An introduction to Lean and Mathlib

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Examples of theorems proved with a computer

Theorem (Helfgott, 2015)

Every odd number ≥ 7 is the sum of three prime numbers.

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Theorem (Clausen-Scholze, 2019)

Let $0 < p < p' \leq 1$, and S profinite, and V a p-Banach space. Denote with $\mathcal{M}_{p'}(S)$ the p'-measures on S. Then, for $i \geqslant 1$,

$$\mathrm{Ext}^i_{\mathrm{Cond(Ab)}}(\mathcal{M}_{p'}(S), V) = 0.$$

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Comparison proof on the board / proof in Lean.

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Roughly $1.5 * 10^6$ lines of code.
Covers all the undergraduate curriculum, goes further in some areas. Still very few things in dynamics (we have rotation number, for instance, but not the Krylov-Bogolyubov theorem that a continuous map on a compact space has an invariant probability measure).

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Example : Gowers, Green, Manners, Tao, On a conjecture of Marton, preprint on arxiv on November 9, 2023. Collaborative formalization project started by Tao. Finished in roughly one month, way before the referee was done with the paper.

Resources:

- Natural number game `https://adam.math.hhu.de/#/g/leanprover-community/nng4`: like a video game
- Mathematics in Lean `https://leanprover-community.github.io/mathematics_in_lean/`
- Downloading and using mathlib: `https://leanprover-community.github.io/index.html`
- Contributing to the library: `https://github.com/leanprover-community/mathlib4`