G. DALL'AGATA (U. PADOVA)

SUPERGRAVITY ON BIEBERBACH MANIFOLDS

INTRODUCTION

- Standard lore views Supergravity as good EFTs for String Theory
    - Incredibly efficient also for Quantum Gravity

      (BH microstates, localization,...)
- However: **When** do we really use it as an EFT?
- The landscape of supergravities seems much larger than ST
    - Example *infinite family of SO(8) N=8 supergravities*

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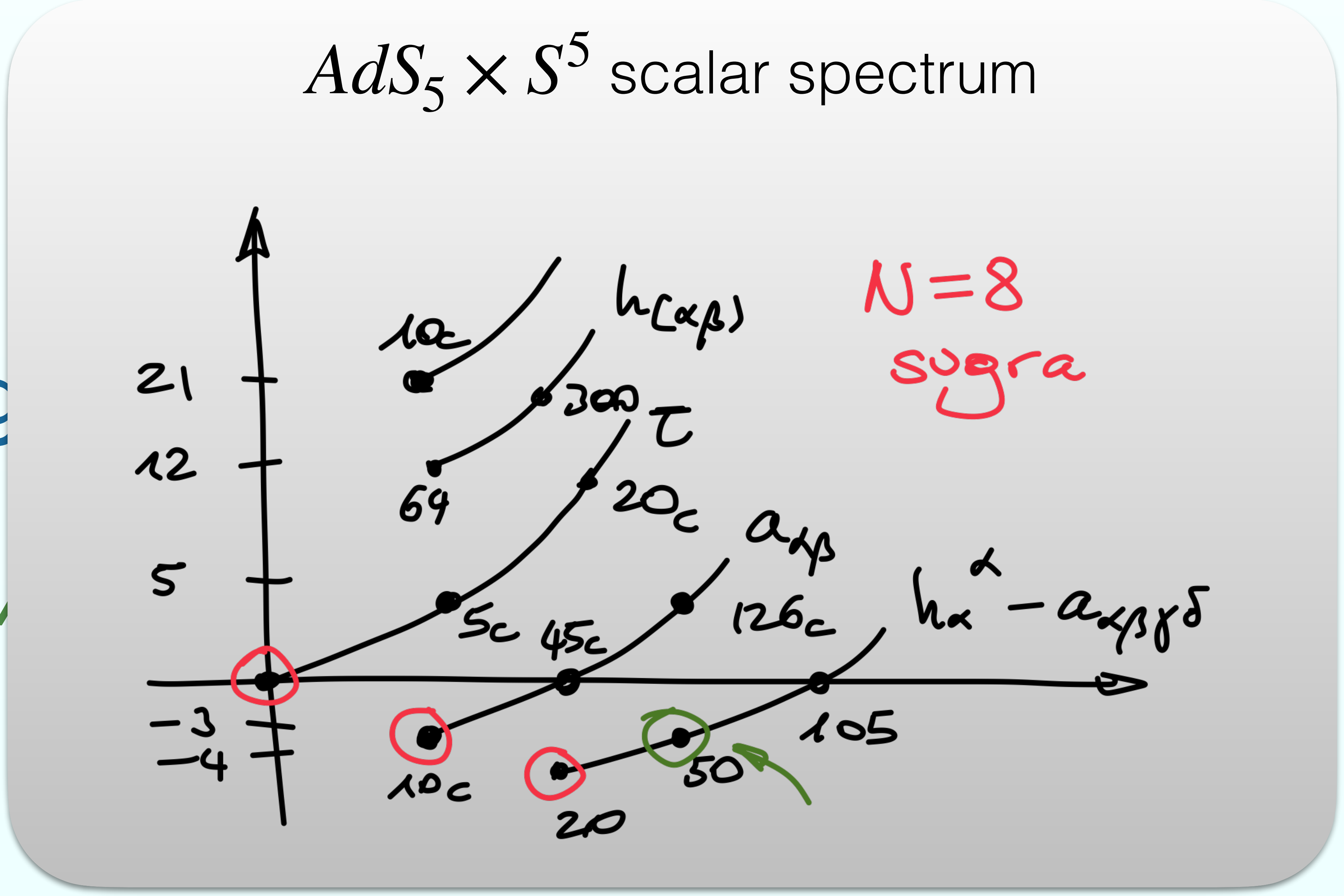

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INTRODUCTION

- Focus on Minkowski vacua of supergravities and their moduli space
  - Simplified setup: maximal SUSY at Lagrangian level
  - Simple models are reductions on tori and twisted tori
    - Intricate web of vacua connected by moduli fields
    - Sliding susy breaking scale
- Finite quantum effects lifting vacuum energy and degeneration?

TWISTED TORI REDUCTIONS AND GAUGED SUPERGRAVITIES

- Cremmer–Scherk–Schwarz reductions from the 4d perspective [FLAT GROUPS]

$$\mathrm{U}(1) \ltimes T^{27} \qquad \left\{ \begin{array}{rcl} [X_0, X^I] & = & Q^I{}_J\, X^J \\ [X^I, X^J] & = & 0 \end{array} \right.$$

- Gives:
  - Minkowski vacua with $N=0,2,4,6$
  - Gravitino masses $2 \times M_i$
  - Overall sliding scale, but M_i/M_j fixed

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$$\mathrm{U}(1) \ltimes T^{27} \qquad \begin{cases} [X_0, X^I] &= Q^I{}_J X^J \\ [X^I, X^J] &= 0 \end{cases}$$

- Simple generalisation:

$$[Z_I, X^J] = Q_I{}^J{}_K X^K, \qquad [Z_I, Z_J] = 0, \qquad X^J \ \textit{Nilpotent}$$

- One can classify “flat groups” (either solvable or nilpotent)

TWISTED TORI REDUCTIONS AND GAUGED SUPERGRAVITIES

- Reduce on circle(s) with periodic coordinates $y \sim y + 1$, twisting fields in a $G \subset GL(d, \mathbb{R})$ representation

