An unusual BPS equation

Costas Bachas

Département de Physique
École normale supérieure

April 3, 2025 (S.N. Pisa)

Based on papers with Zhongwu **Chen**

arXiv: 2404.14998 [hep-th]

& Zhongwu + Lorenzo **Bianchi**

arXiv: 2501.13197 [hep-th]

NB: *motivated by earlier work on* holographic energy transport $\longleftrightarrow$ brane tension

Shira Chapman, Giuseppe Policastro, Dongsheng Ge, Vassilis Papadopoulos, Stefano Baiguera, Tal Schwartzman

2006.11333 [hep-th] , 2107.00965 [hep-th] , 2212.14058 [hep-th]

This talk

1. The BPS equation

2. Gravitational tension = Inertial tension gravity

3. Radiation from accelerating charges CFT

4. The proof

5. Remarks

1. The BPS equation

In this talk I will sketch the proof of an unusual BPS equation, and explain why it is interesting. The context is Defect CFT

Defects are non-local observables that have played a key role in the early development of QFT, and occupy center stage in its modern versions

***Heavy** quarks (Wilson lines), Kondo impurities, BCFT,*

generalized symmetries, quantum devices, . . .

Two universal characteristics of a defect are

(a) the energy-momentum stored in its fields

(b) its resistance to deformations

In a CFT these depend on two parameters that control the 1-point function $\langle T^{\mu\nu} \rangle$

and the 2-point function of the defect-displacement operator $\langle D^j D^k \rangle$

This latter is defined by the Ward identity of broken translation symmetry

$$\partial_\mu T^{\mu j} = \delta^{(d-p)}(x_\perp)\, D^j$$

Here p is the dimension of the defect:

$p = 0$	local operator
$p = 1$	line defect
....	
$p = d-1$	interface/boundary

For a static (hyper)planar defect the unbroken conformal symmetry implies:

$$\langle T^{\alpha\beta}(x)\rangle = a_T \left(\frac{d-p-1}{d}\right)\frac{\eta^{\alpha\beta}}{|x_\perp|^d}\,, \qquad \langle T^{\alpha j}(x)\rangle = 0 \qquad \text{energy}$$

$$\langle T^{ij}(x)\rangle = a_T \left[\frac{x^i x^j}{|x_\perp|^{d+2}} - \left(\frac{p+1}{d}\right)\frac{\delta^{ij}}{|x_\perp|^d}\right] \qquad \text{stress}$$

$$\langle D^j(y)D^k(0)\rangle = \frac{C_D\,\delta^{jk}}{|y|^{2p+2}}$$

C_D is not defined for local operators $(p=0)$

a_T is not defined for boundaries/interfaces $(p=d-1)$

These parameters (part of the DCFT data) enter in many physical observables.

Unitarity $\Longrightarrow$ $C_D > 0$

Positivity of energy $\Longrightarrow$ $a_T < 0$

Other than that the parameters vary independently. When both are defined $(0 < p < d-1)$ the BPS equation reads

$$\boxed{\frac{C_D}{a_T} = -\frac{2(d-1)(p+2)\Gamma(p+1)}{d\,\pi^{p-d/2}\,\Gamma(\frac{p}{2}+1)\Gamma(\frac{d-p}{2})}}$$

Lewkowycz, Maldacena 1312.5682; Bianchi, Lemos, Meineri 1805.04111; $p = 1,\, d = 4$

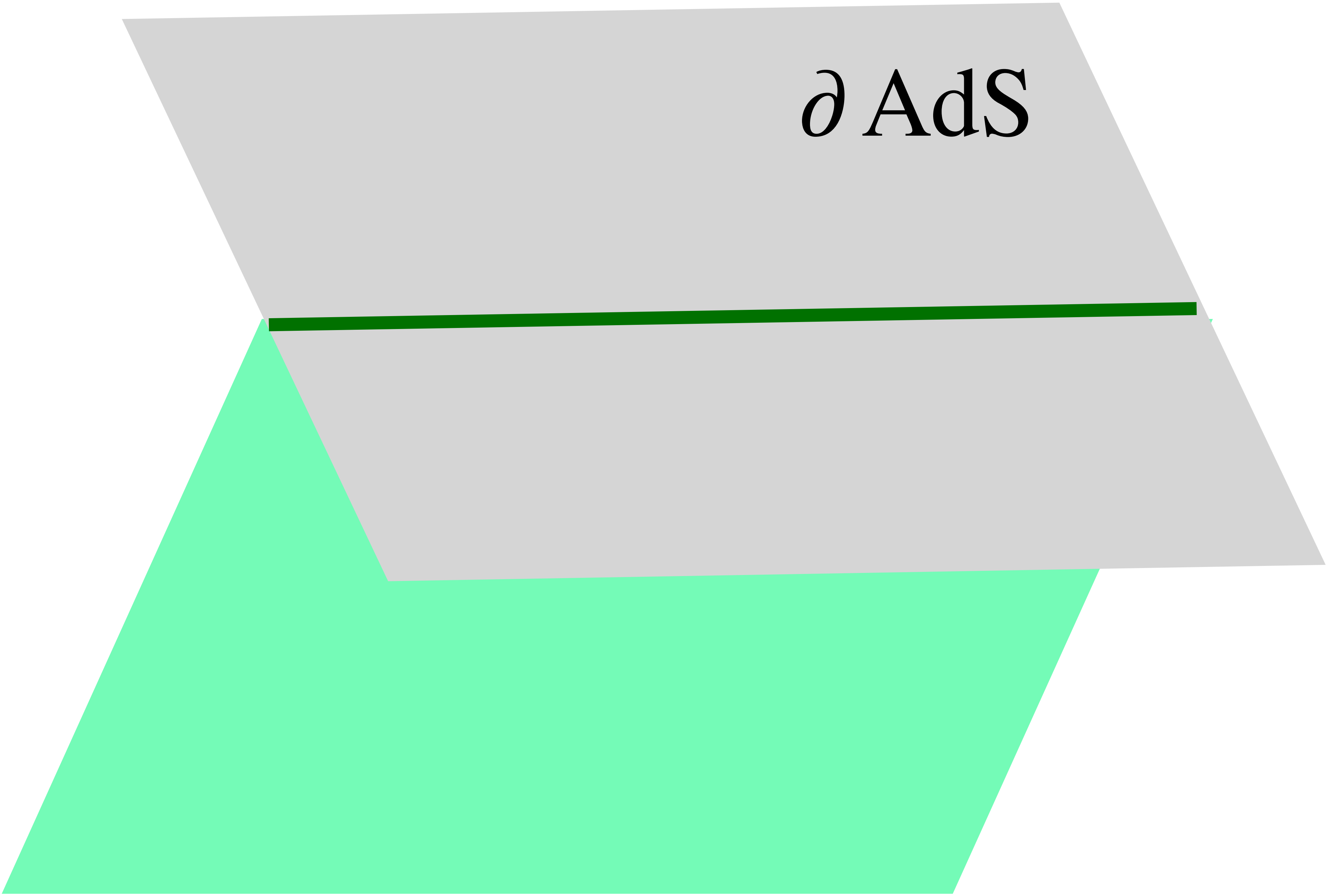
. . . Bianchi, Lemos 1911.05082 conjectured $\forall p, d$

