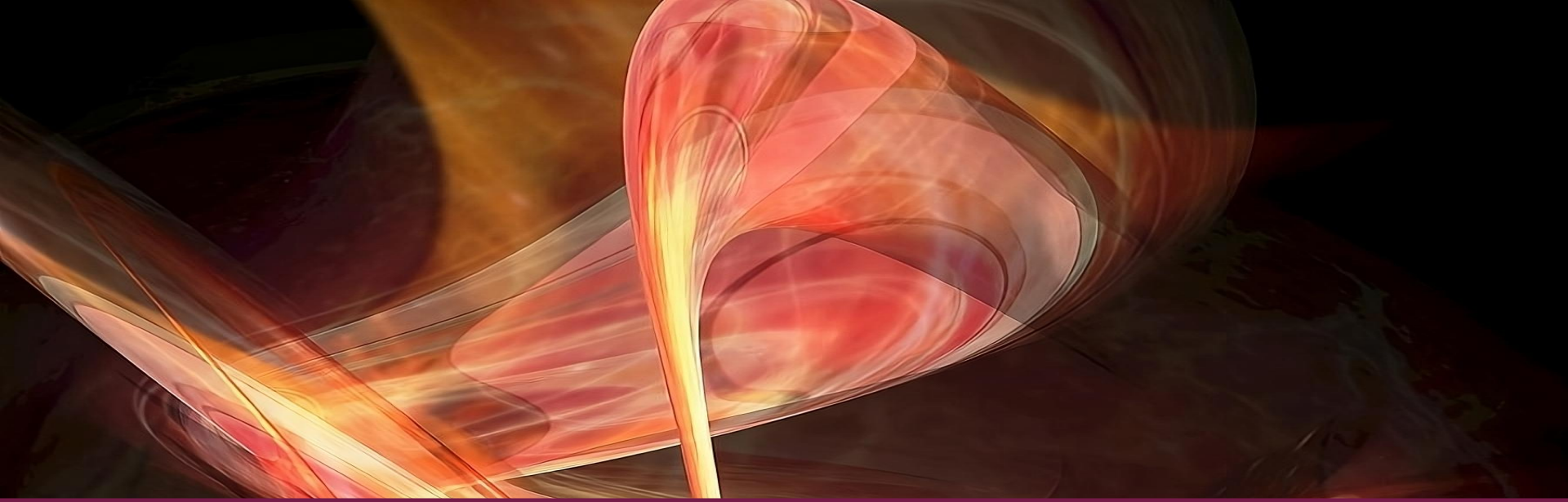
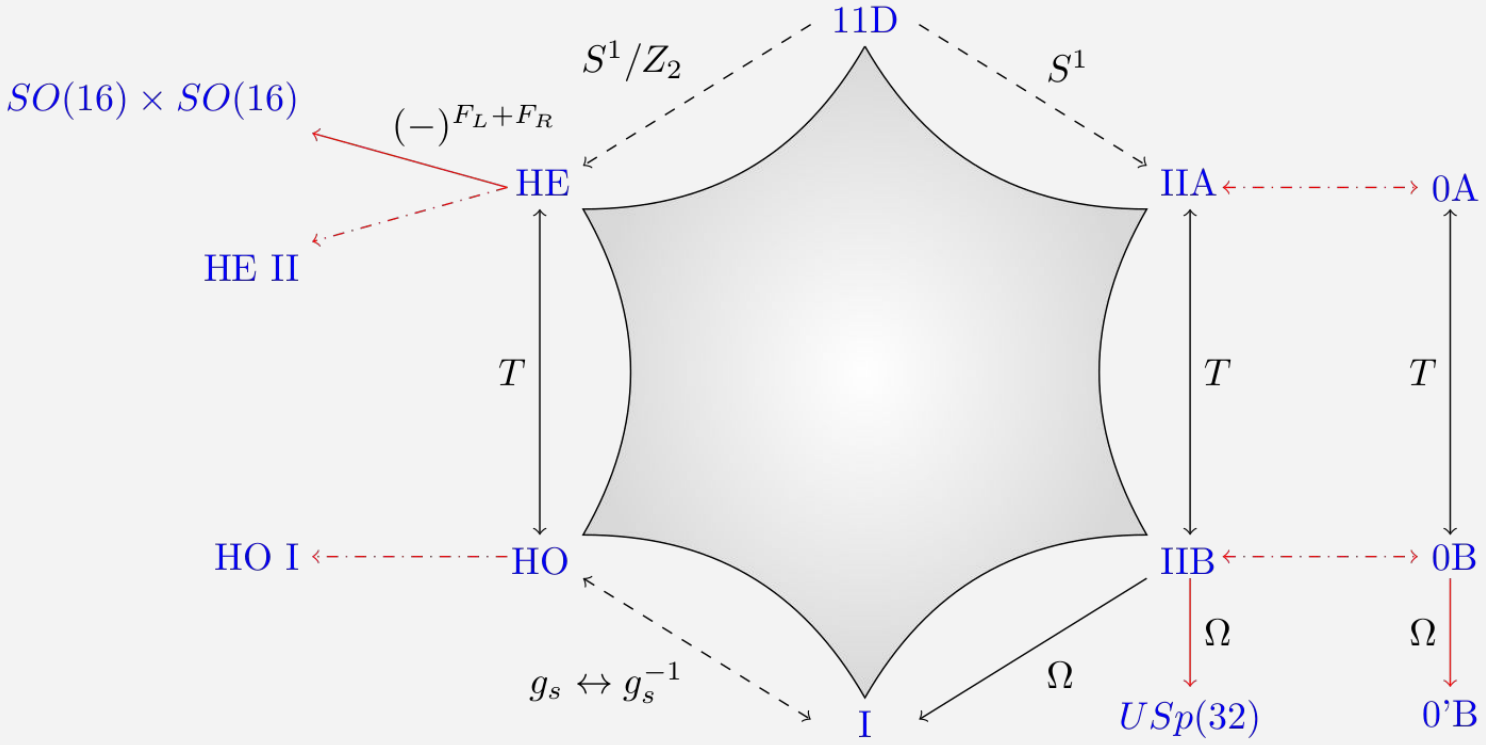


