MICROCAUSALITY WITHOUT LORENTZ

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(see also Creminelli, Janssen, Salehian, Senatore)

MICROCAUSALITY

$$[\hat{O}(x), \hat{O}(y)] = 0 \qquad \text{for} \quad x \times y$$

- Fundamental property of relativistic QFT's
- Operator statement $\Rightarrow$ valid on <u>all</u> states.
- Can it be used to constrain Lorentz-violating EFT's for matter / cosmology ?

ON LORENTZ-VIOLATING STATES

$|\psi\rangle$ (OR $\hat{\rho}$) such that:

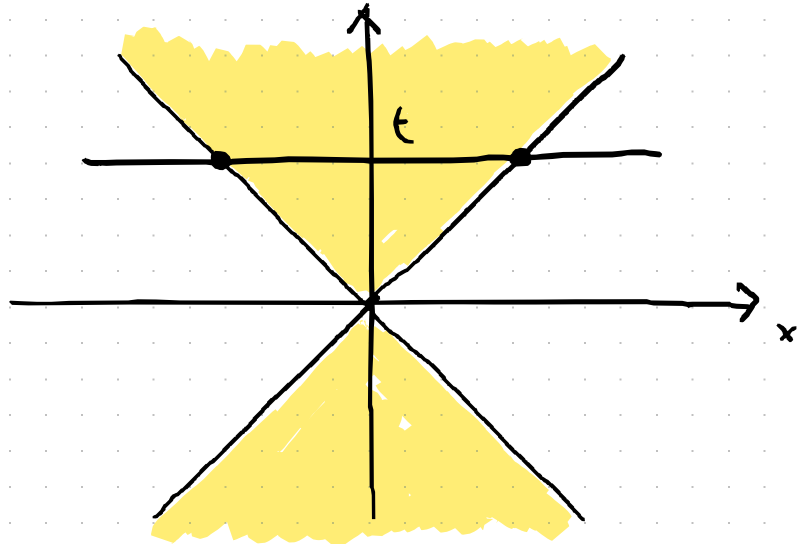
- boosts are BROKEN
- spatial translations/rotations are UNBROKEN

$$\Longrightarrow G_c(\vec{x},t) \equiv \langle [\mathcal{O}(x),\mathcal{O}(0)] \rangle = 0 \quad \text{for} \quad |\vec{x}| > |t|$$

$$\left(\text{but} \quad G_c \neq f(x^2) \right)$$

(Same for G_{ret}, G_{adv})

$\forall t$, compact support



Theorem (Paley-Wiener): $\tilde{G}_c(t, \vec{k})$ is

1) ANALYTIC in $\vec{k}$ everywhere

2) EXP. BOUNDED: $|\tilde{G}_c| \lesssim A(t, \vec{k})\, e^{|\mathrm{Im}\,\vec{k}|\,|t|}$

non-exponential

DATA

1) Phonon-like scalar:

$$S = \int \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}v^2(\vec{\nabla}\phi)^2$$

$$\tilde{G}_c(t,\vec{k}) = \frac{\sin(v|\vec{k}|t)}{v|\vec{k}|}$$

- analyticity: $\frac{\sin(\alpha)}{\alpha} = \sum_n \# \alpha^{2n} \Rightarrow$ analytic in $\vec{k}^2$

  $\Rightarrow$ analytic in $\vec{k}$ ✓

- exp. boundedness: $\left|e^{\pm i v k t}\right| \sim e^{\mp v \operatorname{Im} k\, t} \lesssim e^{|\operatorname{Im} k||t|}$

  $\Longleftrightarrow \quad v \lesssim 1$ ✓

2) Relativistic massive scalar

$$S = \int \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\phi^2$$

$$\tilde{G}_c(t,\vec{k}) = \frac{\sin(\omega_k t)}{\omega_k} \qquad \omega_k \equiv \sqrt{\vec{k}^2 + M^2}$$

- analyticity: as before ✓

- exp. boundedness:

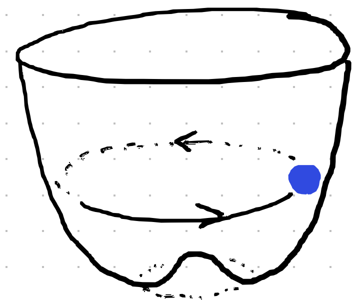
$$\left| e^{\pm i\omega_k t} \right| \sim e^{\mp \mathrm{Im}(\omega_k) t} \overset{?}{\leq} e^{|\mathrm{Im}\, k|\,|t|}$$

