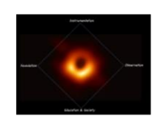
# Gravity and Statistical Physics

Jan de Boer, Amsterdam

Based on work with Alex Belin, Diego Liska, Pranjal Nayak, Tarek Anous, Julian Sonner, Daniel Jafferis, Boris Post, Martin Sasieta, Jildou Hollander, Andrew Rolph, Ramesh Chandra, Igal Arav, Shira Chapman, Joshua James-King ‘20-’25

Gravity, Strings and Supersymmetry Breaking
Pisa, 3 April 2025



# Upshot:

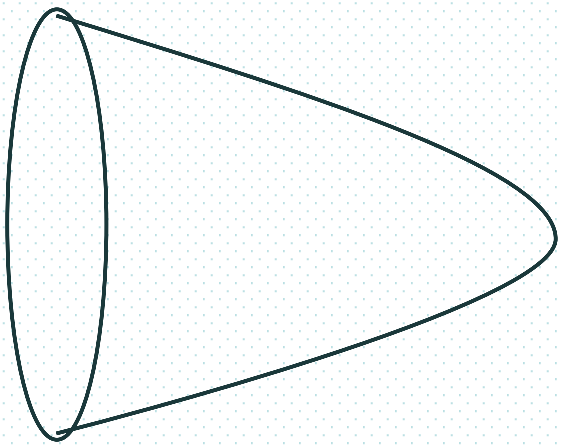
Gravitational path integral = $$\int d\alpha\, \mu[\alpha]\, \mathrm{theory}[\alpha]$$

Here theory[α] are all “theories” which are semi-classically indistinguishable.

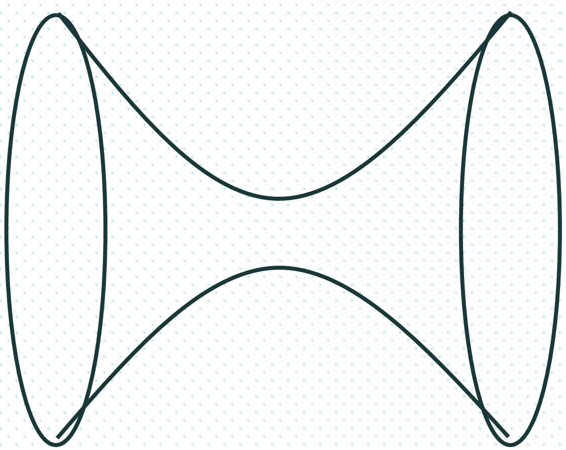
μ[α] is a probability distribution.

At first sight not very different from coarse graining.

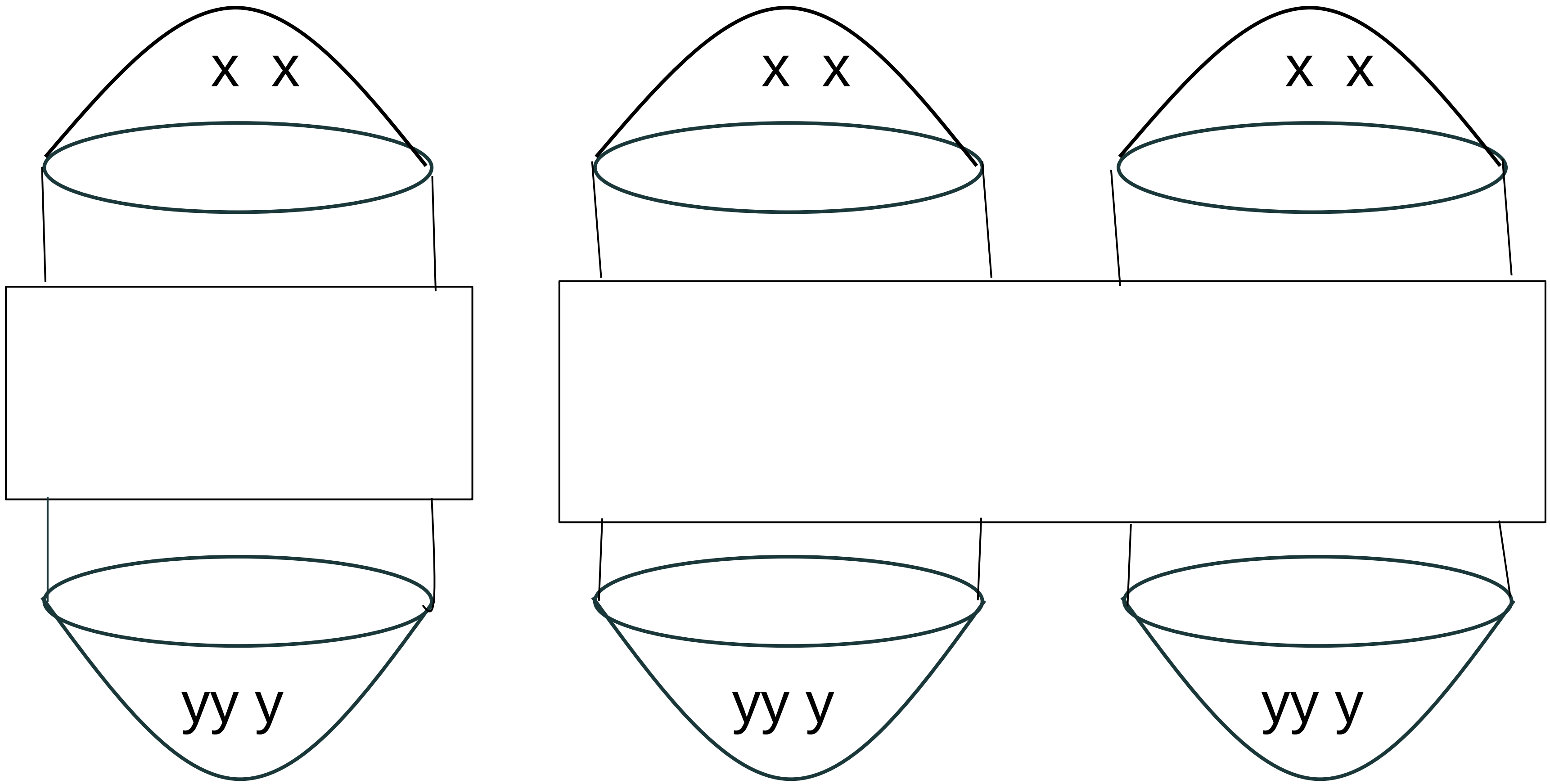
# In AdS/CFT



$$= \int d\alpha\, \mu[\alpha]\, \mathrm{theory}[\alpha]$$



$$= \int d\alpha\, \mu[\alpha]\, \mathrm{theory}[\alpha]\, \mathrm{theory}[\alpha]$$