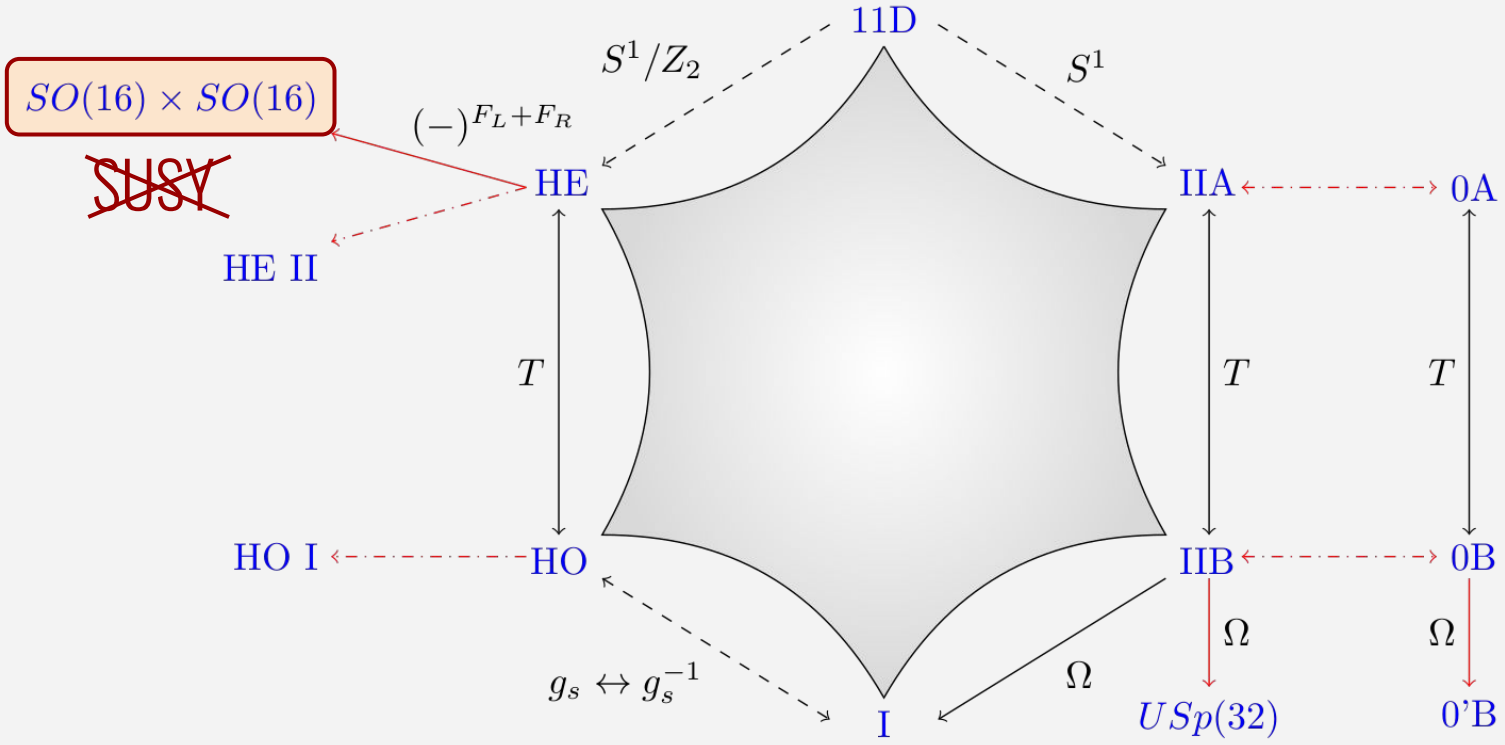
Fivebrane anomalies & discrete θ-angles

Ivano Basile | Max