Subtle: $\omega_k > k$ for $k \in \mathbb{R}$, but $|\mathrm{Im}\,\omega_k| \leq |\mathrm{Im}\,k|$

Notice: for $k \to \mathbb{R}$, $\mathrm{Im}\,\omega_k / \mathrm{Im}\,k = \frac{\partial \omega_k}{\partial k}$ = group velocity! ✓

3) UV - complete superfluid

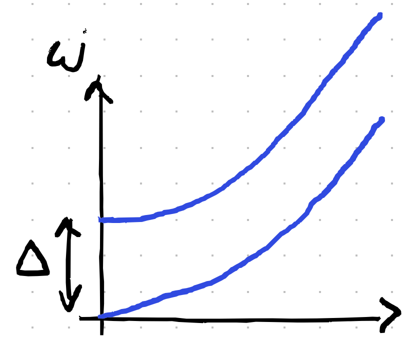
complex $|\Phi|^4$ at finite μ:



$$\tilde{G}_c(t,\vec{k}) = \sum_{\alpha=\pm} |Z_\alpha(\vec{k})|^2 \, \frac{\sin(\omega_\alpha(\vec{k})t)}{\omega_\alpha(\vec{k})}$$

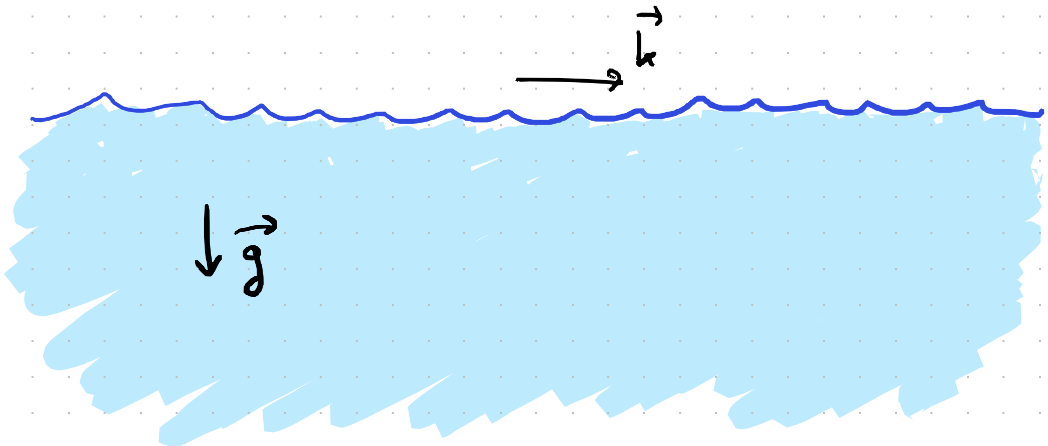
$$\omega_\pm(\vec{k}) = \sqrt{\vec{k}^2 + \frac{1}{2}\Delta^2\left(1 \pm \sqrt{1 + 4(1-c_s^2)\vec{k}^2/\Delta^2}\right)}$$

$$Z_\pm(\vec{k}) = \sqrt{1 \pm \frac{(1-2c_s^2)}{\sqrt{1+4(1-c_s^2)\vec{k}^2/\Delta^2}}}$$



- Individual contributions analytic at low $\vec{k}$
- Highly non-analytic at $\vec{k}^2 \gtrsim \Delta^2$
- Sum analytic everywhere!
- The two particles need each other, w/ finely tuned $\omega(\vec{k})$ and $Z(\vec{k}) = \langle \Omega | \Phi | \vec{k} \rangle$
- Equally non-trivial cancellations for hydrodynamical and inflationary correlators.

4) Ocean waves



Deep, incompressible limit: $\omega = \sqrt{g\,|\vec{k}|} \propto (\vec{k}^2)^{1/4}$

Highly non-analytic. Also: $v_g = \frac{\partial \omega}{\partial k} \sim \frac{1}{\sqrt{k}} \to \infty$

for $\vec{k} \to 0$

They should obey sonic microcausality

For compressible ($c_s < \infty$) ocean:

$$K = g/2c_s^2 \qquad \Omega = c_s K \qquad \left(\sim (200\,\mathrm{km})^{-1}\right)$$

- $k \gg K$, $\omega \gg \Omega$: same as before
- $k \sim K$, $\omega \sim \Omega$: expect major modifications
- Maybe

$$\omega(\vec{k}) = \sqrt{g|\vec{k}|}\; f\left(|\vec{k}|/K\right)$$

s.t. $\Rightarrow$ analyticity & exp. boundedness ?

