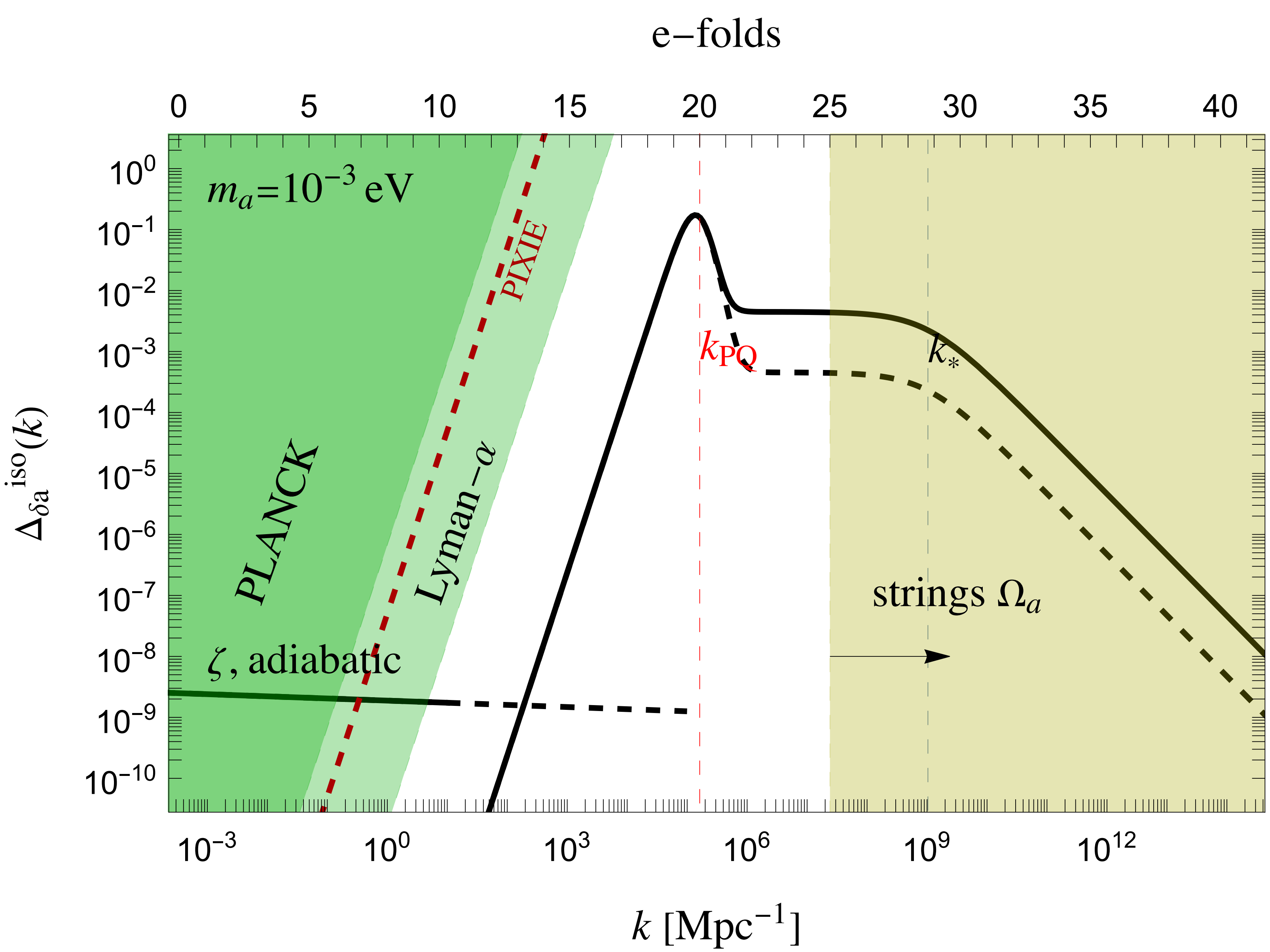
In our universe θ scans all values as in the post-inflationary axion but domains can be macroscopic.

Inhomogeneities re-enter the horizon at a temperature:

$$T_{\rm PQ} = T_{\rm CMB}\, e^{N_{\rm PQ}+4}$$

For $T_{PQ} < \text{GeV}$ ($N_{\rm PQ} < 25$) the string network cannot form as in the post-inflationary axion scenario.

Abundance of DM and its distribution is modified.



