Gauge Theory meets Cosmology

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Talk @ Gravity, Strings and Supersymmetry Breaking
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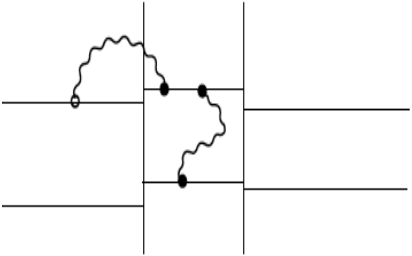
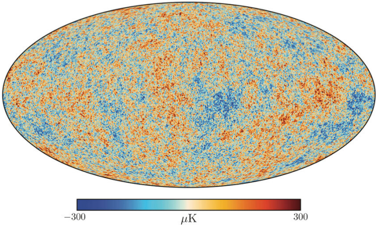
What do they have in common?





Everything ... if M87* were a Kerr BH in AdS and the Universe were filled in only with matter and radiation

Everything ... if $M87^*$ were a Kerr BH in AdS and the Universe were filled in only with matter and radiation

Not much if $M87^*$ is a fuzzball or a Top Star and the Universe contains also (dark) energy

Plan

- "Black-Hole" perturbations and observables
- H-W brane setups, Quantum S-W curves à la N-S and A-G-T correspondence
- Applications:
  - Gauge Theory meets Cosmology with G. Dibitetto, J. F. Morales
  - * Stability and Deformability of Top Stars with G. Di Russo, A. Grillo, J. F. Morales, G. Sudano, ... D. Bini
- Conclusions and outlook

Black Hole, ECO and fuzzball perturbation theory

- *Linearized* wave-equation (e.g. $s = 0$)

$$\Box_g \Phi = \mu^2 \Phi$$

ansatz

$$\Phi = e^{-i\omega t} e^{im_\phi \phi} R(r) S(\theta) \cdot e^{im_\psi \psi} \cdot e^{ipz} \cdot e^{iP_i Z^i}$$

- Separation à la Carter

$$K^2 = \ell(\ell + D - 3) + ...$$

- Schrödinger-like form $Q \sim \mathcal{E} - V$ with $\mathcal{E} \sim \omega^2$

$$\Psi'' + Q\Psi = 0$$

“Black Hole” perturbation theory

- Rotational invariant objects:

$$K^2 = \ell(\ell + D - 3) = K_0^2$$

E.g. Regge-Wheeler-Zerilli equations

- Rotating objects: ‘spheroidal harmonics’

$$K^2 = K_0^2 + \mathcal{O}(a_J^2 \omega^2)$$

Teukolsky equations $\sim$ CHE = Confluent Heun Equation

- one irregular singularity at ∞ (confluence): $R \sim e^{i\omega r}$
- two regular singularities at $r = r_H$ and $r = r_-$: $R \sim (r - r_0)^{\sigma_\pm}$

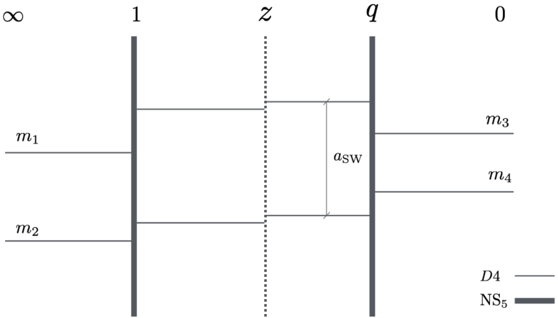
For extremal ‘objects’ $r_+ = r_-$ ... further confluence

Observables

- Waveforms: Inspiral, merger, ring-down ... echoes
- Quasi Normal Modes (QNMs):
  prompt $\sim$ photon-rings, late $\sim$ meta-stable, ... echoes
- Static/Dynamical Tidal Love Numbers ... 'Tidal Love Function'
- Self-Force for Extreme Mass-Ratio Inspirals (EMRI's)
- Grey-body factors, absorption cross sections ... if any
- (Near) Super-radiant (N-SR) modes, amplification $\sim$ Penrose mechanism
- Ergo-region and charge instability
- ...

A new gauge/gravity correspondence?