- No. Resolution more interesting

Integrate out bulk modes

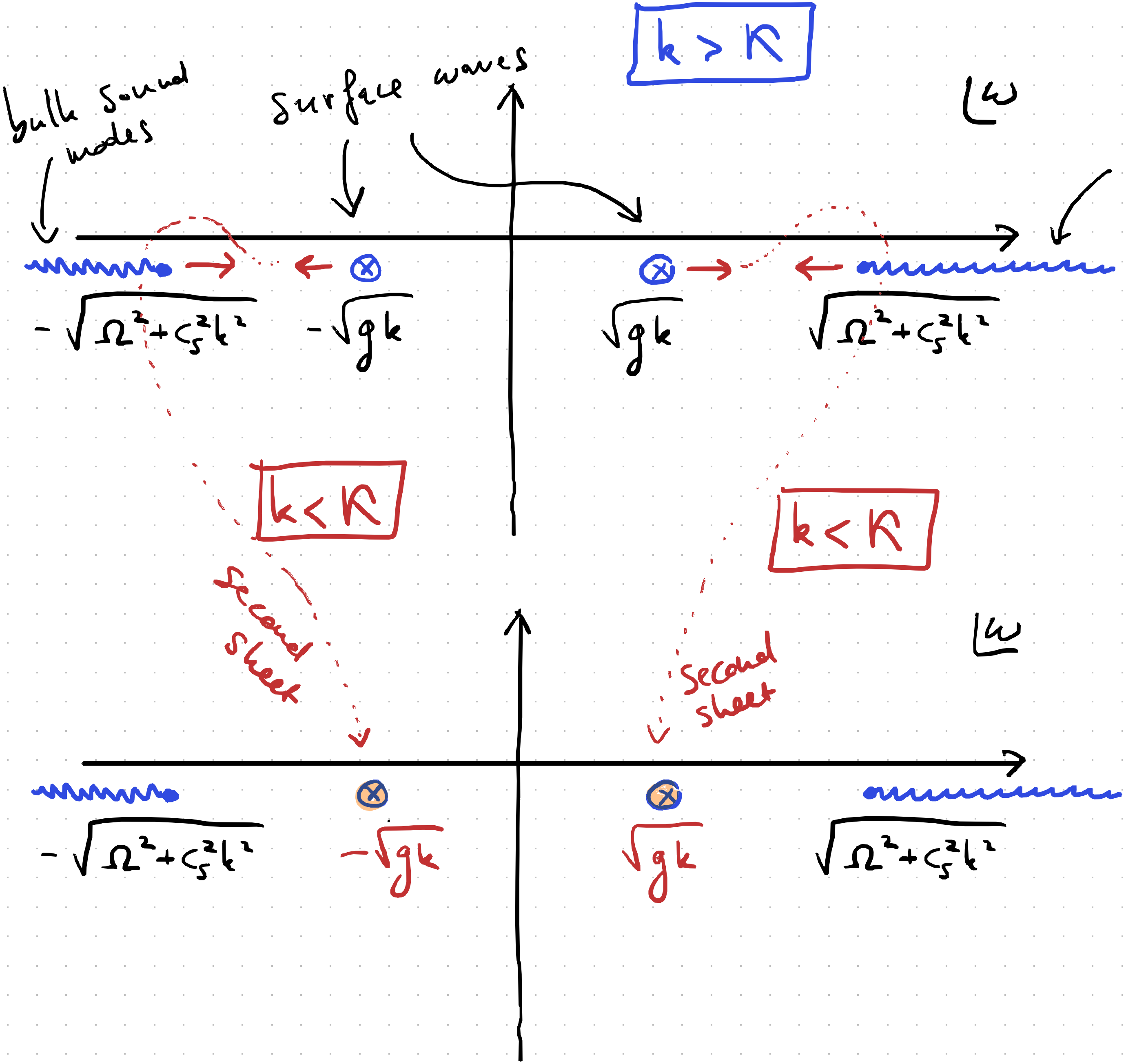
vertical displacement

$$\longrightarrow \quad S_{bdy} = \frac{1}{2}\int_{\omega,\vec{k}} \tilde{f}^*(\omega,\vec{k})\, K(\omega,\vec{k})\, f(\omega,\vec{k})$$