$$\int d\alpha\, \mu[\alpha]\, \langle\psi_x(\alpha)|\psi_y(\alpha)\rangle \Rightarrow \int d\alpha\, \mu[\alpha]\, \langle\psi_x(\alpha)|\psi_y(\alpha)\rangle\langle\psi_x(\alpha)|\psi_y(\alpha)\rangle$$

The gravitational path integral does not just perform some sort of coarse graining, it can also compute higher moments of the probability distribution $\mu[\alpha]$.

This is why gravity:

- Can produce the ramp in the spectral form factor
- Has Euclidean wormholes (factorization puzzle)
- Can count black hole entropy
- Can produce a page curve for black hole evaporation
- Knows that the volume of black hole interiors does not grow forever
- Thinks that the Hilbert space for de Sitter is one dimensional

# The CFT spectrum in AdS/CFT

CFT

$$Z(\beta) = \sum_i e^{-\beta E_i}$$

=

Exact quantum gravity in AdS

???

=

Semi-classical approximation (GPI)

$$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

$$Z(\beta) = \sum_i e^{-\beta E_i}$$

$$\rho(E) = \sum \delta(E - E_i)$$

=

???

CFT

GRAVITY

$$Z(\beta) = \int dE\, \rho(E) e^{-\beta E}$$

$$\log \rho(E) \sim E^{\frac{d-1}{d}}$$

=

$$Z(\beta) \sim \exp(c/\beta^{d-1})$$

Saddles + pert corrections

In the CFT, the exact spectrum gets replaced by a continuous “coarse grained” spectral density.

**Claim**: the right way to do this coarse graining is by replacing the CFT by a statistical average over all sets of CFT data which are semi-classically indistinguishable. Those sets need not obey the axioms of a CFT as long as those violations are not semi-classically detectable.

To make this more precise need to (i) specify which data and (ii) provide a probability distribution (measure) on the space of data.

Here we will label the theory by its Hamiltonian H, and probability measure will therefore be some function $\mu[H]$.

Statistical physics gives us a preferred method to deal with situations like this (Wigner ‘55 Balian ’68).

Maximize ignorance (=entropy) subject to the constraints imposed by the semi-classical approximation:

$$\int dH \;\; -\mu[H]\log\mu[H] + \mu[H]\int d\beta\lambda(\beta)\left(\mathrm{Tr}(e^{-\beta H}) - Z(\beta)\right)$$

$$V'(E) = 2\int d\lambda\,\frac{\rho_0(\lambda)}{E-\lambda}$$

One finds

$$\mu[H] \sim \exp\left(\int d\beta\lambda(\beta)\mathrm{Tr}(e^{-\beta H})\right) \sim \exp(-\mathrm{Tr}V(H))$$

where V is arbitrary but needs to be fixed to yield the right partition function (or spectral density).

JdB, Liska, Post, Sasieta ‘23

This shows that *in the absence of other information* the best description of the Hamiltonian of a theory with a continuous spectral density is in terms of a matrix model.

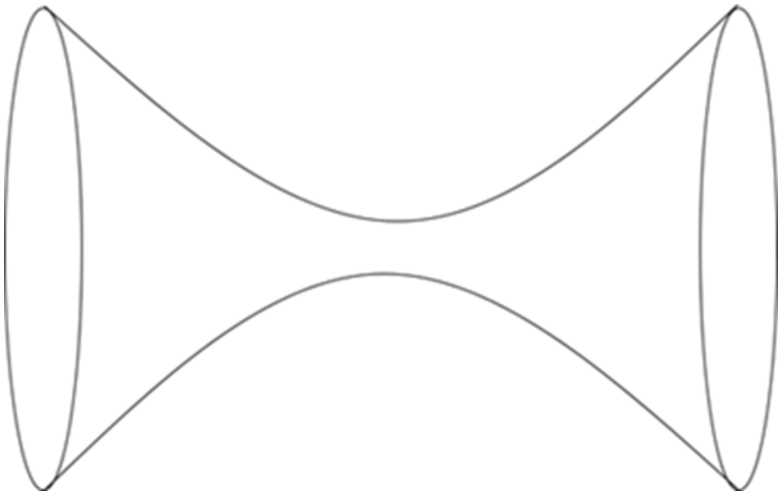
For a chaotic theory, it may be difficult to obtain more detailed information about the spectrum and this may be the best one can do.

(Black holes are very chaotic  Maldacena, Stanford, Shenker ’15)

This would resonate with the Bohigas–Giannoni–Schmit (BGS) conjecture(1984) which asserts that the spectral statistics of quantum systems whose classical counterparts exhibit chaotic behavior are described by random matrix theory.

