S-duality in a non-supersymmetric orientifold

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[with Gabriele Casagrande and Emilian Dudas]

Introduction

Superstring vacua without spacetime supersymmetry

- $\hookrightarrow$ No perturbative vacuum (runaway potential)
- $\hookrightarrow$ Only cosmological solutions
- $\hookrightarrow$ Often develops a tachyon in the descent of the potential

A non-perturbative non-supersymmetric vacuum?

- $\hookrightarrow$ S-duality symmetry without supersymmetry?

Scherk–Schwarz example

Take the orbifold of type IIB by $(-1)^F \delta_{\frac{1}{2}}$ where F is the spacetime fermion number and $\delta_{\frac{1}{2}}$ the half-period shift $X^9 \to X^9 + 2\pi R$.

$\hookrightarrow$ anti-periodic boundary conditions for the spacetime fermions

The one-loop potential energy is

$$V^{\mathrm{SS}} = -2\frac{R}{(2\pi)^9\alpha'^5}\int_0^\infty \frac{d\tau_2}{\tau_2^6}\int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \left|\frac{\vartheta_2(\tau)^4}{\eta(\tau)^{12}}\right|^2 \sum_{n=0}^{\infty} e^{-\frac{4\pi R^2}{\alpha'\tau_2}(n+\frac{1}{2})^2}$$

$$\underset{R\gg\sqrt{\alpha'}}{\sim} -\frac{31}{7560(2\pi)^4 R^9}$$

and drives the radius R to small values until it diverges negatively at $R = \sqrt{2\alpha'}$.

There is a complex scalar of mass $m^2 = \frac{R^2}{\alpha'^2} - \frac{2}{\alpha'}$ that becomes tachyonic at the critical radius.

Scherk–Schwarz example

The SS generator $(-1)^F \delta_{\frac{1}{2}}$ commutes with the duality group $SL(2,\mathbb{Z})$, so one may *wishfully* hope that the theory admits $SL(2,\mathbb{Z})$ duality symmetry such that one could *presomptluously* hope to determine the non-perturbative potential

$$V^{SS}(\mathcal{S} = C_0 + ie^{-\phi}, R = \sqrt{2\alpha'}e^{\varphi})$$

as a real analytic modular function of $\mathcal{S}$ with possibly a metastable minimum at finite $R > \sqrt{2\alpha'}$.

Outline

- Dabholkar-Park and S-duality
- A new Scherk-Schwarz orientifold
- D-brane spectrum and M-theory

[G. Bossard, G. Casagrande and E. Dudas, 2411.00955]

Notations

We use the Bianchi–Sagnotti notation for the superstring partition function [Angelantonj Sagnotti]

$$\frac{\vartheta_3(\tau)^4 - \vartheta_4(\tau)^4 - \vartheta_2(\tau)^4 - \vartheta_1(\tau)^4}{2\eta(\tau)^{12}} = \frac{V_8 - S_8}{\eta^8} \approx 0$$
$$\frac{\vartheta_3(\tau)^4 - \vartheta_4(\tau)^4 + \vartheta_2(\tau)^4 + \vartheta_1(\tau)^4}{2\eta(\tau)^{12}} = \frac{V_8 + S_8}{\eta^8} \approx \frac{\vartheta_2(\tau)^4}{\eta(\tau)^{12}}$$
$$\frac{\vartheta_3(\tau)^4 + \vartheta_4(\tau)^4 - \vartheta_2(\tau)^4 + \vartheta_1(\tau)^4}{2\eta(\tau)^{12}} = \frac{O_8 - C_8}{\eta^8} \approx \frac{\vartheta_4(\tau)^4}{\eta(\tau)^{12}}$$
$$\frac{-\vartheta_3(\tau)^4 - \vartheta_4(\tau)^4 - \vartheta_2(\tau)^4 + \vartheta_1(\tau)^4}{2\eta(\tau)^{12}} = \frac{-O_8 - C_8}{\eta^8} \approx \frac{-\vartheta_3(\tau)^4}{\eta(\tau)^{12}}$$

that captures the spectrum of the theory with O_d and V_d counting bosons and S_d and C_d counting fermions.

