

Duality Defects in String Theory

or

Exploring Utopia

Based on work with:

Noah Braeger, Arun Debray, Jonathan J. Heckman, Miguel Montero

— to appear —

And earlier work with: A. Debray, J.J. Heckman, M. Montero

— Gravity, Strings, and Supersymmetry Breaking —
Pisa — April 3, 2025

Markus Dierigl




What would you do if I give you a theory?

For example the Lagrangian:

$$\mathcal{L} = \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu}$$

Probably:

- **Equations of motion:** $\partial_\mu F^{\mu\nu} = 0$
- **Bianchi identities:** $\partial_{[\mu} F_{\nu\rho]} = 0$
- **Quantize (canonically):** 

(from local definition:
 $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)

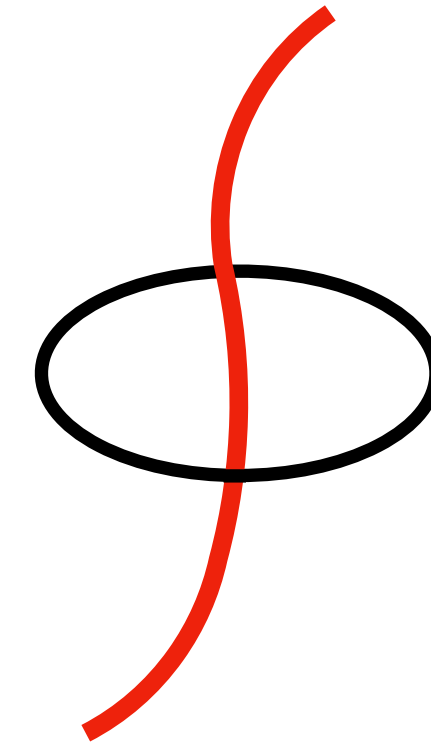
What would you do if I give you a theory?

Maybe: Analyze the symmetries

0-form U(1) gauge symmetry: $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$

1-form U(1) global symmetries:

[Gaiotto, Kapustin, Seiberg, Willett '14]



Maybe: Add charged fields ψ_q and couple, e.g., $\bar{\psi}_q i\gamma^\mu D_\mu \psi_q$

Then: Check for anomalies

Everything pretty action oriented / model dependent

What if instead I give you the following data

- I want to investigate a theory:
- that is D-dimensional
 - allows for fermions
 - has a U(1) gauge theory

Way more general:

$$(F_{\mu\nu}F^{\mu\nu})^n$$
$$j_e^\mu, j_m^\mu, D_\mu \bar{\phi} D^\mu \phi, \bar{\psi}_q i\gamma^\mu D_\mu \psi_q, \dots$$

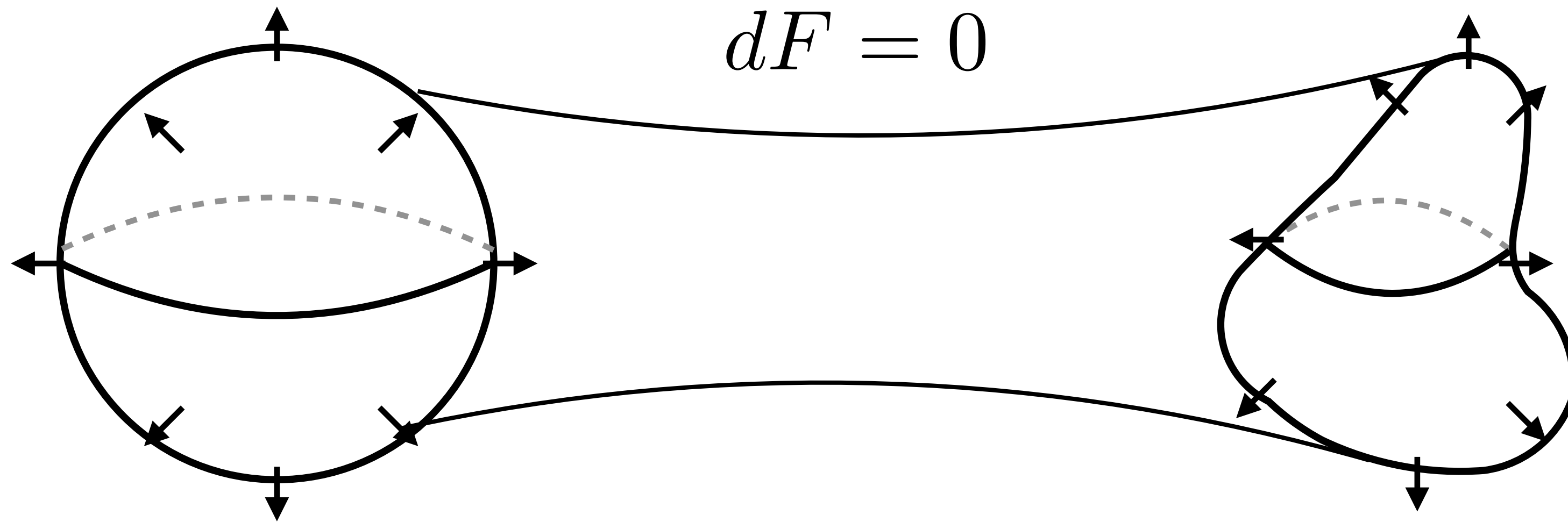
→ **Additionally:** allow for **dynamics of spacetime**
(EFT flavor: **smooth** spacetime)

Topological gauge data

We still have the U(1) gauge field: A

And a Bianchi identity: $dF = d(dA) = 0$

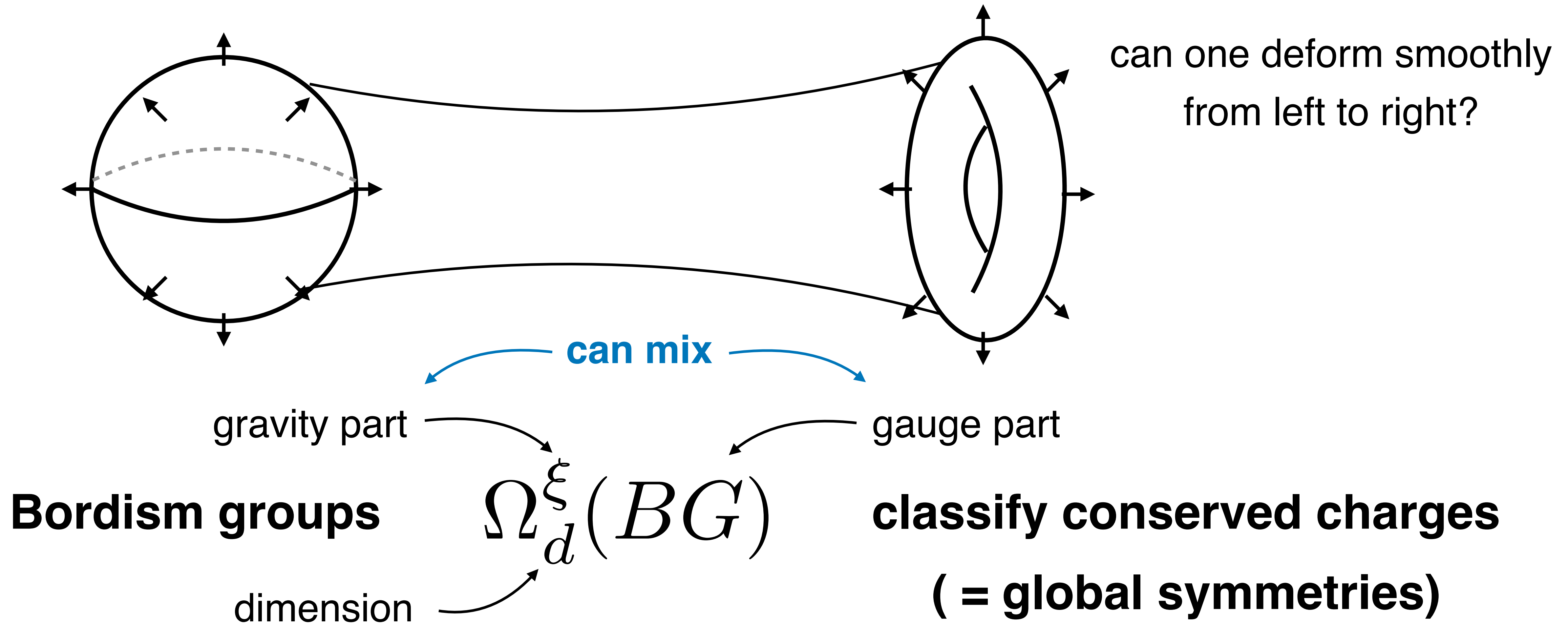
$$\frac{1}{2\pi} \oint F$$



cannot deform away
conserved **topological**
charges

Combine with gravitational data

corresponds to topology of spacetime manifold



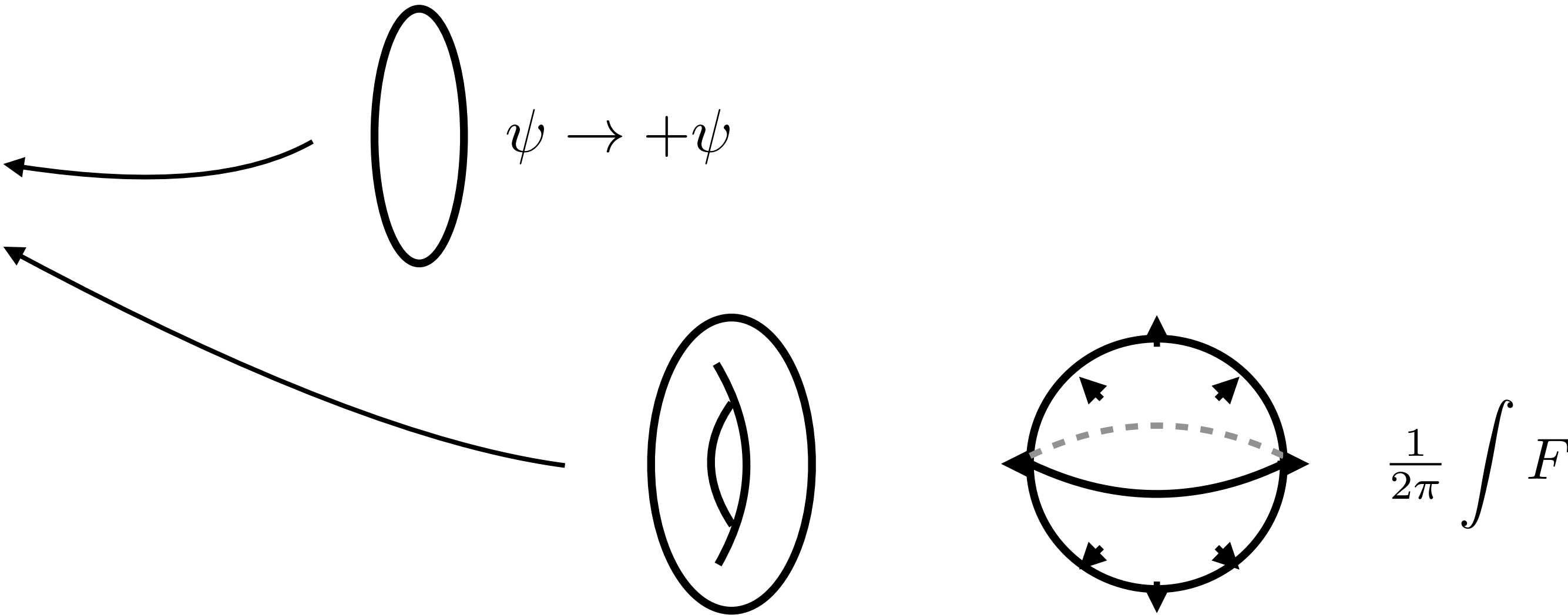
Example

Let's go back to our example:

- that is D-dimensional
- allows for fermions
- has a U(1) gauge theory

$$\Omega_d^{\text{Spin}}(BU(1))$$

d	$\Omega_d^{\text{Spin}}(\text{U}(1))$
1	\mathbb{Z}_2
2	$\mathbb{Z}_2 \oplus \mathbb{Z}$
3	0
4	$\mathbb{Z} \oplus \mathbb{Z}$
5	0
6	$\mathbb{Z} \oplus \mathbb{Z}$



All of this from **minimal input**

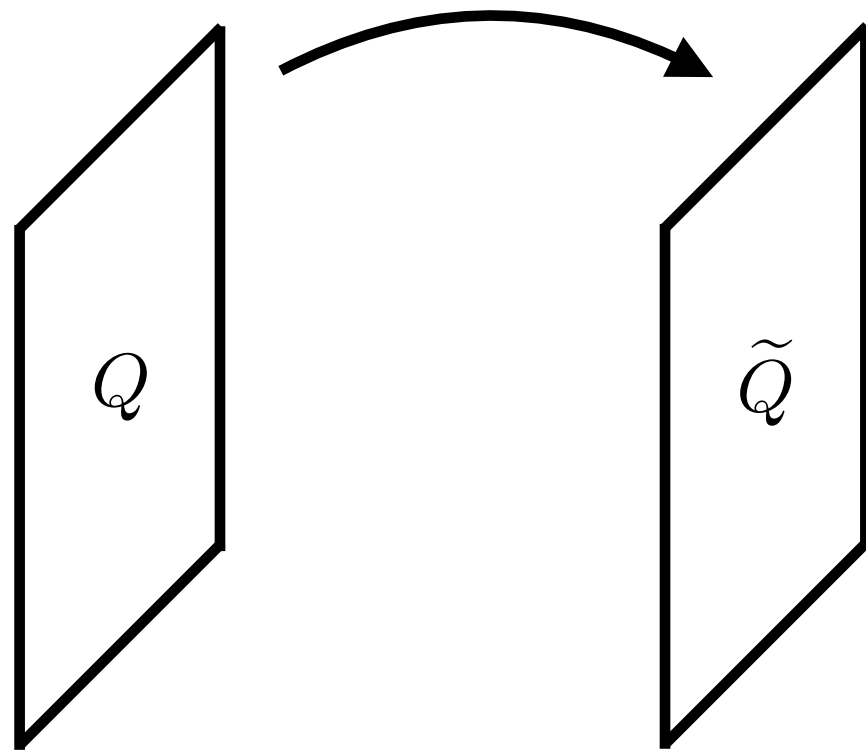
(losing information but sensitive to universal features)

➔ but there is a **problem**

No global symmetries in quantum gravity

See e.g. [Banks, Dixon '88], [Banks, Seiberg '11], [Harlow, Ooguri '18], [Harlow, Shaghoulian '20], [Bah, Chen, Maldacena '22], [Heckman, Hübner, Murdia '24], ...

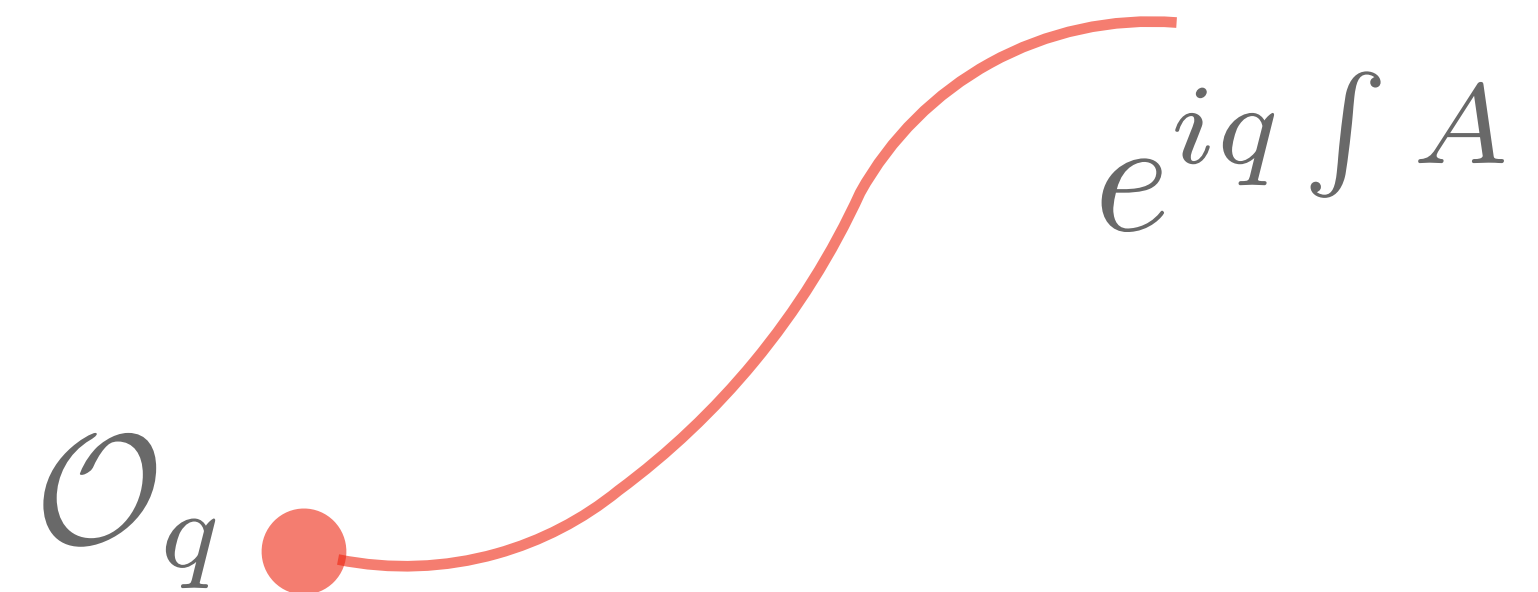
broken



charge not conserved

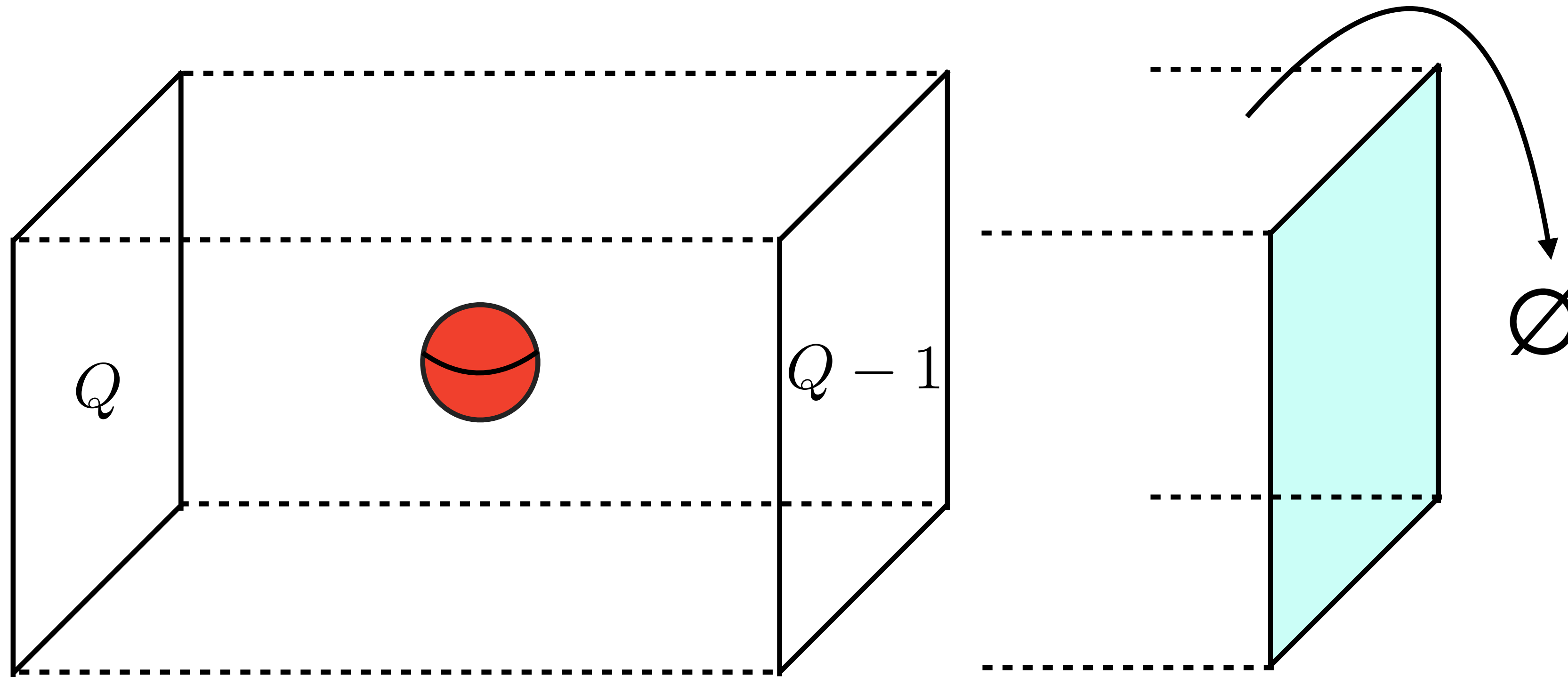
Violation typically at least $e^{-M_{Pl}^2/\Lambda^2}$

see also [Daus, Hebecker,
Leonhardt, March-Russell '20]