Hanany-Witten brane setup



From Hanany-Witten to 'quantum' Seiberg-Witten curve

- Quiver with 3 (4) nodes, $N_c = 2$ colour branes, $N_f = (2,2)$ flavour branes ($\beta = 0$ to start with)
- Degree 2 polynomials

$$P_L = (x - m_1)(x - m_2)\,, \quad P_C = (x - a_1)(x - a_2)\,, \quad P_R = (x - m_3)(x - m_4)$$

- 'Classical' Seiberg-Witten elliptic curve (a torus)

$$q y^2 P_L(x) + y P_C(x) + P_R(x) = 0 \quad , \quad q = \Lambda^\beta$$

- Quantize à la Nekrasov-Shatasvili $\varepsilon_1 = \text{``}\hbar\text{''}, \varepsilon_2 = 0$

$$\hat{x} = \hbar y \partial_y \quad , \quad \hat{y} = y$$

- Get 'quantum' SW curve = 2nd order ODE

$$\left[A(y)\hat{x}^2 + B(y)\hat{x} + C(y)\right] U(y) = 0$$

Another gauge-gravity duality relation?

- Schrödinger-like form

$$\Psi''(y) + Q_{SW}(y)\Psi(y) = 0$$

- Compare with Regge-Wheeler-Zerilli or Teukolsky eqs

$$\Psi''(y) + Q_{BH}(y)\Psi(y) = 0$$

- Same structure ... same physics (wrapped M2-branes?)
- Decouple flavours $N_f \to N_f - 1$ by double scaling limit $q_{N_f} \to 0$, $m_{N_f} \to \infty$ with $q_{N_f - 1} = q_{N_f} m_{N_f}$ fixed ... $\beta = 4 - N_f$

Gauge/gravity encyclopedia (Radial Botany)

Radial equation

- N_f=$(0,0)$ DRDCHE: D3-branes, D1-D5 (D3-D3') small BHs, ...
- N_f=$(0,1)\sim(1,0)$ RDCHE: BMPV ExtBHs in 5-d, ...
- N_f=$(1,1)$ DCHE: Intersecting D3's (4-charge ExtBHs in 4-d), KN, ExtSTURBHs ...
- N_f=$(0,2)\sim(2,0)$ RCHE: CCLP (general 5-d charged, rotating BHs), D1-D5 (D3-D3') fuzzball, JMaRT ...
- N_f=$(2,1)\sim(1,2)$ CHE: K-N BHs in 4-d, STURBHs, Top Stars ...
- N_f=$(2,2)$ HE: K-N BHs in AdS_4 with $\mu_\Phi^2 L^2 = -2$ $(\Delta = 1, 2)$

Legenda: H=Heun, E=Equation, C=confluent, R=reduced, D=doubly
All with $N_c = 2$ and $N_L, N_R \leq 2$!!!!

Gauge/gravity encyclopedia (Angular Zoology)

Angular equation $\rightarrow$ 'spheroidal' harmonics

- All 4-d and 4-d $\times S^1$ geometries (S^2): $N_f = (1,2)$ CHE
- All 5-d and 5-d $\times S^1$ geometries (S^3): $N_f = (0,2)$ RCHE

Dictionary

- RG-scale / instanton counting parameter
  $q = \Lambda^\beta \sim (\omega a_J)^\beta$, $a_J \sim J/M$ BH/fuzz spin
- Coulomb-branch $u = \langle Tr\phi^2 \rangle = a^2 + ... \sim K^2$ Carter constant
- Hyper Masses $m_f \sim m_\phi, m_\psi$ angular momenta
- Quantization condition: $a = \hbar(\ell + \frac{D-3}{2})$
- 'Straightforward' determination of K^2

$$\frac{1}{4}(1+K^2) = u = -q\partial_q \mathcal{F}_{NS}(a = \ell + \frac{1}{2}, m_f, q; \hbar) = a^2 + ...$$

using 'quantum' Matone relation [Matone; Flume, Fucito, Morales] or else [H. Poghosyan]