I will outline the proof of this conjecture at the end of the talk

First I will reexpress this `BPS' equation in two different ways which bring

to light two very different physical implications/interpretations.

2. Gravitational tension = Inertial tension



In AdS/CFT a p-dimensional *defect* is (the boundary anchor) of a p-brane

Maldacena '98, Karch-Randall '01, . . .

Local operators *(p=0)* in particular are endpoints of particle worldlines. For large operators the dual is a **black hole** whose mass is not locally defined

But (an extension) of ADM mass/energy exists in aAdS spacetime

Abbot, Deser '82 , Hawking, Horowitz '96 , . . .

It is the **dilatation charge** of the dual operator, *e.g.* for unit-radius AdS_4

$$\Delta = \boxed{\frac{3}{2} + \sqrt{m_0^{\,2} + \frac{9}{4}} \;+\; \text{GR corrections}} = \boxed{m_0 \left[\, 1 + \frac{3}{2m_0} + \frac{9}{8m_0^2} + \cdots + G_N m_0 + (G_N m_0)^2 + \cdots \,\right]}$$

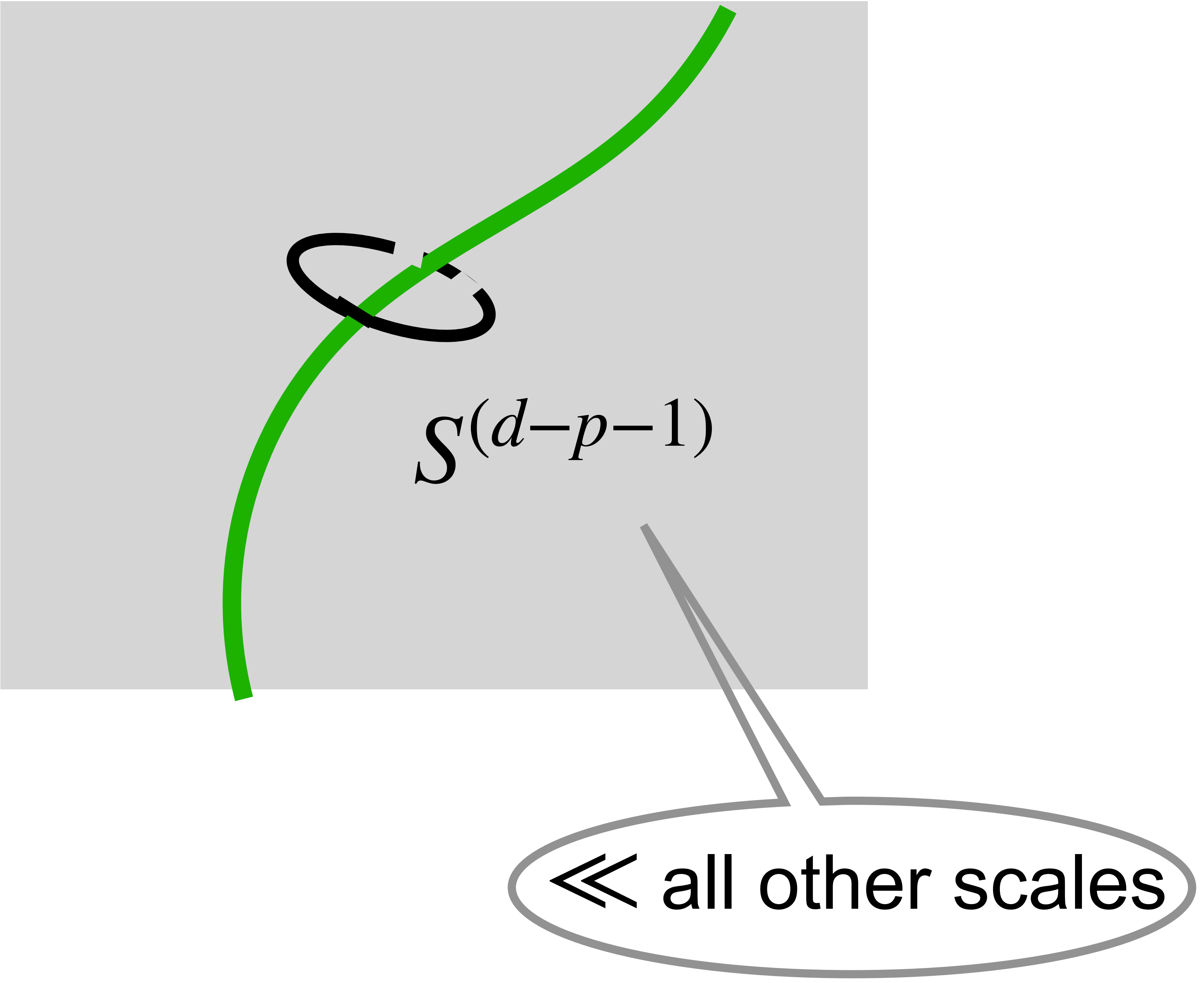
$\lambda_{\text{Compton}} \ll 1$

$r_{\text{Schwarzschild}} \ll 1$

The ADM mass resums the *classical GR corrections*, whereas Δ resums everything *(quantum fluctuations, intermediate scales etc)* . It is exact if one knows the dual microscopic CFT.

