Ghost free theories of multiple spin-2 fields

Fawad Hassan
Stockholm University, Sweden

Workshop on Gravity, Strings and Supersymmetry Breaking
April 3-5, 2025, ENS, Pisa, Italy

Based on work with:

Joakim Flinckman

Angnis Schmidt-May

Mikica Kocic

Rachel. A. Rosen

Mikael von Strauss

Anders Lundkvist

Luis Apolo

Disclaimer: Many important contributors to this field, but I will focus on a subset of the works

Outline of the talk

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Trivia and Motivation

Ghost-free multi spin-2 theories

Ghost-free bimetric theory

Uniqueness and the local structure of spacetime

Discussion

Trivia

1. Fields/particles classified by
   mass: $m^2 \geq 0$, **Spin:** $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots$
   Maximum number of polarizations= $2s + 1$

2. Transform Lorentz group/GCT: $\psi_\alpha, A_\mu, h_{\mu\nu}, \cdots$
   Number of field components $> 2s + 1$

3. Unwanted components lead to ghost instabilities
   $\mathcal{L} \sim -(\dot{\phi})^2 + \cdots$

4. Need to be eliminated by gauge symmetries/constraints
   More difficult for higher spins

5. Absence of ghost + Lorentz/general covariance determine the basic structure of field equations

Motivation: spin based picture of field theories

- $s < 2$: Well known field theories with finite number of fields
- $s > 2$: Local theories with finite field content do not exist
  (cf. Higher spins, String theory)
- $s = 2$: Simplest possible theory is General Relativity
  (*The spin-2 equivalent of* $\Box\phi = 0$ & $\partial_\mu F^{\mu\nu} = 0$)

Is GR unique? Or do theories of *multiple* spin-2 fields exist?

* A less understood corner of the theory space, difficult to probe
* Has features relevant to gravity, dark matter, dark energy, inflation (will not be discussed here)

Historical timeline (Spin-2 fields)

- Einstein (GR and linearized gravity) (1915-17)
- Fierz and Pauli (linearized massive gravity) (1939)
- van Dam, Veltman, Zakharov (1970)
- Vainshtain (1972)
- Boulware, Deser (1972)
- Isham, Salam, Strathdee (1971-79)
- Creminelli, Nicolis, Papucci, Trincherini (2005)
- de Rham, Gabadadze (2010)
- de Rham, Gabadadze, Tolley (2010)
- (post 2010 developments partly discussed here)

Outline of the talk

GR + multiple spin-2 fields

A dynamical theory of the gravitational metric $g_{\mu\nu} = g_{1\mu\nu}$ interacting with spin-2 fields $g_{2\mu\nu}, g_{3\mu\nu}, \cdots g_{n\mu\nu}$

$$\mathcal{L} = m_p^2 \sqrt{|g|} R(g) + \mathcal{L}(g, g_2) + \cdots + \mathcal{L}(g, g_n) - V(g, g_2, \cdots, g_n) + \mathcal{L}_{matter}$$

The No-ghost condition suggests:

1) Derivative part: $\sum_{I=1}^{n} m_I^2 \sqrt{|g_I|} \left[R(g_I) - 2\Lambda_I \right]$

2) Independent matter sectors ψ_I*:* $\sum_{I=1}^{n} \mathcal{L}_{matter}(g_I, \psi_I)$

3) Potential V *is given in terms of vielbeins.*

Ghost-free multi spin-2 theories

[SFH, Angnis Schmidt-May (arXiv:1804.09723)]

Introduce vielbeins $e^A_{I\mu}$ for each metric,

$$g_{I\mu\nu} = (e_I)^A_{\ \mu}(e_I)^B_{\ \nu}\eta_{AB} \qquad (I = 1, \cdots, n)$$

