Some Mathematical Approaches to Climate Change and its Impacts

Report of Contributions

https://indico.sns.it/e/68

Contribution ID: **2** Type: **not specified**

Analysis of a two-layer energy balance climate model

Monday, 22 April 2024 10:00 (30 minutes)

A simple yet extremely valuable approach to the study of the climate system comes from the use of Energy Balance Models (EBMs), which had originally been introduced in the sixties independently by Budyko and Sellers. Such models
describe in a simplified yet effective way the evolution of the zonally averaged
temperature on the Earth's surface, thus reducing the problem to a single 1D field.
The classical EBM can be improved by increasing the vertical resolution. In this talk I will present a two-layer energy balance model that allows for vertical exchanges between a surface layer and the atmosphere. I will analyse stability, long time behaviour of solutions and the sensitivity of our model with respect to parameters which are partly related to the greenhouse effect.

The presented results are the outcome of a joint work with P. Cannarsa, V. Lucarini, P. Martinez, J. Vancostenoble.

Presenter: URBANI, Cristina (Universitas Mercatorum)

Contribution ID: **3** Type: **not specified**

Scale dependent models in climate modelling

Tuesday, 23 April 2024 09:00 (30 minutes)

One of the main features of the global climate is that it incorporates a very wide range of length and time scales and associated physical processes. Therefore it is important to understand which phenomena occur according to the use of single scales or to the interactions of them (i.e. internal gravity waves, Rossby waves, cloud formation). From a mathematical point of view, these various physical behaviors give rise to different singular limits and, consequently to a different analysis of the asymphtotics of the governing equations. In this talk first we will show a very general scaling analyse and then we will work on a simplified model for geophysical fluids
and we will show, according to the values of different scales, that the asymptotic behavior of the model will be those of an incompressible fluid or of a geostrophic flow.

Presenter: DONATELLI, Donatella (Università degli Studi dell'Aquila)

Contribution ID: **4** Type: **not specified**

Welcoming remarks and project presentation

Monday, 22 April 2024 09:40 (20 minutes)

Presenter: COMMITTEE, PRIN PNRR

Contribution ID: **6** Type: **not specified**

Opinion dynamics of two populations with time-delayed coupling

Tuesday, 23 April 2024 11:30 (30 minutes)

We analyze a Hegselmann-Krause type opinion formation model for a system of two populations. The two groups interact with each other via subsets of individuals, namely the leaders, and natural time delay effects are considered. By using careful estimates on the system's trajectories, we prove that all agents asymptotically converge to a consensus state. Some future research directions as well as possible applications are illustrated.
Joint work with Chiara Cicolani, Università di L'Aquila.

Presenter: PIGNOTTI, Cristina (Università degli Studi dell'Aquila)

Contribution ID: **7** Type: **not specified**

Weather, macroweather and statistical properties for a spatially heterogeneous 1D energy balance model

Tuesday, 23 April 2024 12:10 (30 minutes)

A one-dimensional energy balance model (1D-EBM) is a simplified climate model that describes the evolution of Earth's temperature based on the planet's energy budget.

In this study, we examine a 1D-EBM that incorporates a parameter representing the impact of carbon dioxide on the energy balance. Based on empirical studies showing that bistability may occur in Earth's tropics, we consider the planet's ongoing radiation to be latitude-dependent, presenting bistability in low-latitude regions but not in high-latitude ones. This local bistability does not lead to a bifurcation in the entire system, in addition to the classical saddle-node bifurcations between Snowball Earth and the present climate.

We focus on investigating the statistical properties of the system when the model is perturbed with additive noise. Our work is a step towards a clearer understanding of the dynamics in a spatially heterogeneous setting.

Joint ongoing research with F. Flandoli.

Presenter: DEL SARTO, Gianmarco (Scuola Normale Superiore)

Contribution ID: **8** Type: **not specified**

A Mean-Field Game network model for urban planning

Monday, 22 April 2024 17:00 (30 minutes)

We study a mathematical model to describe the evolution of a city, which is determined by the interaction of two large populations of agents, workers and firms. The map of the city is represented by a network with the edges representing at the same time residential areas and communication routes. We obtain a two population Mean-Field Game system coupled with an Optimal Transport problem defined on the network. We prove existence and uniqueness of the solution and several numerical simulations are also provided.

Presenter: MARZUFERO, Luciano (Libera Università di Bolzano)

Contribution ID: **9** Type: **not specified**

Non autonomous degenerate parabolic problems

Monday, 22 April 2024 11:50 (30 minutes)

Inspired by a Budyko-Seller model, we consider non-autonomous degenerate parabolic problems. Using Kato's Theorem, we first prove the well-posedness of such problems. Then, obtaining new Carleman estimates for the non-homogeneous adjoint problems, we deduce null-controllability for the original ones. Some linear and semilinear extensions are also considered, as well as open problem and work in progress.