CREMMER-SCHERK-SCHWARZ
KALOPER-MYERS

$$\Phi(x, y) = \exp(My)[\phi(x)] \qquad M \in \mathfrak{g}$$

- We assume a non-periodic map, with monodromies $\mathscr{M} = e^M \in G(\mathbb{Z})$
  - ***Locally*** the manifold is G/Γ, for some discrete group Γ
- The metric follows from the usual Maurer-Cartan equations for G

$$e^0 = dy \qquad e^a = \exp\left(My\right)^a{}_b\, dz^b \qquad de^a + M^a{}_b e^0 \wedge e^b = 0$$

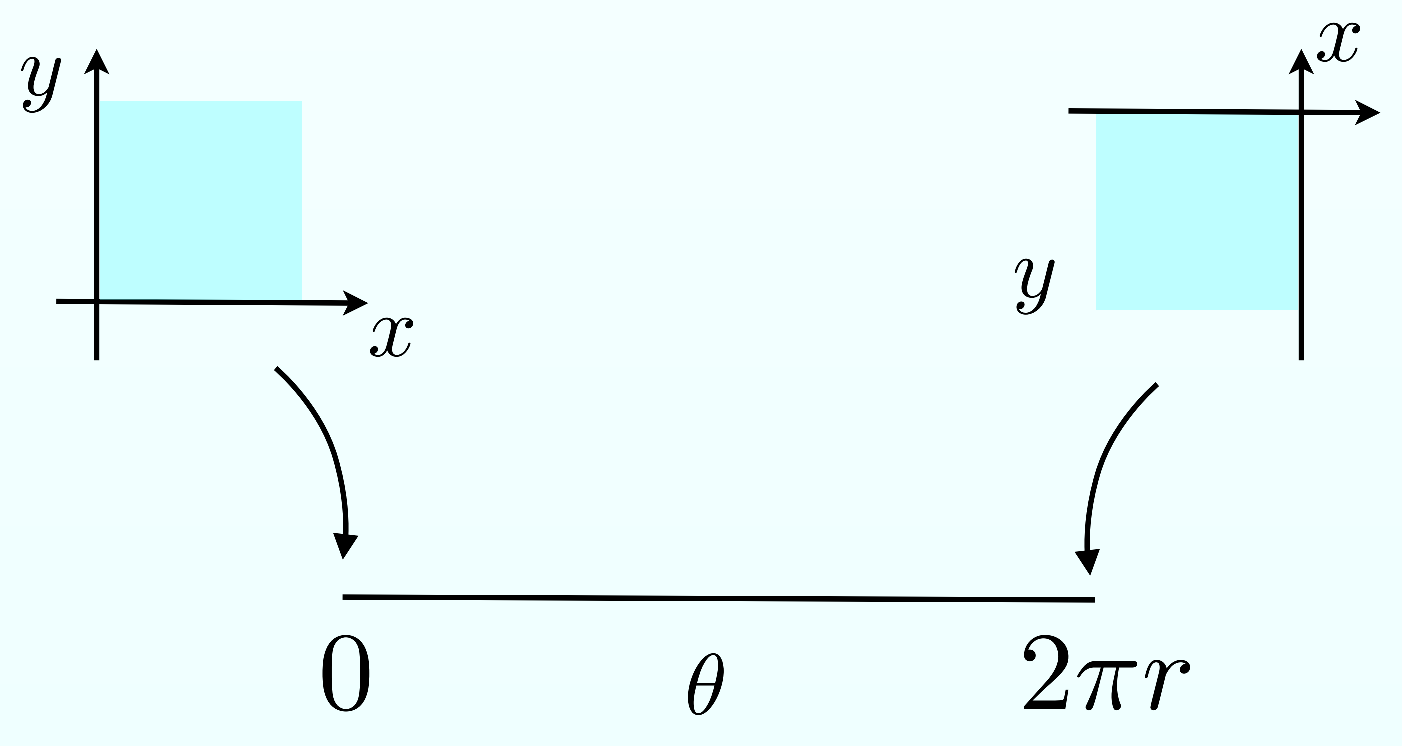
DOUBLED-TWISTED TORI AND GAUGED SUPERGRAVITIES