Symmetry-breaking objects

backgrounds associated with **new symmetry-breaking defects**



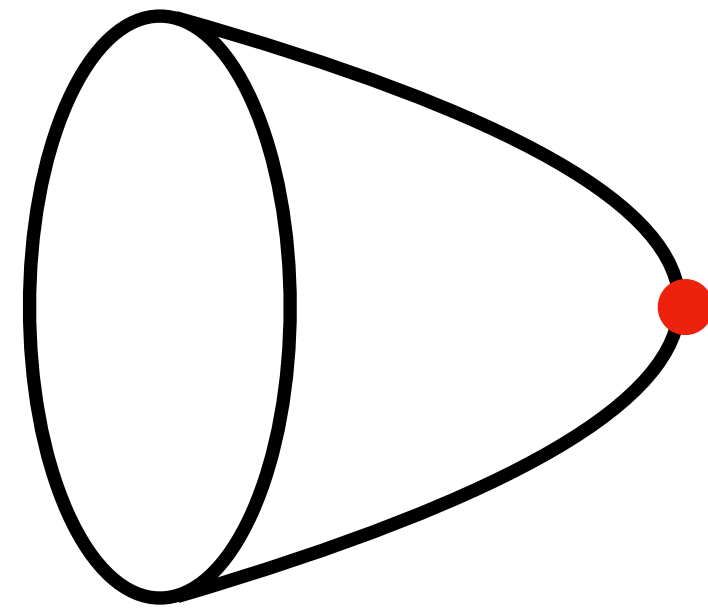
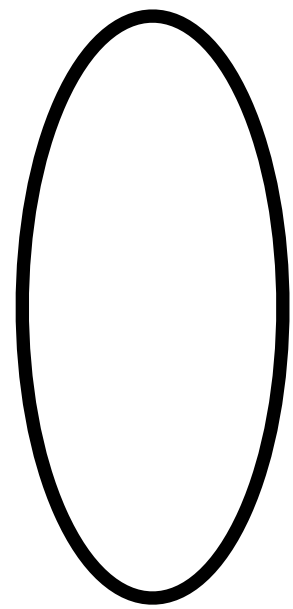
$$\Omega_d^{QG} = 0$$

**Swampland
Cobordism
Conjecture**

[McNamara, Vafa '19]

Can look **singular** at **low-energies**, but have **finite mass** (tension)

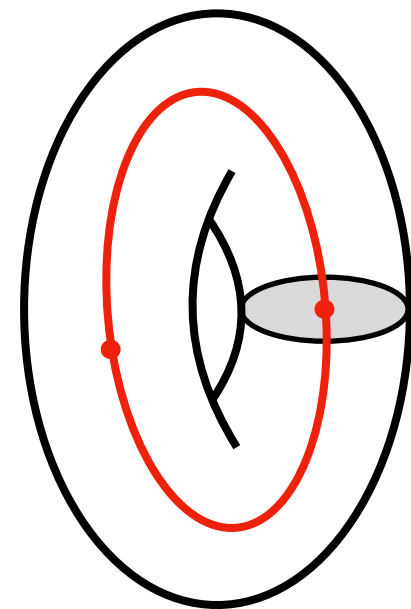
Example



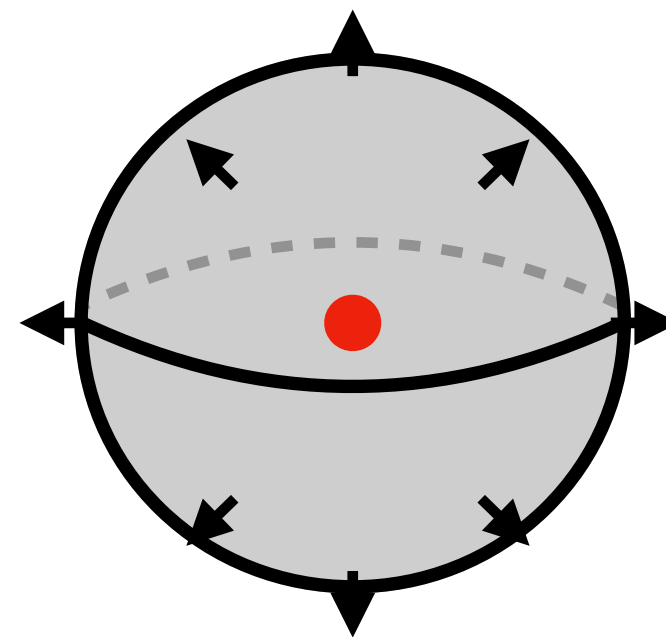
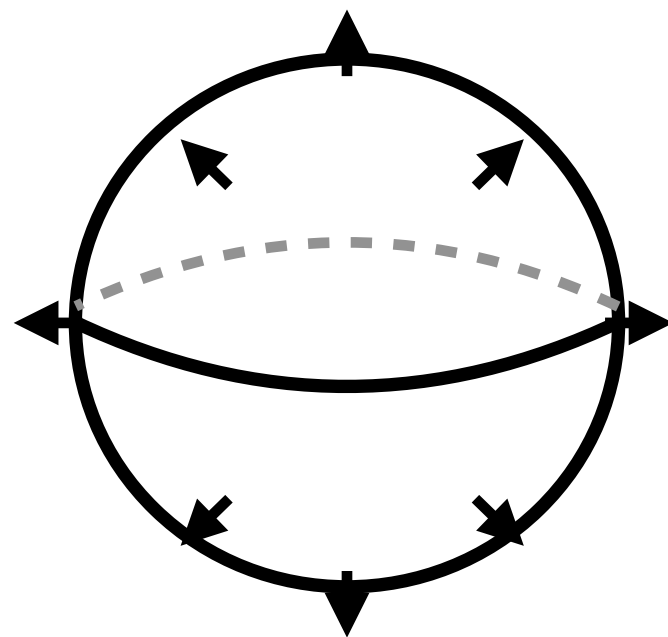
codimension-two Spin defect

[McNamara, Vafa '19], [Hamada, Hamada, Kimura '25]

No issue in IIB: [Debray, MD, Heckman, Montero '23]



also takes care of this
(compactified on circle)



magnetic monopole

Once upon a time in IIBordia

[Debray, MD, Heckman, Montero '21 & '23]

Type IIB supergravity:

- properties of **spacetime** (tangential structure)
- **S-duality**: a gauged discrete strong-weak coupling duality

Expectation:

$$\Omega_d^{\text{IIB}} = 0$$

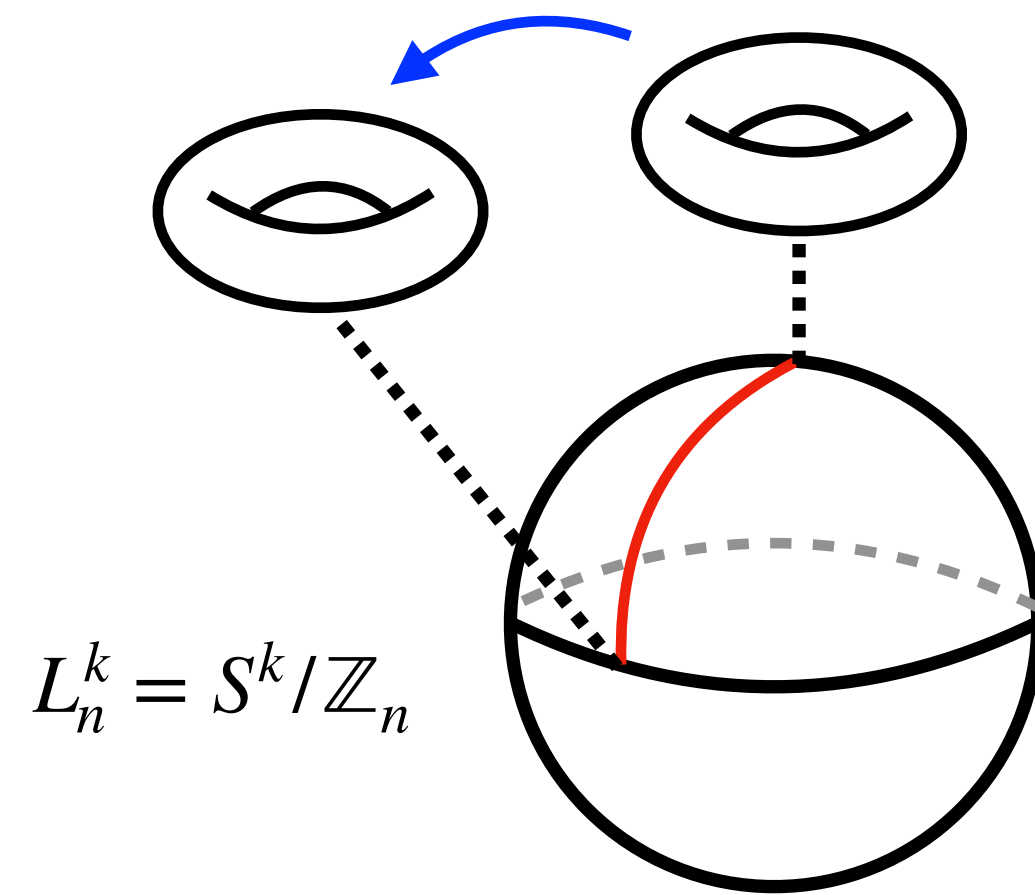
d	$\Omega_d^{\text{Spin}}(BSL(2, \mathbb{Z}))$	$\Omega_d^{\text{Spin-Mp}(2, \mathbb{Z})}$	$\Omega_d^{\text{Spin-GL}^+(2, \mathbb{Z})}$
	§4	§5	§6
0	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	\mathbb{Z}_2
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	\mathbb{Z}	\mathbb{Z}	\mathbb{Z}
5	\mathbb{Z}_{36}	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	\mathbb{Z}_2	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

many potential
global symmetries

[Debray, MD, Heckman, Montero '21 & '23]