Radial dictionary

- RG-scale / instanton counting parameter
  $q = \Lambda^\beta \sim (\omega M)^\beta$, M BH/fuzz mass-scale
- Coulomb-branch $u = \langle Tr\phi^2 \rangle = a^2 + ... \sim K^2 + \delta K^2$ shifted Carter
- Hyper Masses $m_f \sim m_\phi + \delta m_\phi, m_\psi + \delta m_\psi$ shifted angular momenta
- QNM quantization condition: n 'overtone number'

$$a_D\,[+ka] = -\frac{1}{2\pi i}\partial_a \mathcal{F}_{NS}(a, m_f, q; \hbar)\,[+ka] = n\hbar$$

  NOT straightforward ... photon-rings, critical geodesics
- Invert quantum Matone relation or use 'difference' equation ($\hat{x} \leftrightarrow \hat{y}$)

$$u = -q\partial_q \mathcal{F}_{NS}(a, m_f, q; \hbar) = a^2 + ...$$

  to get $a(u) \approx \pm\sqrt{u} + ...$ with $u_R = u_A + \delta u_{AR}$ and then a_D
- Solve for ω in terms of n, ℓ, m's, M, Q, J's ... compare with WKB, Leaver, series of (confluent) hypergeometric, ... connection formulae

AGT: BPZ $\sim$ qSW $\sim$ HW $\sim$ CHE $\sim$ RW-Z-T

AGT: $\mathcal{N}=2$ quiver theories $\sim$ 2-d Liouville CFT [Alday, Gaiotto, Tachikawa]
$c = 1 + 6Q^2$, $Q = b + \frac{1}{b}$, $b = \sqrt{\epsilon_1/\epsilon_2}$
Chiral vertex operators $V_p = e^{2p\phi}$, $\Delta = p(Q-p)$
n-pt conformal blocks $\sim$ (ratios of) $SU(2)^{n-3}$ quiver partition functions
5-pt with level-2 degenerate field $\chi_{2,1}$ with $p_3 = -b/2$ at $z_3 = z$

$$\Psi''(\{z_i\}) + b^2 \sum_{i\neq 3}^{0,4} \left[\frac{\Delta_i}{(z-z_i)^2} + \frac{1}{z-z_i}\partial_{z_i}\right] \Psi(\{z_i\}) = 0$$

BPZ Eq [Belavin, Polyakov, Zamolodchikov] $\sim$ qSW curve for $SU(2)\times SU(2)$ quiver $\sim$
Regge-Wheeler / Zerilli / Teukolsky Eqs $\sim$ (D/R/D/)C/HEq
Connection formulae from Braiding and Fusing matrices [Bonelli, Iossa, Panea-Lichtig, Tanzini; Consoli, Fucito, Morales, Poghossian; ....]

Wavefunction in a nut-shell

$\Psi_\alpha(z) = \lim_{b\to 0} z^{\frac{1}{2}+\alpha\mathfrak{a}}\,(1-z)^{\frac{2k_0-1}{2b^2}}\,\left(1-\frac{q}{z}\right)^{\frac{1}{2}+k_2}\,\frac{Z_{\text{inst}}^{SU(2)\times SU(2)}{}_{p_0}{}^{k_0}{}_{\mathfrak{a}-\alpha}{}^{k_{\text{deg}}}{}_{\mathfrak{a}}{}^{k_2}{}_{p_3}(z,\frac{q}{z})}{Z_{\text{inst}}^{SU(2)}{}_{p_0}{}^{k_0}{}_{\mathfrak{a}}{}^{k_2}{}_{p_3}(q)}$

where $p_0 = \frac{m_1-m_2}{2}$, $k_0 = \frac{m_1+m_2}{2}$, $p_3 = \frac{m_3-m_4}{2}$, $k_2 = \frac{m_3-m_4}{2}$, $k_1 = k_{\text{deg}} = \frac{1}{2} + b^2$,

$p_1 = \mathfrak{a}^\alpha = \mathfrak{a} + \alpha\frac{b^2}{2}$, $p_2 = \mathfrak{a}$ ($\mathfrak{a}$ = quantum a-cycle)

$Z_{\text{inst}}^{SU(2)}{}_{p_0}{}^{k_0}{}_{\mathfrak{a}}{}^{k_2}{}_{p_3}(q) = \sum_W q^{|W|}\frac{z^{\text{bifund}}_{\emptyset,W}(p_0,\mathfrak{a},-k_0)\,z^{\text{bifund}}_{\emptyset,W}(\mathfrak{a},p_3,-k_2)}{z^{\text{bifund}}_{W,W}(\mathfrak{a},\mathfrak{a},\frac{b^2+1}{2})}$