# We now get a *prediction* for the following Euclidean wormhole

$Z(\beta_1)$



$Z(\beta_2)$

$$\int dH\, e^{-V(H)}\, \mathrm{Tr}(e^{-\beta_1 H})\, \mathrm{Tr}(e^{-\beta_1 H})$$

Cotler, Jensen ’21 – see also Di Ubaldo, Perlmutter ’23 and Haehl, Reeves, Rozali ‘23

The off-shell gravity computation agrees to leading order with the universal random matrix theory result

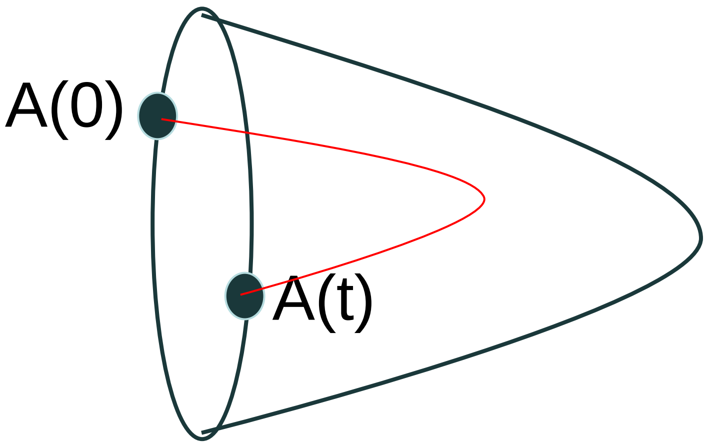
$$\langle Z(\beta_1) Z(\beta_2) \rangle = Z(\beta_1) Z(\beta_2) + \frac{1}{2\pi} \frac{\sqrt{\beta_1 \beta_2}}{\beta_1 + \beta_2} + \ldots$$

Ambjørn, Jurkiewicz, Makeenko ’90
Saad, Shenker, Sanford ’19

# Finite temperature one- and two-point functions in AdS/CFT

$$\langle A(0)\rangle_\beta = \frac{1}{Z}\sum_i \langle E_i|A|E_i\rangle e^{-\beta E_i}$$

$$\langle A(0)A(t)\rangle_\beta = \frac{1}{Z}\sum_{i,j} \langle E_i|A|E_j\rangle\langle E_j|A|E_i\rangle e^{-\beta(E_i+E_j)/2} e^{it(E_i-E_j)}$$



We can then find the classical probability distribution for the matrix elements $A_{ij} = \langle E_i | A | E_j \rangle$ by maximizing the classical entropy with infinitely many constraints.

$$\int \prod_{i,J} dA_{i,j} \; - \mu[\{A_{ij}\}] \log \mu[\{A_{ij}\}] + \mu[\{A_{ij}\}] \int dt d\beta \lambda(t,\beta) \left[\langle A(0) A(t) \rangle_\beta - f(t,\beta)\right]$$

This yields a quadratic matrix model

$$\mu[\{A_{ij}\}] \sim \exp(-\sum_{i,j} c_{i,j} |A_{i,j}|^2)$$

JdB, Liska, Post, Sasieta ‘23

This reproduces the Eigenstate Thermalization Hypothesis (ETH):

JdB, Liska, Post, Sasieta ‘23

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E},\Delta E) R^a_{ij}$$

Deutsch ’91
Srednicki ’94

$f_a(\bar{E})$ : one point functions of simple operators
$g_a(\bar{E},\Delta E)$ : two point functions of simple operators
$R^a_{ij}$ : Gaussian random variables

$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

$$\mu[R] \sim e^{-\mathrm{Tr}(R^2)}$$

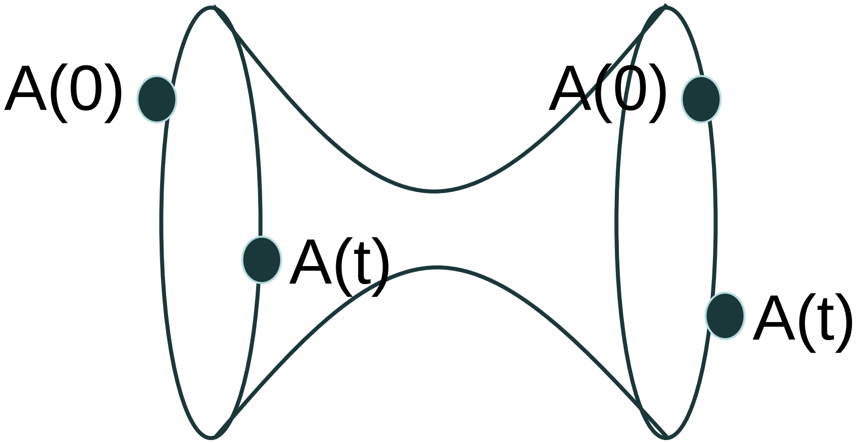
ETH correctly reproduces the thermal one- and two-point functions and implies that typical states look thermal.

Note: this does *not* prove the validity of ETH, nor does ETH require more input that the thermal one- and two-point functions.

One can thus argue that ETH is simply a consequence of applying statistical physic principles to simple finite temperature correlators.

# This yields a *prediction* for the "ETH wormhole"

Chandra, Collier, Hartman, Maloney '22



# More general constructions for CFT’s

One can repeat these stept for other choices of data.

The general picture is one where if one e.g. inputs connected ≤k-point correlators, one gets a “matrix model” with up to k-th order interactions in the exponent.

Lagrange multipliers

$$\int dA d\lambda_i \left( -\mu[A] \log \mu[A] + \sum_i \lambda_i (f_i[A] - c_i) \right)$$