- One can generalise this to *doubled spacetime* HULL-REID EDWARDS

$$\exp M \in O(d, d, \mathbb{Z})$$

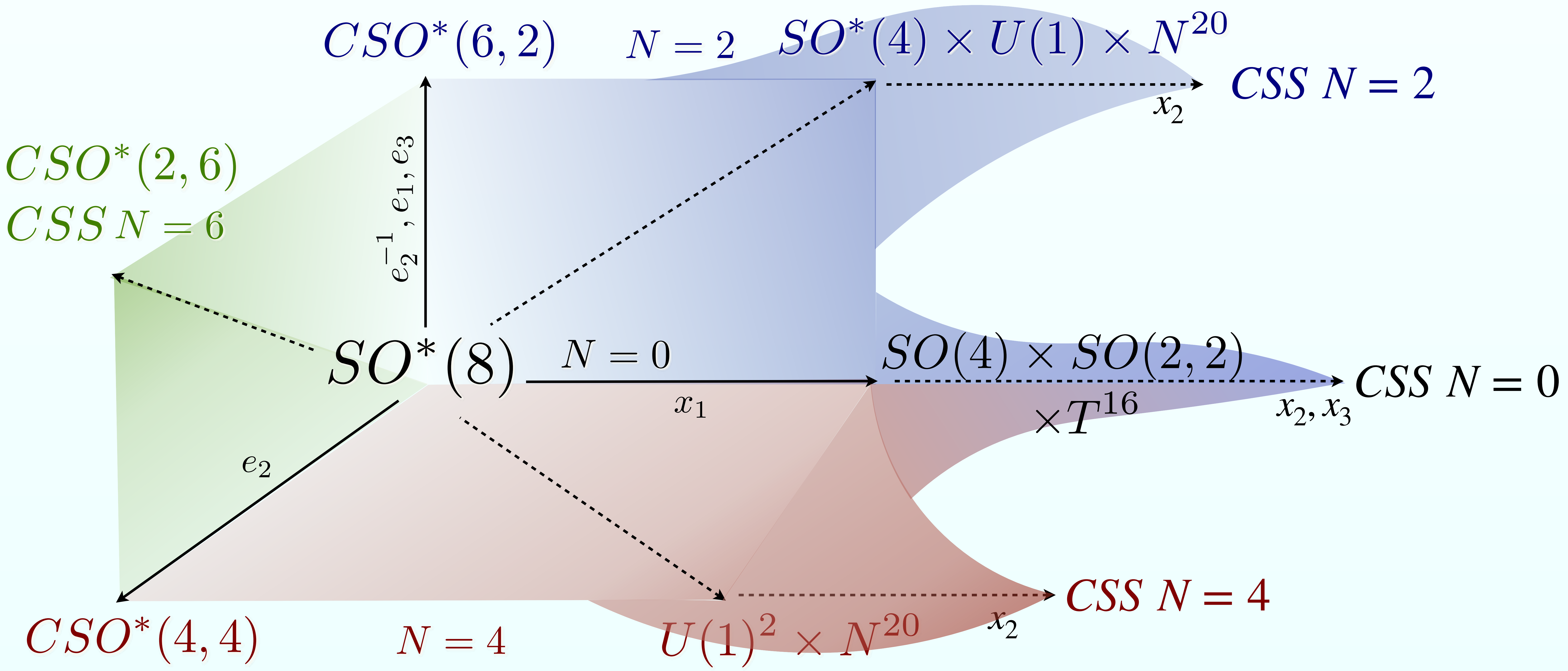
- Double twisted tori, also related to freely acting orbifolds & non-geometric compactifications

CONDEESCU-KOUNNAS-FLORAKIS-LÜST



MAXIMAL SUPERGRAVITY

- Gauging **N=8 supergravity with G=SO*(8)** can lead to Minkowski vacua with an interesting *moduli space* $[SU(1{,}1)/U(1)]^3$



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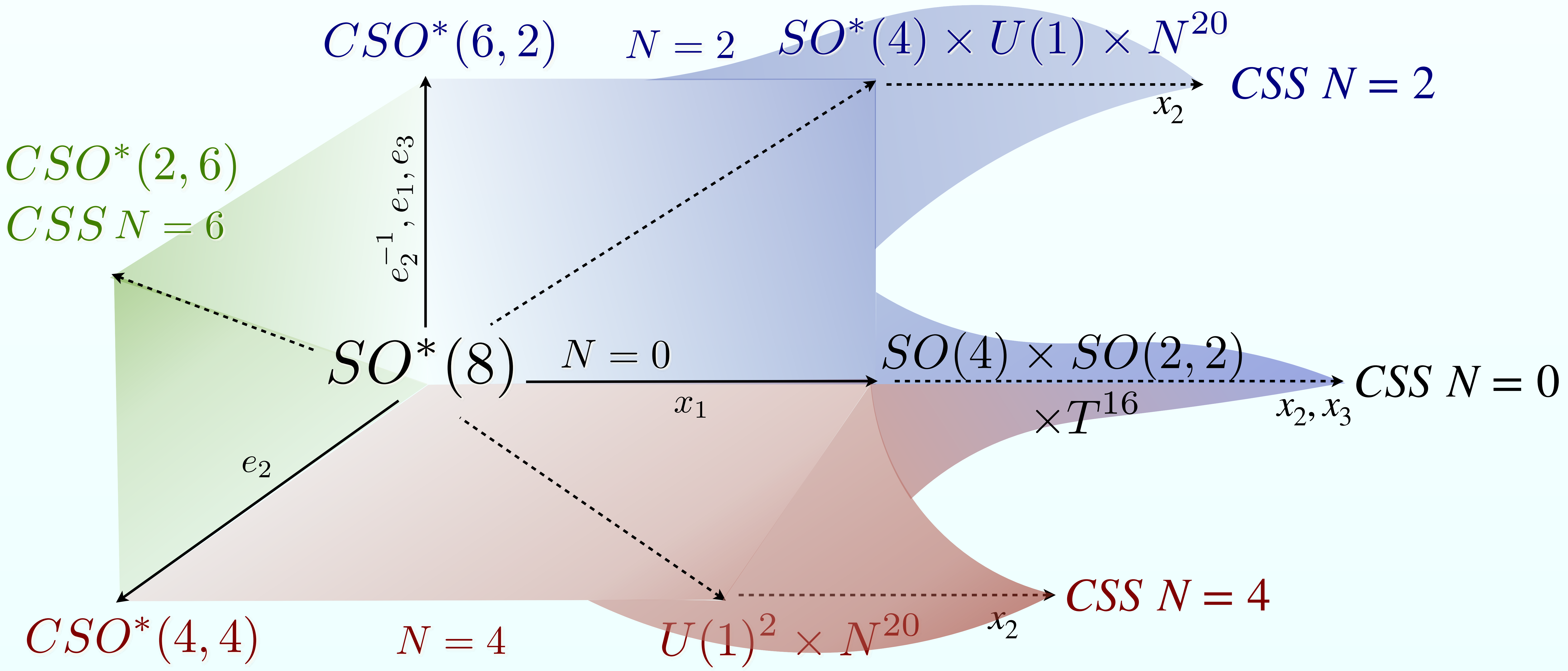
GD-Inverso
Catino-GD-Inverso-Zwirner