Can one define similarly an invariant brane tension ?

The answer is yes. One can define the invariant **gravitational tension** of a *p*-brane in AdS as an integral of the dilatation current around the dual defect on $\partial\,\mathrm{AdS}$



$$\sigma_{\rm gr} \;\propto\; \oint ds^j\, x^\mu \langle T_{\mu j}\rangle$$

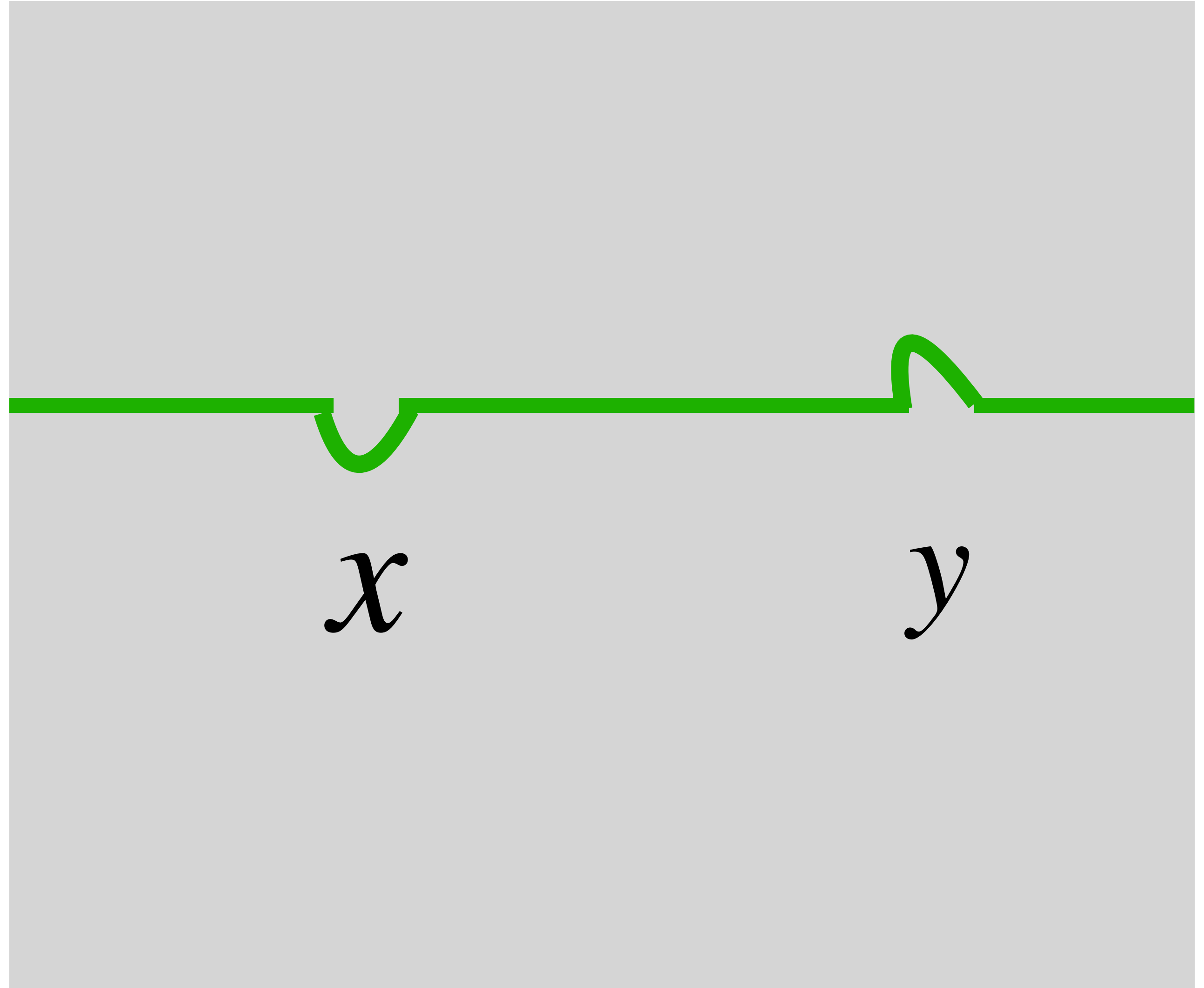
$$\Longrightarrow \quad \boxed{\sigma_{\rm gr} = \gamma_{\rm gr}\, a_T}$$

#

If a DCFT is known this is exact. Otherwise can be used as a proxy to find the ADM definition of tension in GR defined in the early days of holography

Myers '99; Townsend & Zamaklar '01; Traschen, Fox '01; Harmark, Obers '04

But for extended objects ($p>0$) there is another measure of tension, that one would use for a (non-gravitating) violin string. It is an **inertial tension** or **stiffness**



$$\boxed{\sigma_{\rm in} \;=\; \underbrace{\gamma_{\rm in}}_{\#}\, C_D}$$

As opposed to the ADM tension that depends on the metric far from the brane, stiffness depends on the behaviour of the displacement field near the AdS boundary

The universal prefactors $\gamma_{\rm gr}$ and $\gamma_{\rm in}$ can be fixed by requesting that for a classical, probe brane in GR both tensions reduce to the (bare) tension of the effective Nambu-Goto action. For this, one must extract a_T and C_D in Einstein gravity + Nambu-Goto using the appropriate Witten diagrams.