Shannon entropy

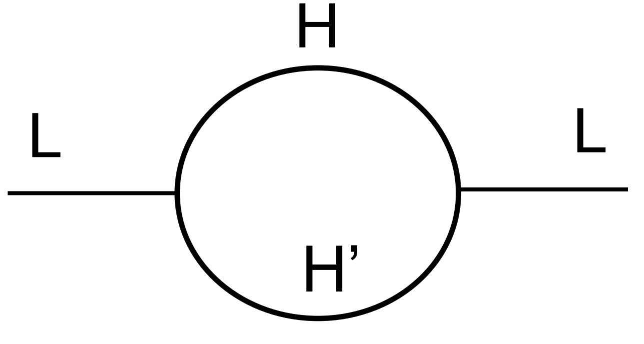
Input observations

$$\Rightarrow \mu[A] \sim e^{-\sum_i \lambda_i f_i[A]}$$

In holographic CFT's, there are two types of degrees of freedom

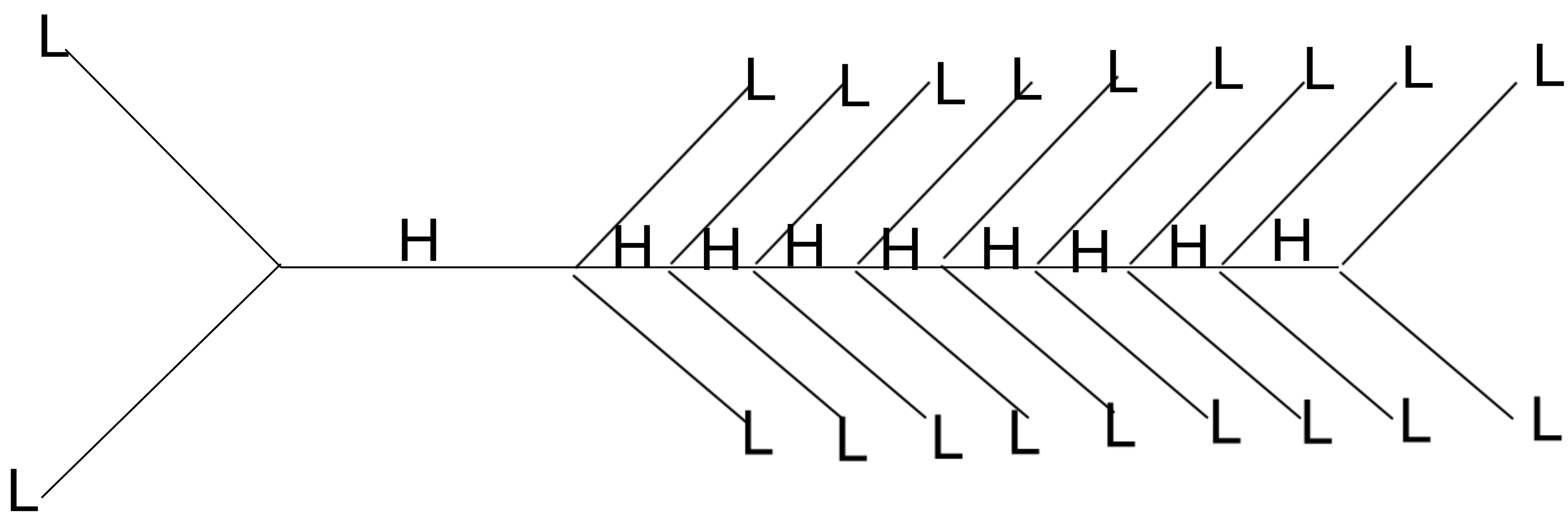
- Simple, observable, low energies: L
- Complicated, chaotic, high energies: H (black hole microstates)

We can write "Feyman diagrams" to describe measurements of observables. Only L can appear on external lines.




(two-point function of simple operators in a black hole background)

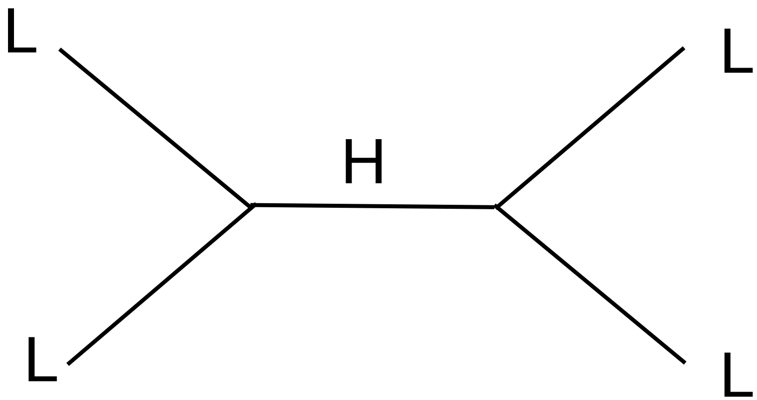
Complete description is in terms of vertices and propagators




Atypical diagram contributing to black hole creation/evaporation

Based on low-energy computations/observations, we can now build a statistical theory as before. We find a statistical theory for both the propagators (spectrum of the theory) as well as the vertices (the operator product expansion coefficients of the theory). As before, the spectrum exhibits random matrix theory statistics.

JdB, Belin ‘20
Belin, JdB, Nayak, Sonner ’20 ‘21
Belin, JdB, Liska ‘21
Anous, Belin, JdB, Liska ‘21
JdB, Liska, Post ‘24

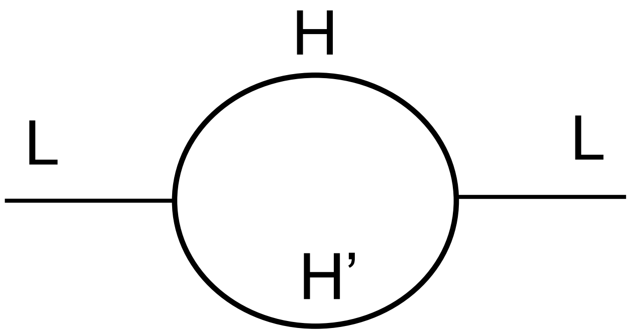



$$\sum_H C^2_{LLH}$$

4 point correlator on $S^d$

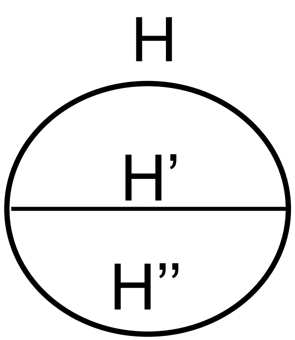
$$\overline{|C_{LLH}|^2} \sim \frac{\Delta_H^{2\Delta_L - 1}}{\Gamma(2\Delta_L)\rho(\Delta_H)}$$