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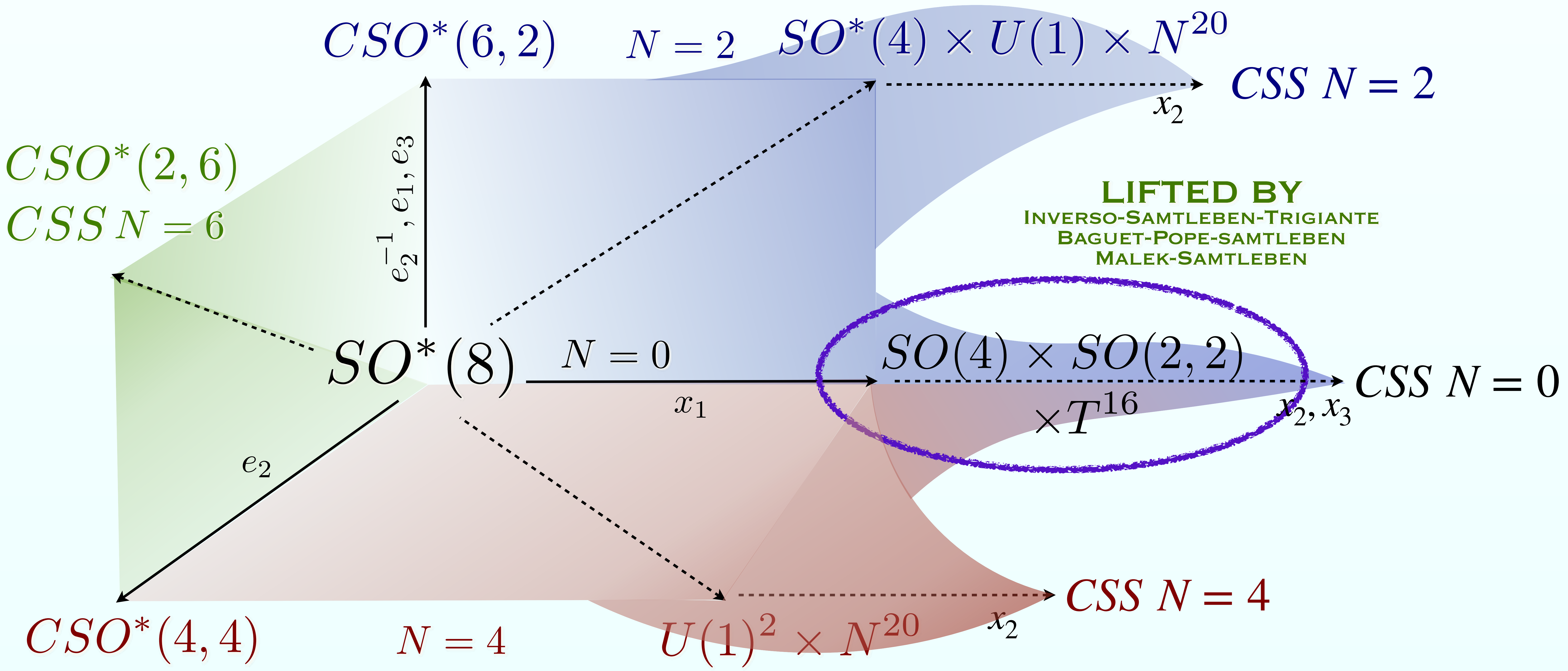
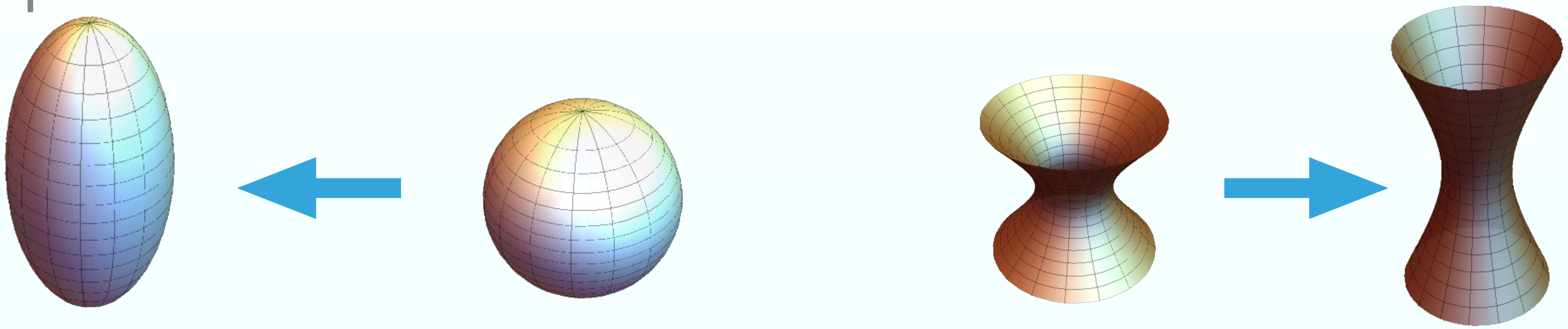


MAXIMAL SUPERGRAVITY & FLAT FOLDS

- The uplift uses as internal manifold $S^3 \times H^{2,2}$



- Trick: CSS reduction of DFT ALDAZABAL-BARON-MARQUÉS-NÚÑEZ GEISSBÜHLER

$$g_{\mu\nu} = e^{4\gamma\varphi(x)} g_{\mu\nu}(x) \quad e^{\phi} = \rho^2(y) e^{\varphi(x)}$$