Notations

For example, the Scherk–Schwarz torus partition function can be written [Rohm]

$$
\begin{aligned}
T = \frac{1}{|\eta|^{16}} \sum_{m,n} \Bigg[& (|V_8|^2 + |S_8|^2)\ q^{\alpha'(\frac{m}{2R}+\frac{Rn}{\alpha'})^2}\,\bar{q}^{\alpha'(\frac{m}{2R}-\frac{Rn}{\alpha'})^2} \\
& - (V_8\bar{S}_8 + \bar{V}_8 S_8)\ q^{\alpha'(\frac{2m+1}{4R}+\frac{Rn}{\alpha'})^2}\,\bar{q}^{\alpha'(\frac{2m+1}{4R}-\frac{Rn}{\alpha'})^2} \\
& + (|O_8|^2 + |C_8|^2)\ q^{\alpha'(\frac{m}{2R}+\frac{R(2n+1)}{2\alpha'})^2}\,\bar{q}^{\alpha'(\frac{m}{2R}-\frac{R(2n+1)}{2\alpha'})^2} \\
& - (O_8\,\bar{C}_8 + \bar{O}_8\,C_8) q^{\alpha'(\frac{2m+1}{4R}+\frac{R(2n+1)}{2\alpha'})^2}\,\bar{q}^{\alpha'(\frac{2m+1}{4R}-\frac{R(2n+1)}{2\alpha'})^2} \Bigg]
\end{aligned}
$$

Dabholkar-Park and dualities

Type IIB $/(\Omega\delta_{\frac{1}{2}})$ with Ω the worldsheet orientation reversal and $\delta_{\frac{1}{2}}$ the half-period translation $X^9 \to X^9 + 2\pi R$.

$$K = \frac{1}{2}\frac{V_8(2i\tau_2) - S_8(2i\tau_2)}{\eta(2i\tau_2)^8}\sum_m (-1)^m e^{-\pi\tau_2\alpha'\frac{m^2}{R^2}}.$$

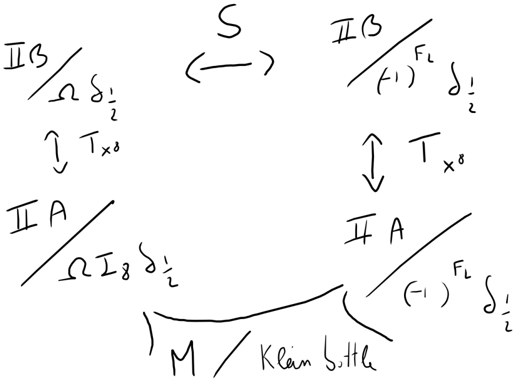
Using $S^{-1}\Omega S = (-1)^{F_L}$ the left-handed spacetime fermion number, one obtains that it is S-dual to the asymmetric orbifold Type IIB $/((-1)^{F_L}\delta_{\frac{1}{2}})$

$$T = \frac{\overline{V_8 - S_8}}{|\eta|^{16}}\sum_{m,n}\Bigg[V_8\; q^{\alpha'(\frac{m}{2R}+\frac{Rn}{\alpha'})^2}\bar{q}^{\alpha'(\frac{m}{2R}-\frac{Rn}{\alpha'})^2} - S_8\; q^{\alpha'(\frac{2m+1}{4R}+\frac{Rn}{\alpha'})^2}\bar{q}^{\alpha'(\frac{2m+1}{4R}-\frac{Rn}{\alpha'})^2}$$
$$- C_8\; q^{\alpha'(\frac{m}{2R}+\frac{R(2n+1)}{2\alpha'})^2}\bar{q}^{\alpha'(\frac{m}{2R}-\frac{R(2n+1)}{2\alpha'})^2} + O_8\; q^{\alpha'(\frac{2m+1}{4R}+\frac{R(2n+1)}{2\alpha'})^2}\bar{q}^{\alpha'(\frac{2m+1}{4R}-\frac{R(2n+1)}{2\alpha'})^2}\Bigg]$$