Pappadopulo, Rychkov, Espin, Ratazzi '12




$$\sum_{H,H'} C^2_{LHH'}$$

2 point correlator on $S^{d-1} x S^1$

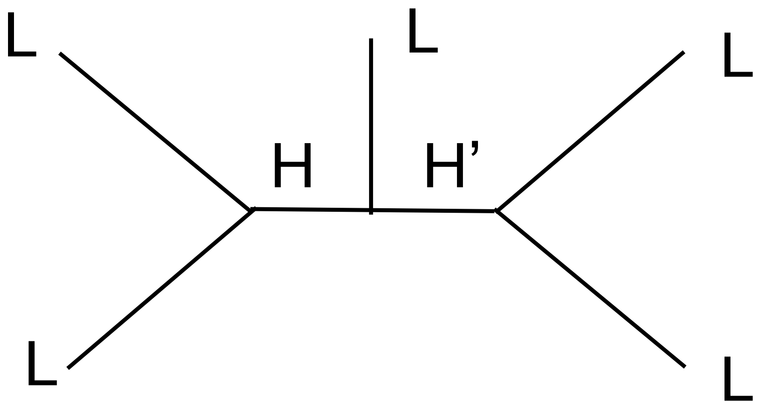



$$\sum_{H,H',H''} C^2_{HH'H''}$$

Connected sum of two times $S^{d-1} x S^1$

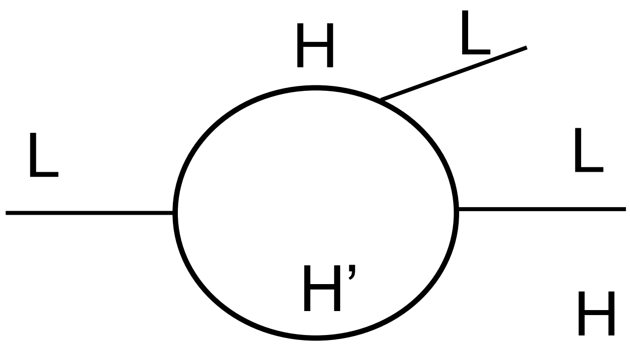
Benjamin, Lee, Ooguri, Simmons-Duffin '23

Input gives rise to quadratic matrix model for the C's



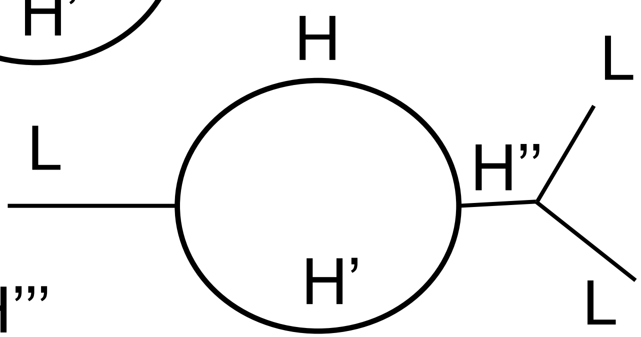

$$\sum_{H,H'} C_{LLH} C_{HLH'} C_{LH'H'}$$

5 point correlator on $S^d$



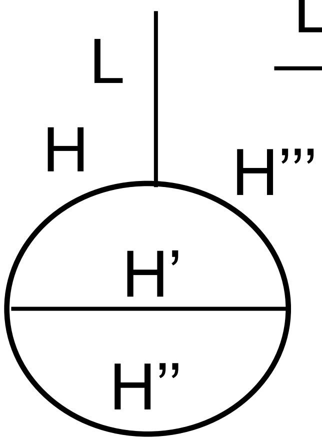

$$\sum_{H,H',H''} C_{LHH'} C_{LH'H''} C_{LH''H}$$

3 point correlator on $S^{d-1} x S^1$




$$\sum_{H,H',H''} C_{LHH'} C_{HH'H''} C_{H''LL}$$

3 point correlator on $S^{d-1} x S^1$



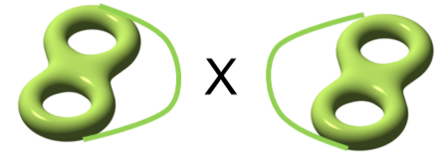

$$\sum_{H,H',H'',H'''} C_{HH'H''} C_{H'H''H'''} C_{HH'''L}$$

1 point function on connected sum of two times $S^{d-1} x S^1$

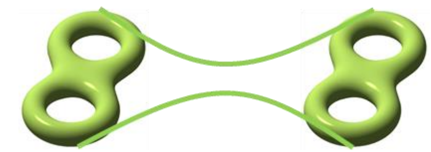
Input gives rise to cubic tems in matrix model for the C’s

# Yet another prediction

$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij}C^{*}_{jkk}e^{-3\beta\Delta}\right)\left(\sum_{lmn} C_{llm}C^{*}_{mmn}e^{-3\beta\Delta}\right)\right\rangle$$

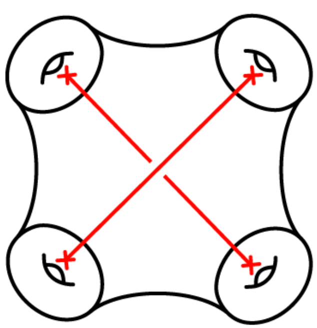



$$Z_{g=2\times g=2} = \left\langle \left(\sum_{i,j,k} C_{iij}C^{*}_{jkk}e^{-3\beta\Delta}\right)\left(\sum_{l,m,n} C_{llm}C^{*}_{mmn}e^{-3\beta\Delta}\right)\right\rangle$$



Belin, JdB, ‘20

# This also works with more boundaries

$Z_{\text{grav}}\Bigg($  $\Bigg)$

$$= \left|\int_0^\infty \mathrm{d}P_4\, C_0(P_4,P_4,P_{\mathcal{O}})^2\, \mathbb{F}_{P_4P_4}\begin{bmatrix} P_4 & P_{\mathcal{O}} \\ P_{\mathcal{O}} & P_4 \end{bmatrix} \prod_{i=1}^{4} \mathcal{F}_{1,1}(P_4;P_{\mathcal{O}};\tau_i)\right|^2$$