$Z_{\text{inst}}^{SU(2)\times SU(2)}{}_{p_0}{}^{k_0}{}_{p_1}{}^{k_1}{}_{p_2}{}^{k_2}{}_{p_3}(q_1,q_2) = \sum_{Y_1,Y_2} q_1^{|Y_1|}q_2^{|Y_2|}\frac{\prod_{i=0}^{2} z^{\text{bifund}}_{Y_i,Y_{i+1}}(p_i,p_{i+1},-k_i)}{\prod_{i=1}^{2} z^{\text{bifund}}_{Y_i,Y_i}(p_i,p_i,\frac{b^2+1}{2})}$

single (W) or pairs $\{Y_{1\pm}\}$, $\{Y_{2\pm}\}$ of Young tableau(x)

$z^{\text{bifund}}_{Y,Y'}(p,p',m) = \prod_{\beta,\beta'=\pm}\prod_{(i,j)\in Y_\beta}\left[E_{Y_\beta,Y'_{\beta'}}(\beta p-\beta' p',i,j)-m\right]$

$\times\prod_{(i',j')\in Y'_{\beta'}}\left[-E_{Y'_{\beta'},Y_\beta}(\beta' p'-\beta p,i',j')-m\right]$

with $E_{Y,Y'}(x,i,j) = x - (\lambda^T_{Y'j} - i) + b^2(\lambda_{Yi} - j + 1) - \frac{b^2+1}{2}$

λ_{Yi} number of boxes in i-th row of tableau Y

$\lambda^T_{Y'j}$ number of boxes in j-th column of tableau Y'

Gauge Theory meets cosmology

Gauge Theory meets cosmology

FLRW metric ($c = \hbar = 1 = 8\pi G$)

$$ds_4^2 = -dt^2 + a(t)^2 \frac{|d\vec{x}|^2}{1 + \kappa |\vec{x}|^2} \quad , \quad \kappa = 0, \pm 1$$

Dibitetto-Sasaki: scale factor a as 'time' coordinate ... $dt = \frac{da}{aH(a)}$

$$H(a)^2 = \frac{\rho(a)}{3} - \frac{\kappa}{a^2} \,, \quad p(a) = -\rho(a) - \frac{1}{3} a \rho'(a) \,, \quad q(a) = -1 - a \frac{H'(a)}{H(a)}$$

Single-component (with $\kappa = 0$): 'equation of state' $w = [\partial p / \partial \rho]_s = c_s^2$

$$\rho = 3 \frac{H_0^2}{a^{3(1+w)}} = 3H^2 \,, \quad p = w\rho \,, \quad t(a) \sim a^{\frac{3}{2}(1+w)} \,, \qquad \text{for } w > -1$$

Multi-component fluid ($\sum_i \Omega_i = 1 - \Omega_\kappa$):

$$\rho = 3H_0^2 \sum_i \frac{\Omega_i}{a^{3(1+w_i)}} \quad , \quad p = 3H_0^2 \sum_i w_i \frac{\Omega_i}{a^{3(1+w_i)}}$$

Linearized perturbations around FLRW

Scalar perturbations ($\rho = \rho_0 + \delta\rho$, ...) $\sim$ 'Newtonian potential' $\Phi(a, \vec{x})$

$$ds_4^2|_s = -(1+2\Phi)\,\frac{da^2}{a^2\,H^2} + a^2\,(1-2\Phi)\,ds_3^2$$

Adiabatic perturbations: neglect coupling with entropy or iso-curvature modes (relevant at transients)

Canonical form $\Phi(a) = \frac{\sqrt{\rho_0'(a)}}{[a^2\rho_0(a)-3\kappa]^{1/4}}\Psi(a)$ with $\triangle_3\Phi = -k^2\Phi$

$$\Psi''(a) + Q_s(a)\Psi(a) = 0$$

$$Q_s(a) = \frac{\rho_0'''}{2\rho_0'} - \frac{3\,(\rho_0'')^2}{4\,(\rho_0')^2} - \frac{k^2\,(a\rho_0'' + 4\rho_0')}{a^2\rho_0'\,(a^2\rho_0 - 3\kappa)} + \frac{\rho_0''\,\left(12\kappa - a^2\rho_0\right)}{2a\rho_0'\,(a^2\rho_0 - 3\kappa)} + 3\,\frac{a^6\,(\rho_0')^2 + 4a^5\rho_0\rho_0' - 4a^4\rho_0^2 + 120a^2\kappa\rho_0 - 288\kappa^2}{16a^2\,(a^2\rho_0 - 3\kappa)^2}$$

