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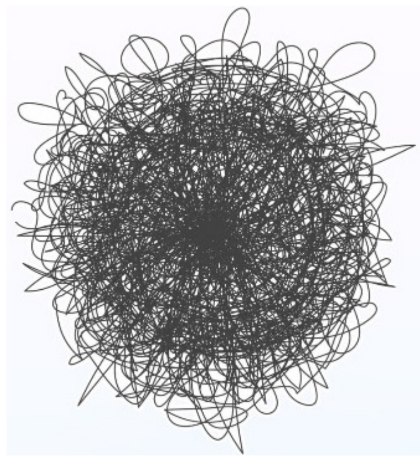
Absorption and emission of light particles from heavy string states



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Bibliography

Ongoing work with:

Massimo Bianchi, Maurizio Firrotta, Vasilis Miarchos

Maurizio Firrotta, Vasilis Miarchos

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Introduction

• Black holes have provided us with some of the deepest puzzles in theoretical physics in the last 50+ years.

♠ The puzzles are always connected with a clash between quantum mechanics and gravity.

♠ It is well known that T-inversion symmetry implies that an absorbing object must also emit. For black holes the absorption is classical but the emission is quantum.

♠ For a long time, the puzzle was between the thermal properties of black holes and the possibility that the underlying state describing them is a pure state.

♠ Recently, we have learned a few more important properties of black holes, that also led to a few more conjectures.

• Black holes have the highest entropy density from any physical system.

Bekenstein

• They are the fastest scramblers of all systems.

Susskind

• They are some of the most chaotic quantum systems in nature as they saturate the quantum Lyapunov exponent bound.

Maldacena+Shenker+Stanford

• They are the most strongly entangled systems in nature.

Maldacena+Susskind

• They are potentially the most powerful quantum computers because of storage capacity and degree of entanglement.

Lloyd, Dvali

• And for (near) extremal black holes, quantum gravitational effects cannot be neglected.

Kitaev, Maldacena,....

Black holes vs String Theory

• String theory has shed important light in black-hole puzzles in several distinct ways:

♠ It has provided concrete calculable examples of explicit microscoping counting of black hole microstates, in agreement with the Bekenstein entropy.

Strominger+Vafa

• The microscopic counting was done at $g_s \ll 1$ when a bound state is loosely coupled, and does not resemble a black hole.

• Supersymmetry transfered this result to $g_s \gg 1$ where the bound state resembles a semiclassical black hole.

♠ Extensions of this counting to black holes with small T gave the hints for the AdS/CFT correspondence.

• The holographic correspondence , provided a breakthrough on the way with think about black holes since.

♠ The black hole state was mapped to a canonical ensemble of a standard (holographic) QFT.

♠ The information paradox was refined and transformed to subtler puzzles associated to firewalls.

♠ The chaotic nature of black holes was understood, and it lead to the quantum Lyapunov exponent bound that is independent of holography.

• String theory on the other hand, provides simpler complex systems, namely High Energy String states (HES) that may serve as proxies for complex systems like black holes.

• Indeed a HES at weak coupling is expected to become a black hole, developing a macroscopic horizon, as one increases the string coupling.

• At the transition point, several quantities agree when computed from the two pictures.

• This is known as the principle of string-black hole complementarity:

Susskind, Horowitz+Polchinski

(black hole gravitational mass) $M_{BH} = \frac{R_{BH}}{G_N}$;   (Area law) $S_{BH} = \frac{R_{BH}^2}{G_N}$

(string states) $$M_{String} = \frac{\sqrt{N}}{\ell_{string}}$$

(multiplicity) $$S_{String} = \ell_{string}\ M_{String} = \sqrt{N}$$

- The string/black-hole transition happens when

$$R_{BH} = \ell_{string} \quad , \quad G_N = \frac{1}{g_s^2 \ell_{string}^2}$$

- At this point:

$$M_{BH} = M_{String} \quad \text{and} \quad g_s^4 = \frac{1}{N}$$

and

$$S_{BH} = S_{String} = \sqrt{N}$$

- It follows that a perturbative string approach to HES is valid iff

$$g_s \ll \frac{1}{N^{\frac{1}{4}}}$$

The thermal nature of pure states

• In his PhD thesis of 1987, Seth Lloyd has studied aspects of the black-hole information paradox.

• He has proven some powerful results that remained largely unknown until recently.

• His central result assumes a finite-dimensional subspace of the Hilbert space of a QFT of dimension n. As an example ,this subspace can be the space of states with $\bar{E} \leq E \leq \bar{E} + d\bar{E}$.

• Consider a typical pure state in this subspace, $|\psi\rangle$.

$$\sqrt{\left(\langle\psi|O|\psi\rangle - \frac{Tr[O]}{n}\right)^2} \leq \frac{\sqrt{\frac{Tr[O^2]}{n} - \left(\frac{Tr[O]}{n}\right)^2}}{\sqrt{1+n}} \leq \frac{\max(O_i)}{\sqrt{n+1}}$$

• When $n \gg 1$, this is the microcanonical ensemble and the traces are the microcanonical thermal correlators.

• Similar bounds hold for standard deviations as well as higher correlators.

• The word typical is again important. For example if $|\psi\rangle$ is an eigenvector of O , or a superposition dominated by an eigenvector of O, then the bound fails.

• The working definition is that $|\psi\rangle$ is a linear superposition of all states in the subspace with complex coefficients drawn at random by a flat distribution.

• The choice of operator is also important: it should not be "too complicated" so that its maximal eigenvalue does increase fast enough with n.

• There is an alternative hypothesis that serves to explain thermalization and quantum chaos, known as the ETH.

Deutche, Srednicki

• It is formulated in terms of the matrix elements of operators in the energy eigenstates' basis, $O_{m,n} \equiv \langle E_m | O | E_n \rangle$.

$$O_{mn} = \bar{O}\ \delta_{m,n} + \sqrt{\frac{\overline{O^2}}{e^S}}\ R_{m,n}$$

• The numbers $R_{m,n}$ are random variables with zero mean and unit variance.

• This is an ansatz, that can explain both thermal properties as well as chaos.

• It is not however general and can fail in many theories.

• Lloyd's theorems are weaker but more robust.

HES as laboratories for complex systems

• It seems like a reasonable idea to use HES as laboratories for complex systems and in particular black holes.

• Beyond the correspondence principle of Horowitz-Polchinski, several other past efforts went in the same direction.

• The computation of string form factors.

Mitchell+Sundborg

• The decay rate of highly-excited fundamental string states.

Amati+Russo, Mañes, Chialva+Iengo+Russo

• However, the topic is technically very complex, as it is difficult to have appropriate vertex operators for such complicated states.

• Previous works have taken shortcuts that looked reasonable at the time.

• The main reason such calculations are doable today is via the use of two formalisms:

♠ One is the old DDF formalism that constructs physical HES vertex operators using a rather counter-intuitive (but efficient) construction.

Di Vecchia+Del Giudice+Fubini

♠ The second is more recent: the construction of the (non-trvial) BRST-invariant coherent-state formalism for strings, motivated by the necessity to study systematically cosmic string interactions.

Skliros+Hindmarsh, Skliros

• This gave rise to the first systematic computations of HES amplitudes recently.