The result is:

$$\gamma_{\rm gr} = -\frac{2(d-1)\pi^{(d-p)/2}}{d\,\Gamma(\frac{d-p}{2})}\,, \qquad \gamma_{\rm in} = \frac{\pi^{p/2}\,\Gamma(\frac{p}{2}+1)}{(p+2)\Gamma(p+1)}$$

Inserting in the (conjectured) BPS equation one finds

$$\boxed{\sigma_{\rm gr} = \sigma_{\rm in}}$$

CB, Chen 2404.14998

Although the relevant Witten diagrams are tree-level, this calculation is non-trivial because there are no global Fefferman-Graham coordinates for both bulk and brane fields. Absolute normalizations matter, and one uses different boundary cutoffs for $\langle T^{\mu\nu}\rangle$ and $\langle D^j D^k\rangle$

To ensure consistency, we calculated also the 2pt function $\langle T^{\mu\nu} D^j\rangle$ and checked that it verifies the Ward identities of broken translation symmetry

$$\boxed{\langle D^j \int_{x_\perp} \partial_\mu T^{\mu k}\rangle \sim \langle D^j D^k\rangle \qquad \text{and} \qquad \langle T^{\mu\nu} \int_{x_\parallel} D^j\rangle \sim \partial^j \langle T^{\mu\nu}\rangle}$$

Schematically $\langle TD\rangle \sim \langle DD\rangle$ and $\langle TD\rangle \sim \langle T\rangle$, so these fix unambiguously both bulk fields

An example: the *F*-string in $\mathrm{AdS}_5 \times \mathrm{S}^5$ $\longleftrightarrow$ line defect in $\mathcal{N} = 4$ SYM

$$\boxed{C_D = -\,18 a_T} \;=\; \frac{6}{\pi^2}\lambda\partial_\lambda \log\left(\frac{1}{N}\, e^{\lambda/8N}\, \underbrace{L^1_{N-1}}_{\text{modified Laguerre}}\left(-\frac{\lambda}{4N}\right)\right)$$

Erickson, Semenoff, Zarembo '00 ; Drukker, Gross '00 ; Pestun '09

Both tensions receive different types of corrections, but they remain equal !

$$\sigma_{\rm gr} = \sigma_{\rm in} \;=\; \underbrace{\frac{\sqrt{\lambda}}{2\pi}}_{\sigma_{\rm NG}} \left[\,1 \;+\; \underbrace{O\!\left(\frac{1}{\sqrt{\lambda}}\right)}_{\text{Compton}} \;+\; \underbrace{O\!\left(\frac{\sqrt{\lambda}}{N}\right)}_{\text{D}-\text{string Compton}} \;+\; \underbrace{O\!\left(\frac{\sqrt{\lambda}}{N^2}\right)}_{\text{BH radius}}\,\right]$$

The equality does not hold for non-supersymmetric Wilson lines

Usually susy equations relate mass or tension to charge.

This one relates two tensions, whence the title of the talk.

3. Radiation from accelerating defects

Let me now recall the problem at the origin of the conjecture. Lewkowycz, Maldacena '13 were trying to reconcile two different calculations of the energy radiated by an accelerating half-BPS quark in $\mathcal{N} = 4 \;\; \mathrm{SYM}$:

(i) from an instanteous ``kick" $\propto$ $\text{(Bremstrahlung function)} \;=\; \frac{1}{12} C_D$

(ii) from a uniformly- accelerating quark which is conformal to a static one

Correa, Henn, Maldacena, Sever '12;

Fiol, Garolera, Lewkowwycz '12

They attributed this disagreement to the problem of separating radiation from the self-energy stored in fields of the quark.

They also noticed that the supersymmetric multiplet of $T^{\mu\nu}$ contains a scalar with dimension $\Delta_{\mathcal{O}} = d - 2$, and proposed that the problem is resolved if the radiation is computed with the modified energy-momentum tensor

$$\boxed{\tilde{T}^{\mu\nu} = T^{\mu\nu} + \xi\,(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\mathcal{O}}$$

conserved but not traceless

The intuition is correct, but the role of supersymmetry remained unclear.

To better understand it we note first that

$$\langle \mathcal{O}(x) \rangle = \frac{a_{\mathcal{O}}}{|x_\perp|^{d-2}} \quad \Longrightarrow \quad \langle \mathcal{O}(x) D^j(y) \rangle = b_{\mathcal{O}} \frac{x^j |x_\perp|^{p-d+2}}{|x-y|^{2(p+1)}}$$

$$\text{with} \quad b_{\mathcal{O}} = 2^p\,\Gamma(\frac{p+1}{2})\pi^{-(p+1)/2}(d-2)\,a_{\mathcal{O}}\ .$$

The 1pt function of the modified stress tensor reads

$$\langle \tilde{T}^{\mu\nu} \rangle = \langle T^{\mu\nu} \rangle + \xi a_{\mathcal{O}} (\eta^{\mu\nu} \Box - \partial^\mu \partial^\nu) |x_\perp|^{2-d}$$

The problem arises because the fields have singular stress, $\langle T^{ik} \rangle \neq 0$

Provided that $a_{\mathcal{O}} \neq 0$ it is possible to remove the stress of the static fields by choosing

$$\boxed{\xi_{\text{defect}} = \frac{a_T}{d(d-2)\, a_{\mathcal{O}}}} \implies \langle \tilde{T}^{ik} \rangle = 0$$

not to confuse with dual brane

The leading singularity of $\tilde{T}^{\alpha\beta}$ can be absorbed in the renormalized **CFT defect** tension,

e.g. for a line defect : $T^{\mu\nu}_{\text{tot}}(x) = m_{\text{ren}} \int d\tau\, \delta^{(d)}(x - Y(\tau))\, \dot{Y}^\mu \dot{Y}^\nu + \tilde{T}^{\mu\nu}_{\text{reg}}(x)$