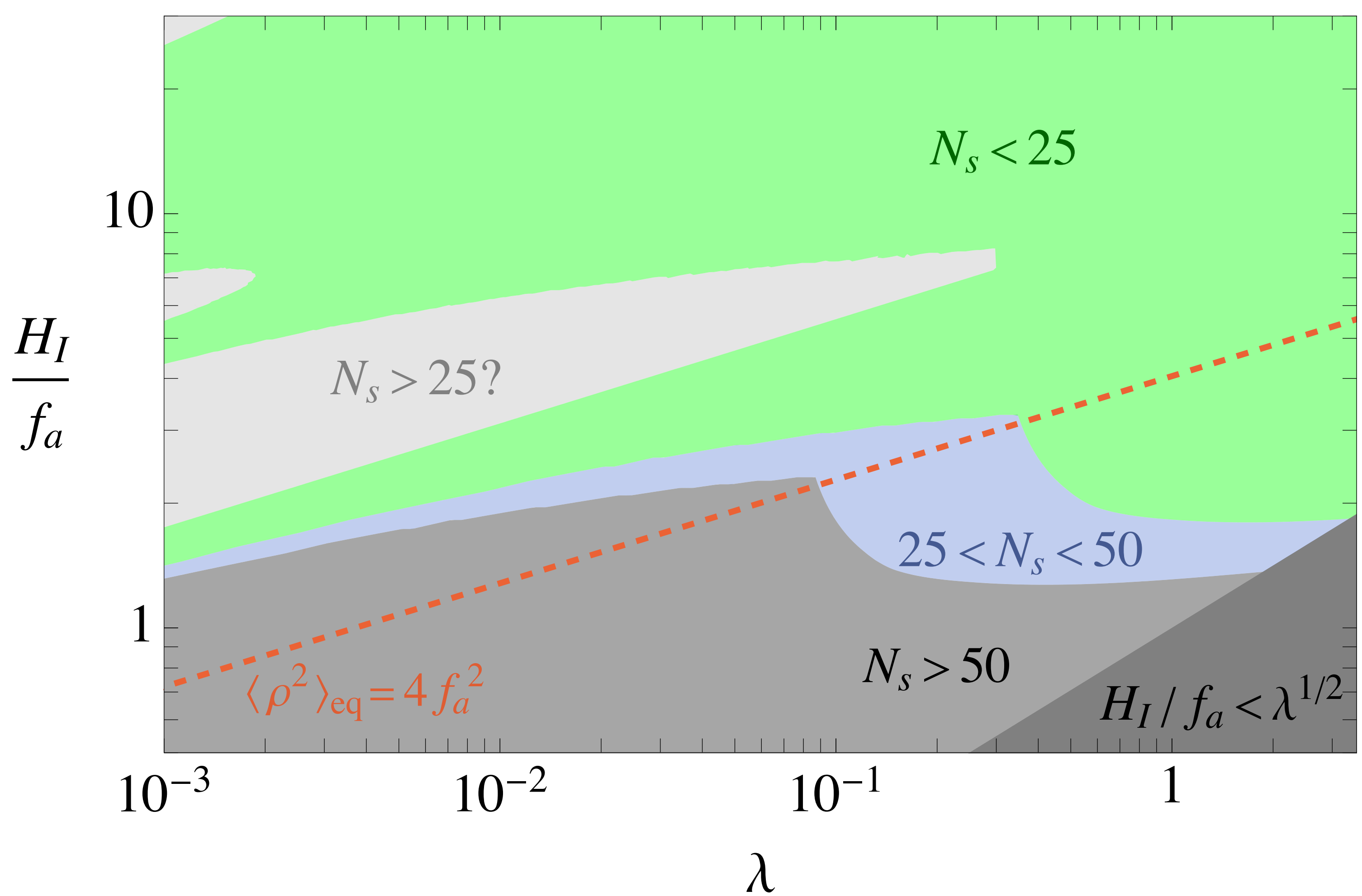
- KSVZ axion:

[with Gorghetto, Hardy, Nicolaescu, Notari]

We find that a very similar scenario is effectively realised even in the most minimal QCD axion (with $N_{DW} = 1$)

$$\mathcal{L} = |\partial_\mu \Phi|^2 - \lambda \left(|\Phi|^2 - \frac{f_a^2}{2} \right)^2 - y(\bar{\Psi}_L \Phi \Psi_R + h.c.)$$

$$U(1)_{\rm PQ}: \quad \Phi \to e^{2i\alpha}\Phi \qquad \Psi_{L,R} \to e^{\pm i\alpha}\Psi_{L,R}$$



[see also Lyth, Stewart '92]

- $H_I \gtrsim f_a > T_{\max}$

PQ symmetry is not restored by thermal effects.

During inflation Φ undergoes a random walk

$$\frac{\partial}{\partial N}\langle \Phi^2 \rangle = \frac{H_I^2}{4\pi^2} - \lambda \frac{2\langle \Phi^2 \rangle (4\langle \Phi^2 \rangle - f_a^2)}{3H_I^2}$$

After a critical number of e-foldings equilibrium is reached:

$$\langle \Phi^2 \rangle_{\rm eq} \approx \sqrt{\frac{3}{32\pi^2\lambda}} H_I^2 \qquad\qquad N_s \sim \sqrt{\frac{3\pi^2}{2\lambda}}$$

After $N > N_s$ the field is randomised so topologically non-trivial configurations will be generated.

N_s is also the number of e-folding over which Φ is coherent so that the symmetry is effectively broken:

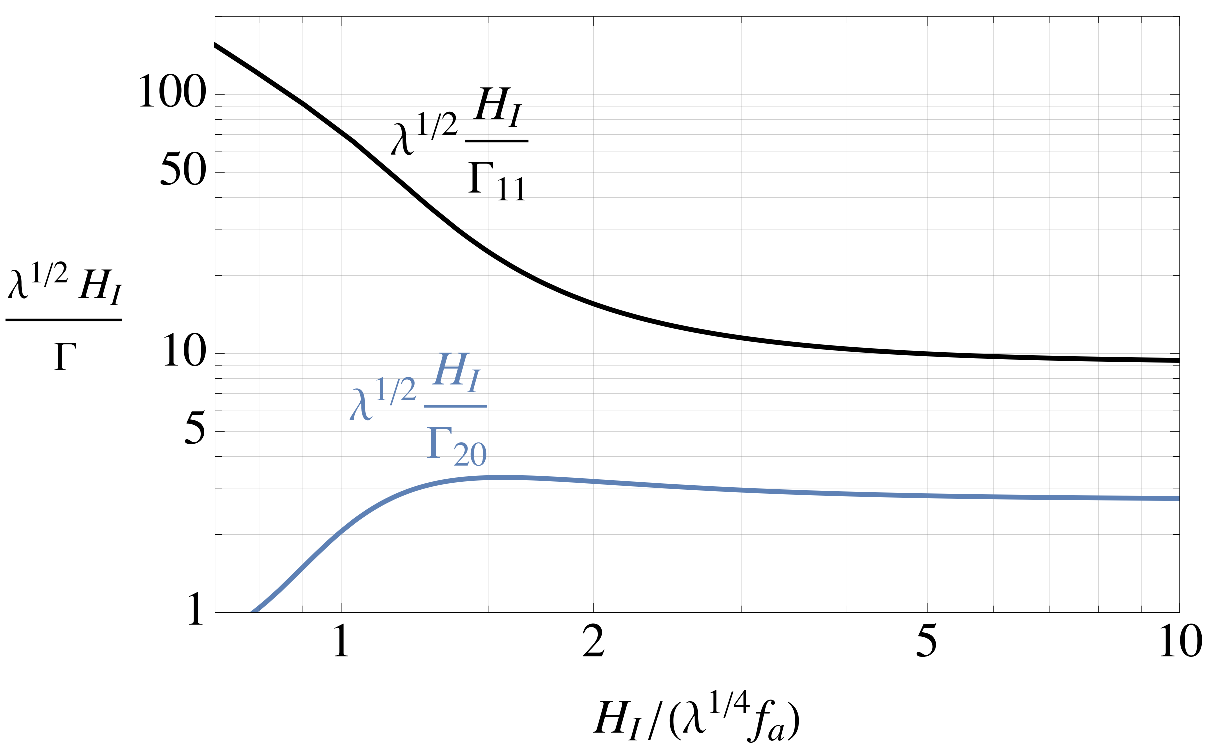
$$N_{\rm PQ} \sim N_{\rm vis} - N_s$$

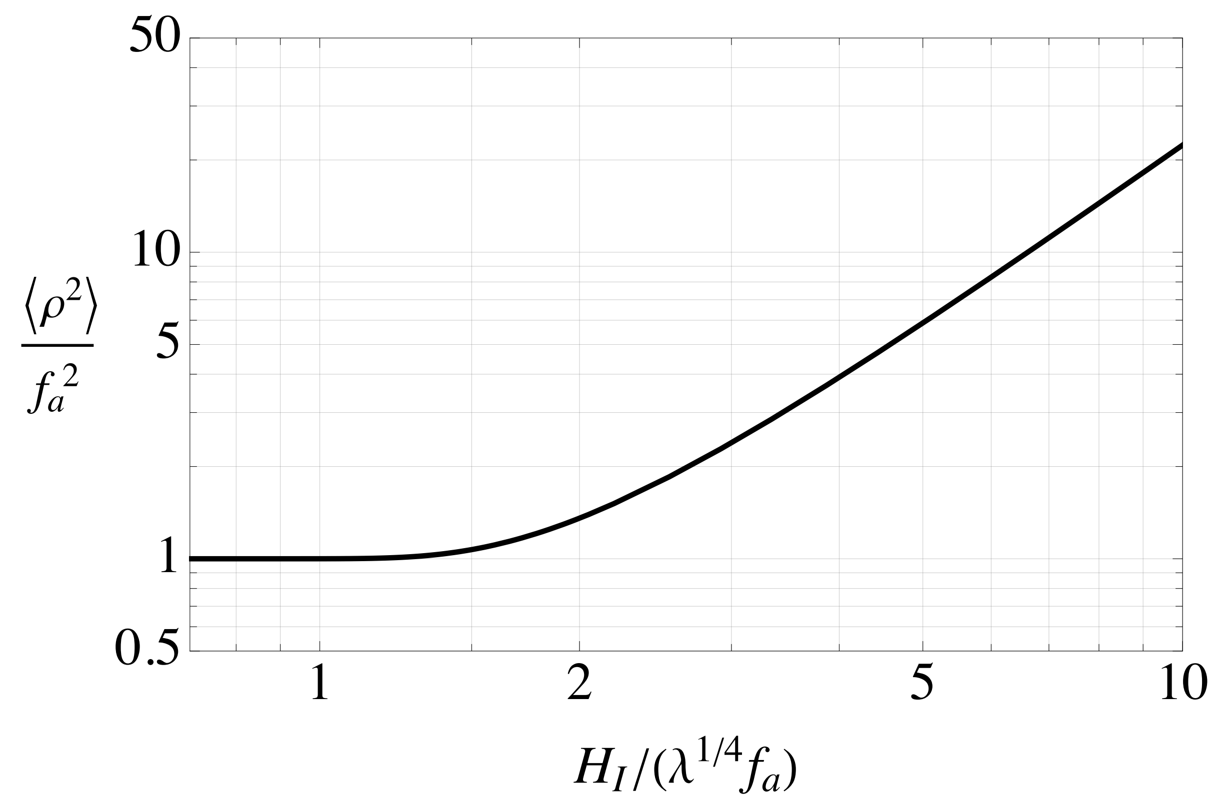
More precisely we want to study the Fokker-Planck equation for the probability distribution of Φ.

$$\frac{\partial P}{\partial t} = \frac{H_I^3}{8\pi^2} \sum_i \left[\partial_i \partial_i P + \frac{8\pi^2}{3H_I^4} \partial_i (P \partial_i V) \right]$$