Bianchi+Firrotta

Chaos in perturbative string theory

• An obvious question is whether there are signals of chaotic behavior in HES states.

• Signals of chaos in quantum-mechanical scattering amplitudes have been found in many cases in past.

• They would be attributed though to judiciously chosen scattering potentials.

• However, recently, signals of chaos have been found in the scattering form factors of HES.

Rosenhaus+Gross, Rosenhaus, Bianchi+Firrotta+Sonneschein+Weissman

• An analogue of the spectral form factor was defined using the angular zeros of the logarithmic derivative of the amplitude.

• For a HES with mass$^2 = \frac{N}{\ell_s^2}$, it was subsequently shown that such a form factor has generically chaotic properties described by the β-ensemble of random matrices with $\beta \simeq 2$.

Bianchi+Firrotta+Sonneschein+Weissman

• The word generically is important because the chaoticity is not there for every state in the subspace of states at level N.

• There are two special states at level N:

♠ The state $\left(a^{\mu}_{-1}\right)^N |p\rangle$ belonging to the leading Regge trajectory.

♠ The state $a^{\mu}_N |p\rangle$.

• These two states do not exhibit chaotic behavior.

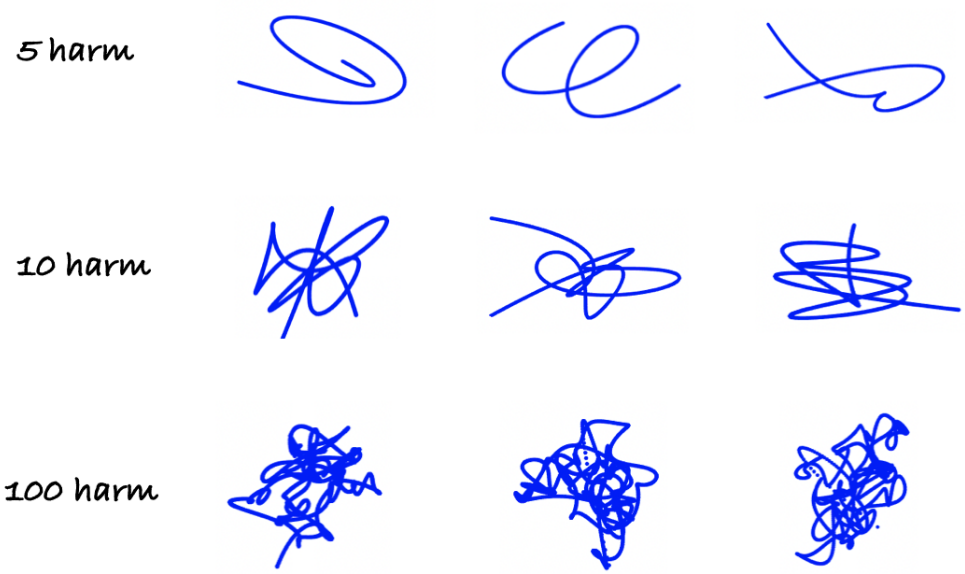
• This is also true for states near them.

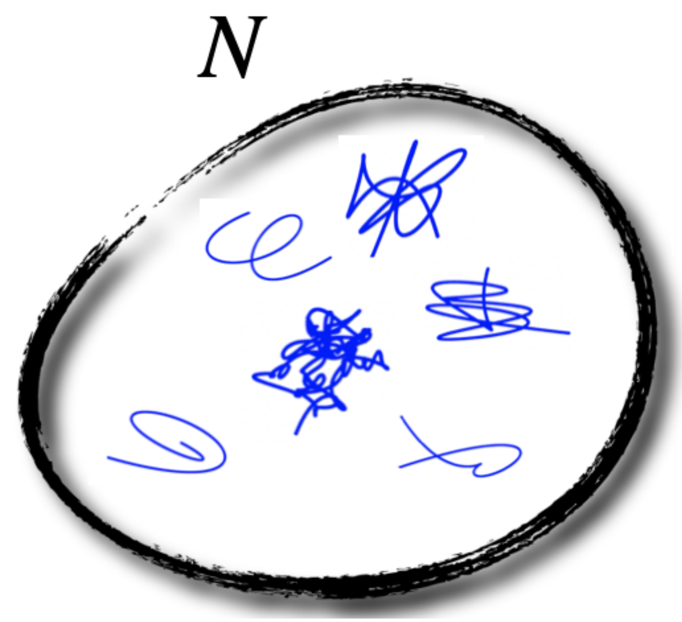
• However , a generic state at level N exhibits chaotic behavior.

Covariant, coherent DDF vertex operators

• Degenerate open string excitations at level $L_0 = N$ can be seen as string microstates.

$$\left(\prod_{i=1}^{g_1} a_{-1}^{\mu_i}\right)\left(\prod_{i=1}^{g_2} a_{-2}^{\mu_i}\right)\cdots\left(\prod_{i=1}^{g_N} a_{-N}^{\mu_i}\right)|p\rangle \quad \Rightarrow \quad \left|g_n\right\rangle_N \quad \Rightarrow \quad N = \sum_{n=1}^{N} n g_n$$





• There are many states for a given mass when $N \gg 1$ and their number scales as $e^{M\ell_s} \sim e^{\sqrt{N}}$.

• The DDF formalism uses as basic operators

$$\mathcal{A}^{\mu}_{-n}(q) = \oint \frac{dz}{2\pi i}\, i\partial X^{\mu}(z)\; e^{-inq\cdot X} \quad , \quad [\mathcal{A}^{\mu}_{m}, \mathcal{A}^{\nu}_{n}] = n\eta^{\mu\nu}\delta_{m,-n}$$

with constraints

$$q \cdot \mathcal{A}_{-n} = 0 \quad , \quad q^2 = 0$$

• The generic state is obtained by acting with DDF oscillators on the tachyon state.

$$|T, g_n, p\rangle = T_{\mu_1\mu_2\cdots} \lim_{z\to 0} \prod_n \frac{1}{\sqrt{n^{g_n} g_n!}} \prod_{r=1}^{g_n} : \mathcal{A}^{\mu^r_n}_{-n}(q) :: e^{ip_t\cdot X}(z) : |0\rangle$$

$$= \lim_{z\to 0} \mathcal{V}(T, \{n, g_n\}, p; z)|0\rangle$$

which is an on-shell state with

$$p = p_t - Nq \quad , \quad p_t^2 = \frac{1}{\ell_s^2} \quad , \quad 2p_t \cdot q = \frac{1}{\ell_s^2} \quad , \quad M^2 = -p^2 = (p_t - Nq)^2$$

• The polarisation tensor T is contracted with matrices and is transverse.

$$R^{\mu}{}_{\nu} = \delta^{\mu}{}_{\nu} - \frac{2}{\ell_s^2}\,(p_t)^{\mu} q_{\nu} \quad , \quad R^{\mu}{}_{\nu}(p_t)^{\nu} = 0 \quad , \quad q_{\mu}R^{\mu}{}_{\nu} = q_{\nu}$$

• Therefore SO(D-2) reps are described.