This does not seem to require susy. But unbroken susy ensures two things:

♠ The modified energy-momentum tensor is the same for all defects

♣ It does not violate the Null Energy Condition

Let's verify these statements with a simple example: $\mathcal{N}=2\ \ \mathrm{SQED}_4$

A general conformal line defect is a charge coupling linearly to the scalar fields

$$W=\exp\Big(\int ds\,(ieA_\mu\,\dot{Y}^\mu+g\,|\,\dot{Y}\,|\,\hat{n}^I\phi_I)\Big)\ , \qquad \text{where} \quad \hat{n}^I\hat{n}_I=1\,.$$

The defect is half-BPS iff $e=\pm g$

The e-m tensor, displacement & classical background fields are:

$$T^{(s)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}|\partial\phi|^2 + \boxed{\frac{1}{6}(\eta^{\mu\nu}\Box - \partial^\mu\partial^\nu)\phi^2},$$

$$T^{(\mathrm{v})}_{\mu\nu} = F_\mu^{\ \rho}F_{\nu\rho} - \frac{1}{4}\eta_{\mu\nu}|F|^2 ,$$

$$D^j = eF^{0j} + g\,\partial^j\phi ,$$

$$\phi_{\mathrm{class}} = \frac{g}{4\pi|x_\perp|} \quad \text{and} \quad F^{0j}_{\mathrm{class}} = \frac{ex^j}{4\pi|x_\perp|^3} .$$

The scalar operator is $\mathcal{O} = \phi^2$ and $\langle\phi^2\rangle = \dfrac{g^2}{16\pi^2|x_\perp|^2}$, so to cancel the static stress

$$\boxed{\xi_{\mathrm{defect}} = \frac{a_T}{8a_{\mathcal{O}}} = -\frac{g^2+3e^2}{24g^2}}$$

The stress can be removed for any e, g . But something special happens for $e = g$:

In this case $\xi = -1/6$, and the improvement precisely removes the $\int R\phi^2$ contribution to $T^{\mu\nu}$ which violates the Null Energy Condition.

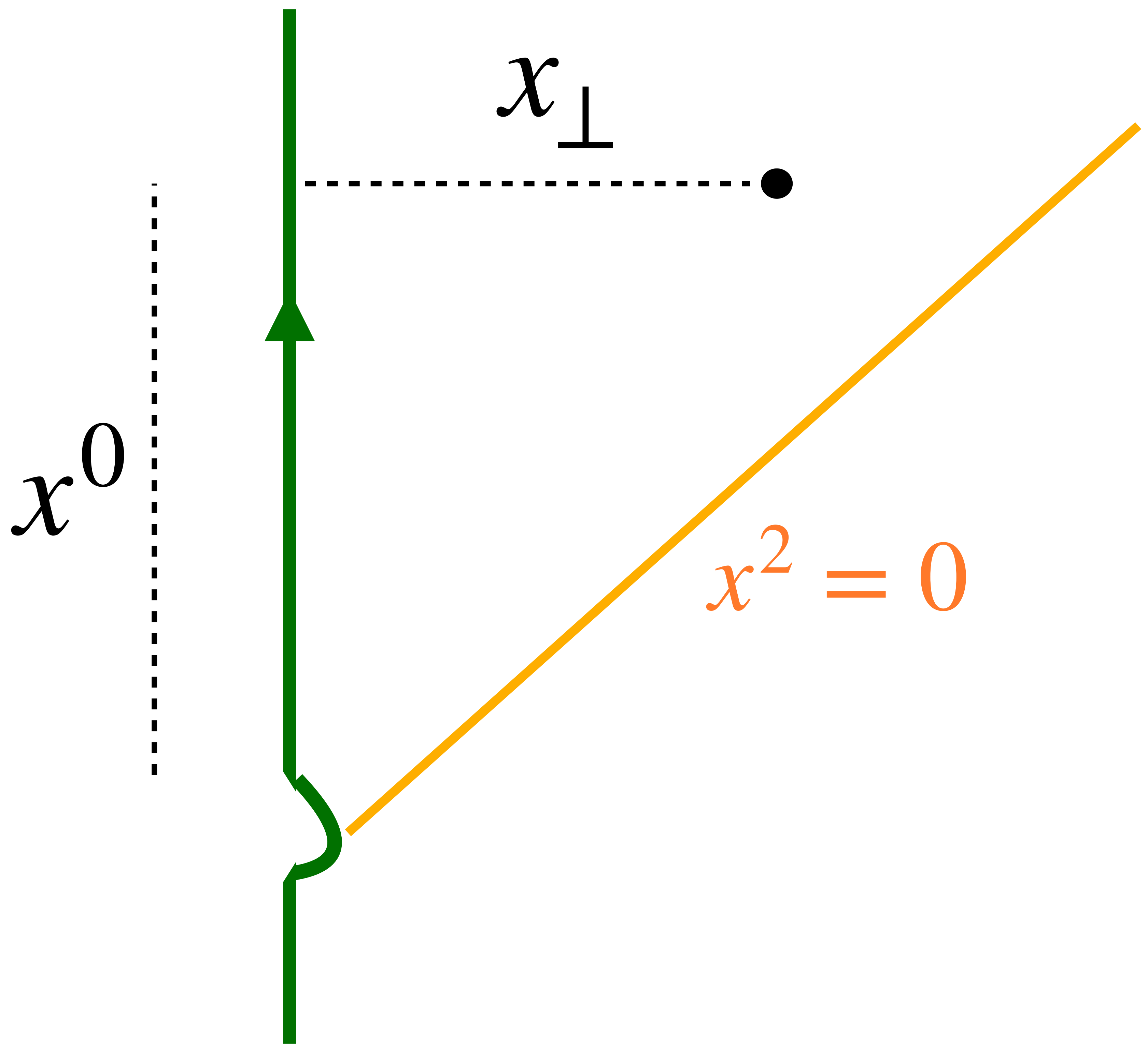
no definite sign for $k^\mu k^\nu \partial_\mu \partial_\nu \phi^2$

This generalizes to all superconformal defects:

Supersymmetry guarantees that the modification of $T^{\mu\nu}$ that cancels the *transverse stress* stored in the static fields, also removes the *NEC-violating radiation*.