Dabholkar-Park and dualities

Compactifying on an additional circle and using T-duality one obtains respectively Type IIA $/(I_8\Omega\delta_{\frac{1}{2}})$ (with I_8 the spacetime reflection) and Type IIA $/((-1)^{F_L}\delta_{\frac{1}{2}})$.

$\hookrightarrow$ At strong coupling M-theory on a Klein bottle.



Orientation reversal as a duality

The metaplectic cover [Pantev Sharpe]

$$\left\{1, (-1)^F\right\} \to Mp(2,\mathbb{Z}) \to GL(2,\mathbb{Z})$$

is defined as the type IIB $GL(2,\mathbb{Z})$ duality symmetry on bosons and the compensating R-symmetry $O(2)$ acting on spacetime fermions [Sen Dabholkar]

$$\Omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_{\mathrm{R}}, \; (-1)^{F_L} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{\mathrm{R}}$$

and

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}_{\mathrm{R}},$$

so that $S^{-1}\Omega S = (-1)^{F_L}$.

Adiabatic argument

One must be careful with the formal identity $S^{-1}\Omega S = (-1)^{F_L}$ in string theory. If there are strong arguments to believe that type IIB $/(\Omega\delta_{\frac{1}{2}})$ is S-dual to type IIB $/((-1)^{F_L}\delta_{\frac{1}{2}})$, it is not true that type IIB $/\Omega =$ type I is S-dual to type IIB $/(-1)^{F_L} =$ type IIA.

- ⋆ Must take into account the open sector
  - $\hookrightarrow$ Type I is S-dual to heterotic $Spin(32)$
- ⋆ Valid if *adiabatic* deformation of type IIB [Vafa Witten]
  - $\hookrightarrow$ In the large radius limit type IIB $/(\Omega\delta_{\frac{1}{2}}) \approx$ type IIB.
- ⋆ Must have an interpretation in M-theory on an elliptic fibration.

Dabholkar-Park stable branes

The stable branes in M-theory are M5 and M9 branes wrapping homology cycles of the Klein bottle times T^d [Gaberdiel Schafer-Nameki]

$$H_0(K_2,\mathbb{Z}) = \mathbb{Z} \ , \quad H_1(K_2,\mathbb{Z}) = \mathbb{Z}\oplus\mathbb{Z}_2 \ , \quad H_2(K_2,\mathbb{Z}) = 0 \ ,$$

or M2 and KK6 branes wrapping the (orientation bundle) twisted homology $\widehat{H}_n(K_2,\hat{\mathbb{Z}}) = H^{2-n}(K_2,\mathbb{Z})$ cycles of the Klein bottle times T^d

$$\widehat{H}_0(K_2,\hat{\mathbb{Z}}) = \mathbb{Z}_2 \ , \quad \widehat{H}_1(K_2,\hat{\mathbb{Z}}) = \mathbb{Z} \ , \quad \widehat{H}_2(K_2,\hat{\mathbb{Z}}) = \mathbb{Z} \ .$$

Predicts a single stable D3-brane orthogonal to the coordinate X^9

$$\mathrm{D3}_{0128} \underset{\text{T-duality}}{\longrightarrow} \mathrm{M2}_{012} \ , \qquad \mathrm{D3}_{0123} \underset{\text{T-duality}}{\longrightarrow} \mathrm{M5}_{0123810} \ .$$

Dabholkar-Park stable branes

For $R > \sqrt{2\alpha'}$, the stable D3 brane is a D3 brane at $X^9 = 0$ and its image $\overline{D3}$ brane at $X^9 = \pi R$ [Bergman Gimon Horava] . A pair of them can rotate such that the D3–$\overline{D3}$ annihilate each others.