• The other ingredient is the coherent vertex operator

$$\mathcal{V}_{\text{coherent}}(\{\lambda_n\},p;z) =: e^{\sum_{n=1}^{\infty}\lambda_n\cdot\mathcal{A}_{-n}(q)} : e^{ip_tX}(z)$$

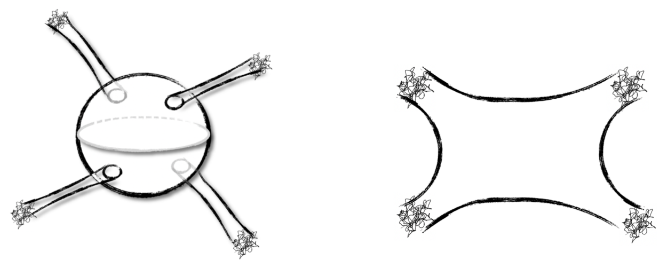
$$=: \exp\left[\sum_{n,m}\frac{\zeta_n\cdot\zeta_m}{2}\mathcal{S}_{n,m}e^{-i(n+m)q\cdot X}+\sum_n\zeta_n\cdot\mathcal{P}_n e^{-inq\cdot X}+ip_t\cdot X\right](z) :$$

with $\zeta_n^\mu = \lambda_n^\nu\ R_\nu{}^\mu$.

• Individual HES states can be obtained by taking λ_n- derivatives on the coherent vertex operator.

The four-point amplitude

- The four-point amplitude of four coherent vertex operators $\mathcal{A}_4(\mathcal{V}_{coh}, \mathcal{V}_{coh}, \mathcal{V}_{coh}, \mathcal{V}_{coh})$



- **Veneziano amplitude with AES**

$$\mathcal{A}^{4,HES}_{gen}(s,t) = \mathcal{A}_{Ven}(s,t)\, e^{\mathcal{K}\left(\{\zeta^{(\ell)}_n\};\partial_{\beta_s},\partial_{\beta_t}\right)}\,\Phi_{\beta_s,\beta_t}(s,t)\Big|_{\beta_{s,t}=0}$$

- **Shapiro-Virasoro amplitude with AES**

$$\mathcal{M}^{4HES}_{gen}(s,t,u) = \mathcal{M}_{SV}(s,t,u)\cdot$$

$$e^{\mathcal{K}\left(\{\zeta^{(\ell)}_n\};\partial_{\beta_s},\partial_{\beta_t}\right)}\Phi_{\beta_s,\beta_t}(s,t)\; e^{\overline{\mathcal{K}}\left(\{\overline{\zeta}^{(\ell)}_n\};\partial_{\overline{\beta}_s},\partial_{\overline{\beta}_t}\right)}\overline{\Phi}_{\overline{\beta}_s,\overline{\beta}_t}(s,t)\Big|_{\beta_{s,t},\overline{\beta}_{s,t}=0}$$

The absorption cross section

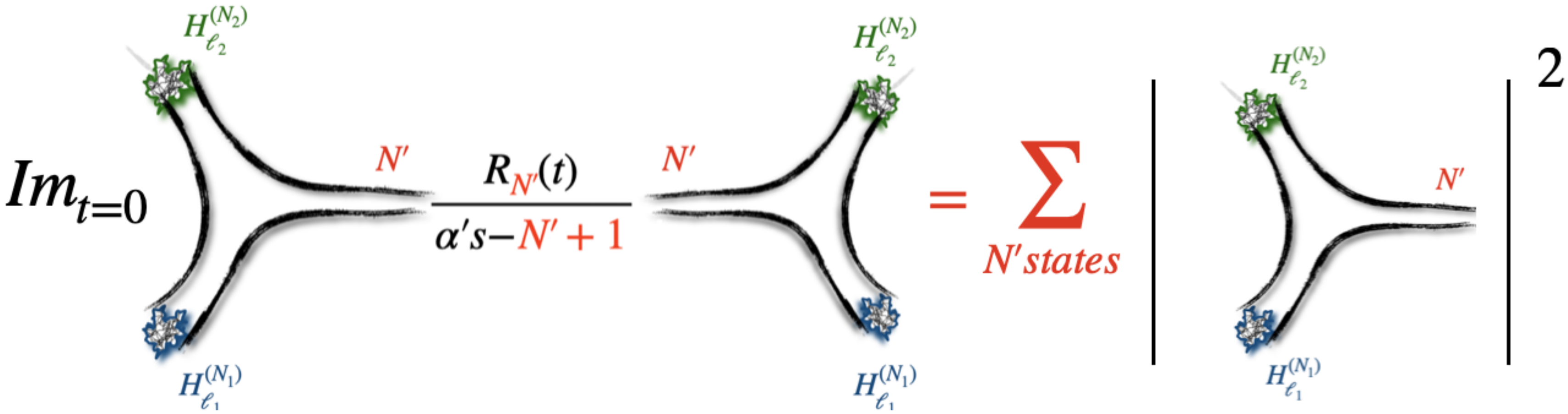
• We are interested in computing the elastic absorption cross section, where a general state $H^{N_1}_{\ell_1}$ absorbs another $H^{N_2}_{\ell_2}$ to become a third state.

$$H^{N_1}_{\ell_1} + H^{N_2}_{\ell_2} \quad \rightarrow \quad \text{anything} \quad \text{in level} \quad \mathrm{N}'$$

where "anything" is any one-particle state, to leading order in perturbation theory.

• We do not average over ANY initial state: they are prepared to be fixed microstates in their respective levels.

• The respective absorption cross section can be computed using the optical theorem = the appropriate imaginary part of the forward four-point amplitude.



$$\sigma_{abs}^{H_{\ell_1}^{(N_1)}+H_{\ell_2}^{(N_2)}\to N'} = \frac{g_s^2 {\ell_s}^{d-4}}{F_{H_1H_2}^{\phi}} \sum_{\ell'} \left| \mathcal{A}_{H_{\ell_1}^{(N_1)}+H_{\ell_2}^{(N_2)}\to H_{\ell'}^{(N')}} \right|^2 =$$

$$= \frac{g_s^2 {\ell_s}^{d-4}}{F_{H_1H_2}^{\phi}} Im\mathcal{A}_{H_{\ell_1}^{(N_1)}+H_{\ell_2}^{(N_2)}\to H_{\ell_1}^{(N_1)}+H_{\ell_2}^{(N_2)}}(s = M_{N'}^2, t = 0)\Big|_{\text{cut}}$$

$$= \frac{g_s^2 {\ell_s}^{d-4}}{F_{H_1H_2}^{\phi}} \mathcal{F}_{s=M_{N'}^2, t=0}(H_{\ell_1}^{(N_1)}, H_{\ell_2}^{(N_2)})\ \mathcal{R}_{N'}(t=0)\ \delta(s - M_{N'}^2)$$

$$\alpha' F_{H_1H_2}^{\phi} = 2M_{H_1}|\vec{p}_2| \quad , \quad \mathcal{R}_{N'}(t=0) = 1 + N'$$

- Elastic Absorption **Universality** : only the levels matter.

$$\mathcal{F}_{s=M^2_{N'},t=0}(H^{(N_1)}_{\ell_1},H^{(N_2)}_{\ell_2}) = 1 - \frac{N_1+N_2}{N'+1}$$

- There is no dependence on ℓ_1, ℓ_2 .