Symmetry-breaking defects

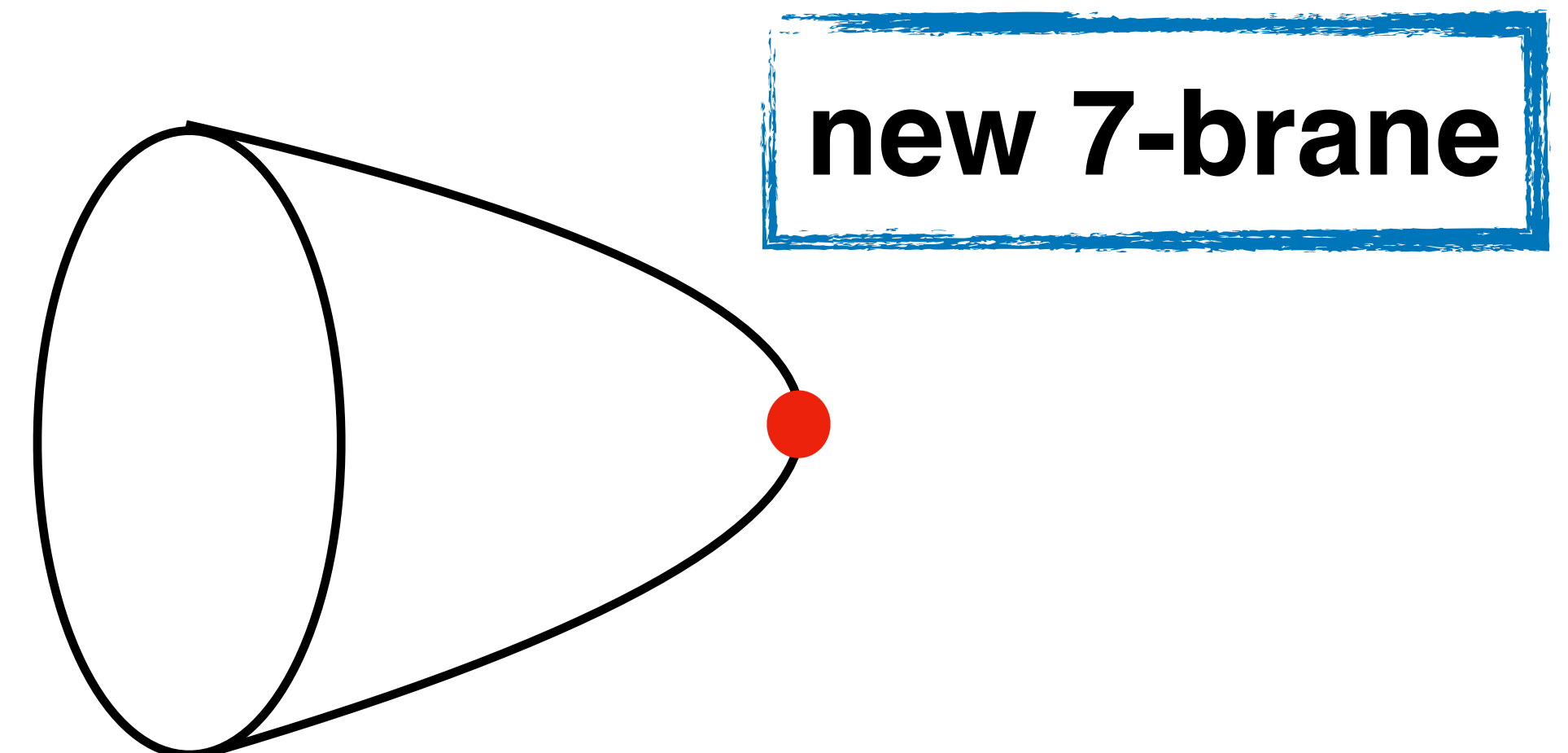
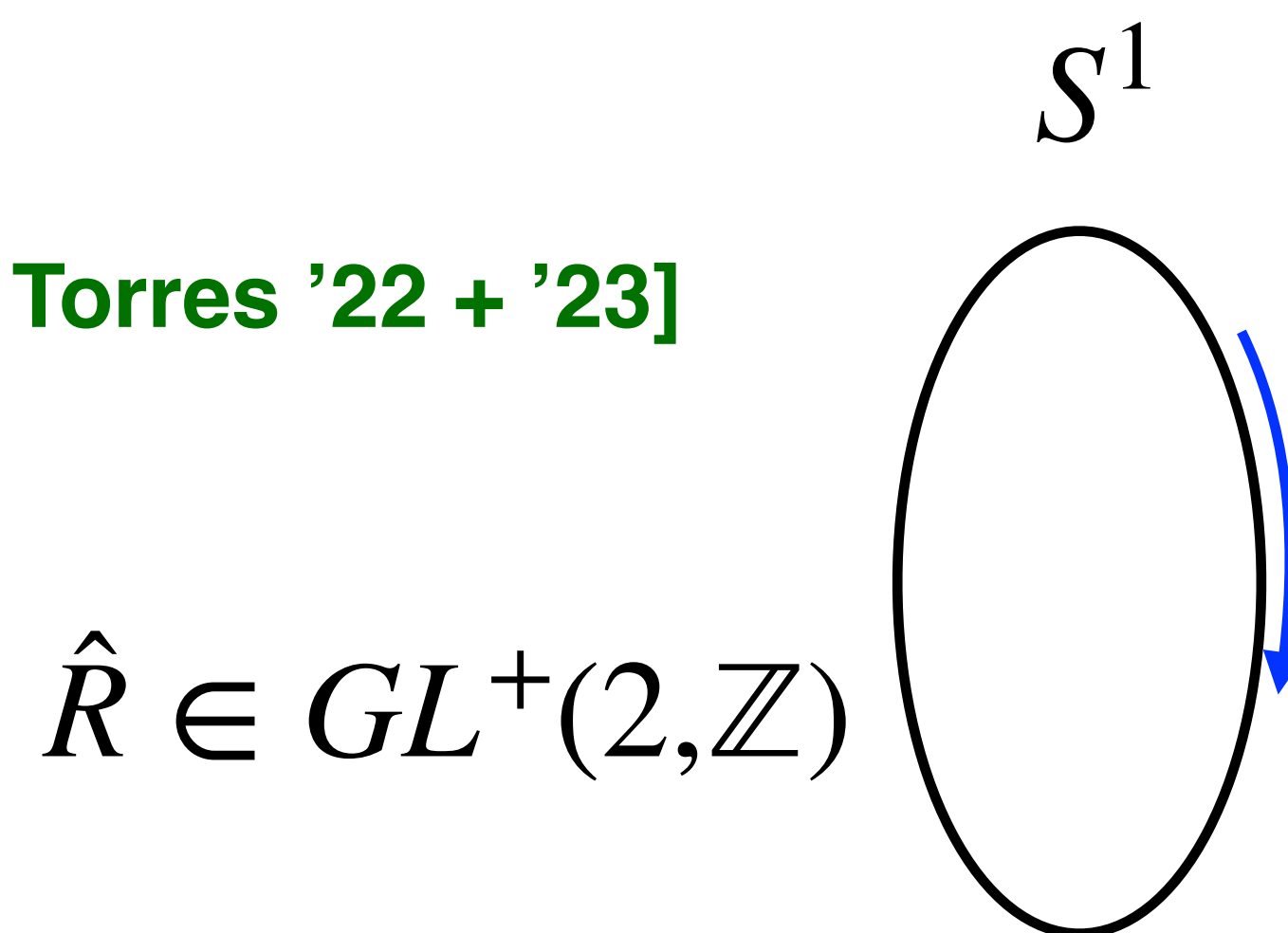
String theory takes care of it → interesting corners



- Non-Higgsable clusters
- $\mathcal{N} = 3$ S-folds
- $[p,q]$ -7-branes

One remains

[MD, Heckman, Montero, Torres '22 + '23]



Let's explore Utopia

Supergravity with 32 supercharges and U-duality

U-duality

See e.g. [Obers, Pioline '98]

U-duality in (11-k) dimensions:

$$\mathrm{SL}(k; \mathbb{Z}) \bowtie \mathrm{SO}(k-1, k-1; \mathbb{Z})$$

D	G_U^{Dd}
10	1
9	$\mathrm{SL}(2, \mathbb{Z})$
8	$\mathrm{SL}(3, \mathbb{Z}) \times \mathrm{SL}(2, \mathbb{Z})$
7	$\mathrm{SL}(5, \mathbb{Z})$
6	$\mathrm{SO}(5, 5, \mathbb{Z})$
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$

comes from:

M-theory on T^k

\wr

type IIB on T^{k-1}

Surprise I: Sparse codimension-two

In general one has:

$$\Omega_1^{\text{Spin}}(BG_U) = \mathbb{Z}_2 \oplus \text{Ab}(G_U)$$

Abelianization

$$\text{Ab}(G_U) = \frac{G_U}{[G_U, G_U]}$$

$$\text{Ab}(\text{SL}(3, \mathbb{Z})) = 0$$

no extra defects needed in codim-two

$$\text{Ab}(G_U^{D^d}) = 0, \quad D < 8$$

Still interesting exotic branes

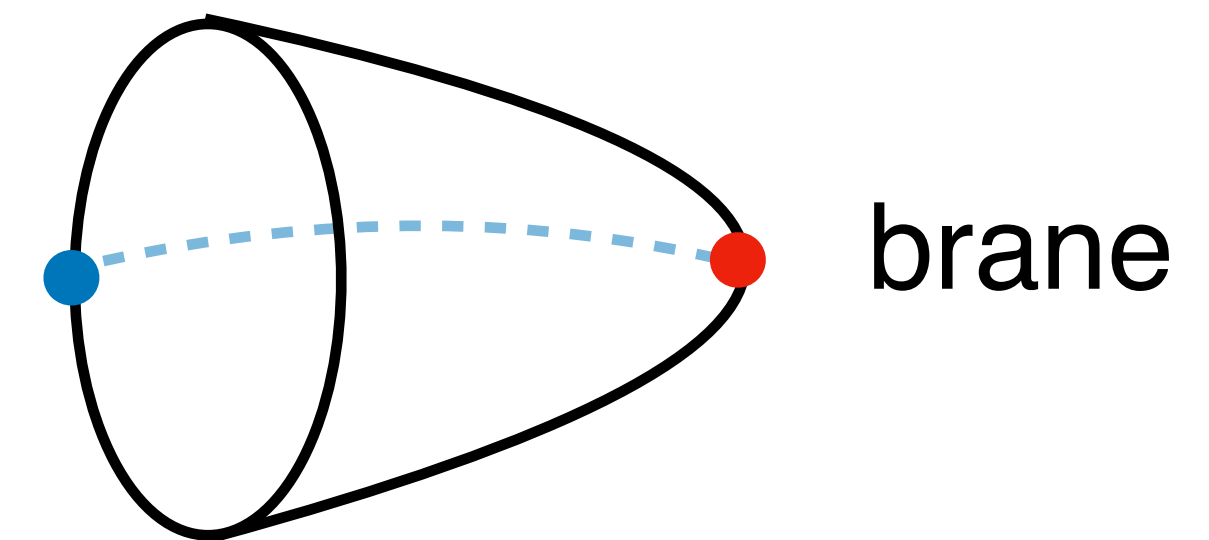
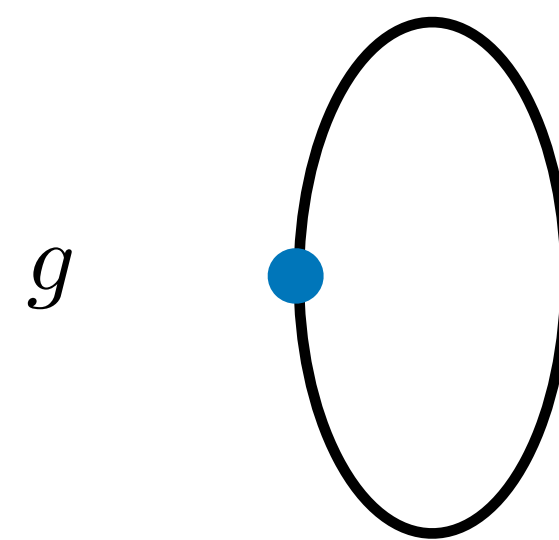
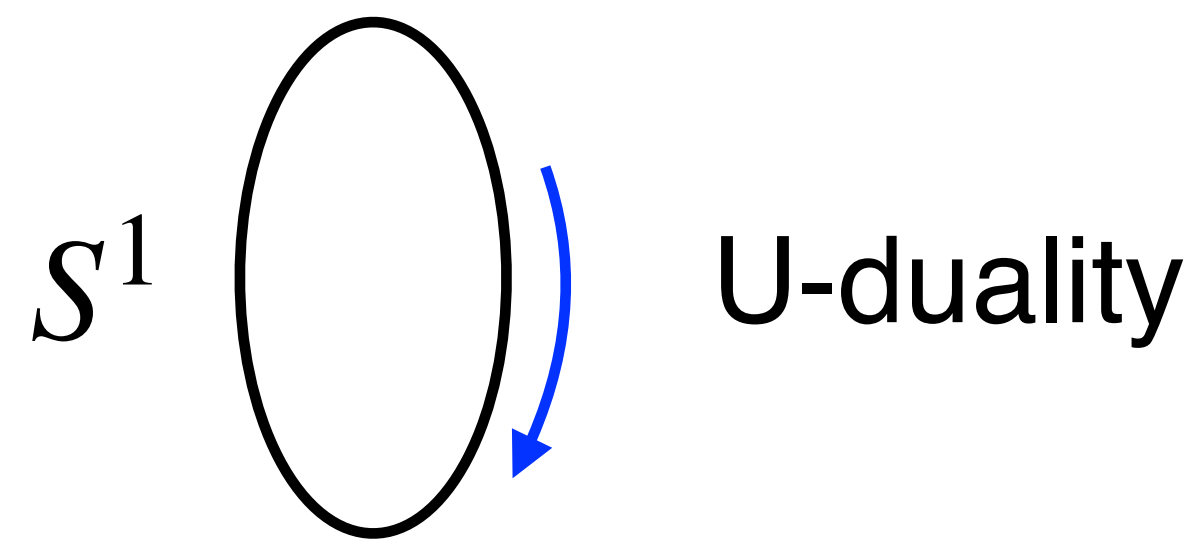
[deBoer, Shigemori '12]

Why are they not required?

Gravitational solitons

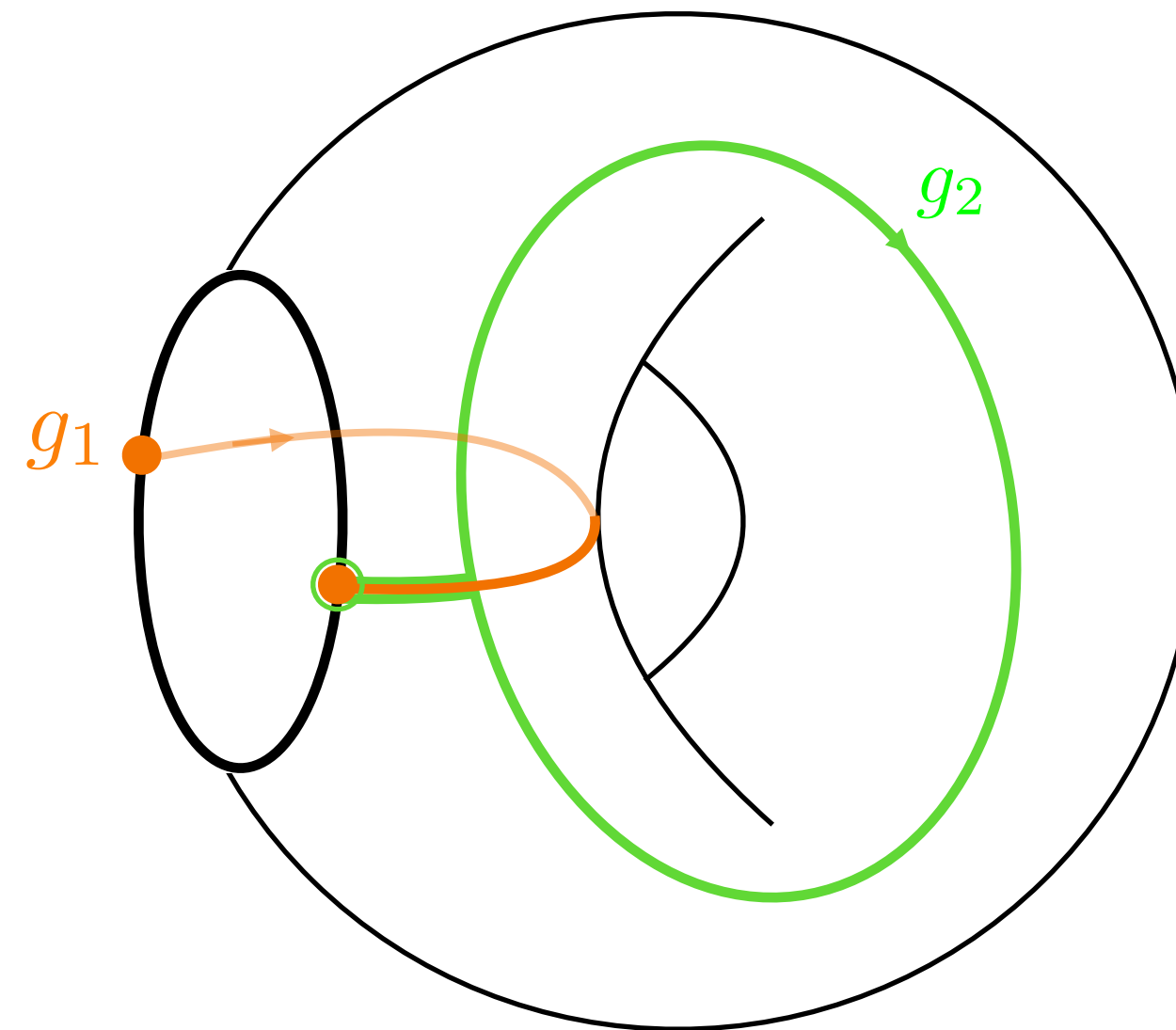
[McNamara '21], [Debray, MD, Heckman, Montero '23]

Gravitational solitons do the job (smooth geometries)



If

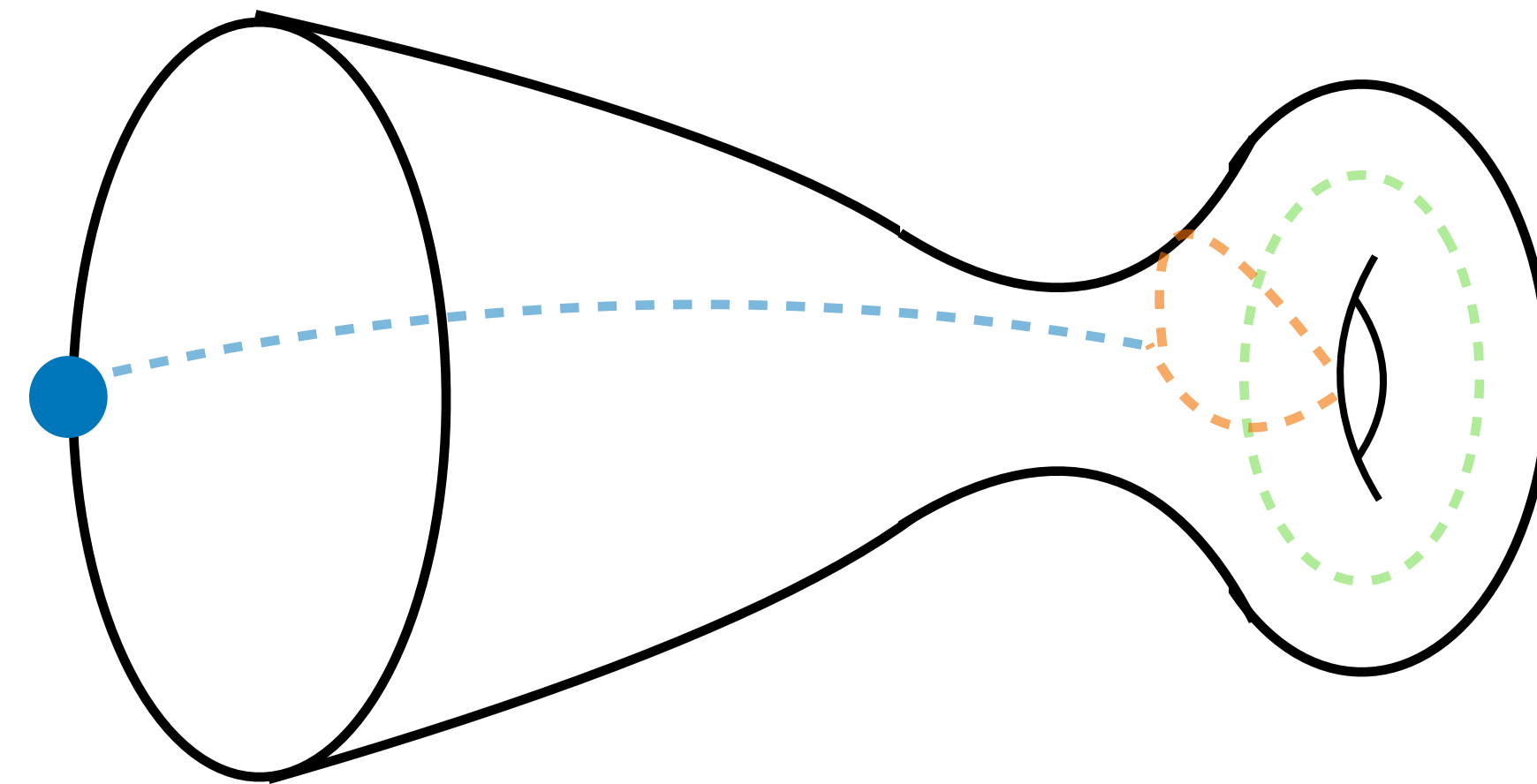
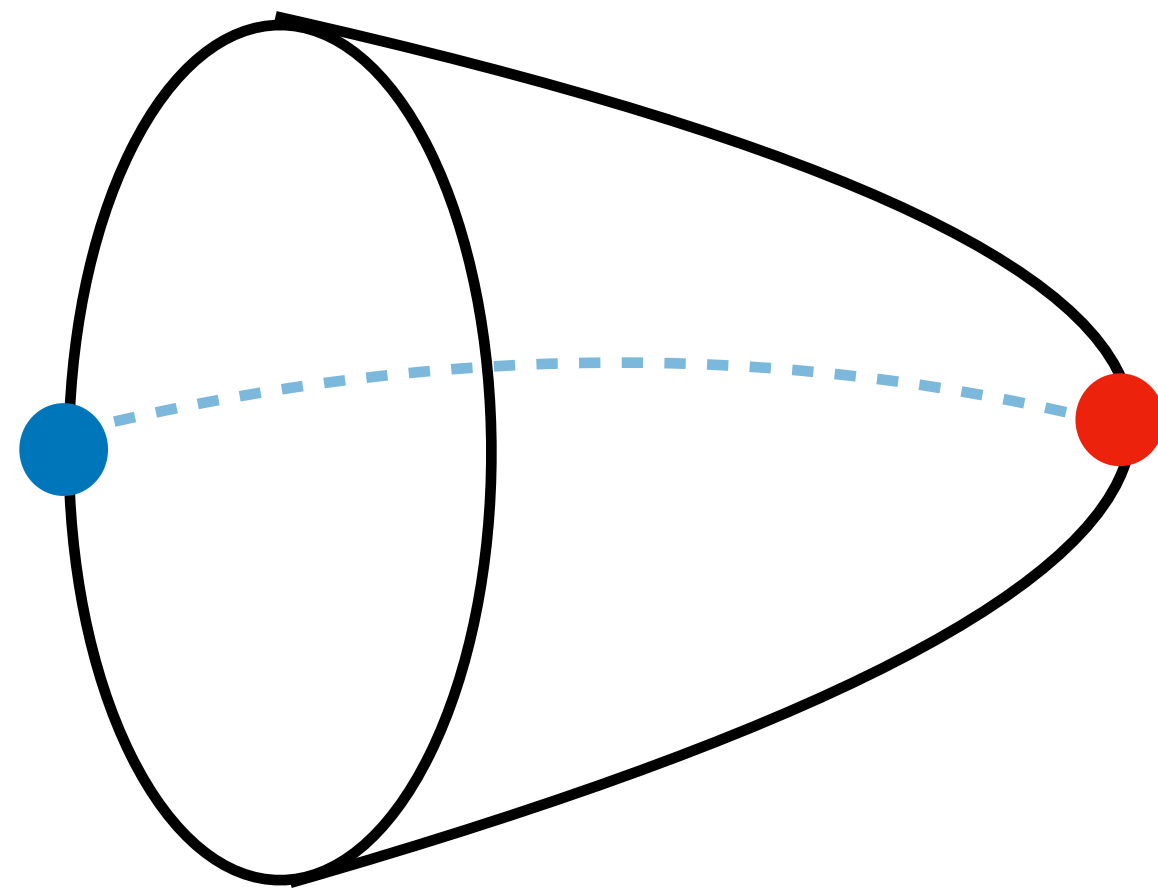
$g = [g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}$
in commutator subgroup