Presenter: FRAGNELLI, Genni

Contribution ID: **10** Type: **not specified**

A very brief introduction to Data Assimilation

Monday, 22 April 2024 15:10 (30 minutes)

The term data assimilation from the geosciences refers to reconstructing the current state of a dynamical system from current and past observations of the system. Here "current" and "past" refer to the time in which the system evolves.
By "observation" we mean the value of a function taken at the current state of the system; these observation functions are typically not injective. The system itself as well as the observations might be subject to noise; furthermore, the underlying dynamics as well as the observation function are either completely specified as part of the problem statement, or contain unknown parameters which might be of interest as well. In geophysical applications, the dynamical systems under concern are typically infinite dimensional, while observations are finite dimensional (albeit with very large dimensions). Data assimilation is necessary as a precursory step to every weather forecast and thus part of every operational weather forecasting system; data assimilation is therefore of great practical interest. In this contribution, I will try to give a brief overview over data assimilation from a mathematical perspective. A wide range of different approaches exist, due to a large variety of paradigms, criteria of optimality, objectives, and operational constraints. These approaches have been analysed mathematically to a varying degree. I will list what I believe to be the most interesting mathematical questions in relation to data assimilation (this list will clearly be highly subjective), and subsequently discuss some existing results concerning these questions.

Presenter: BRÖCKER, Jochen (University of Reading)

Contribution ID: **11** Type: **not specified**

Quasi-geostrophic models

Monday, 22 April 2024 16:20 (30 minutes)

The QG equations incorporate the most important balances in rotating geophysical fluid dynamics, the hydrostatic and the geostrophic balance. Furthermore, they reflect some thermodynamical aspects of the fluid as well. These equations form the simplest models that contain most important dynamical phenomena of ocean and atmosphere dynamics at mid-latitude. After the systematical derivation by Charney in 1948 they were also one of the first models used for computer based weather prediction in the 50ties. Today, they are frequently used in qualitative studies and conceptional works. Mathematically, the equations are very similar to 2D Navier-Stokes in vorticity formulation, though the physical interpretation is quite different. I will derive the model formally from 3D Navier-Stokes on the sphere. Finally, I will shortly discuss results we obtained for layered versions of the model.

Presenter: KUNA, Tobias (Università degli Studi dell'Aquila)

Contribution ID: **12** Type: **not specified**

Mass transport via flows of control-affine systems

Tuesday, 23 April 2024 09:40 (30 minutes)

We study the controllability of the continuity equation where the dynamics follows a control-affine system without drift. Under suitable regularity conditions, the controllability of the system is a sufficient condition for achieving controllability of the continuity equation. Moreover, we show the existence of controls such that the flow of the control system is the optimal transport map, for the 2-Wasserstein distance, between two given probability measures.

Presenter: CAPONIGRO, Marco (Università degli Studi di Roma Tor Vergata)

Contribution ID: **13** Type: **not specified**

Controllability of evolution equations with applications to Energy Balance Climate Models

Monday, 22 April 2024 11:10 (30 minutes)

In this talk we introduce the Energy Balance Climate Models (EBCM), then we discuss some results concerning the approximate multiplicative controllability of reaction-diffusion equations, also degenerate with the application to a particular EBCM, that is the Budyko-Sellers model. We also present some interesting open problems and new research directions.

Presenter: FLORIDIA, Giuseppe (Università degli Studi di Roma La Sapienza)

Contribution ID: **14** Type: **not specified**

A model for pollution on Network

Tuesday, 23 April 2024 10:50 (30 minutes)

We study the joint determination of optimal investment and optimal depollution in a spatiotemporal framework where pollution is transboundary and spatial component is described by a network structure. Pollution is controlled at a global level by a central planner. The problem is solved explicitly and the optimal investment and depollution are found. In conclusione, some investigations on the impact of heterogeneity on the optimal path are performed.

Presenter: LEOCATA, Marta (LUISS Guido Carli)

Contribution ID: **15** Type: **not specified**

On response theory for climate models

Monday, 22 April 2024 14:30 (30 minutes)

In this talk I will give an overview on response theory for stochastic PDEs, with a particular interest on its potential for energy balance models. With response theory we mean in this context the following: studying the regularity of averages of observables against the stationary distribution with respect to changes in relevant system parameters, in particular their differentiability (linear response). We will also discuss the role of exponential ergodicity to establish linear response.

Presenter: CARIGI, Giulia (Università degli Studi dell'Aquila)