- This is because of the sum over final states.

- Further averaging over the initial states makes no difference.

- Open string elastic absorption

$$\sigma^{\langle N_1\rangle+\langle N_2\rangle\to N'}_{abs,op}(E_{cm})\Big|_{CoM} = \pi\ell_s^{d-2}g_o^2\frac{(1+N'-N_1-N_2)}{\ell_s|\vec{p}_{cm}|\ell_s E_{cm}}$$

- Closed string elastic absorption

$$\sigma^{\langle N_1\rangle+\langle N_2\rangle\to N'}_{abs,cl}(E_{cm})\Big|_{CoM} = \pi\ell_s^{d-2}g_c^2\frac{(1+N'-N_1-N_2)^2}{\ell_s|\vec{p}_{cm}|\ell_s E_{cm}}$$

Special cases

• Photon absoption by any heavy open string state (no averaging over initial states)

$$\sigma_{abs}^{(\text{open})} = \pi \ell_s^{d-2} g_o^2 \ ,$$

• If interpret this as the "size" of the HES as seen by the photon, then the size is independent of mass.

• The corresponding absorption cross-section for massless states in closed string theory is

$$\sigma_{abs}^{(\text{closed})} = 2\pi \ell_s^{d} g_c^2 M\omega \ ,$$

• Closed HES have a non-trivial grey-body factor at tree-level.

• Both results are exact to leading order in string perturbation theory.

T-invariance and emission rates

• In a T-invariant theory we always have

$$\mathcal{A}_{H^{N_1}_{\ell_1}+H^{N_2}_{\ell_2}\to H^{N_3}_{\ell_3}} = \mathcal{A}^*_{H^{N_3}_{\ell_3}\to H^{N_1}_{\ell_1}+H^{N_2}_{\ell_2}}$$

• This is distinct from "detailed balance" (that assumes equilibrium).

• We shall use this to compute the emission rate.

• As we have previously averaged over H^{N_3}, the emission rate will be averaged over initial states.

• We must also average over $H^{N_2}_{\ell_2}$ (if we assume this is the "heavy" final state, $H^{N_1}_{\ell_1}$ being a graviton or photon.)

- The microscopic rate is

$$\frac{\delta\Gamma^{H_{\ell_3}^{N_3}\to H_{\ell_1}^{N_1}+H_{\ell_2}^{N_2}}}{\delta E_1\ d\Omega_{\text{solid}}^{d-2}} = \frac{g_s^2}{8(2\pi)^{d-2}}\frac{E_1^{d-3}}{E_{N_2}}M_{N_3}^2\left(1-\frac{M_{N_1}^2}{E_1^2}\right)^{\frac{d-3}{2}}\left|\mathcal{A}_{H_{\ell_3}^{N_3}\to H_{\ell_1}^{N_1}+H_{\ell_2}^{N_2}}\right|^2\times$$

$$\times\delta(E_3-E_1-E_2)$$

- We now sum over all external states

$$\frac{1}{\rho(d,N')}\sum_{\ell'}\ \sum_{\ell_1}\sum_{\ell_2}\left|\mathcal{A}_{H_{\ell'}^{(N')}\to H_{\ell_1}^{(N_1)}+H_{\ell_2}^{(N_2)}}\right|^2 \quad\Rightarrow$$

- For photon emission (open strings)

$$\frac{d\Gamma^{N'\to\gamma+N}}{d\omega\ d\Omega_{\text{solid}}^{d-2}} = \frac{\pi g_o^2}{(2\pi)^{d-1}} \frac{(\ell_s\omega)^{d-2}}{1-\frac{\ell_s\omega}{\sqrt{N-1}}} \cdot \frac{\rho\left(N'-1-2\ell_s\omega\sqrt{N'-1}\right)}{\rho(N'-1)}$$

- For a heavy initial state, $N' \gg 1$ and a low-energy photon, $\ell_s\omega \ll N'$:

$$\frac{\delta\Gamma^{N'\to\gamma+N}}{\delta\omega\ d\Omega_{\text{solid}}^{d-2}} \simeq \frac{\pi g_o^2}{(2\pi)^{d-1}}(\ell_s\omega)^{d-2}\ M_{N'}\ e^{-\frac{\omega}{T_H}} \quad , \quad T_H = \frac{1}{2\pi\ell_s}\sqrt{\frac{6}{d-2}}$$

- This is in agreement with $\omega \gg T_{\text{Hawk}}$ limit of the black-hole rate.

• For the emission of gravitons from a closed string HES

$$\frac{d\Gamma^{N'\to g+N}}{d\omega\ d\Omega^{d-2}_{\text{solid}}} = \frac{\pi g_c^2}{(2\pi)^{d-1}} \frac{(\ell_s M'_N)(\ell_s\omega)^{d-1}}{1-\frac{\ell_s\omega}{\sqrt{N-1}}} \cdot \frac{\rho^2\left(\frac{N'-1}{4} - \frac{\ell_s\omega}{2}\sqrt{N'-1}\right)}{\rho^2\left(\frac{N'}{4}\right)}$$

• For a heavy initial state, $N' \gg 1$ and a low-energy graviton, $\ell_s\omega \ll N'$:

$$\frac{d\Gamma^{N'\to g+N}}{d\omega\ d\Omega^{d-2}_{\text{solid}}} = \frac{g_c^2}{(2\pi)^{d-2}}(\ell_s M'_N)^2(\ell_s\omega)^{d-1}\ e^{-\frac{\omega}{T_H}}$$

• This is again in agreement with $\omega \gg T_{\text{Hawk}}$ limit of the black hole rate.

Conclusions

• We have computed general four-point amplitude of four coherent string vertex operators.

• We have computed the absorption cross sections for various open and closed string states from HES closed and open string states.

• The cross section do not depend on the details on the initial states.

• Using T-invariance, we have also computed the emission cross sections for various open and closed string states.

• When the degeneracies are large, our results are in agreement withLloyd's theorems, as well as with black-hole emission rates (at $\omega \gg T_H$)

• For $\omega \ell_s \ll 1$ our results differ from previous (indirect) calculations.

Open Ends

- There are several other computations that are interesting to do:

♠ Emission and absorption of open strings from closed HES.

♠ Emission and absorption of closed strings from open HES.

- Improving the eikonal high-energy scattering of HES states.

- Computing the long range fields of HES.

- Study the entanglement properties of final states.

- Compare extremal black holes with leading Regge trajectory states.

THANK YOU!