Single-component perfect fluid: Bessel functions

Multi-component Perfect Fluid

N	Type	Components
3	Hypergeometric	Λ κ, Λ m, κ m, κ γ
4	Heun	Λ γ, mγ, Λ κ γ
5	Generalized Heun	κ m Λ, κ m γ
7	Generalized Heun	Λ m γ, Λ κ m γ

Scalar perturbations: Number of singularities N and type of ODE

N	Type	Fluid Components
4	Heun	Λ γ, mγ
5	Generalized Heun	Λ m
7	Generalized Heun	Λ m γ

Tensor perturbations: Number of singularities N and type of ODE

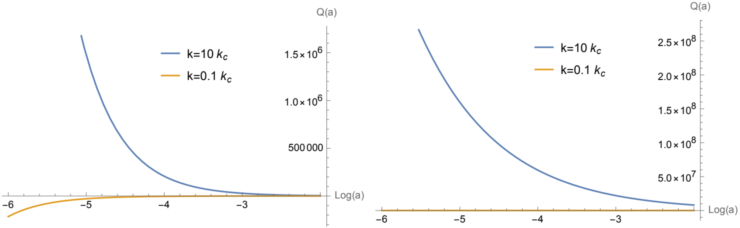
Energy density

$$\rho_0(a) = 3(M_{\rm Pl}\, H_0)^2\ \left(\Omega_\Lambda + \Omega_{\rm m} a^{-3} + \Omega_\gamma a^{-4}\right)$$

Deceleration

$$q(a) = \frac{\Omega_\gamma a^{-4} + \frac{1}{2}\Omega_{\rm m} a^{-3} - \Omega_\Lambda}{\Omega_\gamma a^{-4} + \Omega_{\rm m} a^{-3} - \Omega_\kappa a^{-2} + \Omega_\Lambda}$$

ΛCDM: $\Omega_\Lambda = 0.6889\ ,\ \Omega_{\rm m} = 0.3111\ ,\ \Omega_\gamma = 4.635{\cdot}10^{-5}\ ,\ \Omega_\kappa = 0$



Effective 'potential' $-Q(a)$ for $k > k_c$ (blue) and $k < k_c$ (orange)
L) Scalar perturbations R) Tensor perturbations

Scalar vs Tensor perturbations in ΛCDM

Set $\zeta_{\rm I} = \Omega_\gamma/\Omega_m \approx 1.5{\cdot}10^{-4}$ and $\zeta_{\rm II} = \Omega_\Lambda/\Omega_m \approx 2.21$
Scalar: for $k = 0$
$\Phi_{\rm II}(a) = \sqrt{a^3\zeta_{\rm II}+1}\left[\frac{2c_3}{5}\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-a^3\zeta_{\rm II}\right)+c_4a^{-\frac{5}{2}}\right]$
$\Phi_{\rm I}(a) = \frac{c_1\sqrt{a+\zeta_{\rm I}}}{a^3} + \frac{2c_2\left(9a^3+2a^2\zeta_{\rm I}-8a\zeta_{\rm I}^2-16\zeta_{\rm I}^3\right)}{15a^3}$
regularity and matching: $c_1 = \frac{8}{5}\zeta_{\rm I}^{5/2}$, $c_2 = \frac{3}{4}$, $c_3 = \frac{9}{4}$, $c_4 = 0$
For $k \neq 0$ Heun in domain I, two regions $Q(a)$ positive/negative ... critical
$a_c = \frac{16}{3}\zeta_I$, $\hat{k}_c = \frac{3}{8}\sqrt{\frac{23}{2\zeta_I}}$
Tensor: for $k = 0$ $\hat{h}_{\rm I} = \hat{c}_1 + \hat{c}_2\left[\operatorname{arctanh}\sqrt{1+a\zeta_{\rm I}^{-1}} - \frac{\zeta_{\rm I}}{a}\sqrt{1+a\zeta_{\rm I}^{-1}}\right]$
$\hat{h}_{\rm II} = \hat{c}_3 + \hat{c}_4 a^{-\frac{3}{2}}\sqrt{1+a^3\zeta_{\rm II}}$
only $\hat{h}(a) =$ const solution satisfies gluing conditions
For $k \neq 0$ Heun in domain I, two regions $Q(a)$ positive/negative ... critical
$a_c = \frac{2}{3}\left(\sqrt{22}-4\right)\zeta_{\rm I}$, $\hat{k}_c = \frac{\sqrt{5+4\sqrt{22}}}{4\sqrt{\zeta_{\rm I}}}$