$$A_{33} = \sum_n \left[N\overline{N} \left(\frac{V_8 - S_8}{\eta^8} \right) q^{\frac{R^2}{4\alpha'} n^2} + \frac{N^2 + \overline{N}^2}{2} \left(\frac{O_8 - C_8}{\eta^8} \right) q^{\frac{R^2}{4\alpha'} (n+\frac{1}{2})^2} \right] .$$

For $R < \sqrt{2\alpha'}$ the configuration becomes unstable and dilutes into a D4 brane wrapping the ninth circle.

$$A_{44} = \frac{N^2}{2} \frac{1}{\eta^8} \left[(O_3 + V_3)(O_5 + V_5) - 2 S_5' S_5' \right] \sum_m q^{\frac{\alpha'}{R^2} m^2}$$

$$M_4 = - \frac{N}{2} \frac{2^{\frac{5}{2}}}{\hat{\vartheta}_2^{\frac{5}{2}} \hat{\eta}^{\frac{1}{2}}} \left[\hat{O}_3 \hat{O}_5 + \hat{V}_3 \hat{V}_5 - \hat{O}_3 \hat{V}_5 + \hat{V}_3 \hat{O}_5 \right] \sum_m (-1)^m q^{\frac{\alpha'}{R^2} m^2}$$

Dabholkar-Park stable branes

For a stack of N_1 D3-branes at distance $L = 2\pi a R$ of a stack of N_2 D3-branes, the tree-level amplitude is

$$\tilde{A}_{33} = \frac{\sqrt{\alpha'}}{8R}\frac{1}{\eta^8}\sum_m q^{\frac{\alpha'}{R^2}m^2}\Bigg[\Big(N_1\overline{N}_1+N_2\overline{N}_2+\cos(2\pi am)(N_1\overline{N}_2+N_2\overline{N}_1)\Big)(V_8-S_8)$$
$$+\Big((-1)^m\frac{N_1^2+\overline{N}_1^2+N_2^2+\overline{N}_2^2}{2}+(-1)^m\cos(2\pi am)(N_1N_2+\overline{N}_1\overline{N}_2)\Big)(V_8+S_8)\Bigg]$$

which gives minimum potential energy at $a=\frac{1}{2}$, where the N_1 branes are at the position of the N_2 anti-branes.

There is a complex (N_1, N_2) tachyon if $|L-\pi R| < 4\pi\sqrt{\alpha'}$

$$A_{33} = \sum_n\Bigg[\Bigg(\Big(N_1\overline{N}_1+N_2\overline{N}_2\Big)q^{\frac{R^2}{4\alpha'}n^2}+\frac{N_1\overline{N}_2+N_2\overline{N}_1}{2}\big(q^{\frac{R^2}{4\alpha'}(n-a)^2}+q^{\frac{R^2}{4\alpha'}(n+a)^2}\big)\Bigg)\frac{V_8-S_8}{\eta^8}$$
$$+\Bigg(\frac{N_1^2+\overline{N}_1^2+N_2^2+\overline{N}_2^2}{2}q^{\frac{R^2}{4\alpha'}(n+\frac{1}{2})^2}+\frac{N_1N_2+\overline{N}_1\overline{N}_2}{2}\big(q^{\frac{R^2}{4\alpha'}(n+\frac{1}{2}-a)^2}+q^{\frac{R^2}{4\alpha'}(n+\frac{1}{2}+a)^2}\big)\Bigg)\frac{O_8-C_8}{\eta^8}\Bigg]$$

A new Scherk–Schwarz string

One takes the orientifold of type IIB by $\Omega(-1)^{F_L}\delta_{\frac{1}{4}}$. This generates a $\mathbb{Z}_4$ group

$$\widehat{G} = \left\{1, \Omega(-1)^{F_L}\delta_{\frac{1}{4}}, (-1)^F\delta_{\frac{1}{2}}, \Omega(-1)^{F_R}\delta_{\frac{3}{4}}\right\} ,$$

including the Scherk–Schwarz generator $(-1)^F\delta_{\frac{1}{2}}$.