Claim: A ghost-free multi spin-2 theory is,

$$\mathcal{L} = \sum_{I=1}^{n} m_I^2 \sqrt{|g_I|}\left[R(g_I) - 2\Lambda_I\right] - V(e_1, \cdots, e_n) + \sum_{I=1}^{n} \mathcal{L}_{matter}(e_I, \psi_I)$$

where

$$V = 2M^4 \det\left(\beta_1 e_1 + \beta_2 e_2 + \cdots + \beta_n e_n\right)$$

- Are there enough constraints to eliminate the ghosts? Yes
  1 massless + (n-1) massive modes

[SFH, Joakim Flinckman (to appear)]

EoM's and symmetrization conditions

Let $u^A_\mu = \beta_1 e^A_{1\mu} + \beta_2 e^A_{2\mu} + \cdots + \beta_n e^A_{n\mu}$

$$\mathcal{L} = \sum_{I=1}^{n} m_I^2 \sqrt{|g_I|}\left[R(g_I) - 2\Lambda_I\right] - 2M^4 \det(u) + \mathcal{L}_{matter}$$

Vielbein EoMs:

$$R_{I\mu\nu} - \frac{1}{2} g_{I\mu\nu} R_I + V^I_{(\mu\nu)} + V^I_{[\mu\nu]} = m_I^{-2} T^I_{\mu\nu}$$

Finally,

$$V^I_{[\mu\nu]} = 0 \quad \Longleftrightarrow \quad (e_I)^A{}_{[\mu} \eta_{AB} u^B{}_{\nu]} = 0 \quad \forall I$$

This structure is crucial for avoiding the earlier no-go statements!

Mass eigenvalues

[SFH, Joakim Flinckman (arxiv:2410.09439)]

Mass eigenstates exist around proportional backgrounds

$$(\bar{e}_I)^A_{\ \mu} = c_I\, \bar{e}^A_{\ \mu} \qquad (\textit{Einstein \ spacetimes})$$

Cosmological constant: $\Lambda = c_I^2 \Lambda_I + M^4 \frac{\beta_I}{m_I^2 c_I} (\sum_J^N c_J \beta_J)^3$

Parametrization of fluctuations (computable to all orders):

$$(e_I)^A_{\ \mu} = L^A_{IB}\, (\hat{e}_I)^A_{\ \mu} = (\eta + A_I)^{-1} (\eta - A_I) \left(c_I\, \bar{e}^A_{\ \mu} + E^A_{I\mu}(\delta g_I) \right)$$

$$\textit{Mass \ matrix} : \qquad M_{IJ} = M^4 k^2 \left(k \frac{\beta_I}{m_I^2 c_I} \delta_{IJ} - \frac{\beta_I \beta_J}{m_I m_J} \right)$$

Mass eigenvalues

Structure of mass matrix

$$M_{IJ} = d_I \delta_{IJ} - v_I v_J$$

The mass eigenvalues $\mu_1^2, \cdots, \mu_n^2$ cannot be determined exactly for $n > 3$ but are bounded as:

$$0 = \mu_1^2 \leq \Lambda - \Lambda_1 c_1^2 \leq \mu_2^2 \leq \Lambda - \Lambda_2 c_2^2 \leq \cdots \leq \mu_n^2 \leq \Lambda - \Lambda_n c_n^2$$

Healthy non-tachyonic mass spectrum

[SFH, Joakim Flinckman (arxiv:2410.09439)]

Some generalizations and specializations:

In terms of 1-forms $e_I^A = e_{I\mu}^A dx^\mu$, in $d = 3 + 1$,

$$V = \det\left(\sum_{I=1}^{n} \beta_I e_I\right) = \sum_{I,J,K,L=1}^{n} \beta_I \beta_J \beta_K \beta_L e_I^A \wedge e_J^B \wedge e_K^C \wedge e_L^D \, \epsilon_{ABCD}$$

Consider:

$$\beta_I \beta_J \beta_K \beta_L \to \beta_{IJKL}$$

- $n = 2$: Allowed $\to$ the ghost-free bimetric theory of
  [SFH, Rosen (1109.3515,1111.2070)]
- $n \geq 3$: Not allowed, ghosts re-emerge
  [K. Hinterbichler, R. A. Rosen (arXiv:1203.5783)]
- Simple generalizations possible, general structure?