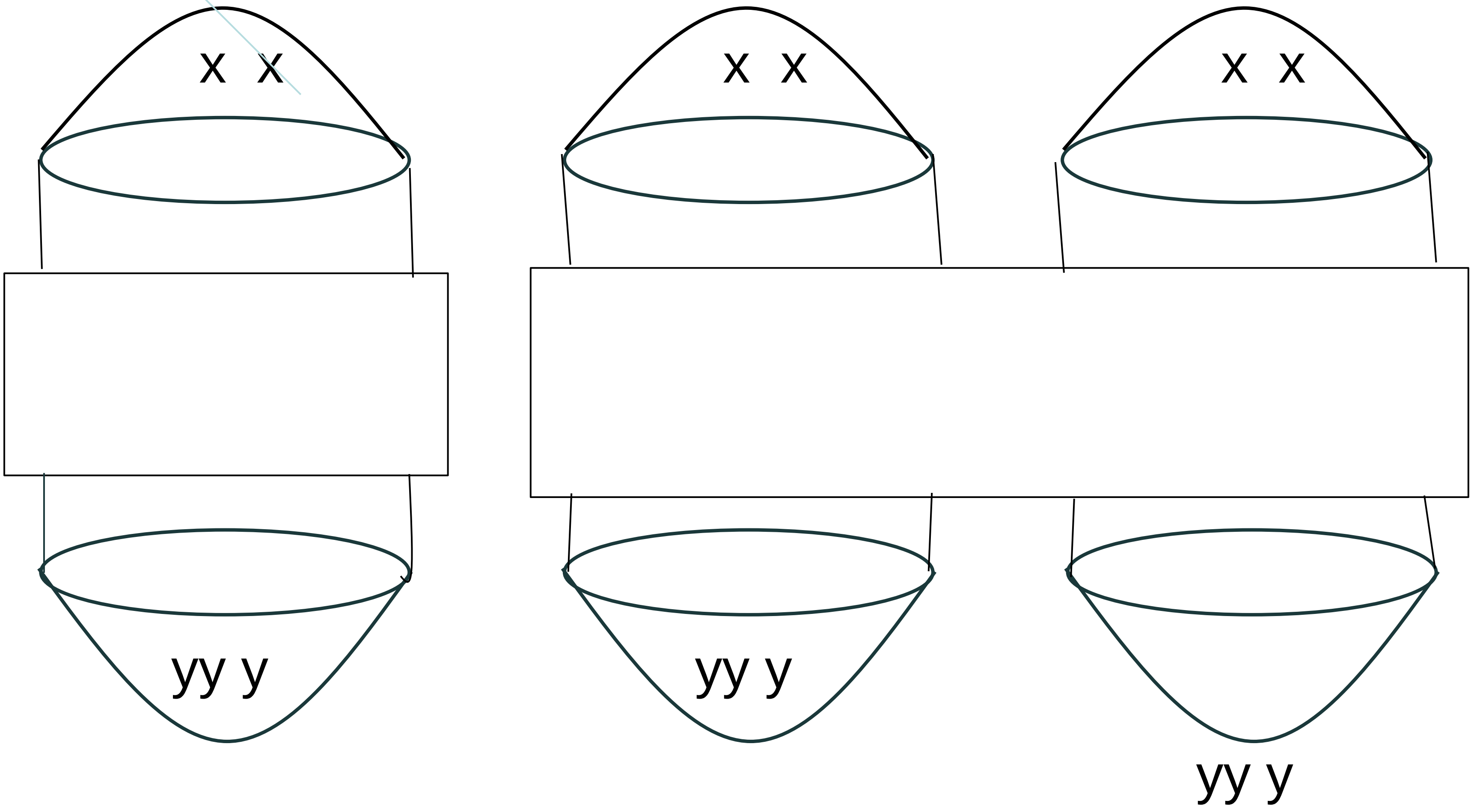
JdB, Liska, Post ‘24

Main conjecture:

Wormhole solutions in semiclassical gravity are simply a consequence of the statistical description associated to semiclassical gravity. They contain no new information.

This conjecture has been tested quite extensively (e.g. Alex Belin, JdB ’20; Chandra, Collier, Hartman, Maloney ’22, JdB, Liska, Post, Sasieta ’23; JdB, Liska, Post ’24; Post Tsiares ‘24; ) for computations involving OPE coefficients in AdS3. More general understanding for pure 3d gravity follows from the Virasoro TQFT (Collier, Eberhardt, Zhang ’23 ‘24).

# Statistics of States




$$\langle\psi_x|\psi_y\rangle = \delta_{x,y} \Longrightarrow \langle|\psi_x|\psi_y\rangle|^2 = e^{-S} \quad x \neq y$$

Prediction if known that Hilbert space is dimension $e^S$
– otherwise a determination of the dimension
(Balasubramanian, Lawrence, Magan, Sasieta ‘22)

# Information recovery in the semiclassical approximation

JdB, Hollander, Rolph '23 + WIP

Time evolution of an initial state

$$\rho_0 \Rightarrow \overline{\rho(t)} = \int dH\, \mu[H]\, e^{-iHt} \rho e^{iHt}$$