Planck Institut für Physik

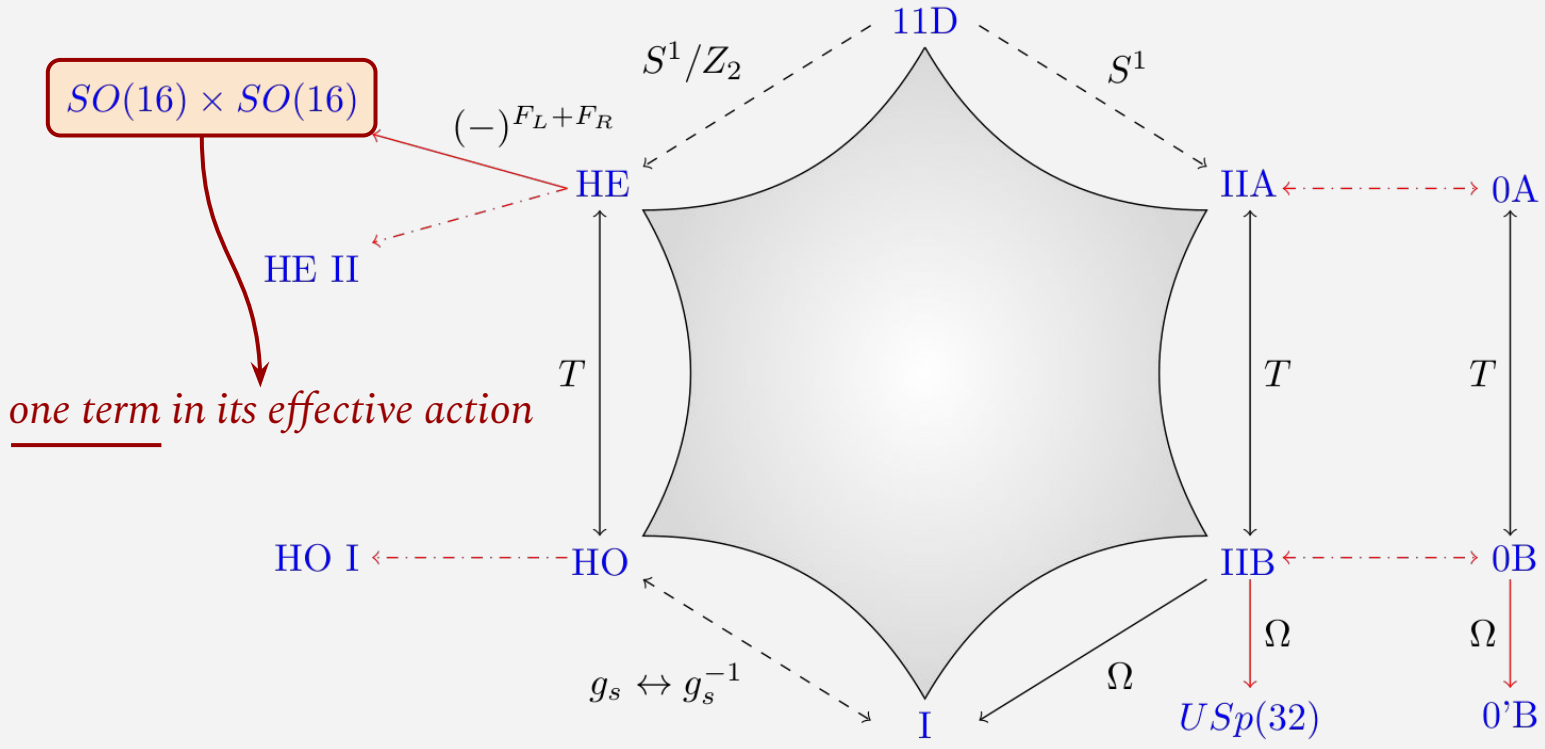
“What is string theory?”