(non-local)

$$G_R(\omega,\vec{k}) = K(\omega+i\varepsilon,\vec{k})^{-1} = \frac{\omega^2-k^2}{\omega^2\left(1+\sqrt{1+\omega^2-k^2}\right)-2k^2}$$

(in K, Ω units)



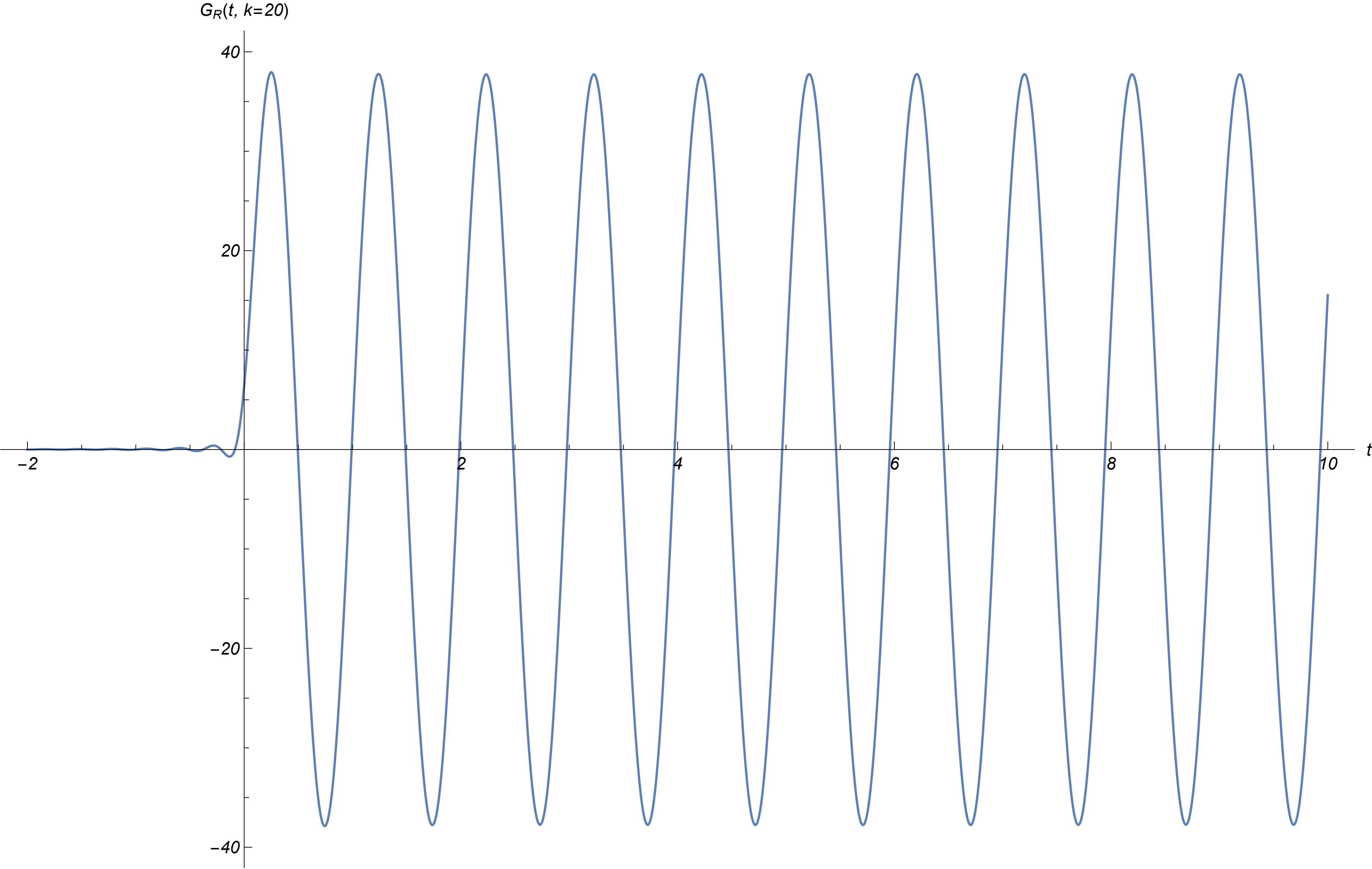
k > K
ω
bulk sound modes
surface waves
bulk sound modes
$-\sqrt{\Omega^2+c_s^2k^2}$
$-\sqrt{gk}$
$\sqrt{gk}$
$\sqrt{\Omega^2+c_s^2k^2}$
k < K
k < K
second sheet
second sheet
ω
$-\sqrt{\Omega^2+c_s^2k^2}$
$-\sqrt{gk}$
$\sqrt{gk}$
$\sqrt{\Omega^2+c_s^2k^2}$