The $\mathcal{S}$ and $\mathcal{B}$ polynomials in the coherent vertex operator

• $\mathcal{P}_m$, $\mathcal{S}_{\ell,m}$ are suitable polynomials of the world-sheet operators $\partial_z^s X$ of the form

$$\zeta_n{\cdot}\mathcal{P}_n(z) := \sum_{k=1}^{n} \frac{\zeta_n{\cdot}i\partial_z^k X(z)}{(k-1)!}\mathcal{Z}_{n-k}(a_s^{(n)}) \ , \quad a_s^{(n)} \equiv -in\frac{q{\cdot}\partial_z^s X}{(s-1)!}$$

$$\mathcal{S}_{n,m} := \sum_{r=1}^{m} r\,\mathcal{Z}_{n+r}(a_s^{(n)})\mathcal{Z}_{m-r}(a_s^{(m)})$$

and $\mathcal{Z}_n$ is the cycle index polynomial

$$\mathcal{Z}_n(x) = \frac{1}{2\pi i}\oint \frac{dw}{w^{n+1}} e^{\sum_{s=1}^{\infty} \frac{x}{s} w^s}$$

Anatomy of the four-point amplitude

- Anatomy of the result: open string case

- typical Veneziano factor

$$\mathcal{A}_{Ven}(s,t) = g_o^2 \frac{\Gamma(-\ell_s^2 s - 1)\Gamma(-\ell_s^2 t - 1)}{\Gamma(-\ell_s^2 s - \ell_s^2 t - 2)}$$

- Dressing of external arbitrarily excited states = polynomials V, W, I.

$$\mathcal{K}\left(\partial_{\beta_s}, \partial_{\beta_t}\right) = \sum_{\ell=1}^{4} \left(\sum_{n=1}^{\infty} \zeta_n^{(\ell)} \cdot V_n^{(\ell)}(\partial_{\beta_s}, \partial_{\beta_t}) + \sum_{n,m=1}^{\infty} \zeta_n^{(\ell)} \cdot \zeta_m^{(\ell)} W_{n,m}^{(\ell)}(\partial_{\beta_s}, \partial_{\beta_t}) \right) +$$

$$+ \sum_{v<f=1}^{4} \sum_{n,m=1}^{\infty} \zeta_n^{(v)} \cdot \zeta_m^{(f)} I_{n,m}^{(v,f)}(\partial_{\beta_s}, \partial_{\beta t})$$

- The pole-free (s,t)-symmetric Φ-function

$$\Phi_{\beta_s,\beta_t}(s,t) = \sum_{r=0}^{\infty} \sum_{v=0}^{\infty} \frac{\beta_s^r}{r!} \frac{\beta_t^v}{v!} \frac{(-\ell_s^2 s - 1)_r (-\ell_s^2 t - 1)_v}{(-\ell_s^2 s - \ell_s^2 t - 2)_{r+v}}$$

where $(x)_n \equiv \Gamma[x+n]/\Gamma[x]$ (Polchammer).

• The anatomy of the contractions

$$\mathcal{V}_{\text{coherent}} =: \exp\left[\sum_{n,m}\frac{\zeta_n\cdot\zeta_m}{2}\mathcal{S}_{n,m}e^{-i(n+m)q\cdot X}+\sum_n\zeta_n\cdot\mathcal{P}_n e^{-inq\cdot X}+ip_t\cdot X\right](z):$$

• $\mathcal{S}$ from any given V_ℓ contracts only with the tachyon vertex of other operators giving $W^{(\ell)}_{m,n}$.

• $\mathcal{P}$ from any given V_ℓ contracts with the tachyon vertex of other operators giving $V^{(\ell)}_m$.

• $\mathcal{P}$ from any given V_ℓ can contract with the $\mathcal{P}$ from another V_f giving $I^{(\ell,f)}_{m,n}$.

• V, I, W are combinations of Jacobi Polynomials $P^{a,b}_N(\partial_{\beta_s},\partial_{\beta_t})$.

• For the special case that 2 and 3 are massless states and 1,4 are HES we have:

$$V_n^{(1)\mu}(z) = \sqrt{2\alpha'}p_2^\mu P_{n-1}^{(\alpha_1^{(n)},\beta_1^{(n)})}(1-2z) + z\sqrt{2\alpha'}p_3^\mu P_{n-1}^{(\alpha_1^{(n)}+1,\beta_1^{(n)})}(1-2z)$$

$$V_n^{(4)\mu}(z) = \sqrt{2\alpha'}p_1^\mu P_{n-1}^{(\alpha_4^{(n)},\beta_4^{(n)})}(2z-1) + (1-z)\sqrt{2\alpha'}p_2^\mu P_{n-1}^{(\alpha_4^{(n)}+1,\beta_4^{(n)})}(2z-1)$$

$$W_{n,m}^{(1)}(z) = \sum_{r=1}^{m} r\, P_{n+r}^{(\alpha_1^{(n)}-r,\beta_1^{(n)}-1)}(1-2z)\; P_{m-r}^{(\alpha_1^{(m)}+r,\beta_1^{(m)}-1)}(1-2z)$$

$$W_{n,m}^{(4)}(z) = \sum_{r=1}^{m} r\, P_{n+r}^{(\alpha_4^{(n)}-r,\beta_4^{(n)}-1)}(2z-1)\; P_{m-r}^{(\alpha_4^{(m)}+r,\beta_4^{(m)}-1)}(2z-1)$$

$$I_{n_1,m_4}^{(1,4)}(z) = (-)^{m_4+1}\sum_{r,s=0}^{n_1,m_4} z^{r+s+1} P_{n_1-r-s-1}^{(\alpha_1^{(n_1)}+r+s+1;\,\beta_1^{(n_1)}-1)}(1-2z)\cdot$$

$$\cdot P_{m_4-r-s-1}^{(\beta_4^{(m_4)}+r+s+1;\,\alpha_4^{(m_4)}-1)}(1-2z)$$

- The map $z \leftrightarrow \beta_s, \beta_t$ comes from the combination

$$\beta_s z + \beta_t(1-z) \to \partial_{\beta_s} = -\partial_{\beta_t}$$

- The parameters $\alpha_1, \alpha_4, \beta_1, \beta_4$ are related to the momenta as follows

$$\alpha_1^{(n)} = -n - 2\alpha' n q_1 \cdot p_2 \,, \quad \beta_1^{(n)} = -n - 2\alpha' n q_1 \cdot p_4 \;,$$
$$\alpha_4^{(n)} = -n - 2\alpha' n q_4 \cdot p_1 \,, \quad \beta_4^{(n)} = -n - 2\alpha' n q_4 \cdot p_3 \;.$$

- The Jacobi Polynomials:

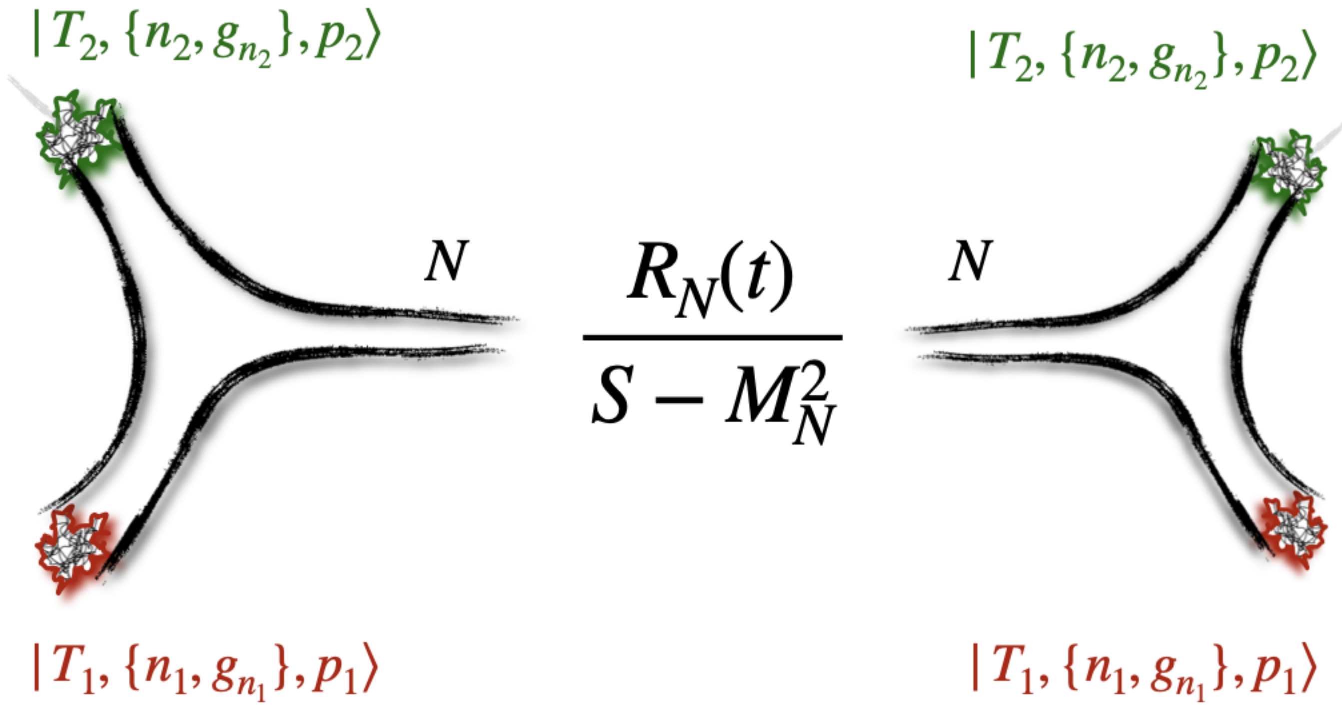
$$P_N^{(\alpha,\beta)}(x) = \sum_{r=0}^{N} \binom{N+\alpha}{N-r} \binom{N+\beta}{r} \left(\frac{x-1}{2}\right)^r \left(\frac{x+1}{2}\right)^{N-r}$$

- The following are the properties of the amplitude

- **Cyclicity and Crossing symmetry enforced by** $\Phi_{\beta_s,\beta_t}(s,t)$

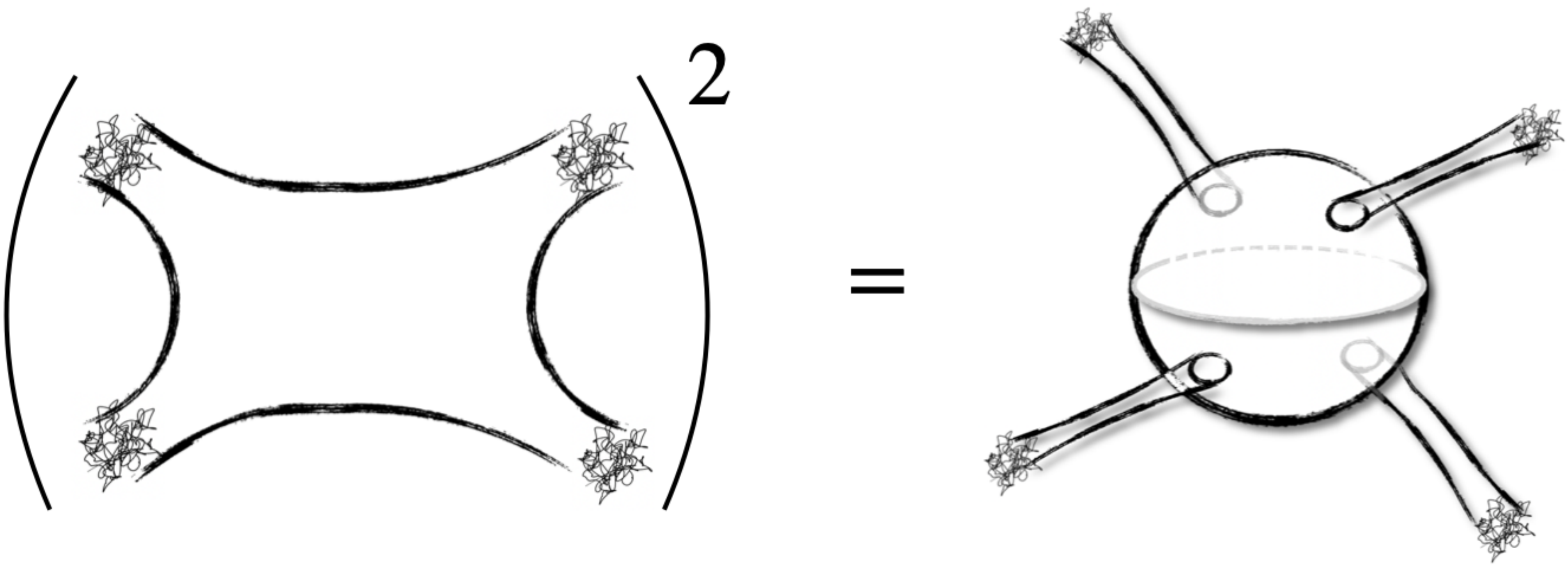
- **Correct factorization properties**

$$\mathcal{A}^{4HES}_{gen}(s,t)\Big|_{P_N^2\sim N-1} = \mathcal{A}^{2HES+N}_{gen}(p_1,p_2,P_N)\frac{R_N(t)}{P_N^2-M_N^2}\mathcal{A}^{2HES+N}_{gen}(p_3,p_4,-P_N)$$



- **Generalized KLT**

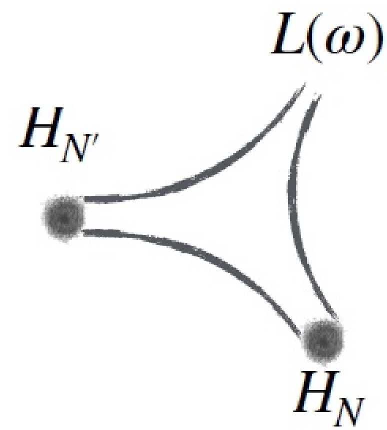
$$\mathcal{M}^{4HES}_{gen}(s,t,u) = \sin\frac{\pi\ell_s^2 t}{4}\mathcal{A}^{4HES}_{gen}(s,t)\mathcal{A}^{4HES}_{gen}(t,u)$$



The Amati-Russo calculation

- Amati and Russo in particular, computed the average of the inclusive emission rate of massless strings from highly excited string states in light-cone gauge.

$$\frac{d\Gamma}{d\omega} = \frac{1}{\mathcal{N}_{H_{N'}}} \sum_{H_{N'}} \sum_{H_N} \left| \quad \right|^2$$



number of states at level N'

- They found for open strings (in d dimensions)

$$\frac{d\Gamma(\omega)}{d\omega} = (\text{constant}) \frac{\omega^{d-2}}{e^{\frac{\omega}{T_H}} - 1}$$

and for closed strings

$$\frac{d\Gamma(\omega)}{d\omega} = (\text{constant})\frac{\omega^{d-1}\ e^{-\frac{\omega}{T}}}{\left(e^{\frac{\omega}{T_L}}-1\right)\left(e^{\frac{\omega}{T_R}}-1\right)} \quad , \quad \frac{1}{T}=\frac{1}{T_L}+\frac{1}{T_R}$$

recovering the familiar expressions of emission rates from black holes!!