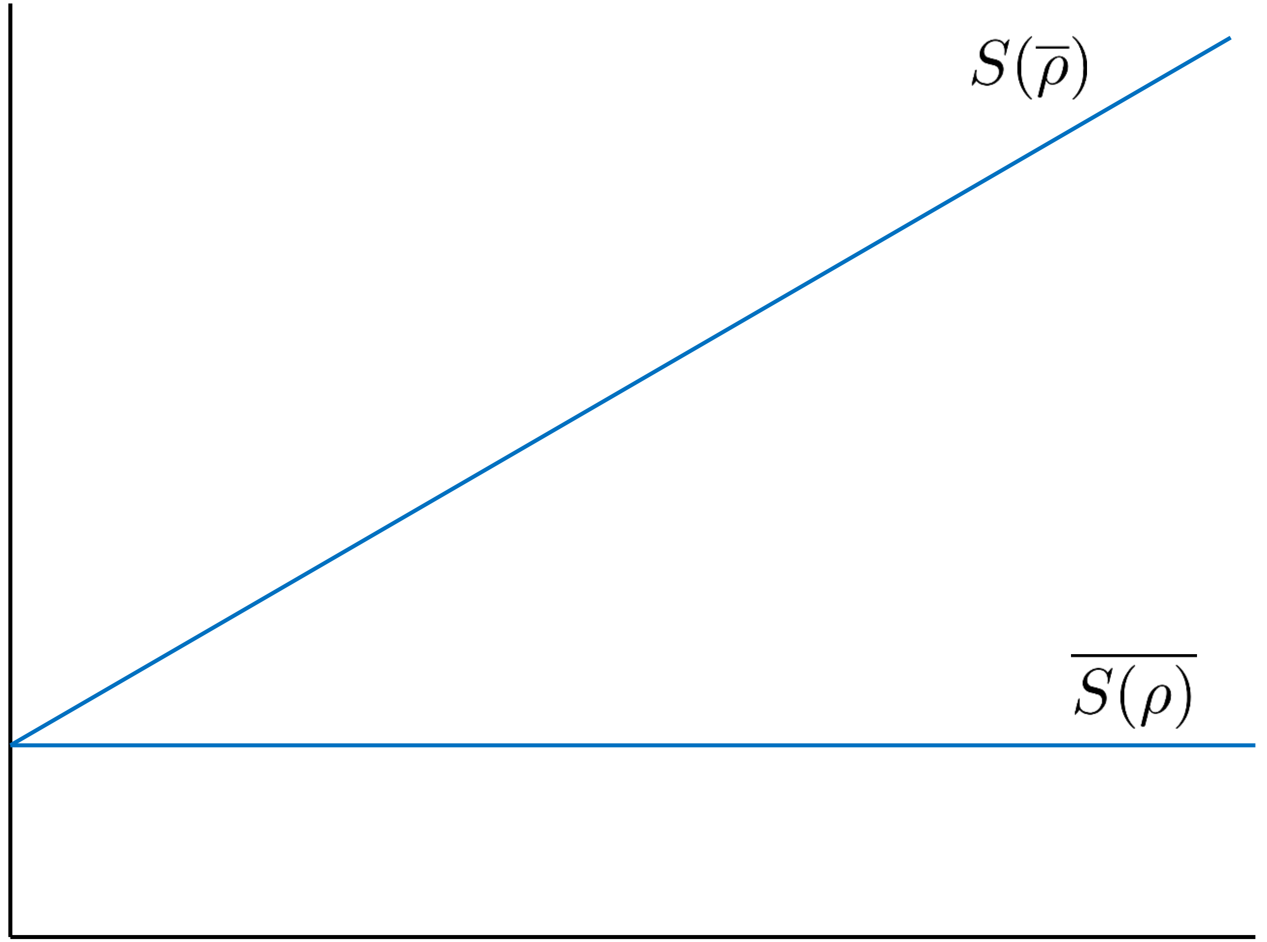
produces a *classical statistical* mixture of states.

In general $S(\overline{\rho(t)})$ will increase: information loss. But since

$$\mathrm{Tr}(\overline{\rho(t)^n}) = \mathrm{Tr}(\rho_0^n) \Rightarrow \overline{S(\rho(t))} = S(\rho_0)$$

a suitable semi-classical replica computation knows that information is actually not lost. Wormholes are crucial.

Penington '19
Almheiri, Engelhardt, Marolf, Maxfield '19
Penington, Shenker, Stanford, Yang '19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19


$S(\overline{\rho})$
$\overline{S(\rho)}$

# Questions/puzzles/conclusions

# 1. Main conjecture

Can one prove the main conjecture?

What is the physics intuition for it?

Should it include only the leading saddle with one boundary or also higher topologies with one boundary?

## 2. Restoring factorization?

It is an interesting question what the minimum number of ingredients are that we need to add to semiclassical gravity in order to uncover more detailed features of the UV and restore factorization.

Several suggestions exist in the literature, like half-wormholes, various branes, non-local interactions, ….

A simple universal explanation could be that wormholes are unstable due to brane creation by an analogue of Schwinger pair production. The Swampland cobordism conjecture suggests that such branes always exist.

But cf Marolf Santos ‘21

Or overcounting? Eberhardt ‘20’21

See e.g.
Gao, Jafferis, Kolckmeyer ‘21
Saad, Shenker, Stanford, Yao ’21
Blommaert, Kruthoff ‘21
Mukhametzhanov ‘21
Blommaert, Iliesiu, Kruthoff ‘21

Alternative: gauging higher-form symmetries.

Benini, Copetti, Di Pietro ‘22

# 3. Why are off-shell configurations needed to make this work?

Spectral correlations of the matrix model are obtained from geometries with two $S^{d-1} \times S^1$ boundaries. There are no on-shell geometries except the on-shell double cone (identify $t \sim t + T$ in a two-sided black hole geometry)

To reproduce the matrix model result we need to integrate over some off-shell configurations with the "constrained instanton" method (Cotler, Jensen '21)

Due to their topological nature, can do something more precise in JT gravity (Saad, Shenker, Stanford '19) and in 3d gravity (Cotler, Jensen '20)

Heuristic observation: OPE correlations are on-shell, spectral correlations are off-shell – why?

One can glue in spectral correlations “RMT surgery” but this always leads to off-shell configurations.

JdB, Joshua-King, Post, WIP

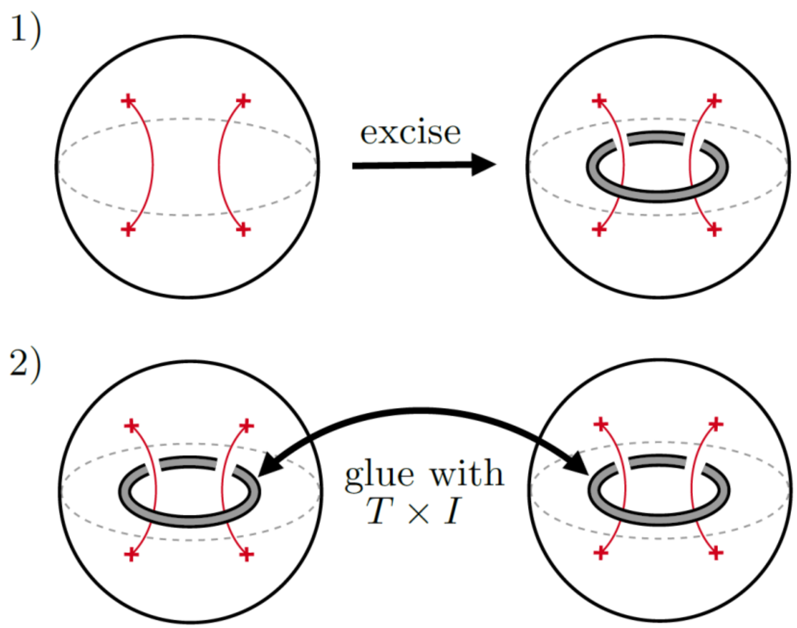



FIG. 2: RMT surgery.