So, from the poles:

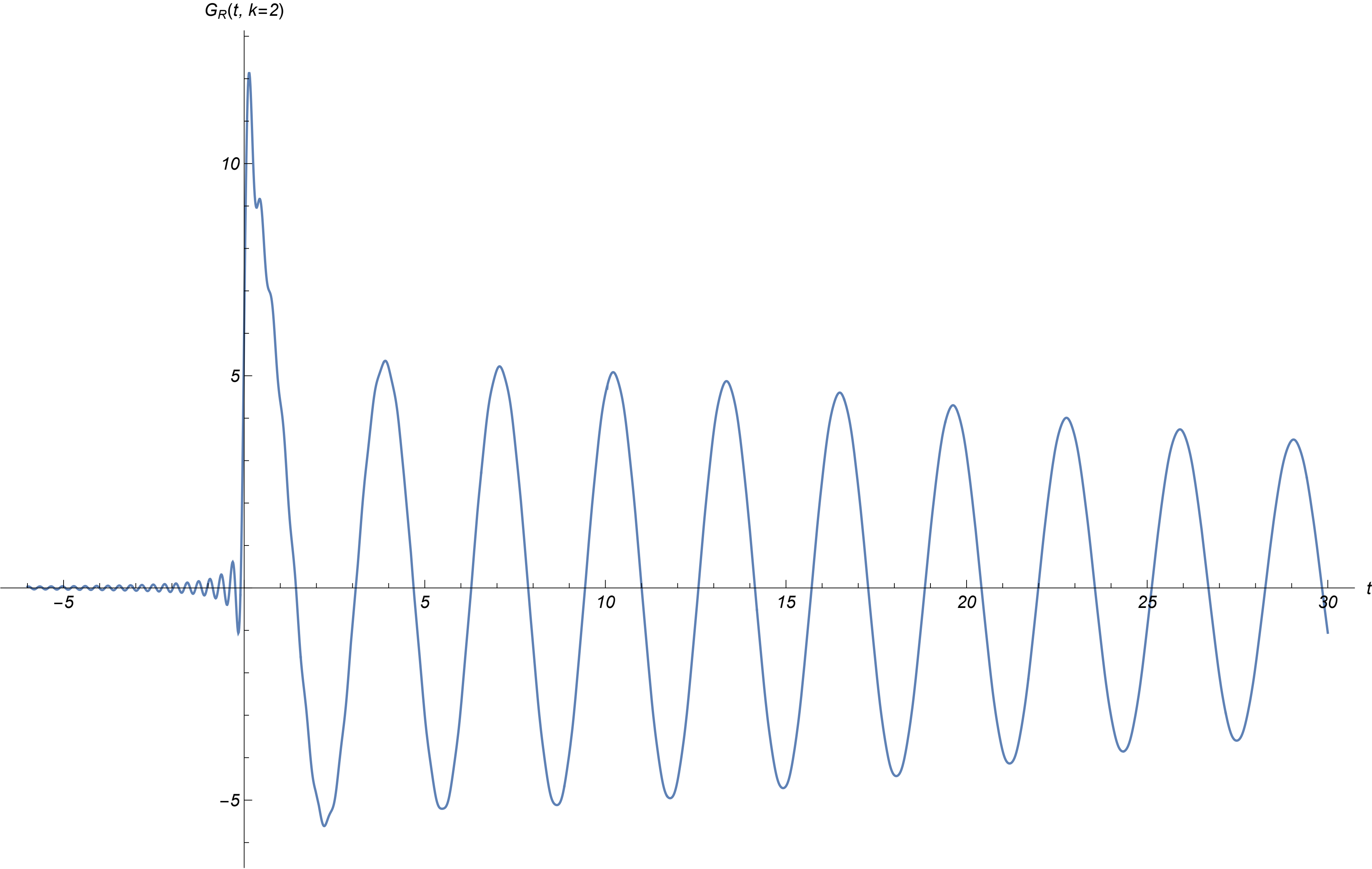
$$\tilde{G}(t, \vec{k}) \propto \Theta(|\vec{k}| - \kappa) \frac{\sin(\sqrt{g|\vec{k}|}\, t)}{\sqrt{g|\vec{k}|}}$$

but there are NO POLES for $k < \kappa$!