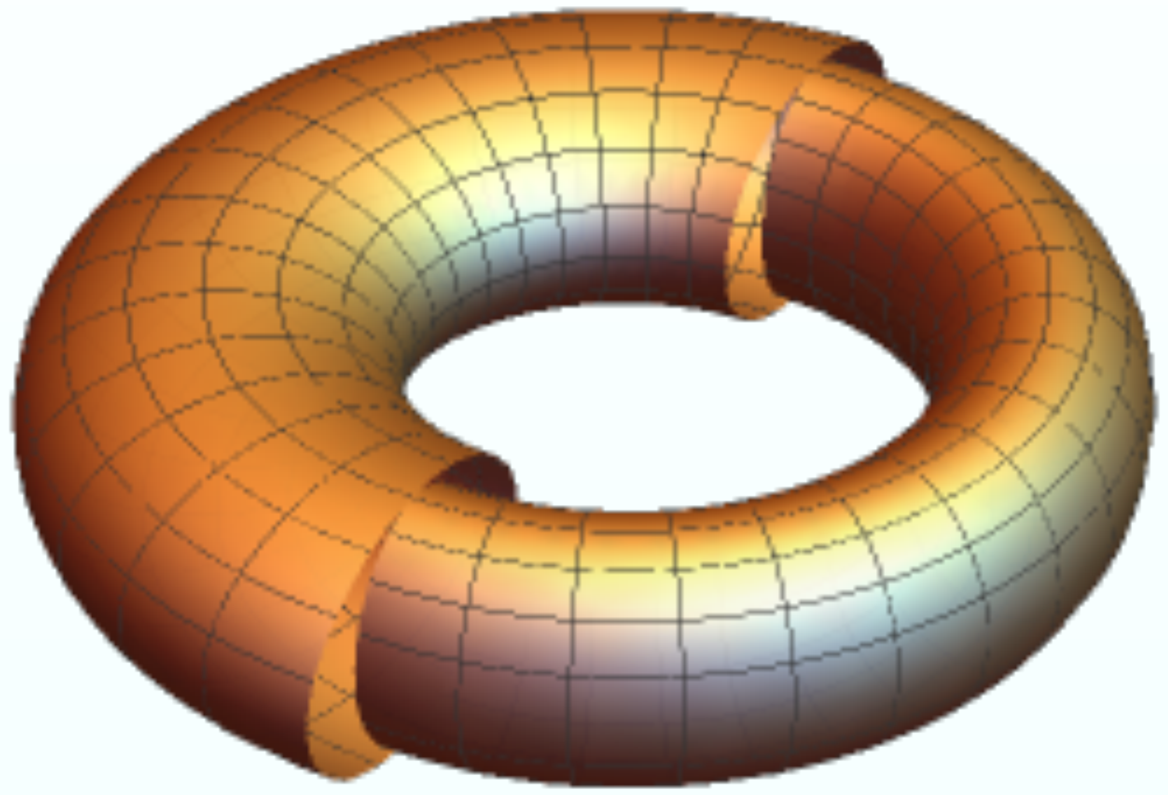
$$\mathcal{H}_{MN} = U_M{}^A(y) M_{AB}(x) U_N{}^B(y)$$

$$\mathcal{A}_\mu^M = U^{-1}{}_A{}^M(y)\, A_\mu^A(x)$$

$$M_{AB}(x) \in \frac{\mathrm{SO}(6,6)}{\mathrm{SO}(6) \times \mathrm{SO}(6)}$$

MAXIMAL SUPERGRAVITY & FLAT FOLDS

GD-Inverso-Spezzati



- The result is a space with T-duality patching
- Example:

$$ds^2 = dx_1^2 + dx_2^2 + d\theta^2 + dy_1^2 + dy_2^2 + d\psi^2$$

- With patching conditions

$$\theta \sim \theta + \alpha$$

$$z_L \sim e^{-i\alpha} z_L$$

$$z_R \sim e^{i\alpha} z_R$$

$$w \sim e^{i\alpha} w$$

$$\psi \sim \psi + \delta$$

$$w_L \sim i\, e^{-i\delta} w_L$$

$$w_R \sim -i\, e^{i\delta} w_R$$

$$z \sim e^{i\delta} z$$

STABILITY

- *Ungauged sugra is finite up to 4 loops (possibly 7)*
- Gauging = new couplings
- One-loop divergencies governed by super traces

$$Str\left(\mathcal{M}^{2k}\right) = \sum_J (-1)^{2J}(2J+1)tr(\mathcal{M}_J)^{2k}$$

- Example: 1-loop potential

$$\begin{aligned} V_{eff} &= \frac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^0\Lambda^4\log\frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2}\mathrm{Str}\mathcal{M}^2\Lambda^2 - \frac{1}{64\pi^2}\mathrm{Str}\mathcal{M}^4\log\Lambda^2 \\ &+ \frac{1}{64\pi^2}\mathrm{Str}\left(\mathcal{M}^4\log\mathcal{M}^2\right) \end{aligned}$$

STABILITY

- Using only general identities and the vacuum condition GD-Zwirner

$$\boxed{Str\,(\mathscr{M}^2) = Str\,(\mathscr{M}^4) = Str\,(\mathscr{M}^6) = 0}$$