Outline of the talk

Ghost-free bimetric theory ($n = 2$)

$$g_{\mu\nu} = (e_1)^A_{\ \mu}(e_1)^B_{\ \nu}\,\eta_{AB}\,, \qquad f_{\mu\nu} = (e_2)^A_{\ \mu}(e_2)^B_{\ \nu}\,\eta_{AB}$$

Symmetrization condition:

$$(e_1)^A_{\ [\mu}\,\eta_{AB}\,(e_2)^B_{\ \nu]} = 0 \quad \Longleftrightarrow \quad (e_1^{-1}e_2)^\mu_{\ \nu} = (\sqrt{g^{-1}f})^\mu_{\ \nu}$$

(solving symmetrization condition $\equiv$ finding matrix square root)

$$V(e_1, e_2) = V(\sqrt{g^{-1}f}) = \sqrt{|\det g|}\sum_{i=0}^{4} b_i\, E_i(\sqrt{g^{-1}f})$$

- Fully writable in terms of the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$
- Absence of ghost rigorously established

[SFH, Rosen(1109.3515,1111.2070),SFH, M.Kocic(arXiv:1706.07806)]

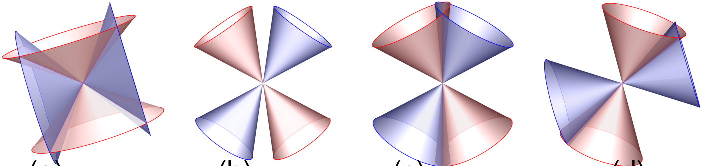
[SFH, A.Lundkvist (arXiv:1802.07267)]

Outline of the talk

Two Potential problems and solutions

Potential problem 1: Incompatible spacetimes

$g_{\mu\nu}$ & $f_{\mu\nu}$ may not admit compatible notions of *space* and *time*
(3+1 splits)



(a) (b) (c) (d)

Then, no consistent time evolution, no Hamiltonian formulation, ghost proof fails.

Two potential problems and solutions

Potential problem 2: Uniqueness, Reality, Covariance

Matrix square root: $S^{\mu}_{\ \nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}_{\ \nu}$

- Not unique: Multiple *primary & non-primary* roots
- Possibly non-real
- May not transform as a $(1,1)$ tensor $\Rightarrow$ breakdown of general covariance!

Both problems have a common solution

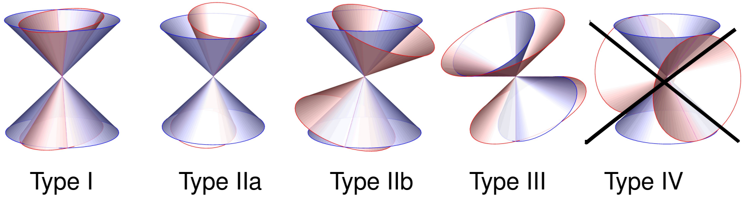
Uniqueness and the local structure of spacetime

Solution: *[SFH, M. Kocic (arXiv:1706.07806)]*

2) General Covariance: $\Rightarrow$ $S =$ *principal root* $\Rightarrow$ Unique.

1) Reality:

Theorem: S is real *iff* the null cones of $g_{\mu\nu}$ and $f_{\mu\nu}$ intersect



Types I-III: Allowed, proper 3+1 decompositions possible.

Type IV: Non-primary, excluded by general covariance

(Implication for accausality arguments in the literature)

Space-time in multi metric theory

Similar features but less understood

Outline of the talk

Discussion

The beginnings of understanding spin-2 fields beyond General Relativity

- Causality
- Superluminality? (yes, not necessarily harmful)
- Systematics of multi spin-2 interactions? Is there a purely metric formulation?
- Extra symmetries $\Rightarrow$ Modified kinetic terms? MacDowell-Mansouri type theories. More interesting but less understood.
- Theoretically unavoidable mixings of mass eigenstates (unlike neutrino mixings)
- Spin-2 dark matter, Dynamical dark energy candidates

Thank you!