“What is string theory?” — a tiny step



“What is string theory?” — a tiny step



How to make progress?

- *dynamical crutch:* ***supersymmetry***
  - *protects some quantities from corrections, fixes their form, …*
  - *may ultimately be necessary for full vacuum stability and/or holography*
- *kinematic crutch:* ***topology***
  - *anomalies, θ-terms: robust against continuous deformations*
  - *need not rely on supersymmetry*

this talk: ***θ-term*** *in 10d heterotic*

The gravitational θ-term

- *phase in Euclidean path integral*
  - *invariant under* ***cobordism****: classify evaluating on* ***generators***

$$M_d \to \exp\left(2\pi i S_{\text{top}}[M_d]\right)$$

- *ignore YM fields. spacetime is a* ***string manifold****: "*$\mathrm{dH} = \mathrm{p}_1/2$*".*
  - *cobordism generated by "exotic spheres" (Milnor).* *simple case:* Sp(2) *manifold*

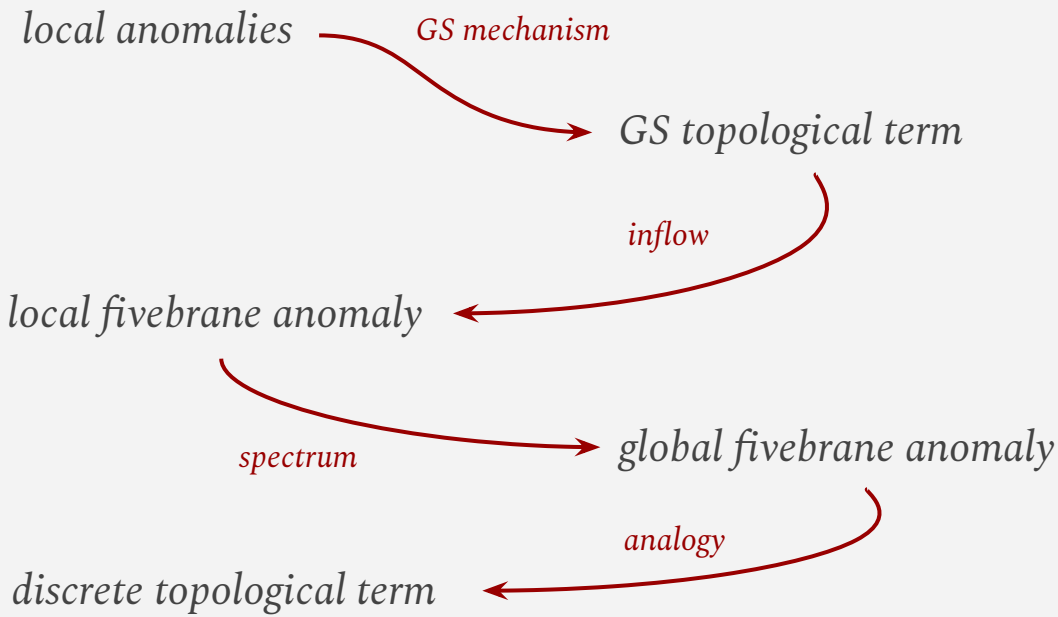
$$\Omega_{10}^{\text{string}} = \mathbb{Z}_2 \times \mathbb{Z}_3$$