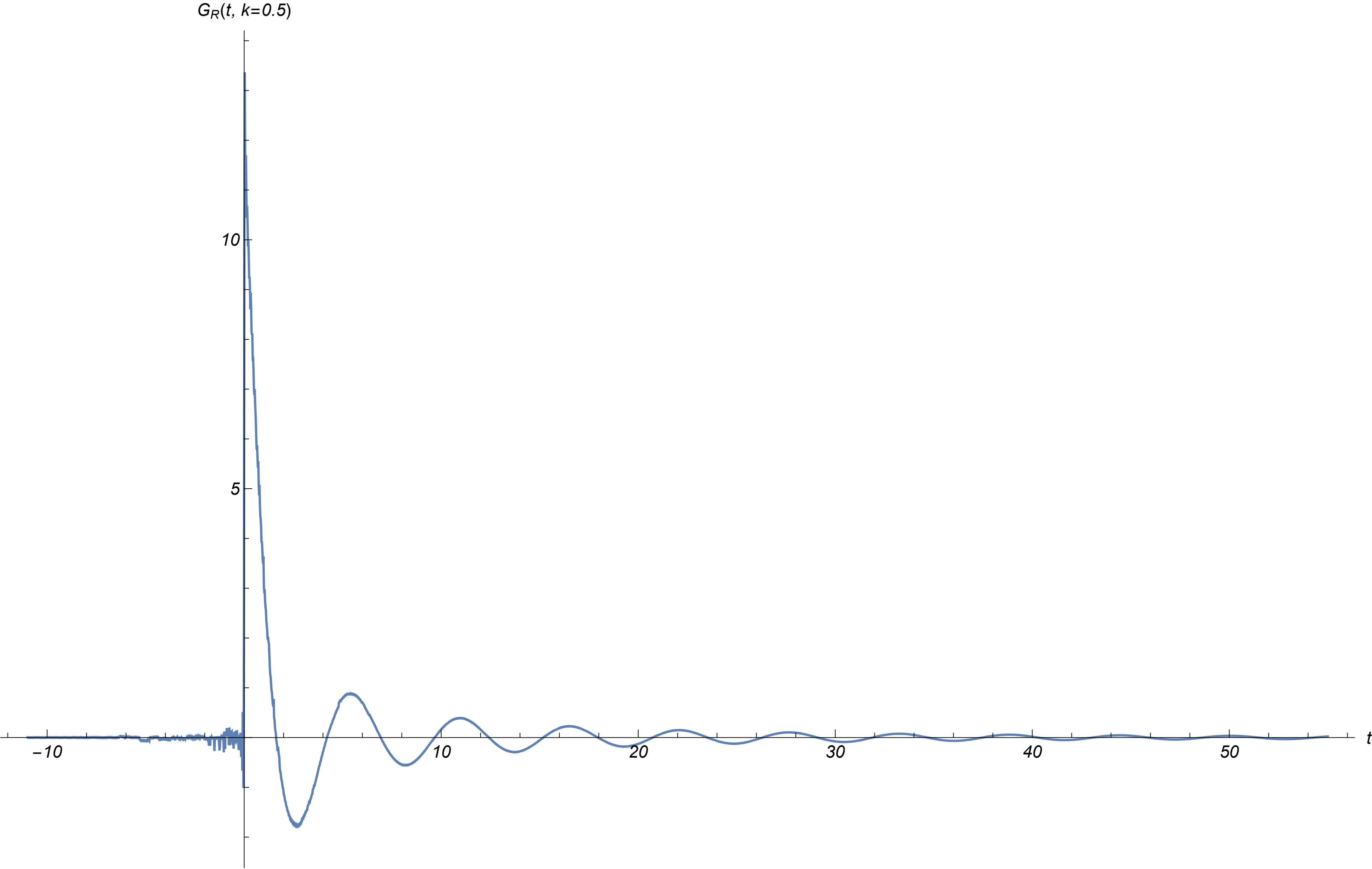
(Usually excitations better defined at smaller $\vec{k}$!)



G_R(t, k=20)
40
20
−20
−40
−2
2
4
6
8
10
t



$G_R(t, k=2)$
10
5
−5
t
−5
5
10
15
20
25
30



$G_R(t,\ k{=}0.5)$
10
5
−10
10
20
30
40
50
t

CONCLUSIONS

- Microcausality implies very precise properties of Lorentz-breaking correlators
- Different systems obey them in different ways.
- The Devil is in the details
- How do we use all this to constrain effective field theories for matter / cosmology ?