The partition function is the sum of 1/2 the Scherk–Schwarz torus partition function and the Klein bottle

$$K = \frac{1}{2}\frac{V_8(2i\tau_2)+S_8(2i\tau_2)}{\eta(2i\tau_2)^8}\sum_m (-1)^m e^{-\pi\tau_2\alpha'\frac{m^2}{R^2}} .$$

A new Scherk–Schwarz string

The Klein bottle contribution to the tree-level vacuum vacuum amplitude gives

$$\tilde{K} = \frac{2^5 R}{2\sqrt{\alpha'}} \frac{O_8(i\ell) - C_8(i\ell)}{\eta(i\ell)^8} \sum_n e^{-2\pi\ell \frac{R^2(2n+1)^2}{4\alpha'}}$$

with no coupling to the untwisted sector.

- $\hookrightarrow$ A twisted O-plane, no coupling to gravity
- $\hookrightarrow$ No interpretation in type IIA because $R > \sqrt{2\alpha'}$.

There is no tadpole and therefore no open sector.

Twisted O-planes

Example of twisted O-plane in a supersymmetric theory:

Type IIB on T^4 with the orientifold action ΩZ_4 where $Z_4(z_1, z_2) = (iz_1, -iz_2)$ on T^4. $(\Omega Z_4)^2 = Z_2$.

- ⋆ The twisted O-plane only couples to the twisted sector
- ⋆ There is no open string and no gravitational anomaly

$(n_H = 12, n_T = 9)$

Among the 16 $\mathbb{Z}_2$ fixed points, 4 are $\mathbb{Z}_4$ fixed points. Blowing up the sixteen $\mathbb{Z}_2$ fixed points by giving expectation values to the twisted hyper-multiplets, the $\mathbb{Z}_4$ fixed points are blown up to two $\mathbb{Z}_2$ fixed points at the north and south poles of the blown up $\mathbb{CP}^1$. We get $\Omega\sigma$ with σ an automorphism with eight fixed points of K3.

Twisted O-planes

Example of twisted O-plane in a supersymmetric theory:

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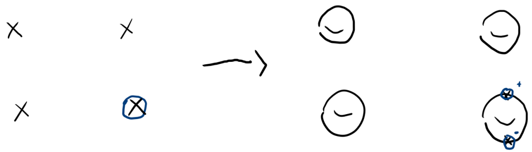


There is a $O5_+$ at the north pole and a $O5_-$ at the south pole with a B-field on the $\mathbb{CP}^1$. In the limit of zero size one obtains a twisted O5-plane.

In type IIB supergravity

Using the basis introduced earlier one can write

$$\Omega(-1)^{F_L} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}_{\mathrm{R}} = S^2\,,$$

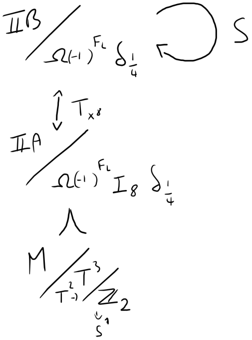
and define this orientifold as type IIB $/(S^2\delta_{\frac{1}{4}})$ with $S^4 = (-1)^F$. We have the Fourier expansion

$$\begin{aligned}
(g_{\mu\nu}, \mathcal{S}, C_4^+)(x^9, \boldsymbol{x}) &= \sum_m e^{2mi\frac{x^9}{R}} (g_{\mu\nu}, \mathcal{S}, C_4^+)^{(m)}(\boldsymbol{x}), \\
(B_2, C_2)(x^9, \boldsymbol{x}) &= \sum_m e^{(2m+1)i\frac{x^9}{R}} (B_2, C_2)^{(m)}(\boldsymbol{x}), \\
\Psi_\pm(x^9, \boldsymbol{x}) &= \sum_m e^{(2m\pm\frac{1}{2})i\frac{x^9}{R}} \Psi_\pm^{(m)}(\boldsymbol{x}),
\end{aligned}$$