• Absorption cross-sections from the detailed balance condition (that assumes thermodynamic equlibrium)

$$\frac{d\Gamma}{d\omega} = \sigma\ \frac{\Omega^{d-2}_{\text{solid}}\ \omega^{d-2}}{e^{\frac{\omega}{T}}-1}$$

with σ being the absorption cross section.

• We note that the calculation of emission rates in Amati+Russo was based on the average of the inclusive rate of emission of massless states using a direct diagonal sum of squared 3-point amplitudes in the light-cone gauge

• The resulting expression is claimed to be equivalent to a trace (and, therefore, basis-independent), but this statement does not seem to be correct, since the Fock space basis in the light-cone gauge is not orthonormal.

A toy summation over states

• A simple quantum mechanical example of calculation of probabilities: a a simple reminder of the issues that appear when we use non-orthogonal bases.

• We consider an initial state $|\psi_0\rangle$ in the Hilbert space, that we assume normalized, $\langle\psi_0|\psi_0\rangle = 1$.

• We also consider a final state that belongs to a finite dimensional subspace V_n that is spanned by an orthonormal basis ψ_i with $\langle\psi_i|\psi_j\rangle = \delta_{ij}$.

• A generic (normalized) vector in V_n can be written as

$$|\psi(\vec{a})\rangle \equiv \sum_{i=1}^{n} a_i|\psi_i\rangle \quad , \quad \sum_{i=1}^{n} |a_i|^2 = 1$$

• Therefore, the manifold of normalized states of V_n is isometric to S^{2n-1}.

• The amplitude for $|\psi_0\rangle \to |\psi(\vec{a})\rangle$ is

$$A(\vec{a}) = \langle\psi_0|\psi(\vec{a})\rangle = \sum_{i=1}^{n} a_i A_{0i} \quad , \quad A_{0i} \equiv \langle\psi_0|\psi_i\rangle$$

• The probability of finding any state of V_n in $|\psi_0\rangle$ is given by the sum of probabilities $P(\vec{a}) = |A(\vec{a})|^2$ of ending in any vector of V_n.

• The sum is performed by the natural metric on V_n that of S^{2n-1}.

$$P_{0\to V_n} = \frac{1}{\Omega_{2n-1}} \int_{S^{2n-1}} d\Omega_{2n-1} |A(\vec{a})|^2 =$$

$$= \frac{1}{\Omega_{2n-1}} \sum_{i,j=1}^{n} A_{0i}^* A_{0j} \int_{S^{2n-1}} d\Omega_{2n-1}\ a_i^* a_j =$$

$$= \sum_{i,j=1}^{n} A_{0i}^* A_{0j} \delta^{ij} = \sum_{i=1}^{n} |A_{0i}|^2$$

• above, $d\Omega_{2n-1}$ is the measure on the unit S^{2n-1}, and $\Omega_{2n-1} = \int_{S^{2n-1}} d\Omega_{2n-1}$ is the volume of the unit S^{2n-1}.

• The end result is the standard sum of squared amplitudes formulae that is valid as we see in an orthonormal basis.

• We now translate the same calculation in a non-orthogonal basis of final states.

• To do this we start from the orthonormal basis above and we rotate it to generic basis by an GL(C,n) rotation M_{ij},

$$|\psi_i\rangle = \sum_{j=1}^{n} M_{ij}\ |\bar{\psi}_j\rangle \quad , \quad detM \neq 0$$

• Now the inner products of the new basis have a nontrivial metric

$$G_{ij} \equiv \langle\bar{\psi}_i|\bar{\psi}_j\rangle = \sum_{k,l=1}^{n} M^*_{ik} M_{jl} \langle\psi_i|\psi_j\rangle = \sum_{k=1}^{n} M^*_{ik} M_{jk} = (M \cdot M^\dagger)_{ji}$$

• We also obtain

$$\bar{A}_{0i} \equiv \langle\psi_0|\bar{\psi}_i\rangle = M_{ij} A_{0j} \quad \Rightarrow \quad A_{0i} = M^{-1}_{ij} \bar{A}_{0j}$$

• The probability can be written as

$$P_{0\to V_n} = \sum_{i=1}^{n} |A_{0i}|^2 = \sum_{i=1}^{n} A^*_{0i} A_{0i} = \sum_{i=1}^{n} \sum_{k,l=1}^{n} (M^*)^{-1}_{il} A^*_{0l} M^{-1}_{ik} \bar{A}_{0k} =$$

$$= \sum_{k,l=1}^{n} (M \cdot M^\dagger)^{-1}_{lk} A^*_{0l} \bar{A}_{0k} = \sum_{k,l=1}^{n} G^{kl} A^*_{0l} \bar{A}_{0k}$$

where G^{ij} is the inverse metric of G_{ij}.

T-invariance versus detailed balance.

• Detailed balance contains, beyond the assumption of T-invariance also the asumption of equlibrium.

• IN the emission/absorption case, it implies that the black body is equilibrium with the emitted radiation.

• In that case there is an extra contribution to the emission rate coming from stimulated emission.

• Assuming this we obtain in the HES case:

• Open string emission:

$$\frac{d\Gamma_{em}{}^{N'\to\gamma+N}}{d\omega\ d\Omega_{solid}^{(d-2)}} = \frac{1}{2}\frac{g_o^2}{(2\pi)^{d-2}}(\ell_s M_{N'})\ (\ell_s\omega)^{d-2}\ \frac{1}{e^{\frac{\omega}{T_H}}-1}$$

• Closed string emission

$$\frac{d\Gamma_{em}^{N'\to g+N}}{d\omega\ d\Omega_{solid}^{(d-2)}} = \frac{g_c^2}{(2\pi)^{d-2}}(\ell_s^2 M_{N'})^2\ (\ell_s\omega)^{d-1}\ \frac{1}{e^{\frac{\omega}{T_H}}-1}\ .$$