To see why consider the energy flux in the background of a moving defect:

$$\langle\, T^{0k}(x)\, e^{\int y^j D^j} \,\rangle \;=\; \langle\, T^{0k} \,\rangle \;+\; \int y^j \langle\, T^{0k}(x)\, D^j(0) \,\rangle \;+\; \cdots$$



The 2pt function is fixed by Ward identities

in terms of $\langle T^{\mu\nu}\rangle$ and $\langle D^j D^k\rangle$;

it has singularities at $x^2 = 0$ and $x_\perp = 0$.

Explicitly:

$$\langle T^{0k}(x) D^j(y)\rangle = \frac{(x-y)^0}{|x_\perp|^{d-p}|x-y|^{2p+4}}\left(-\mathfrak{b}_3\,\delta^{kj}|x_\perp|^2 + (\mathfrak{b}_3 - 2\mathfrak{b}_1)\,x^k x^j + \boxed{4\mathfrak{b}_1\,\frac{x^k x^j |x_\perp|^2}{|x-y|^2}}\right)$$

where the Ward identities of broken translation symmetry impose

$$(p+1)\mathfrak{b}_2 + \mathfrak{b}_1 = \frac{d}{2}\mathfrak{b}_3\,, \qquad \mathfrak{b}_3 = 2^{p+2}\pi^{-(p+1)/2}\,\Gamma(\frac{p+3}{2})\,a_{\mathrm{T}}\,,$$

and $$2p\,\mathfrak{b}_2 - (2d-p-2)\,\mathfrak{b}_3 = \frac{(d-p)\Gamma(\frac{d-p}{2})}{\pi^{(d-p)/2}}\,C_D\,.$$

Billo, Goncalves, Lauria, Meineri '16

In terms of the above parameters the BPS equation reads

$$\boxed{2d\,\mathfrak{b}_1 = (p+2)\mathfrak{b}_3}$$

The NEC-violating radiation comes from the leading lightcone singularity:

$$\langle T^{0k}(x) D^j(y)\rangle = -\frac{32\,\mathfrak{b}_1\,|x_\perp|^{p+2-d}}{p(p+1)(p+2)}\,\partial^0\partial^k\partial^j\left(\frac{1}{|x-y|^{2p}}\right) + \text{subleading}\,.$$

For 4d line defects,* replacing the Green function by the retarded propagator

$$\frac{1}{4\pi^2|x-y|^2} \to \Delta_R(x-y) = \frac{i\theta(x^0-y^0)}{2\pi}\,\delta\big(|x-y|^2\big)$$

and integrating by parts, as in the Lienard-Wiechert calculation, gives a flux $\propto\ \dot{a}$

which has no definite sign (contrary to the usual $\propto\ a^2$)

derivative of acceleration

* *This generalizes using the analytic continuation of Euclidean-space CFT correlators*

This term is absent in Maxwell theory for which $\mathfrak{b}_1 = 0$.

It can be removed in from the modified e-m tensor $\langle \tilde{T}^{i0} D^j \rangle$ more generally

by choosing the subtraction parameter

$$\xi_{\rm NEC}\, b_{\mathcal{O}}(p+1)(p+2) = \mathfrak{b}_1$$

To remove the static stress, on the other hand, requires

$$\xi_{\rm stress}\, b_{\mathcal{O}}(p+1) = \frac{a_T}{d}\,(p+1)\,2^p\,\Gamma(\frac{p+1}{2})\pi^{-(p+1)/2} = \frac{\mathfrak{b}_3}{2d}\,.$$

Thus $\boxed{\xi_{\rm NEC} = \xi_{\rm defect}}$ iff the BPS condition is obeyed, as claimed.

Choosing ξ to remove the NEC-violating term gives more generally

$$\langle \tilde{T}^{0k}(x) D^j(y) \rangle = -\, \mathfrak{b}_3 (1 - \frac{1}{d}) \frac{|x_\perp|^{p-d+2}}{|x-y|^{2p+4}} (x^0 - y^0) \delta^{kj} + [\text{non susy}]$$

fixed by boosting the static fields

vanishes iff BPS

i.e. susy cancels all $k \neq j$ terms which arise from `*dragging of the stress*'. If *z* is a complex transverse coordinate we then have $\langle \tilde{T}^{0z} D^z \rangle = 0$. Equivalently, the BPS equation reads

$$\boxed{\langle T^{0z}(x) D^z(y) \rangle \propto \frac{\partial}{\partial x^0} \frac{\partial}{\partial \bar{z}} \left(\frac{z\,|x_\perp|^{p-d+2}}{|x-y|^{2p+2}} \right)}$$

This is the starting point of the general proof

4. The proof

. . . . or rather its close cousin

$$\boxed{\langle T_{zz}(x)\, D_z(y)\rangle \;\propto\; \frac{\partial^2}{\partial z^2}\left(\frac{\bar{z}\,|x_\perp|^{p-d+2}}{|x-y|^{2p+2}}\right) \quad \text{if superconformal}\ ,}$$

I sketch here the main steps, and refer to CB, Bianchi, Chen 2501.13197 for details.

It suffices to prove this relation for all *minimally-superconformal* defects, i.e. given a pair (d, p) for the minimal bulk susy that admits susy p-defects. We assume that these latter preserve transverse-rotation symmetry.