M-theory compactification

After a T-duality along an additional circle of radius R_{B}, one obtains M-theory on $\mathcal{M}_3 = T^3/\mathbb{Z}_2$ with the freely acting $\mathbb{Z}_2$ involution defined as $I_{810}\delta_{\frac{1}{4}}$

$$(X^8, X^9, X^{10}) \rightarrow (-X^8, X^9 + \pi R, -X^{10})$$



M-theory compactification

$\mathcal{M}_3 = T^3/\mathbb{Z}_2$ is defined as the quotient of $\mathbb{R}^3$ by

$$(X^8, X^9, X^{10}) \approx \left(X^8 + 2\pi\alpha'/R_{\mathrm{B}}, X^9 + 2\pi R, X^{10} + 2\pi e^{\frac{2}{3}\Phi_{\mathrm{A}}}\sqrt{\alpha'}\right)$$

and the freely acting $\mathbb{Z}_2$ involution defined as

$$(X^8, X^9, X^{10}) \to (-X^8, X^9 + \pi R, -X^{10}) \ .$$

This space is orientable with the homology

$$H_0(\mathcal{M}_3, \mathbb{Z}) = \mathbb{Z}\ ,\ H_1(\mathcal{M}_3, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2\ ,\ H_2(\mathcal{M}_3, \mathbb{Z}) = \mathbb{Z}\ ,\ H_3(\mathcal{M}_3, \mathbb{Z}) = \mathbb{Z}\ .$$

Stable $\mathbb{Z}_2$-branes

One checks that all the perturbative stable branes correspond to membranes wrapping cycles in $\mathcal{M}_3$.

For example a single D5-brane orthogonal to the SS circle of radius R is stable as a D3-brane in Dabholkar–Park

$$A_{55} = \frac{1}{\eta^8} \sum_n \left[N\overline{N}(V_8 - S_8) q^{\frac{R^2}{4\alpha'} n^2} + \frac{N^2 + \overline{N}^2}{2} (O_8 - C_8) q^{\frac{R^2}{4\alpha'} (n+\frac{1}{2})^2} \right]$$

They correspond to

$$\mathrm{D5}_{012348} \underset{\text{T-duality}}{\rightarrow} \mathrm{M5}_{0123410} \,, \quad \mathrm{D5}_{012345} \underset{\text{T-duality}}{\rightarrow} \mathrm{KK6}_{0123458} \,,$$

wrapping a $\mathbb{Z}_2$ cycle along X^8 or X^{10} in $\mathcal{M}_3$.
The same for a single D1-brane orthogonal to the SS circle with

$$\mathrm{D1}_{01} \underset{\text{T-duality}}{\rightarrow} \mathrm{M2}_{018} \,, \qquad \mathrm{D1}_{08} \underset{\text{T-duality}}{\rightarrow} \mathrm{KK}(X^{10})$$

Stable D3-branes

N D3-branes orthogonal to the SS circle are stables and give $\mathcal{N} = 4$ super Yang-Mills at low energy. They correspond in M-theory to

$$\mathrm{D3}_{0128} \underset{\text{T-duality}}{\rightarrow} \mathrm{M2}_{012}\,, \qquad \mathrm{D3}_{0123} \underset{\text{T-duality}}{\rightarrow} \mathrm{M5}_{0123810}\,.$$

N D3-branes wrapping the SS circle are stables and give a $\mathbb{Z}_4$ orbifold of $\mathcal{N} = 4$ super Yang-Mills with gauge group U($2N$) on S^1 at low energy. They correspond in M-theory to

$$\mathrm{D3}_{0189} \underset{\text{T-duality}}{\rightarrow} \mathrm{M2}_{019}\,, \qquad \mathrm{D3}_{0129} \underset{\text{T-duality}}{\rightarrow} \mathrm{M5}_{0128910}\,.$$