Solution:

$$P(t,x,\theta) = P_{\rm eq}(x) + \sum_{(n,m)\neq(1,0)} a_{nm} e^{-v(x)} \psi_{nm}(x) \frac{e^{im\theta}}{\sqrt{2\pi}} e^{-\Gamma_{nm} t} \qquad P_{\rm eq}(\Phi) \equiv A\, e^{\frac{-8\pi^2 V(\Phi)}{3H_I^4}}$$

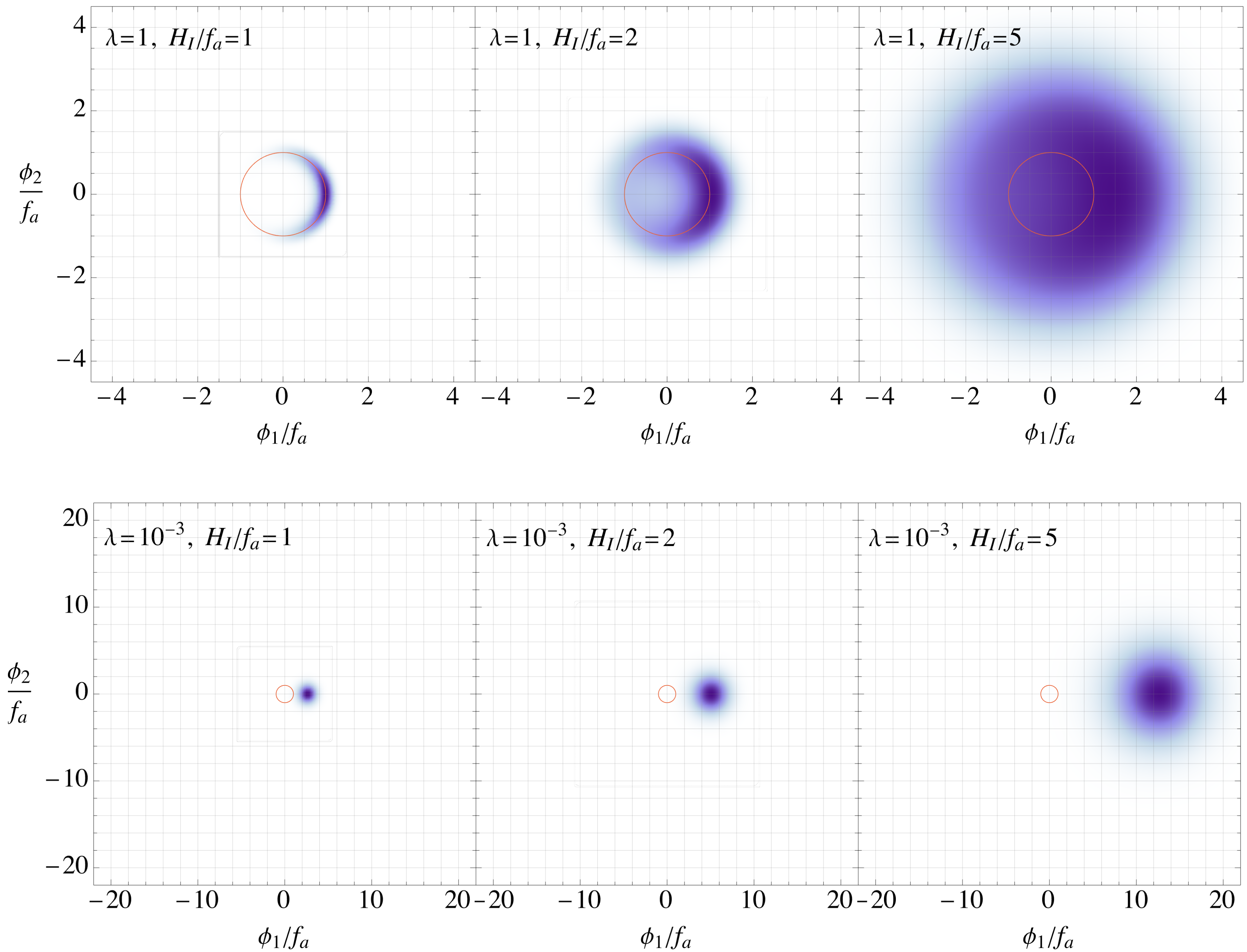




Time scale is set by lowest eigenvalue:

$$N_s \sim \frac{H_I}{\Gamma_{11}} \sim \frac{10}{\sqrt{\lambda}}$$

For $N \gtrsim N_s$ the field spreads over the top of the potential. Simulations confirm that if $O(1)$ fraction of points spreads strings form.