Minimally-superconformal defects

defect	$(d,\mathcal{N})$	superalgebra	p-embedding
line	(3,2)	$\mathfrak{osp}(2\|4;\mathbb{R})$	$\mathfrak{su}(1,1\|1)\oplus\mathfrak{u}(1)_{\mathrm{c}}$
	(4,2)	$\mathfrak{su}(2,2\|2)$	$\mathfrak{osp}(4^*\|2)$
	(5,1)	$F(4;2)$	$D(2,1;2;0)\oplus\mathfrak{su}(2)_{\mathrm{c}}$
surface	(4,1)	$\mathfrak{su}(2,2\|1)$	$\mathfrak{su}(1,1\|1)\oplus\mathfrak{su}(1,1)_{\mathrm{c}}\oplus\mathfrak{u}(1)_{\mathrm{c}}$
	(5,1)	$F(4;2)$	$D(2,1;2;0)\oplus\mathfrak{so}(2,1)_{\mathrm{c}}$
	(6,1)	$\mathfrak{osp}(8^*\|2)$	$\mathfrak{osp}(4^*\|2)\oplus\mathfrak{so}(2,1)_{\mathrm{c}}\oplus\mathfrak{so}(3)_{\mathrm{c}}$
$p=3$	(5,1)	$F(4;2)$	$\mathfrak{osp}(2\|4;\mathbb{R})\oplus\mathfrak{u}(1)_{\mathrm{c}}$
$p=4$	(6,1)	$\mathfrak{osp}(8^*\|2)$	$\mathfrak{su}(2,2\|1)\oplus\mathfrak{u}(1)_{\mathrm{c}}$

Table 1. The minimal supersymmetric DCFTs discussed in the paper. The second column gives the smallest $\mathcal{N}$ that a d-dimensional SCFT must have to admit p-dimensional superconformal defects. The corresponding bulk superalgebras and maximal p-embeddings are given in the third and fourth columns. The subscript 'c' denotes bosonic subalgebras that commute with the preserved supercharges (for details see section 2). For missing (p,d) pairs superconformal and rotation-invariant defects do not exist.

The table was compiled by examining all possible superalgebra embeddings.

We have adapted to our purposes and extended the methods in

D'Hoker, Estes, Gutperle, Krym, Sorba '08; Gutperle, Kaidi, Raaj '17; Agmon, Wang '20

cf. also M. Duff's 'superconforrmal brane scan'

Note that the 6*d* theory has no odd-dimensional susy defects.

Note also that in all cases other than 4*d* $\mathcal{N} = 1$ and 3*d* $\mathcal{N} = 2$ the stress-tensor multiplet contains a scalar that can be used to define $\tilde{T}^{\mu\nu}$. When it does not the discussion of radiation does not apply. But the BPS equation makes no reference to a scalar or to $\tilde{T}^{\mu\nu}$, so our proof holds in all cases.

In order to unify the proofs we chose special bases of γ - matrices and of the defect orientation. The transverse coordinate is $z = x^1 + ix^2$

d	p	defect directions	preserved supercharges
6	2	3,4	$Q^1_1,\ Q^1_3,\ Q^2_1,\ Q^2_3$
	4	3,4,5,6	$Q^1_1,\ Q^1_2,\ Q^2_3,\ Q^2_4$
5	1	5	$Q^1_1,\ Q^1_4,\ Q^2_1,\ Q^2_4$
	2	3,4	$Q^1_1,\ Q^1_3,\ Q^2_1,\ Q^2_3$
	3	3,4,5	$Q^1_1,\ Q^1_2,\ Q^2_3,\ Q^2_4$
4	1	4	$\mathrm{Re}\{\mathcal{Q}^1_1,\ \mathcal{Q}^1_2,\ \mathcal{Q}^2_1,\ \mathcal{Q}^2_2\}$
	2	3,4	$Q_1,\ \bar{Q}_2$
3	1	3	$Q_1,\ \bar{Q}_2$

R-sym

Q^A_α

spinor

Table 2. The preserved supercharges for all the DCFTs of table 1. The directions 1,2 are always transverse to the defect worldvolume, while the parallel directions are shown in the third column above. For $(d, p) = (4, 1)$, the supercharges are complex and only the real parts shown in the table are unbroken. Our proof uses Ward identities of the first preserved supercharge in each case.

Let me show the proof for $d=6$, all other dimensions work in the same way.

One needs the susy transformations of the $T_{\mu\nu}$ supermultiplet, schematically

$$O \xrightarrow{Q} \chi^A_\alpha \xrightarrow{Q} j^I_\mu \oplus H_{\mu\nu\rho} \xrightarrow{Q} J^A_{\mu\alpha} \xrightarrow{Q} T_{\mu\nu}$$

$$Q^A_\alpha(O) = \chi^A_\alpha$$

$$Q^A_\alpha(\chi^B_\beta) = j^I_\mu\,(\gamma^\mu)_{\alpha\beta}(\sigma_I)^{AB} + H_{\mu\nu\rho}\,(\gamma^{\mu\nu\rho})_{\alpha\beta}\,\varepsilon^{AB} + (\partial_\mu O)\,(\gamma^\mu)_{\alpha\beta}\,\varepsilon^{AB}$$

$$Q^A_\alpha(j^I_\mu) = \frac{1}{2}J^B_{\mu\alpha}\,(\sigma^I)_B{}^A - \frac{1}{5}(\partial_\nu\chi^B_\beta)\,(\sigma^I)_B{}^A(\gamma_\mu{}^\nu)_\alpha{}^\beta$$

$$Q^A_\alpha(H_{\mu\nu\rho}) = -\frac{1}{48}J^A_{[\mu|\beta|}\,(\gamma_{\nu\rho]})_\alpha{}^\beta + \frac{1}{30}(\partial_\sigma\chi^A_\beta)\,(\gamma^\sigma\bar{\gamma}_{\mu\nu\rho})_\alpha{}^\beta$$