Stable D3-branes

N D3-branes wrapping the SS circle are stables and give a $\mathbb{Z}_4$ orbifold of $\mathcal{N}=4$ super Yang-Mills with gauge group U($2N$) on S^1 at low energy, define with the unitary matrix

$$\varsigma = \begin{pmatrix} 0 & i\mathbb{1}_{N\times N} \\ \mathbb{1}_{N\times N} & 0 \end{pmatrix} ,$$

by the automorphism

$$\begin{aligned} Z_4 A_\mu(X) &= -\varsigma A_\mu(X+\pi R)^{\intercal}\varsigma^\dagger\,, \quad Z_4\phi_{ij}(X) = \varsigma\phi_{ij}(X+\pi R)^{\intercal}\varsigma^\dagger\,, \\ Z_4\lambda_{\alpha i}(X) &= i\varsigma\lambda_{\alpha i}(X+\pi R)^{\intercal}\varsigma^\dagger\,, \quad Z_4\lambda^i_{\dot\alpha}(X) = -i\varsigma\lambda^i_{\dot\alpha}(X+\pi R)^{\intercal}\varsigma^\dagger\,. \end{aligned}$$

The one-loop beta function vanish and the coupling $\mathcal{S} = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2_{\rm YM}}$ may be well defined.

A non-pertubative extension of the one-loop potential

A naive extrapolation of the one-loop potential gives

$$\hat{V} = -\frac{403}{189(4\pi)^4}\frac{1}{R^9}$$

$$-\frac{1}{2\ell_{10}^5 R^4}\sum_{m,n}{}' \sum_{k\geq 0}\frac{(e^{\Phi/2}|n+\tau m|)^{\frac{5}{2}}}{(k+\frac{1}{2})^5}\left[c(\gcd(m,n))^2 K_5\left(8\pi\frac{R}{\ell_{10}}\sqrt{e^{\Phi/2}|n+\tau m|}(k+\tfrac{1}{2})\right)\right.$$

$$\left. +2c(\gcd(m,n))K_5\left(4\pi\frac{R}{\ell_{10}}\sqrt{e^{\Phi/2}|n+\tau m|}(k+\tfrac{1}{2})\right)\right]$$

with $\sum_{n=0}^{\infty} c(n)q^n = \prod_{n\geq 1}\frac{(1+q^n)^8}{(1-q^n)^8}$. Predicting "tachyonic" closed string scalar with the (m,n)-string mass

$$M^2_{\text{F1-D1}} = -2\frac{|n+\mathcal{S}m|}{\alpha'} + \frac{R^2|n+\mathcal{S}m|^2}{\alpha'^2} \,.$$

For an unstable D1 wrapping the SS circle this predicts

$$M^2_{\text{D1}} = -2\frac{\sqrt{\frac{1}{g_s^2}+C_0^2}}{\alpha'} + \frac{R^2}{\alpha'^2}\left(\frac{1}{g_s^2}+C_0^2\right) \approx \frac{R^2}{\alpha'^2 g_s^2} + \mathcal{O}(g_s^{-1}) \,,$$

in agreement with the perturbative computation.

Conclusion

- Define a fourth non-supersymmetric SS orientifold
  - $\hookrightarrow$ completing the three SS orientifolds

[Blum Dienes Antoniadis Dudas Sagnotti Mourad]

- Analysed the perturbative probe branes spectrum
- M-theory interpretation of stable branes
  - $\hookrightarrow$ Argument for S-duality symmetry

- ⋆ Check S-duality at higher order
- ⋆ Can one determine the exact potential
  - $\hookrightarrow$ stabilise the axio-dilaton and the radius moduli
- ⋆ New orientifolds in lower dimensions with twisted O-planes