Punchline to captivate audience *(Tachikawa, Zhang, 2024) (IB, Larotonda, WIP)*

$$\theta = \pm\frac{1}{3}$$

- ***this θ-term is non-zero.***
  - *tracking conventions, looks like Tachikawa-Zhang got opposite sign*
  - ***different method:*** *"direct" (us) vs "indirect" (them)*
  - *same physics? worldsheet analog: "Arf stacking"* *(Smith, Lin, Tachikawa, Zheng, 2023)*

Roadmap


local anomalies
GS mechanism
GS topological term
inflow
local fivebrane anomaly
spectrum
global fivebrane anomaly
analogy
discrete topological term

Previously, on “strings & SUSY breaking”...

anomaly cancellation: *anomaly factorization + H-field*

- ***type I & heterotic:*** *Green-Schwarz mechanism* *(Green, Schwarz, 1984) (Sagnotti, 1992)*

$$I_{12} = X_4 X_8 \qquad dH_3 = X_4$$

  - *worldsheet modular invariance* *(Schellekens, Warner, 1986) (Lerche, Nilsson, Schellekens, Warner, 1988)*

- ***~~SUSY~~ Sugimoto & heterotic:*** *same story*

  - *type 0′B: richer story w/ many RR fields*

  (Dixon, Harvey, 1986) (Alvarez-Gaume, Ginsparg, Moore, Vafa, 1986) (Sagnotti, 1995-1997) (Sugimoto, 1999)

- ***last episode:*** *“Dai-Freed” anomalies* ***also cancel*** *(IB, Debray, Delgado, Montero, 2023)*

  - *byproduct: proposal for* ***fivebrane chiral spectrum***

Fivebrane anomaly inflow

- ***fivebranes couple to H-field & provide solitons:***
  - *induce new gauge variation of eff. action* localized on worldvolume
  - ***inflow:*** *variation cancelled by chiral d.o.f. on the brane* (Callan, Harvey, 1985)
  - ***upshot:*** *its anomaly polynomial is* X_8 *+ other stuff* (Dixon, Duff, Plefka, 1992) (Mourad, 1997)

soliton analysis *led us to propose a chiral spectrum* see also (Blaszczyk, Nibbelink, Loukas, Ruehle, 2015)

$$(16,1) \oplus (1,16) \ominus 4 \times (1,1) \ominus \mathrm{SD}$$

From local to global

- *fivebranes flux transverse S^3*
  - *global anomaly captured by 7d TFT*
  - ***inflow:*** *take spacetime as fibration* $S^3 \to M_{10} \to N_7$

in particular: *Witten's anomaly for normal SU(2) bundle captured by* (Tachikawa, Zhang, 2024)

$$S^3 \to Sp(2) \to S^7$$

10d θ-term = fivebrane anomaly!