no singular object, but
non-trivial topology

Gravitational solitons

Gravitational solitons = wormholes



Non-perturbative breaking in quantum gravity $e^{-M_{pl}^2}$

8d Supergravity

$\mathcal{N} = 2$ **supergravity** in eight dimensions with **U-duality**

$$\mathrm{SL}(2, \mathbb{Z}) \times \mathrm{SL}(3, \mathbb{Z})$$

$$\rightarrow \Omega_d^{\mathrm{Spin}} \left(B(\mathrm{SL}(2, \mathbb{Z}) \times \mathrm{SL}(3, \mathbb{Z})) \right)$$

d	$\Omega_d^{\text{Spin}}(BSL(2, \mathbb{Z}) \times BSL(3, \mathbb{Z}))$
1	$(\mathbb{Z}_2 \oplus) \mathbb{Z}_3 \oplus \mathbb{Z}_4$
2	$(\mathbb{Z}_2 \oplus) \mathbb{Z}_2^{\oplus 3}$
3	$\mathbb{Z}_3^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_8^{\oplus 3}$
4	$(\mathbb{Z} \oplus) \mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
5	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_2^{\oplus 5} \oplus \mathbb{Z}_4$
6	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
7	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 6} \oplus \mathbb{Z}_8^{\oplus 2} \oplus \mathbb{Z}_{16}^{\oplus 2} \oplus \mathbb{Z}_{32}$

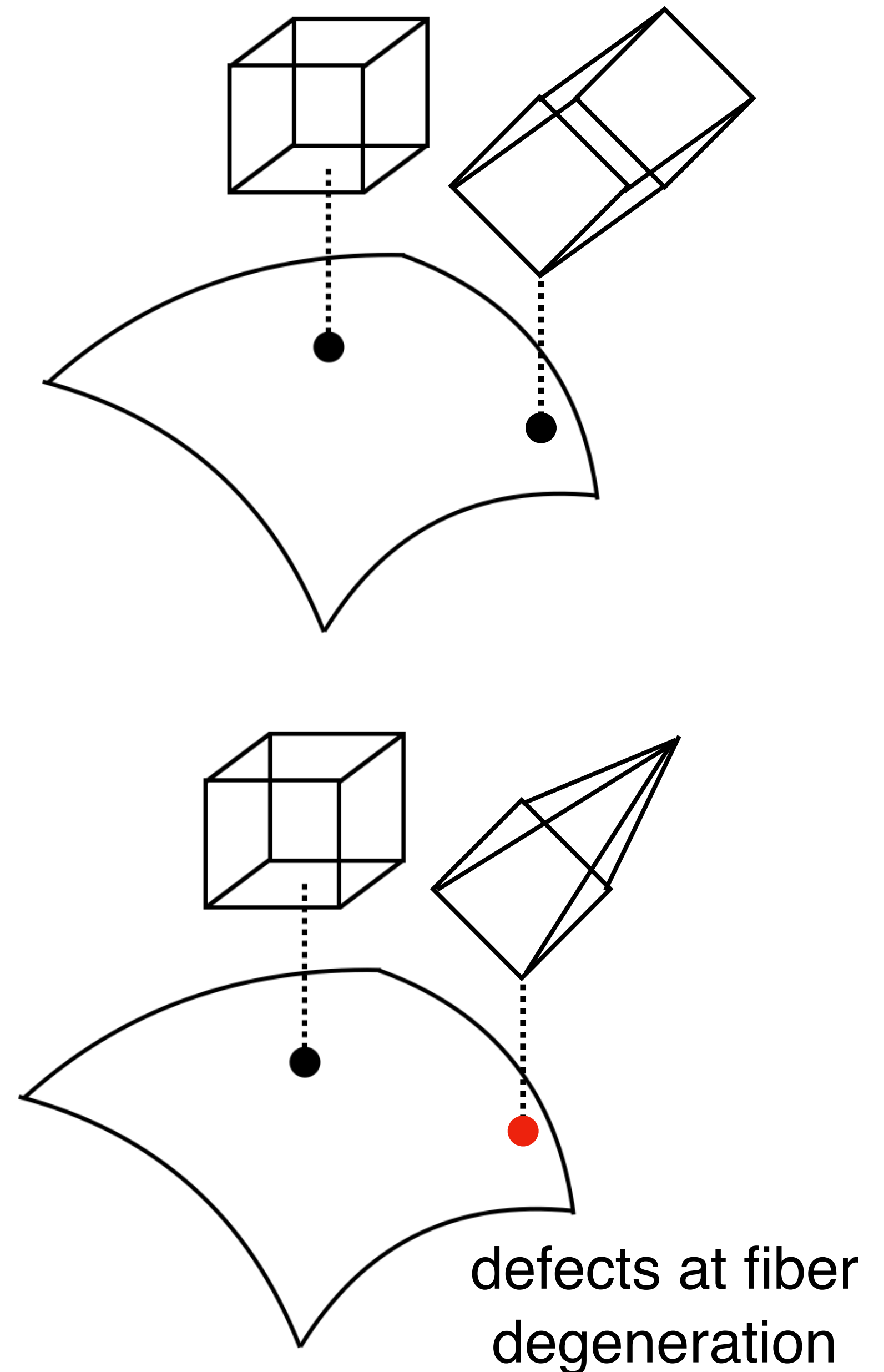
**a LOT of
topological charges**

What are the objects?

What do we expect?

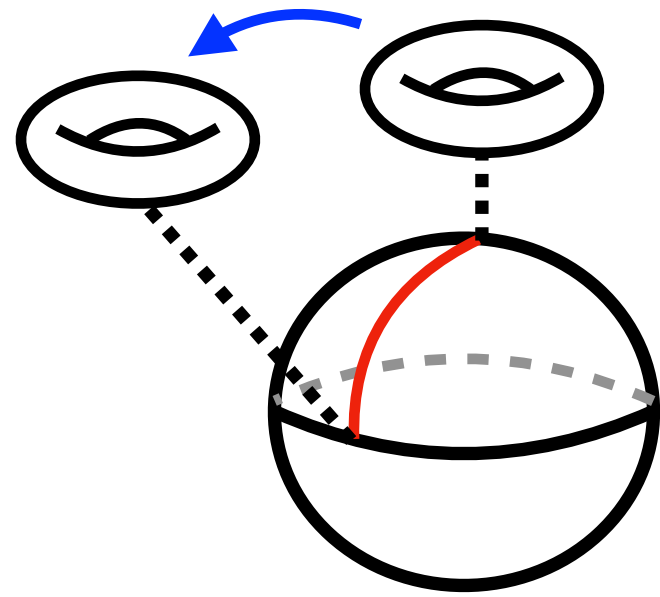
We do know some ‘geometric’ UV completions:

- **Type IIB and F-theory configurations**
- **M-theory**
- **Compactifications of defects**
- **Non-geometric defects**
mixing diffeos and T-duality



What we find (examples)

Type IIB:



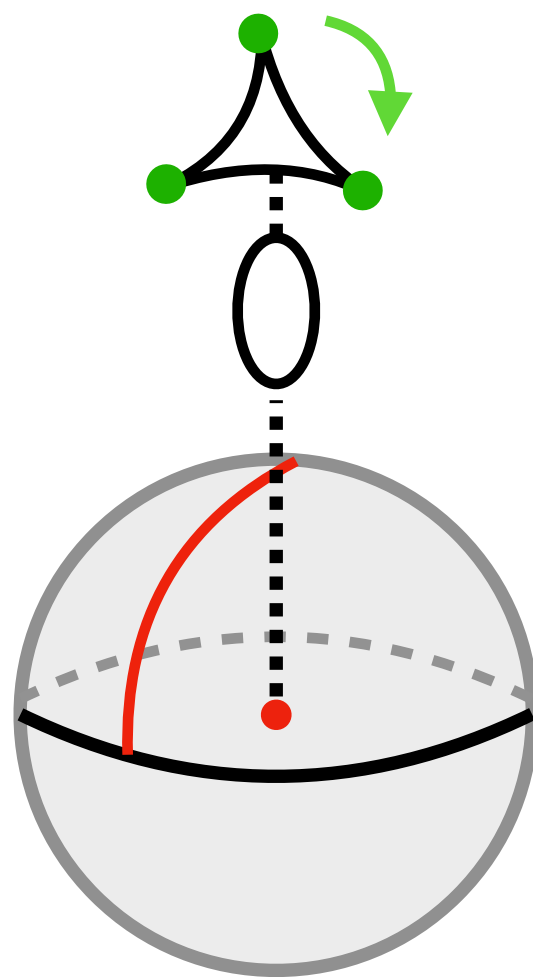
$$\mathbb{Z}_3 \in \Omega_3^{\text{Spin}}(BG_U)$$

non-Higgsable cluster compactified on torus
(a descendant of IIBorida)

➔ **Argyres-Douglas theory** $D_3(\text{SU}(2))$

[del Zotto, Vafa, Xie '15], [Carta, Giacomelli, Mekareeya, Mininno '23]

M-theory:



$$\mathbb{Z}_3 \in \Omega_3^{\text{Spin}}(BG_U)$$

three sector of E_0 theory (M-theory on $\mathbb{C}^3/\mathbb{Z}_3$)

[Seiberg '96], [Morrison, Seiberg '97]

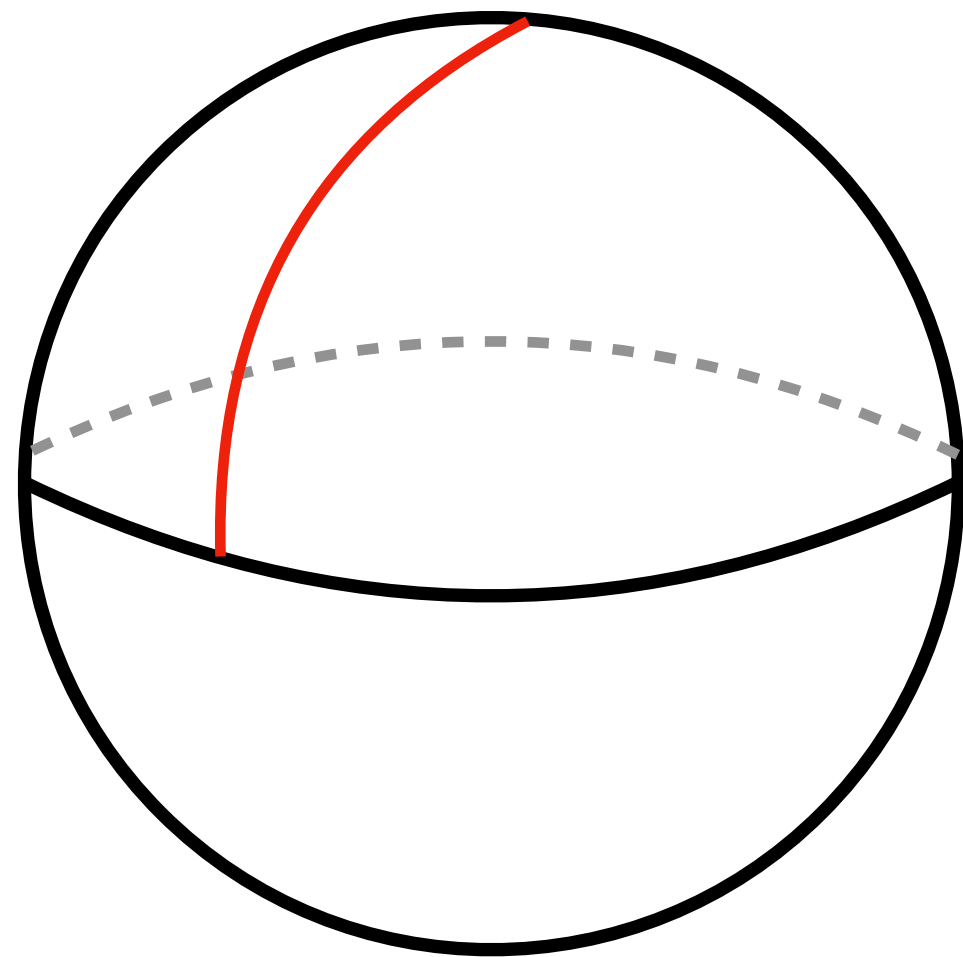
$$(\mathbb{C}^3/\mathbb{Z}_3) \oplus (\mathbb{C}^3/\mathbb{Z}_3) \oplus (\mathbb{C}^3/\mathbb{Z}_3)$$

See also U-folds, e.g., [Kumar, Vafa '96], [Liu, Minasian '97], ...

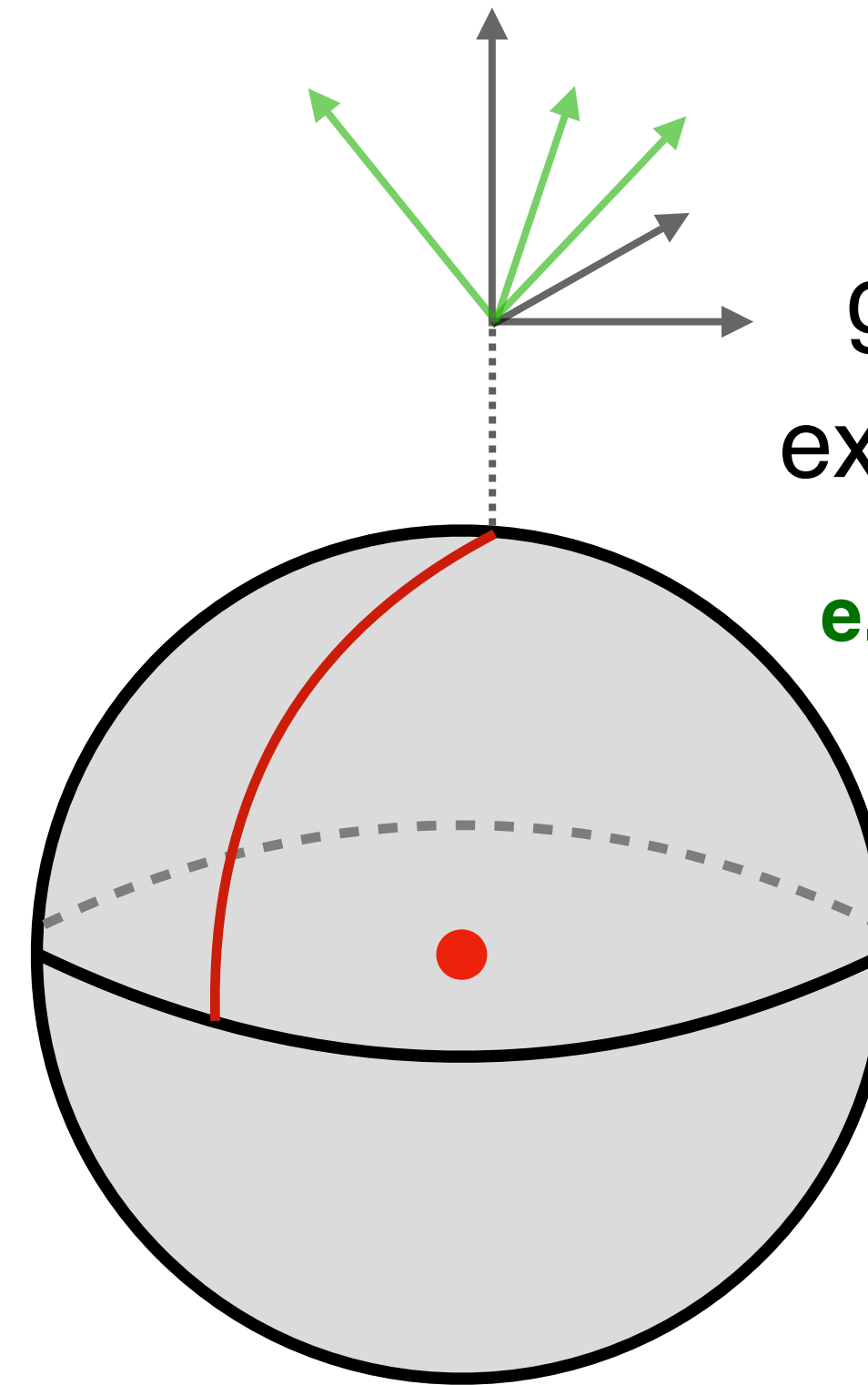
What we find: Non-geometric defects

At $d = 5$ we find non-geometric defects:

$$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ cannot be conjugated to a geometric subgroup}$$



$$L_3^5 = \partial(\mathbb{C}^3 / \mathbb{Z}_3)$$



geometrization within exceptional field theory?

e.g. [Hohm, Samtleben '13]