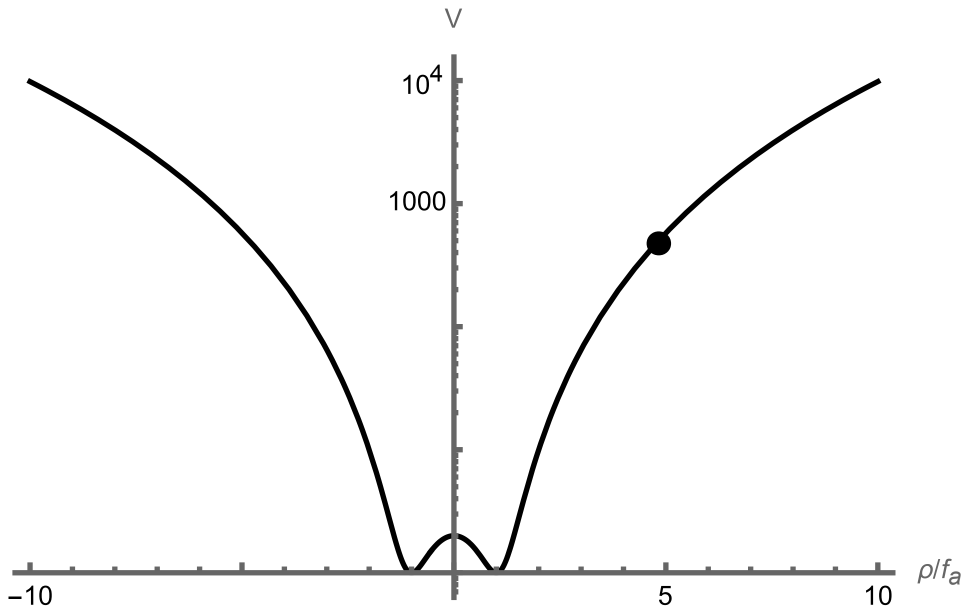


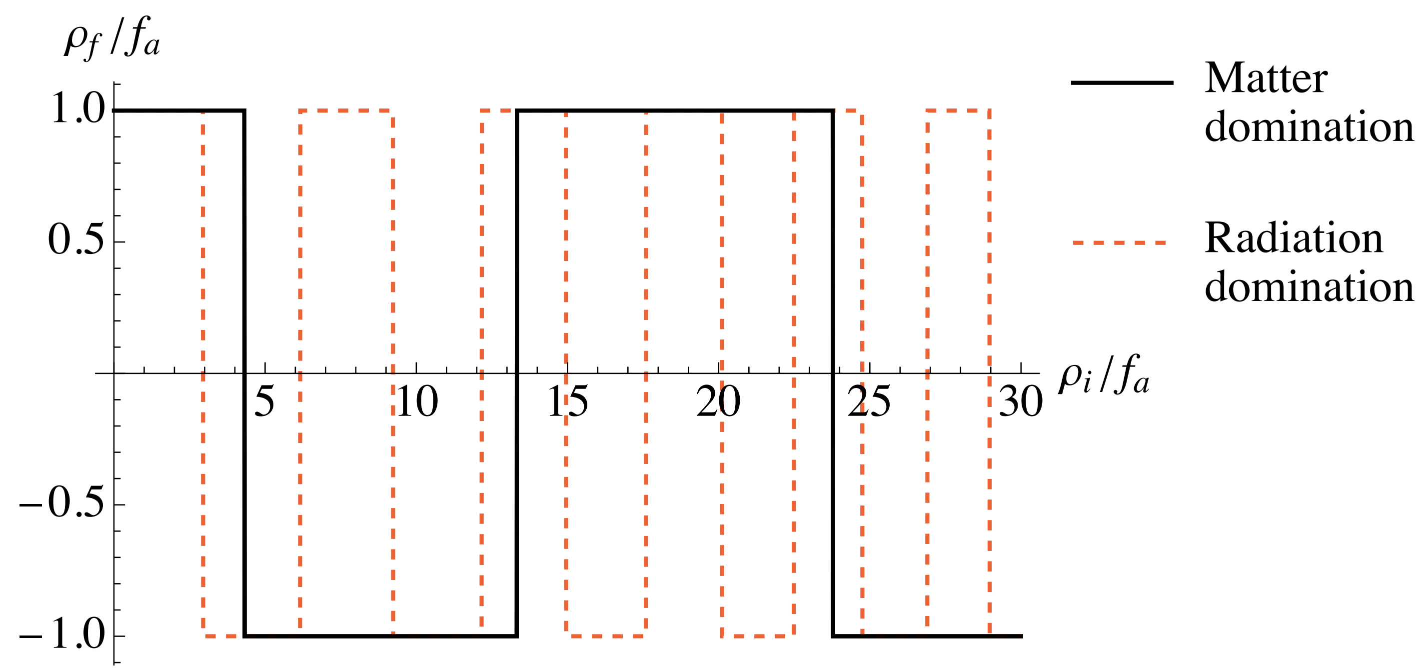
N=25

- Overshoot mechanism:

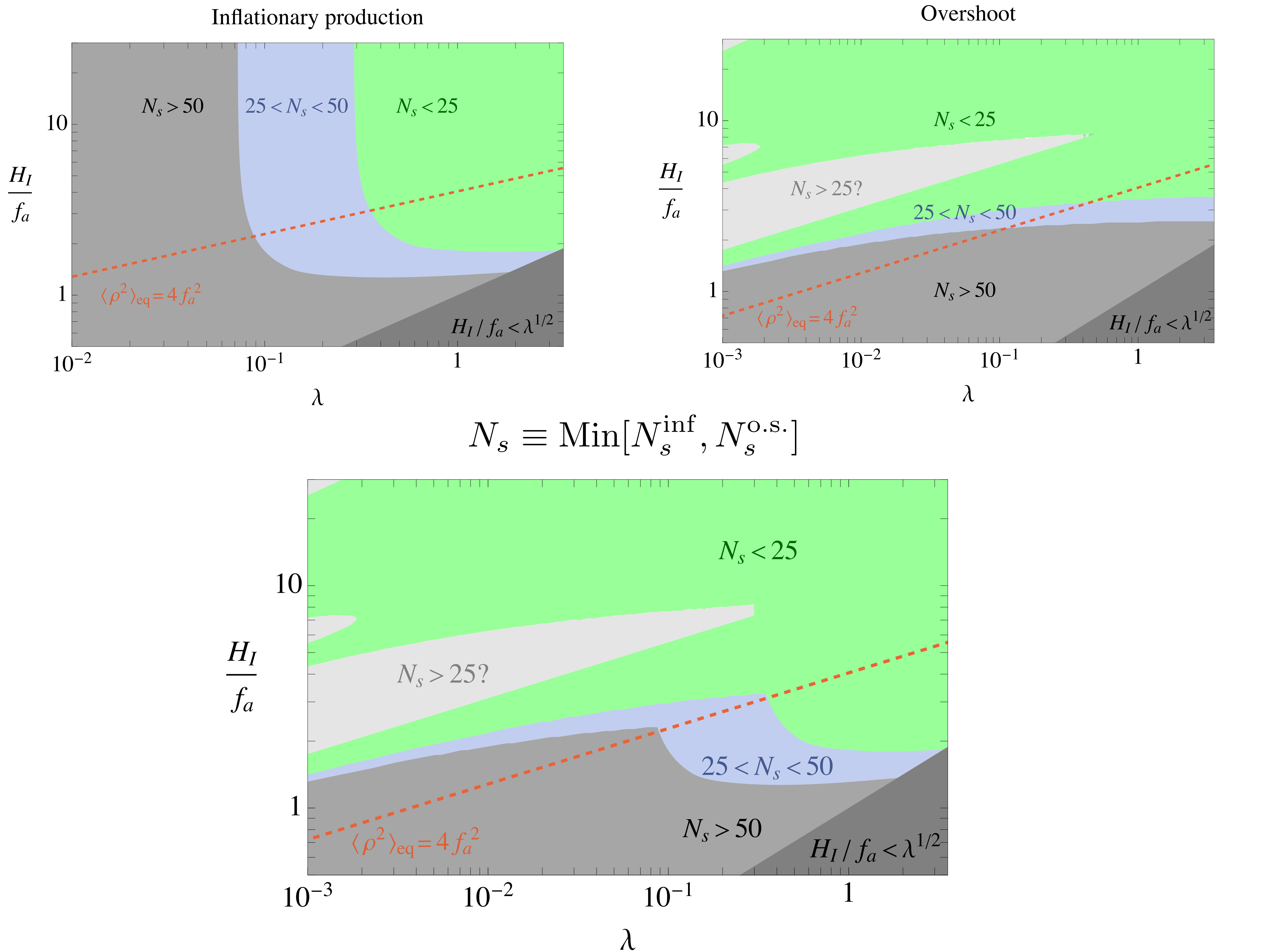
Strings can also form during reheating if the field oscillates over the top of the potential even if it is homogeneous at the end of inflation.

$$\ddot{\rho} + \frac{2}{t}\dot{\rho} + \lambda(\rho^2 - f_a^2)\rho = 0$$





Simulation show that when O(1) of points in the patch overshoots to the other side a string network forms. This provides a new coherence scale $N_s^{o.s.}$ that can be important for small λ.



Inflationary production
Overshoot
$N_s > 50$
$25 < N_s < 50$
$N_s < 25$
$N_s > 25?$
$\frac{H_I}{f_a}$
λ
$\langle \rho^2 \rangle_{\rm eq} = 4 f_a^2$
$H_I / f_a < \lambda^{1/2}$
$N_s \equiv \mathrm{Min}[N_s^{\rm inf}, N_s^{\rm o.s.}]$

- Pre/Post/Late:

- $N_s \gtrsim 50$

PQ symmetry is always broken. Pre-inflationary scenario produced.

- $N_s \lesssim 25$

PQ symmetry is restored on large scales and a network of strings can form at $T > \mathrm{GeV}$. Ordinary post-inflationary axion pheno expected.

- $25 \lesssim N_s \lesssim 50$

PQ is restored on macroscopic scales however the string network cannot form and reach a scaling solution as in the post-inflationary scenario.

Axion starts to oscillate before inhomogeneities re-enter horizon.

The abundance from misalignment is obtained averaging:

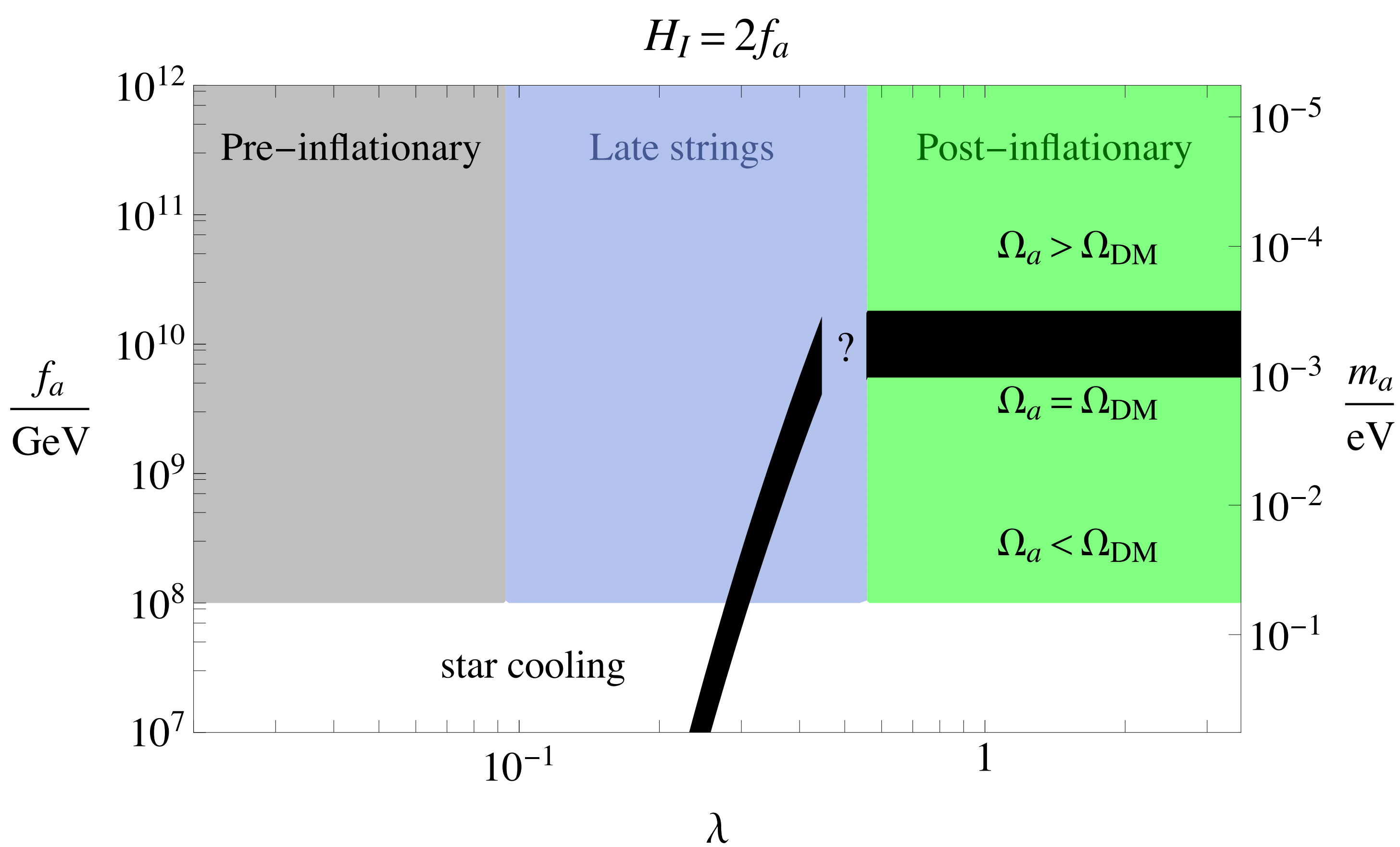
$$\frac{\Omega_a^{\rm mis}}{\Omega_{\rm dm}} \approx \left[\frac{f_a}{2\cdot 10^{11}\,{\rm GeV}}\right]^{\frac{7}{6}}$$

The abundance is dominated by the annihilation of the network.

We estimate this as 1 domain wall per Hubble volume,

$$\rho_{\rm dw} \approx \sigma H \approx 9 m_a f^2 H \longrightarrow \frac{\Omega_a^{\rm dw}}{\Omega_{\rm dm}} \sim \left[\frac{f_a}{10^8\,{\rm GeV}}\right]\left[\frac{1\,{\rm MeV}}{T_{\rm PQ}}\right]$$

DM can be reproduced up to the astrophysical bound!



When the network annihilates $O(1)$ inhomogeneities exist in the axion field at that scale.

$$\Delta_a \sim \left(\frac{k}{k_{\rm PQ}}\right)^3 \qquad\qquad k_{\rm PQ} \sim 10^7\,{\rm Mpc}^{-1}\,\frac{T_{\rm PQ}}{\rm GeV}$$

- Miniclusters

The peak in the power spectrum generates axion mini-clusters that will be larger than in the post-inflationary scenario:

$$M_{\rm MC} \sim \frac{4\pi}{3}\frac{\rho_{DM}}{(k_{\rm PQ}/a)^3}\bigg|_{a=a_{\rm eq}} \sim 10^{-11}\left(\frac{\Lambda_{\rm QCD}}{T_{\rm PQ}}\right)^3 M_\odot$$

Larger miniclusters would be easier to observe in particular through photometric lensing.

[unpublished with Garani, Tesi]

- Large scale structure

Close to the astrophysical bound the tail of the power spectrum might be testable through Lyman-α forest.

- Isocurvature:

The misalignment contribution to DM though subleading inherits isocurvature perturbations. Using stochastic approach to inflation:

$$\langle \cos(\theta(\mathbf{x}))\cos(\theta(0))\rangle \simeq \frac{\kappa_{11}^2}{|R_e H_I \mathbf{x}|^{\frac{2\Gamma_{11}}{H_I}}}$$

$$\Delta^2_{\cos\theta}(k) \approx \beta \left(\frac{k}{R_e H_I}\right)^{\beta}, \qquad \beta = \frac{2\Gamma_{11}}{H_I}$$

Because the misalignment contribution depends on $\theta(x)$ this translates in isocurvature perturbations (not cancelled by strings because axion is massive before network can form):

$$\Delta^2_{\rm iso}(k_{\rm CMB}) \sim \frac{\Omega^2_{\rm mis}}{\Omega^2_{\rm wall}} e^{-2N_{\rm vis}/N_s}$$

This can be a strong constraint even though uncertainties are large. More work (simulations) needed.

SUMMARY

- A new regime exists at the boundary between post-inflationary and inflationary scenario, even in the most minimal QCD axion. It is very difficult to determine precisely the region of parameter space but it looks unavoidable.