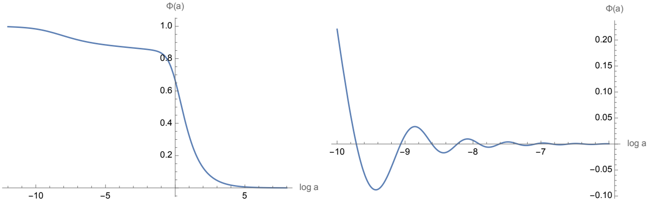


Figure: *Numerical solution for scalar $\Phi(a)$: L) $\hat{k} = 0.1\hat{k}_c$; R) $\hat{k} = 10\hat{k}_c$.*

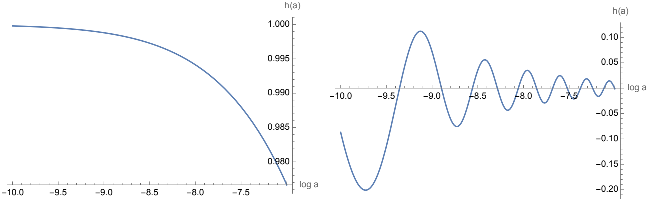


Figure: *Numerical solution for tensor $\hat{h}(a)$: L) $\hat{k} = 0.1\hat{k}_c$; R) $\hat{k} = 10\hat{k}_c$.*

A promise is a promise ...

Universe with only matter and radiation: $\zeta = \Omega_\gamma/\Omega_m$, $\hat{k} = k/H_0\sqrt{\Omega_m}$
Scalar perturbations ($\beta = 0$! $N_f = (2,2) \sim$ HE)

$$Q_S(a) = \frac{64a^2\zeta k^2\left(3a^2+7a\zeta+4\zeta^2\right) - 3\left(189a^4+924a^3\zeta+1820a^2\zeta^2+1600a\zeta^3+512\zeta^4\right)}{48a^2(a+\zeta)^2(3a+4\zeta)^2}$$

gauge/cosmology dictionary $z = -\frac{\zeta}{a}$, $q = \frac{3}{4}$, $u = \frac{4\hat{k}^2\zeta^2}{3} + \frac{33}{16}$,
$m_1 = \frac{7}{4}$, $m_2 = -\frac{5}{4}$, $m_{3,4} = 1 \pm \frac{1}{12}\sqrt{225 - 64\hat{k}^2\zeta}$
Tensor perturbations ($\beta = 2$! $N_f = (2,0) \sim$ RCHE)

$$Q_T(a) = \frac{\hat{k}^2}{(a+\zeta)} - \frac{5a+8\zeta}{16a(a+\zeta)^2}$$

dictionary: $z = -\frac{\zeta}{a}$, $q_2 = \hat{k}^2\zeta$, $u = \hat{k}^2\zeta + \frac{9}{16}$, $m_1 = \frac{3}{4}$, $m_2 = -\frac{1}{4}$

Top(ological) Stars

Top(ological) Stars 1: basic properties

Smooth horizonless solutions of 5-D Einstein-Maxwell [Bah, Heidemann]
Astonishingly simple!!!

$$ds^2 = -f_s(r)dt^2 + \frac{dr^2}{f_s(r)f_b(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + f_b(r)dy^2$$