The H_2TH_2T amplitude

$$\mathcal{A}_{H_2^1+T\to H_2^1+T}(s,t) = \frac{\mathcal{A}_{Ven}(s,t)}{(\alpha's+\alpha't-1)(\alpha's+\alpha't)(1+\alpha's+\alpha't)(2+\alpha's+\alpha't)}$$

$$\Big(-2\zeta_2^{(1)}\cdot\zeta_2^{(4)}(1+\alpha's)(\alpha's+\alpha't-1)\Big((3+4\alpha'q_4\cdot p_1+4\alpha'q_1\cdot p_4(1+2\alpha'q_4\cdot p_1))(\alpha's)^2$$

$$+4\alpha'q_1\cdot p_2 q_4\cdot p_3\alpha't(1+\alpha't)+\alpha's\Big(-2-4\alpha'q_4\cdot p_1+4\alpha'q_4\cdot p_3+\alpha't+4\alpha'q_4\cdot p_3\alpha't$$

$$+4\alpha'q_1\cdot p_2(1+2\alpha'q_4\cdot p_1)(1+\alpha't)-4\alpha'q_1\cdot p_4(1+2\alpha'q_4\cdot p_1-2\alpha'q_4\cdot p_3(1+\alpha't))\Big)\Big)$$

$$+\sqrt{2\alpha'}\zeta_2^{(1)}\cdot p_3(1+\alpha's)\Big(\sqrt{2\alpha'}\zeta_2^{(4)}\cdot p_2(1+\alpha't)\Big((1+4\alpha'q_4\cdot p_3)(\alpha's+4\alpha'q_1\cdot p_4\alpha's$$

$$+4\alpha'q_1\cdot p_2(-1+\alpha't))\alpha't+4\alpha'q_4\cdot p_1\alpha's(-1+4\alpha'q_1\cdot p_4(-1+\alpha's)+\alpha's+4q_1\cdot p_2\alpha't)\Big)$$

$$+\sqrt{2\alpha'}\zeta_2^{(4)}\cdot p_1(\alpha's+\alpha't-1)\Big(4\alpha'q_1\cdot p_2(1+\alpha't)(\alpha's+4\alpha'q_4\cdot p_1\alpha's+\alpha't+4\alpha'q_4\cdot p_3\alpha't)$$

$$+(1+4\alpha'q_1\cdot p_4)\alpha's\Big(4\alpha'q_4\cdot p_1(\alpha's-1)+\alpha's+\alpha't+4\alpha'q_4\cdot p_3(1+\alpha't)\Big)\Big)\Big)$$

$$+\sqrt{2\alpha'}\zeta_2^{(1)}\cdot p_2(\alpha's+\alpha't-1)\Big(\sqrt{2\alpha'}\zeta_2^{(4)}\cdot p_2(1+\alpha't)\Big((1+4\alpha'q_4\cdot p_3)\alpha't\Big(\alpha's+4q_1\cdot p_4(1+\alpha's)$$

$$+4\alpha'q_1\cdot p_2(\alpha't-1)+\alpha't\Big)+4\alpha'q_4\cdot p_1(1+\alpha's)(\alpha's+4\alpha'q_1\cdot p_4\alpha's+\alpha't+4\alpha'q_1\cdot p_2\alpha't)\Big)$$

$$+\sqrt{2\alpha'}\zeta_2^{(4)}\cdot p_1(\alpha's+\alpha't)\Big(4\alpha'q_1\cdot p_2(1+\alpha't)(1+\alpha's+4\alpha'q_4\cdot p_1(1+\alpha's)+\alpha't+4\alpha'q_4\cdot p_3\alpha't)$$

$$+4\alpha'q_1\cdot p_4(1+\alpha's)(1+\alpha's+4\alpha'q_4\cdot p_1\alpha's+\alpha't+4\alpha'q_4\cdot p_3(1+\alpha't))+(1+\alpha's+\alpha't)$$

$$(2+\alpha's+4\alpha'q_4\cdot p_1(1+\alpha's)+\alpha't+4\alpha'q_4\cdot p_3(1+\alpha't))\Big)\Big)\Big)\ .$$

The $H_N T H_N T$ amplitude

$$\mathcal{A}_{H_N^1+T\to H_N^1+T} = A_{Ven}(s,t)$$

$$\Bigg(2\alpha'\zeta_N^{(1)}\cdot p_2 \zeta_N^{(4)}\cdot p_1 \sum_{r_1,r_4=0}^{N-1} \binom{N-1+\alpha_1}{N-1-r_1}\binom{N-1+\alpha_4}{N-1-r_4}\binom{N-1+\beta_1}{r_1}\binom{N-1+\beta_4}{r_4} \mathcal{Q}^{[s;N-1+r_1-r_4]}_{[t;N-1+r_4-r_1]}$$

$$+ 2\alpha'\zeta_N^{(1)}\cdot p_2 \zeta_N^{(4)}\cdot p_2 \sum_{r_1,r_4=0}^{N-1} \binom{N-1+\alpha_1}{N-1-r_1}\binom{N+\alpha_4}{N-1-r_4}\binom{N-1+\beta_1}{r_1}\binom{N-1+\beta_4}{r_4} \mathcal{Q}^{[s;N-1+r_1-r_4]}_{[t;N+r_4-r_1]}$$

$$+ 2\alpha'\zeta_N^{(1)}\cdot p_3 \zeta_N^{(4)}\cdot p_1 \sum_{r_1,r_4=0}^{N-1} \binom{N+\alpha_1}{N-1-r_1}\binom{N-1+\alpha_4}{N-1-r_4}\binom{N-1+\beta_1}{r_1}\binom{N-1+\beta_4}{r_4} \mathcal{Q}^{[s;N+r_1-r_4]}_{[t;N-1+r_4-r_1]}$$

$$+ 2\alpha'\zeta_N^{(1)}\cdot p_3 \zeta_N^{(4)}\cdot p_2 \sum_{r_1,r_4=0}^{N-1} \binom{N+\alpha_1}{N-1-r_1}\binom{N+\alpha_4}{N-1-r_4}\binom{N-1+\beta_1}{r_1}\binom{N-1+\beta_4}{r_4} \mathcal{Q}^{[s;N+r_1-r_4]}_{[t;N+r_4-r_1]}$$

$$+ \zeta_N^{(1)}\cdot\zeta_N^{(4)}(-)^{N-1} \sum_{v_1,v_4=0}^{N-1}\sum_{r_1,r_4=0}^{N-1} \binom{N+\alpha_1}{N-v_1-v_4-1-r_1}\binom{N-v_1-v_4-2+\beta_1}{r_1}$$

$$\binom{N+\beta_4}{N-v_1-v_4-1-r_4}\binom{N-v_1-v_4-2+\alpha_4}{r_4} \mathcal{Q}^{[s;N+r_1+r_4]}_{[t;2(N-1-v_1-v_4)+r_1+r_4]} \Bigg) \quad (1)$$

where

$$\mathcal{Q}^{[s;c_s]}_{[t;c_t]} := \frac{(-\alpha' s-1)_{c_s}(-\alpha' t-1)_{c_t}}{(-\alpha' s-\alpha' t-2)_{c_s+c_t}} \quad (2)$$

and

$$\alpha_1 = -N - 2\alpha' N q_1{\cdot}p_2\,, \quad \beta_1 = -N - 2\alpha' N q_1{\cdot}p_4\ , \tag{3}$$

$$\alpha_4 = -N - 2\alpha' N q_4{\cdot}p_1\,, \quad \beta_4 = -N - 2\alpha' N q_4{\cdot}p_3\ . \tag{4}$$

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 0 minutes
- Introduction 2 minutes
- Black holes vs String Theory 6 minutes
- The thermal nature of pure states 9 minutes
- HES as laboratories for complex systems 11 minutes
- Chaos in Perturbative String Theory 13 minutes
- Covariant, Coherent, DDF Vertex operators 17 minutes
- The four-point amplitude 19 minutes
- The absorption cross section 23 minutes
- Special cases 24 minutes
- T-invariance and emission rates 29 minutes
- Conlusions 30 minutes
- Open ends 31 minutes