- **1-loop finiteness**
- 1-loop potential

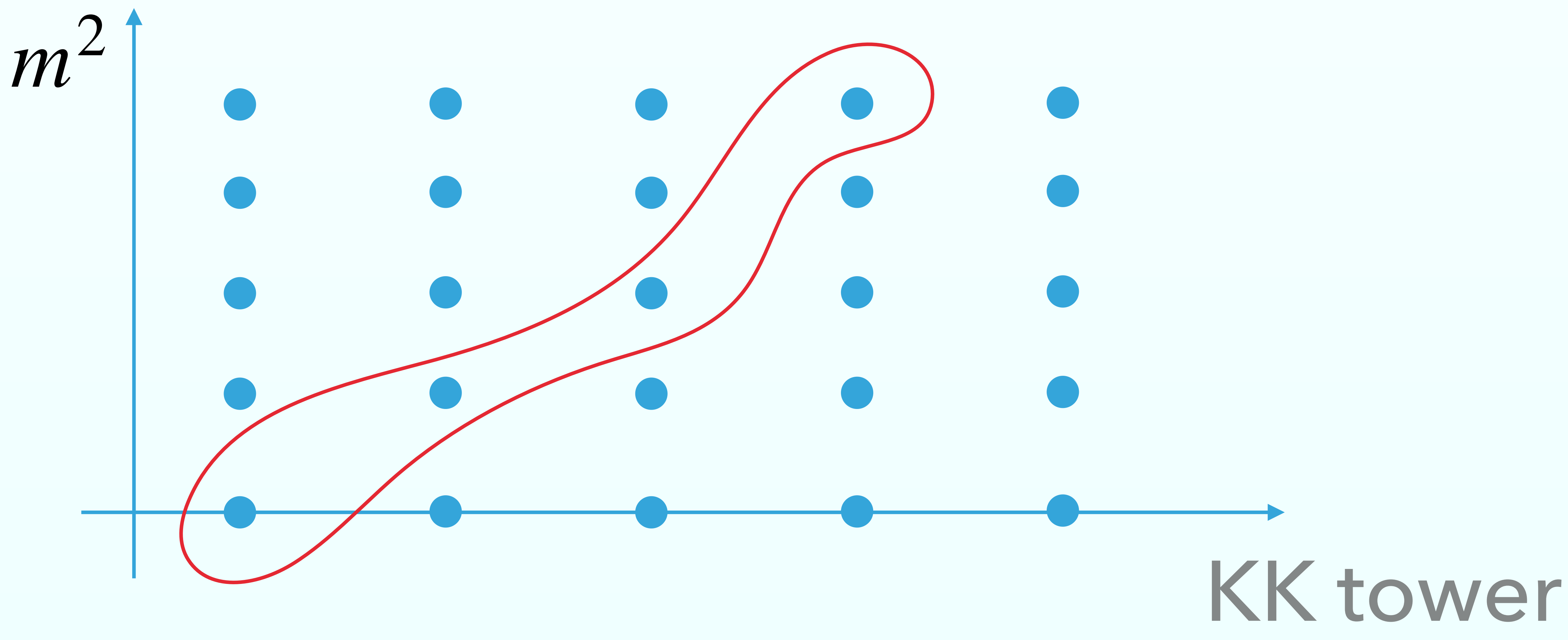
$$V = \frac{1}{64\pi^2} Str\left(\mathscr{M}^4 \log \mathscr{M}^2\right) < 0$$

- Non supersymmetric AdS should decay

WHAT ABOUT TWISTED TORI?

GD-Prezas
Graña-Minasian-Triendl-Van Riet

- Wolf: *Any Riemanniann homogeneous flat space is the direct product of the Euclidean plane with the torus*
- Consistent truncations vs EFT



WHAT ABOUT TWISTED TORI?

GD-Prezas
Graña-Minasian-Triendl-Van Riet

$$q = 2\pi\left(k + \frac{1}{n}\right)$$

- For a 3-torus

Consistent truncation

$$y \sim y + m$$

$$x_1 \sim \cos(qy)\, x_1 - \sin(qy)\, x_2$$

$$x_2 \sim \sin(qy)\, x_1 + \cos(qy)\, x_2$$

Homogeneous

Good EFT

$$y \sim y + m$$

$$x_1 \sim \cos(qy)\, x_1 - \sin(qy)\, x_2 + n - \frac{p}{2}$$

$$x_2 \sim \sin(qy)\, x_1 + \cos(qy)\, x_2 + \sqrt{3}\frac{p}{2}$$

Non-Homogeneous

WHAT ABOUT KK STATES? GD-ZWIRNER

- 5d supergravity to 4d SS reduction

$$V_1 = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{+\infty} \sum_i (-1)^{2J_\alpha}(2J_\alpha + 1)\log\left(p^2 + m^2_{n,\alpha}\right) \qquad m^2_{n,\alpha} = \frac{(n+s_\alpha)^2}{R^2}$$

- Interesting super trace relations for n fixed:
  - $Str\,\mathcal{M}_n^{q<N} = 0,$
  - $Str\,\mathcal{M}_n^{q=N}$ fixed and n-independent

WHAT ABOUT KK STATES? GD-Zwirner

- Resumming

$$V_1 = -\frac{3}{128\,\pi^6 R^4} \sum_\alpha (-1)^{2J_\alpha} (2J_\alpha + 1) \left[\mathrm{Li}_5(e^{-2\pi i s_\alpha}) + \mathrm{Li}_5(e^{2\pi i s_\alpha})\right] \quad m^2_{n,\alpha} = \frac{(n+s_\alpha)^2}{R^2}$$

- For small deformation parameters we have corrections to 1-loop eff. Theory
  - Example, N=8

$$V_1 = -\frac{93\,\zeta(5)}{8\,\pi^6 R^4} \simeq -\frac{0.0125}{R^4} \qquad V_{1,red} \simeq -\frac{0.0184}{R^4}$$

WHAT ABOUT KK STATES? GD-ZWIRNER

- Higher-dimensional twisted tori related to Bieberbach manifolds
- Bieberbach = smooth flat quotients: $\mathbb{R}^d/\Gamma$
- $\mathbb{R}^2/\Gamma$: 17 wallpaper groups
- $\mathbb{R}^3/\Gamma$: 219 affine space groups as orbifolds CONWAY-FRIEDRICHS-HUSON-THURSTON
- Only 10 freely acting and they are related to wallpaper groups
    - $\Gamma \subset \mathbb{E}_2 = U(1) \ltimes \mathbb{R}^2$

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- Bieberbach = smooth flat quotients: $\mathbb{R}^d/\Gamma$
- $\mathbb{R}^6/\Gamma$: CID-SCHULZ LUTOWSKY-PUTRYCZ
  - 28927992 orbifolds (crystals)
  - 38746 flat manifolds (Bieberbach)
  - 3314 orientable
  - 717 spin!