$$Q^A_\alpha(J^B_{\mu\beta}) = 2\,T_{\mu\nu}\,(\gamma^\nu)_{\alpha\beta}\,\varepsilon^{AB} - \frac{2}{5}(\partial_\nu j^I_\rho)\,(\gamma_\mu{}^{\nu\rho} - 4\,\delta_\mu{}^\rho\gamma^\nu)_{\alpha\beta}\,\varepsilon^{AC}(\sigma_I)_C{}^B - \frac{2}{5}(\partial_\nu H_{\rho\sigma\lambda})\,(-\delta_\mu{}^\nu\gamma^{\rho\sigma\lambda} + 6\,\delta_\mu{}^\rho\gamma^{\nu\sigma\lambda} + 18\,\delta_\mu{}^\rho\eta^{\nu\sigma}\gamma^\lambda)_{\alpha\beta}\,\varepsilon^{AB}$$

$$Q^A_\alpha(T_{\mu\nu}) = \frac{1}{4}(\partial_\rho J^A_{\mu\beta})\,(\gamma^\rho{}_\nu)_\alpha{}^\beta + (\mu \leftrightarrow \nu)$$

NB: To derive these one begins with the most general ansatz for Q(A) consistent with the field content of the multiplet and all bosonic symmetries. One imposes the conservation and zero- trace conditions of A, if any. Finally the coefficients for each tensor structure are fixed by requiring that {Q,Q} = 2P, that is {Q,Q}(A) = 2∂A.

see e.g. M. Trépanier, KCL thesis (2021)

The choice of γ-matrices and defect orientation simplifies the action of the supercharge $Q \equiv Q_1^1$ on T_{zz} :

$$Q(T_{zz}) = \frac{1}{2}\partial_z J^1_{z1} \ , \quad Q(j^3_z) = \frac{1}{2} J^1_{z1} + \frac{1}{5}\,\partial_z \chi^1_1 \ , \quad Q(O) = \chi^1_1$$

It is also such that there exists a fermionic defect operator Λ such that $Q(\Lambda) = D_z$, without a derivative of the scalar partner, when one exists.

Goldstino of ~~susy~~

Now combining the Ward identities

$$Q\,\langle T_{zz}(x)\,\Lambda(y)\rangle = 0 \quad \Longrightarrow \quad \langle T_{zz}(x)\,D_z(y)\rangle + \frac{1}{2}\partial_z\,\langle J^1_{1z}(x)\,\Lambda(y)\rangle = 0$$

$$Q\,\langle j^3_z(x)\,\Lambda(y)\rangle = 0 \quad \Longrightarrow \quad \langle j^3_{1z}(x)\,D_z(y)\rangle + \frac{1}{2}\langle J^1_{1z}(x)\,\Lambda(y)\rangle + \frac{1}{5}\partial_z\,\langle \chi^1_1(x)\,\Lambda(y)\rangle = 0$$

$$Q\,\langle O(x)\,\Lambda(y)\rangle = 0 \quad \Longrightarrow \quad \langle O(x)\,D_z(y)\rangle + \langle \chi^1_1(x)\,\Lambda(y)\rangle = 0$$

we find

$$\langle T_{zz}(x)D_z(y)\rangle \;=\; \frac{\partial}{\partial z}\langle j^3_z(x)D_z(y)\rangle - \frac{1}{5}\,\frac{\partial^2}{\partial z^2}\langle O(x)D_z(y)\rangle \;\propto\; \frac{\partial^2}{\partial z^2}\left(\frac{\bar{z}\,|\,x_\perp|^{p-d+2}}{|\,x-y\,|^{2p+2}}\right)$$

only codim=2

where the last last step follows from the standard conformal identities ***qed***.

With minor variations the same proof works for all superconformal defects.

5. Conclusion

<u>Take away lesson</u>: Susy leads to a linear relation between the energy stored in the fields of a defect and its displacement norm. This implies:

- Equality of the inertial and gravitational tensions
- NEC restoration also removes the static stress

<u>Remarks</u>:

- For $p = \text{even}$ defects it relates the coefficients of the first two (Graham-Witten) B-type anomalies. For surface defects

$$T^\mu_\mu\Big|_{\text{Defect}} = \frac{1}{24\pi}\left(\mathbf{a}^{(2)}R + \underbrace{\mathbf{d}_1^{(2)}}_{\frac{3\pi^2}{4}C_D}\bar{K}^i_{ab}\bar{K}^{ab}_i - \underbrace{\mathbf{d}_2^{(2)}}_{-\frac{12(d-1)\pi^{d/2}}{d\,\Gamma(d/2-1)}a_T}W^{ab}_{ab}\right) + \text{odd}$$

For $p = 4$ there are 22 B-type parity-even anomalies. The coefficients of the first two are $\mathbf{d}_1^{(4)} = -\frac{\pi^4}{72} C_D$, $\mathbf{d}_2^{(4)} = \frac{5\pi^3}{6} a_T$. A holographic probe calculation *(Wilmore energy of submanifolds)* gives $\mathbf{d}_1^{(4)} = 2\mathbf{d}_2^{(4)} = -\pi^2 \sigma_0$ in agreement with the leading-order Witten diagrams, but little is known about quantum & gravitational corrections.

Graham, Reichert '17; Chalabi, Herzog, O'Bannon, Robinson, Sisti '21

- Revisit the (academic?) problem of radiation reaction for a moving charge.

  The radiation-reaction force contains the Schott term $f^\mu = \frac{2}{3} e^2 (a^2 u_\mu + \dot{a}^\mu) \Longleftarrow a_\mu u^\mu$ whose ab initio calculation has been the subject of controversy.

  Dirac '38; Landau, Lifshitz 62; Rohrlich '90; Teitelboim '70

  Could susy help ? (in progress)

Thank you for your attention