$f_{s,b}(r) = 1 - \frac{r_{s,b}}{r}$, $y \sim y + 2\pi R_y$, 'magnetic' F_2 / 'electric' H_3
Regularity condition

$$r_s < r_b < 2r_s$$

'cap' at $r = r_b$ NO singularity, NO horizon, 4-d mass ($G_4 = G_5/2\pi R_y = 1$)

$$M_{TS} = \frac{1}{2}r_s + \frac{1}{4}r_b$$

Two classes: TS1 $r_b > 3r_s/2$; TS2 $r_b < 3r_s/2$

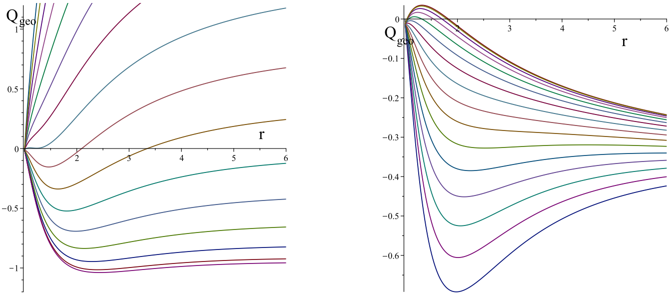
Top Stars 2: (critical) geodesics and ISCO

Spherical symmetry: planar (equatorial) geodesics
E, p_y, J conserved, radial momentum $P_r^2 = Q_{\rm geo}(r)$ (see plot)
Depending on impact parameter $b = J/|p_\infty|$: unbound or bound orbits
Critical geodesics $P_r = 0 = P_r'$: photon-spheres / light-rings, ...

$$r_{c1} = r_b < r_{c2} < r_{c3}$$

For massive probes ISCO at $r_0 = r_{c3} > 3r_s/2$ with

$$\Omega = \sqrt{\frac{r_s}{2r_0^3}}\,,\ \gamma = \frac{1}{\sqrt{1-\frac{3r_s}{2r_0}}}\,,\ E = \mu\frac{1-\frac{r_s}{r_0}}{\sqrt{1-\frac{3r_s}{2r_0}}}\,,\ J = \frac{\mu M}{\sqrt{\frac{r_s}{2r_0}\left(1-\frac{3r_s}{2r_0}\right)}}$$



(a) (b)

$P_r^2 = Q_{\rm geo}(r) = E^2 - V_{\rm eff}(r)$ ($M_{TS} = 0.65$) with
$2r_s = 1.6 > 3r_s/2 = 1.2 > r_b = 1 > r_s = 0.8$, $p_y = 1/4$, $\mu = 1$
(a) fixed $J = 3$, varying E (b) fixed $E = 0.8$, varying J

Top Stars 4: Remarkable results

Various approaches ... perfect agreement:

- Two branches of stable ($\mathrm{Im}\omega < 0$!!!) QNMs:
  - 'prompt' ring-down $\sim$ BH: $\omega_{\mathrm{prompt}} = \omega_c(\ell) - i(2n+1)\lambda$
  - long-live 'metastable' $\neq$ BH: $\omega_{\mathrm{meta}} = \omega_{int}(\ell) - ie^{-\mathcal{S}}$
- Non-zero (static) tidal deformability $\neq$ BH:

$$\mathcal{L}(\omega) = \frac{\Gamma(-2a)\Gamma(\frac{1}{2}+m_1+a)\Gamma(\frac{1}{2}+m_2+a)}{\Gamma(2a)\Gamma(\frac{1}{2}+m_1-a)\Gamma(\frac{1}{2}+m_2-a)} \cdot e^{\partial_a \mathcal{F}_I(a)}$$

  $a \sim$ 'renormalized' angular momentum, poles $\sim$ QNMs

- 'Self-force' $\sim \mu << M$: massless (scalar) radiation from bound (EMRI) and unbound orbits [MB, D. Bini, G. Di Russo]

Built-in PN and PM expansions: $\omega r \sim v$, $M/r \sim v^2$, $\omega M \sim v^3$

Conclusions and outlook

- New gauge / gravity correspondence (?) ... integrability
- Many applications: BHs, branes, fuzzballs, (rotating) Top Stars, ... FLRW cosmology
- Natural PN or PM expansions ... Self-force ... analytic / combinatorial formulae for connection coefficients
- Cosmological perturbations ... natural 'time' variable a
- ...