$\pi_6(SU(2)) = Z_{12}$

(Witten, 1982) (Elitzur, Nair, 1984) (Kiritsis, 1986) (Bershadsky, Vafa, 1997) (Garcia-Etxebarria, Hayashi, Ohmori, Tachikawa, Yonekura, 2017)

Recap

- ***Z_3-valued 10d gravitational θ-term*** *captured by the phase of* $Z[Sp(2)]$
  - *well-defined in non-susy heterotic where* $X_8 = 0$ *w/o YM fields*
  - ***inflow:*** *captured by global normal-bundle anomaly of fivebranes on* S^7

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computation: *"global SU(2) anomaly = local Sp(2) anomaly"* *(Tachikawa, Zhang, 2024)*

$$\pi_7(SU(2)) \to \pi_7(Sp(2)) \to \pi_7(Sp(2)/SU(2)) \to \pi_6(SU(2)) \to \pi_6(Sp(2)) \to \dots$$
$$\mathbb{Z}_2 \qquad \mathbb{Z} \qquad \mathbb{Z} \qquad \mathbb{Z}_{12} \qquad 0$$

Getting down to business

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Sp(2) *extension*

$$\int_{D^8} (I_8^{Sp(2)} - Z_4 W_4)$$

worldvolume GS term (Mourad, 1997)

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maps to ***trivial*** **SU(2)** ***configuration*** = 12 × *generator of* Z_{12}

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$$12\mathcal{A}(1) = \mathcal{A}(12) = \int_{D^8} (I_8^{Sp(2)} - Z_4 W_4) = \int_{S^8} (I_8^{Sp(2)} - Z_4 W_4) = \int_{S^8} \frac{t}{48} \mathrm{Tr} F^4 = t$$

generator Sp(2) *bundle on* S^8

add hemisphere w/ ***trivial*** **SU(2)** ***extension***

only surviving term *from* Sp(2) *extension*

Getting down to business

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only surviving term *from* Sp(2) *extension*

upshot: **θ = t/12** *determined by* Sp(2) *extension*

Direct vs indirect

- ***Tachikawa-Zhang:*** *difference of SUSY anomalies* (Schellekens, Warner, 1987)
  - "SO(32)": *Sp gauge theory;* "$E_8 \times E_8$": *E-string theory*
  - *the latter computed via* ***M-theory inflow*** (Ohmori, Shimizu, Tachikawa, 2014)
  - *result:* t = -4. *we tracked their chirality conventions*

- ***our method:*** *test fivebrane chiral spectrum* (IB, Debray, Delgado, Montero, 2023)
  - *determined normal-bundle representations imposing inflow à la* (Mourad, 1997)
  - *result:* t = 4. *sign can hide in many places/mistakes. if correct:*

same physics? chiral spectrum vs M-theory inflow?

Going a bit deeper

- ***why Z_3-valued?***
  - *in heterotic vacua, topological data of internal CFT = "TMF class"*

    (Hopkins, 1995) (Stolz, Teichner, 2008)
  - ***invariants:*** *elliptic genera + "torsional stuff"*

global anomaly cancellation à la Schellekens-Warner

(Tachikawa, (Yamashita), 2021)

torsional bilinear pairing *(Tachikawa, Yamashita, 2023)*

$$\mathrm{TMF}_d \times \mathrm{TMF}_{-22-d} \supset \mathbb{A}_d \times \mathbb{A}_{-22-d} \to \mathbb{R}/\mathbb{Z}$$

$$(M_d, \mathrm{CFT}_{\mathrm{int}}) \mapsto \exp 2\pi i S_{\mathrm{top}}[M_d]$$

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Z_3 *for* d=10

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- ***only other*** *non-trivial case of the story:* ***8d non-SUSY***
  - *we performed a bottom-up analysis, found formula for θ-term*
  - *future work — 8d non-SUSY heterotic landscape? patterns in discrete data?*

Outlook

thank you!

- ***chipping away** at non-perturbative strings*
  - *anomalies & **topology**: beyond SUSY (?)*
  - ***fivebranes** play a pivotal role*
  - *extend to **more general** anomalies*
- ***deeper insight** on the landscape?*
  - ***TMF** & bordism: "global Schellekens-Warner"*
  - ***wishlist:** new aspects of dualities?*