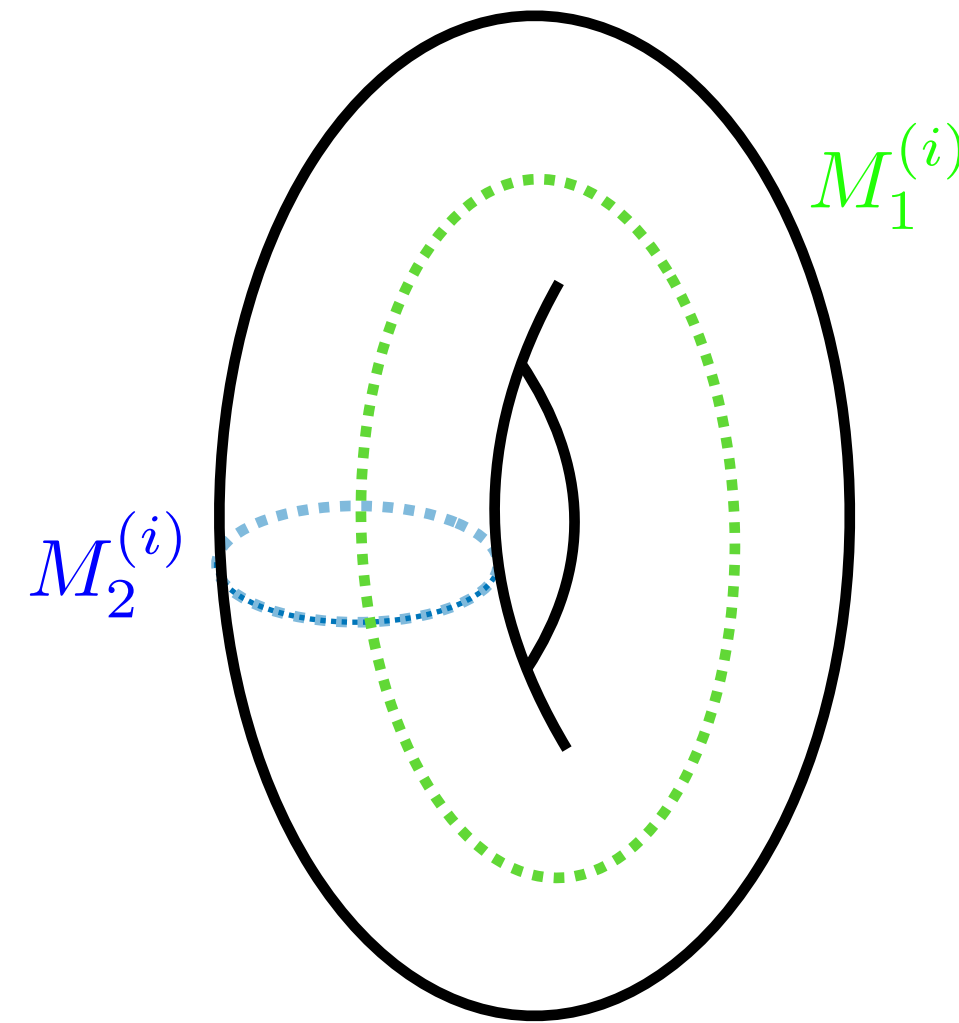
Codimension-six **string defect** with **action on geometry and other moduli**

Non-perturbative generalization of T-fold geometry

[Hull '04, '06]

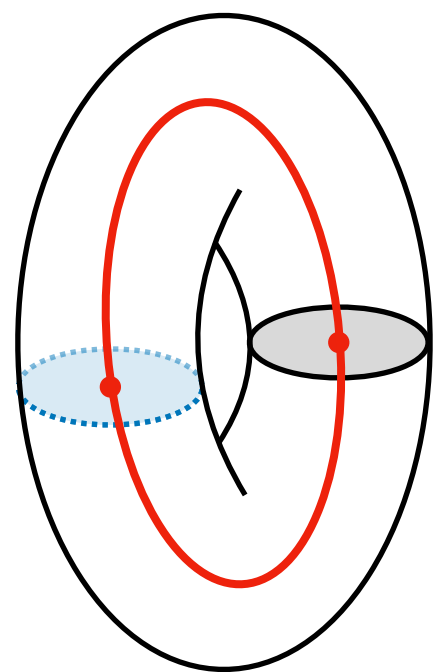
Surprises II: Resurrection of Codim-2

In $d = 2$ we find:



$$M_1^{(i)}, M_2^{(i)} \in \mathrm{SL}(3, \mathbb{Z}), \quad [M_1^{(i)}, M_2^{(i)}] = 0$$

Our usual approach would tell us: **New codimension-two brane**
compactified on circle

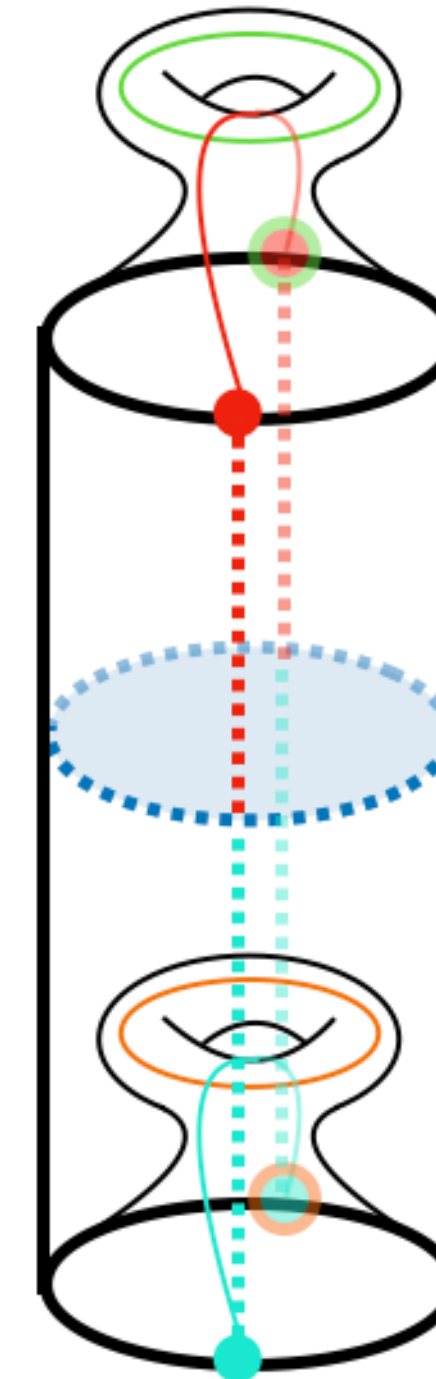
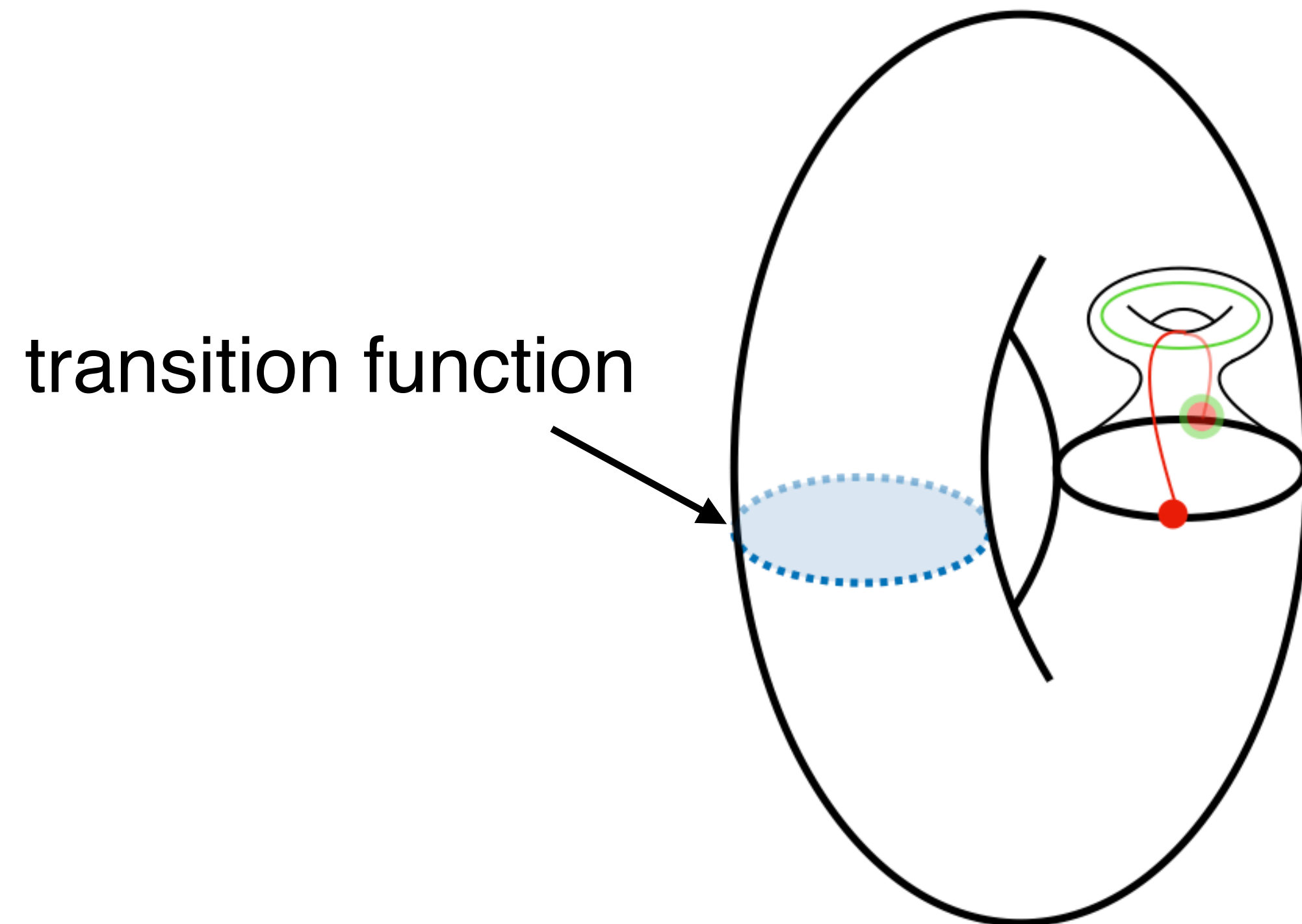


no problem with transition function
since monodromies commute

Why no soliton solution?

Surprises II: Gravitational solitons fail

Try to use the soliton



action incompatible with

$$g_1, g_2$$

for any choice

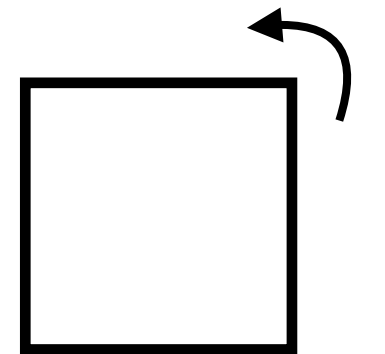
→ does not lead to well-defined background

Subtlety of fermions

We use the bosonic version of the duality group

$$\begin{array}{ccc} & \text{SL}(2, \mathbb{Z}) \times \text{SL}(3, \mathbb{Z}) & \\ \nearrow T^2 & & \nwarrow T^3 \end{array}$$

geometric action in certain duality frames $T^2, \tau = i : S$



there are fermions: $S^4 \psi = -\psi$

should affect the full (fermionic) U-duality

tricky to combine
Spin-GL⁺ of type IIB
Pin⁺ of M-theory

Summary & Outlook

(As for type IIB in 10d) this program is able to predict:

- **Many symmetry-breaking defects imposed by U-duality**
- **Interesting worldvolume theories (SCFTs, twists)**
- **Very stringy corners (non-geometry)**
 - ➔ **very efficient machine to learn something new**

- **Fermionic lift (BPS nature)**

- Many things to do:**
- **Properties of the defects (applications, supergravity)**
 - **Physics in maths**