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS GD-Zwirner

- 3d Twisted tori (wallpapers without reflections)

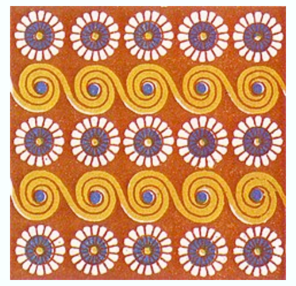
$$O_1^3 \qquad \text{(flat torus)} : \Gamma = \mathbb{Z}^3$$

$$O_2^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z}_2 = \langle a, b, c, r \,|\, r^2 = c, rar^{-1} = a^{-1}, rbr^{-1} = b^{-1} \rangle$$

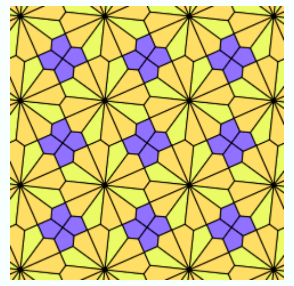
$$O_3^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z}_3 = \langle a, b, c, r \,|\, r^3 = c, rar^{-1} = b, rbr^{-1} = a^{-1}b^{-1} \rangle$$

$$O_4^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z}_4 = \langle a, b, c, r \,|\, r^4 = c, rar^{-1} = b, rbr^{-1} = a^{-1} \rangle$$

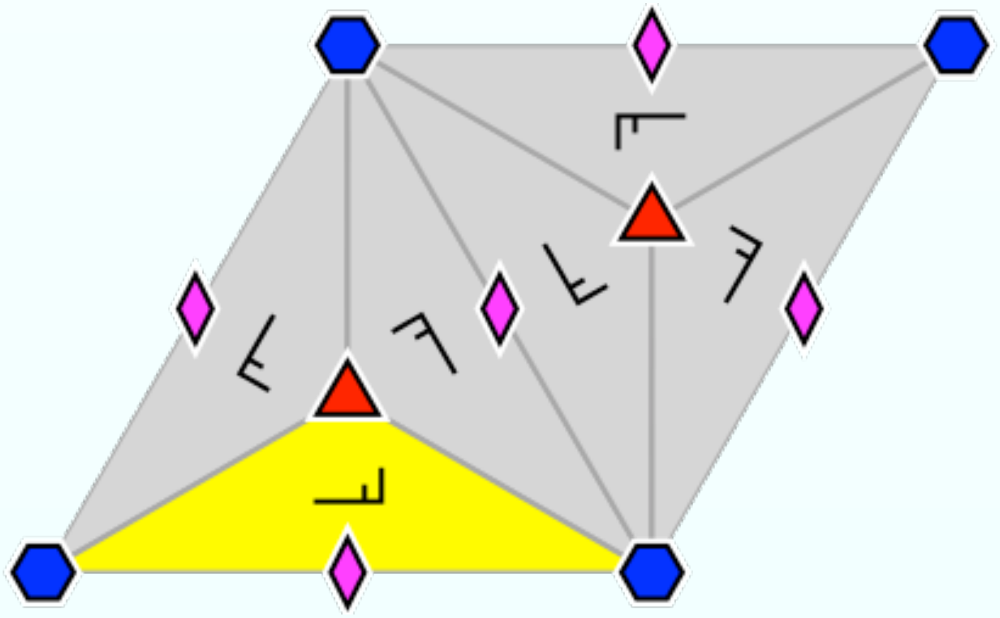
$$O_5^3 : \Gamma = \mathbb{Z}^2 \rtimes \mathbb{Z}_6 = \langle a, b, c, r \,|\, r^6 = c, rar^{-1} = b, rbr^{-1} = a^{-1}b \rangle$$