- Very exiting phenomenology with heavier axion masses (up to the astrophysical bound), larger miniclusters (possibly observable with photometric lensing), potential cosmological bounds from Lya.

- The main issue is isocurvature perturbations.
  Even in the standard post-inflationary axion ($T_{\max} < f_a$) cancellation of isocurvature perturbations has not been proven though it is expected. Simulations are needed.

We use as criterium the fraction of space that spreads over the top of the potential:

$$F(N) \equiv \frac{1}{2}\left[1 - \text{Abs}\int_{-\infty}^{\infty} d\phi_1 \int_{-\infty}^{\infty} d\phi_2 \; \text{sgn}\,(\phi_1) \;\; B\left(\sqrt{\phi_1^2+\phi_2^2}/f_a\right) \;\; P(\phi_1,\phi_2,N)\right]$$

B is the activation function for overshoot.
Using the Fokker-Planck equation (B=1):

$$N(F) = \frac{H_I}{\Gamma_{11}} \log\left[\frac{2a_{11}b_{11}}{2F-1}\right] \propto N_{11}$$

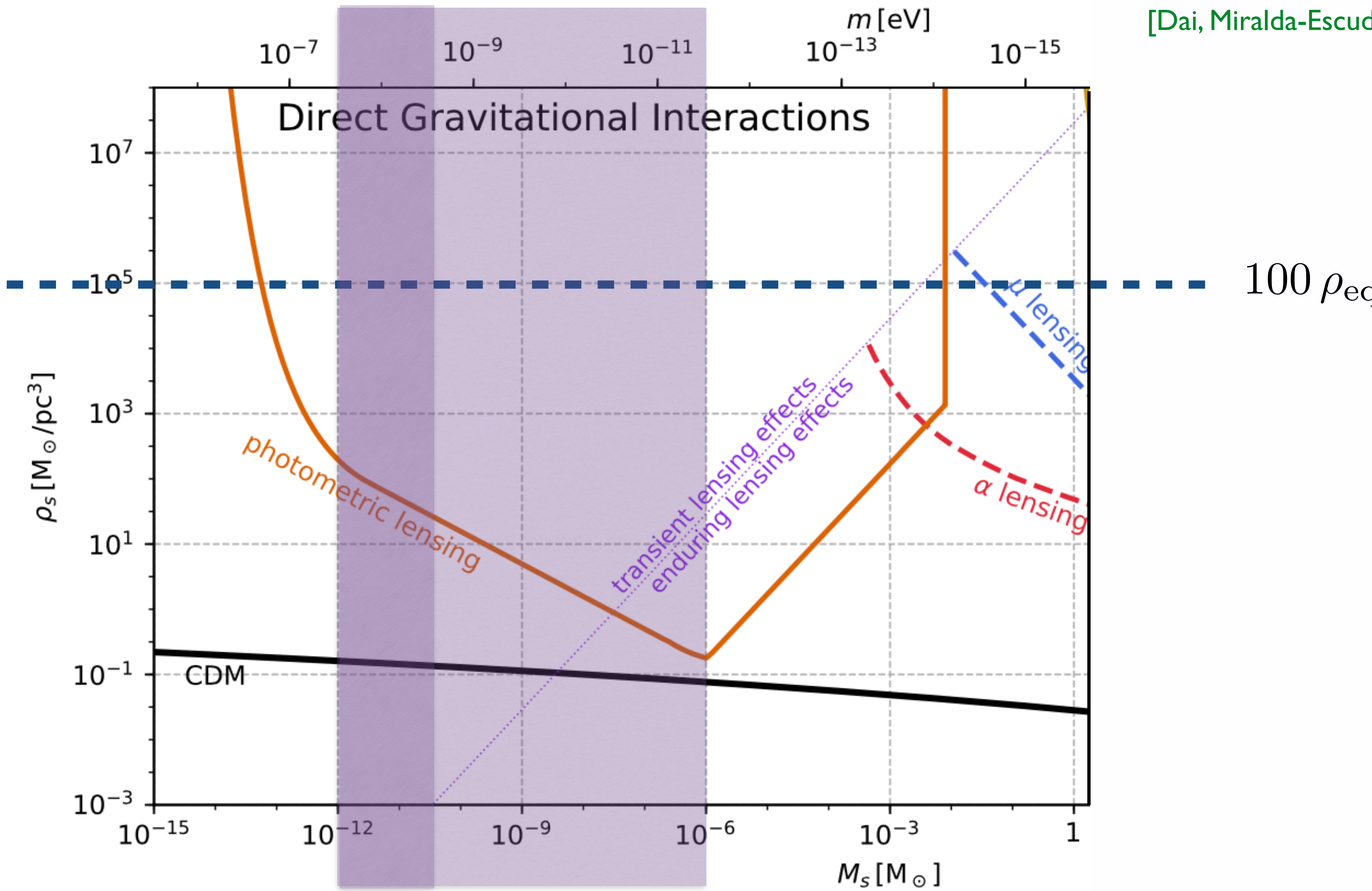
For overshoot we determine the N_s from the equation:

$$\frac{H_I}{2\pi}\sqrt{N_s} = \sigma(F) \qquad\qquad \langle B(\sqrt{\phi_1^2+\phi_2^2}/f_a)\rangle = 1-2F$$

Simulations suggest **F=0.35-0.45** as critical values.

Mini-clusters could be probed through photometric lensing.

[Dai, Miralda-Escude' '19]



[from 1909.11665
Arvanitaki et. al.'19]