## 4. What about all those other wormholes? Chandra, JdB, WIP

- Axionic wormholes (Giddings, Strominger ‘88 + many more) – boundary $S^d, T^d, S^1 \times S^{d-1}, \ldots$
- Wormholes supported by complex scalar (multiplets) (Marolf, Santos ‘21 +..) – boundary $S^3, T^3, \ldots$
- Meron wormholes, e.g with SU(2) gauge fields and boundaries $S^3$ (Maldacena, Maoz ’04 + ..)
- AdS3 wormholes with hyperbolic boundaries (Maldacena, Maoz ’04 + ..)
- Thin shell wormholes
- Double cone wormholes
- Bra-ket wormholes
- Off-shell wormholes

For axionic wormholes, the axion difference between the two boundaries depends on the boundary curvature:

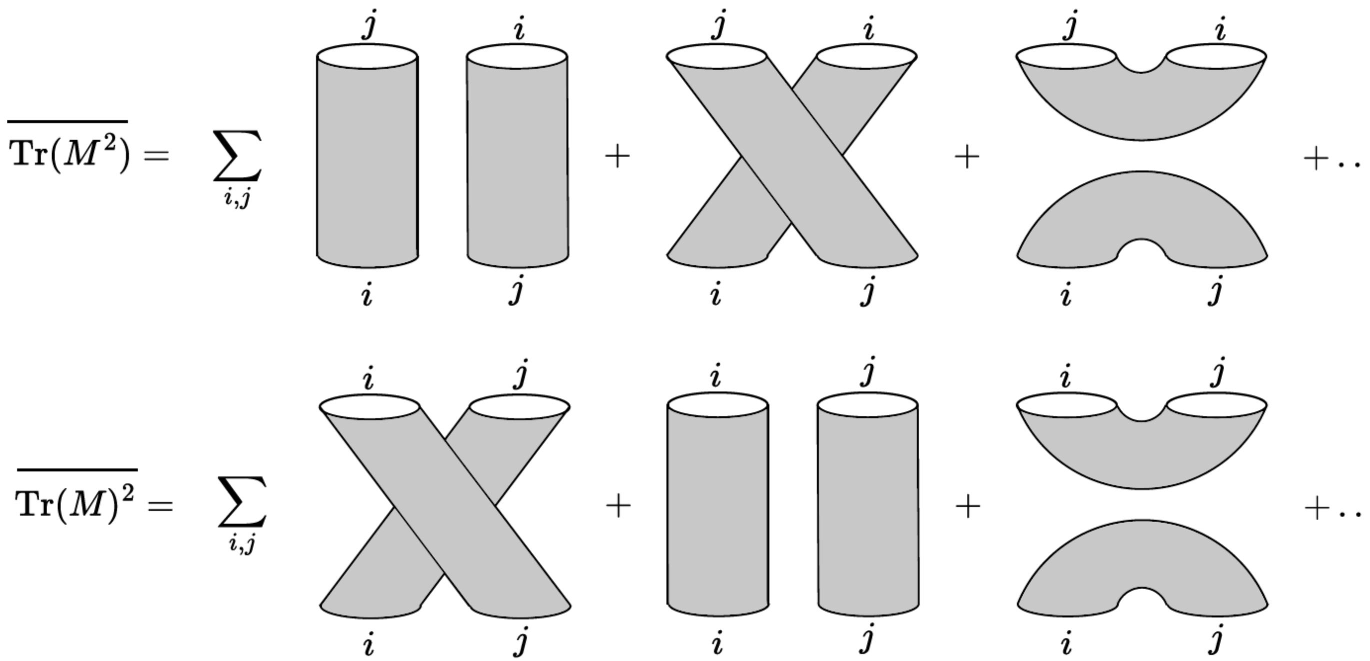
$$k < 0 \implies 0 \leq (\Delta\phi)^2 < \infty$$

$$k = 0 \implies -(\Delta\phi)^2 = \frac{2(d-1)}{d}\pi^2$$

$$k > 0 \implies \frac{2(d-1)}{d}\pi^2 < -(\Delta\phi)^2 < \frac{2d}{(d-1)}\pi^2$$

How to get such a result from a statistical point perspective?

# 5. What about other spacetimes like de Sitter (Harlow, Usatyuk, Zhao ‘25; Abdala, Antonini, Iliesiu, Levine ‘25)?




Picture from Harlow, Usatyuk, Zhao, arXiv:2501.02359

Conclusion: de Sitter is an average over pure states

$$\int d\alpha \mu[\alpha] |\psi_\alpha\rangle\langle\psi_\alpha|$$

and it is hard to say more without adding additional structure. If one adds eternal observers with internal degrees of freedom then we end up with the previous situation

$$\langle\psi_i|\psi_j\rangle = \delta_{ij} + e^{-S/2} R_{ij}$$

# Summary

This statistical physics interpretation of semi-classical gravitational physics is useful and puts many results in a single conceptual framework.

## Finally:

- What to make of topological gravitational theories which are UV complete by themselves (like 2d and 3d gravity)?
- Is this a useful perspective for quantum many body systems? For the bootstrap?
- What is the precise connection to α-vacua and baby universes?
- Did not describe another approach for CFT's by approximately imposing CFT axioms (Belin, JdB, Jafferis, Nayak, Sonner '23; Jafferis, Rozenberg, Wong '24)