p2


p3


p4

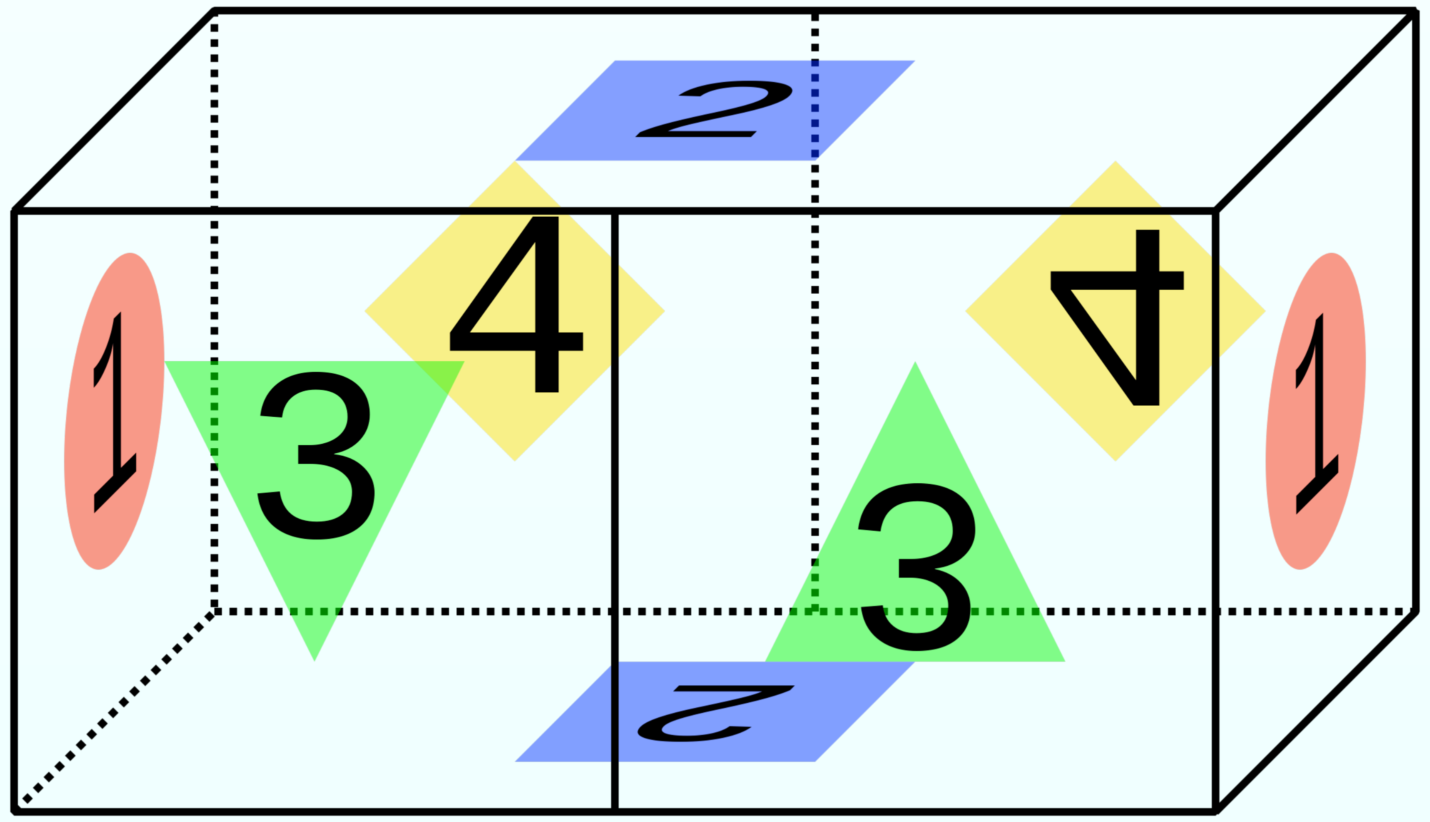

p6


p6 lattice example

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS GD-ZWIRNER

- Presentations (Hantzsche-Wendt 3-manifold):

$$O_6^3 : \Gamma = \langle a, b, c, r, \sigma \quad | \quad r^2 = c, \sigma^2 = a, \sigma r \sigma^{-1} = abr^{-1}, rar^{-1} = a^{-1}, \\ rbr^{-1} = b^{-1}, \sigma a \sigma^{-1} = a^{-1}, \sigma c \sigma^{-1} = c^{-1} \rangle \,.$$



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Non-Homogeneous

WALLPAPER GROUPS AND FREELY ACTING ORBIFOLDS

GD-Zwirner

- To compute the KK spectrum we need harmonics
- Use unitary reps of $\mathbb{E}_2$ with appropriate boundary conditions?

$$T_R(L)[F(\psi)] = \exp(i\,R\,(y\cos\psi + x\sin\psi))F\left(\psi + \kappa\,z\right)$$

- The matrix elements

$$Y^R_{mn}(x,y,z) = \frac{1}{2\pi}\int d\psi\ \exp(-i\,n\,\psi)T_R(L)[\exp(i\,m\,\psi)]$$

- Are harmonic functions, but difficult construction of invariant ones. For instance on $\mathbb{T}^3$

$(-1)^{(m-n)}e^{i\left[m\,\kappa\,z + (n-m)\arctan\left(\frac{x}{y}\right)\right]}J_{n-m}\left(R\sqrt{x^2+y^2}\right)$ should go into $e^{2\pi i(mx+ny+pz)}$

BIEBERBACH MANIFOLDS

- Orientable, freely acting orbifolds

$$M_n = \mathbb{R}^n/\Gamma, \qquad \Gamma \in \mathrm{SO}(n) \ltimes \mathbb{R}^n$$

- All elements $\gamma \in \Gamma$ can be represented as matrices $\Gamma \ni \gamma = (R, \vec{b})$

$$\gamma = \begin{pmatrix} R & \vec{b} \\ 0 & 1 \end{pmatrix}$$

- by which we construct a lattice $\Lambda = \Gamma \cap \mathbb{R}^n$ and the dual

$$\Lambda^* \equiv \{\vec{m} \in \mathbb{R}^n \,|\, \vec{v} \cdot \vec{m} \in \mathbb{Z}, \forall v \in \mathbb{R}^n\}$$

BIEBERBACH MANIFOLDS

- Hantzsche-Wendt

$$\alpha = \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \beta = \begin{pmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- The holonomy is $\mathbb{Z}_2 \times \mathbb{Z}_2$ and the lattice is generated by

$$e_1 = (\mathbb{I}, \overrightarrow{e_1}) = \alpha^2, \quad e_2 = (\mathbb{I}, \overrightarrow{e_2}) = \beta^2, \quad e_3 = (\mathbb{I}, \overrightarrow{e_3}) = (\alpha\beta)^2,$$

BIEBERBACH MANIFOLDS

- Bieberbach Spin manifolds are less, but we can have multiple spin structures, depending on how

$$\varepsilon = \Gamma \to \mathrm{Spin}(n)$$

- In fact one can have

$$\varepsilon(e_i) = \pm \mathbb{1}$$

- and spinors transform as

$$\psi(\vec{x} + \vec{e_i}) = \pm\, \psi(\vec{x})$$

BIEBERBACH MANIFOLDS

- After some work, we found the proper harmonics and computed the KK spectrum GD-ZWIRNER
- For instance, for the scalar harmonics

$$Y_{\vec{m}} = \sum_{\gamma \in \Gamma/\Lambda} \exp\left[2\pi i \vec{m} \cdot (R_\gamma \vec{x} + \vec{b}_\gamma)\right], \qquad \vec{m} = m_i \vec{e}_i^{\,*} \in \Lambda^*$$

- we have

$$\Box Y_{\vec{m}} = -4\pi^2 ||\vec{m}||^2 Y_{\vec{m}}$$

BIEBERBACH MANIFOLDS

GD-Zwirner

- II-A on Bieberbach manifolds fully computed
- For M-theory we miss a classification of 7d Bieberbach!
- Preliminary results keep showing:
    - **Supertrace cancellations at fixed level**
        - (Upon redefinition of the masses w.r.t. quantum numbers)
    - Negative Casimir-energy contribution ⇒ unstable non-susy AdS

SUMMARY

- Minkowski vacua of fully broken sugra theories have a very interesting moduli space (generalised SS)
- *1-loop finite, but unstable?*
- Check supergravity reductions on twisted tori
  - EFT vs consistent truncations
- ***Casimir from KK still negative***
  - *Supertrace cancellations order by order!*
- Full string theory analysis? Brane/orbifold necessary?