
ACCELERATED EXPANSION, CASIMIR ENERGIES, & RIEMANN-FLAT MANIFOLDS

Miguel Montero

IFT Madrid

Gravity, Strings & SUSY-breaking

Pisa, April 4th 2025



~~ACCELERATED EXPANSION, CASIMIR ENERGIES, & RIEMANN-FLAT MANIFOLDS~~

Miguel Montero

IFT Madrid

Gravity, Strings & SUSY-breaking

Pisa, April 4th 2025



ACCELERATED EXPANSION, CASIMIR ENERGIES, & RIEMANN-FLAT MANIFOLDS

Miguel Montero

IFT Madrid

Gravity, Strings & SUSY-breaking

Pisa, April 4th, 2025



**A top-down de Sitter maximum
in M-theory**

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

about computing
Casimir energies in
Riemann-flat
manifolds

[Bruno Bento, MM, 2504.xxxxx]



We are exploring this idea in the context of
Riemann-flat manifolds

$\frac{T^n}{\Gamma}$ Acts freely (+ duality bundle)
Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds
(a.k.a. Bieberbach manifolds)



We are exploring this idea in the context of
Riemann-flat manifolds

$\frac{T^n}{\Gamma}$ Acts freely (+ duality bundle)
Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See Gianguido's talk]



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)

Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See **Gianguido's talk**]

as a means to evade the
Maldacena-Nuñez no go
theorem



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)

Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See **Gianguido's talk**]

as a means to evade the
Maldacena-Nuñez no go
theorem



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)

Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

and obtain **fully explicit**, controlled de Sitter vacua in ST

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See **Gianguido's talk**]

as a means to evade the
Maldacena-Nuñez no go
theorem



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)

Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

and obtain **fully explicit**, controlled de Sitter vacua in ST

Today: Will do the first, and discuss the second

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See **Gianguido's talk**]

as a means to evade the
Maldacena-Nuñez no go
theorem



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)

Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima

In many cases, we have no-go due to “conspiracies”

(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

and obtain **fully explicit**, controlled de Sitter vacua in ST

Today: Will do the first, and discuss the second for a **dS maximum**

This talk is about a project I presented briefly in KITP March' 24
and StringPheno 2024 (Padova)

[Bruno Bento, MM, 2504.xxxxxx]

about computing
Casimir energies in
Riemann-flat
manifolds

(a.k.a. Bieberbach manifolds)

[See **Gianguido's talk**]

as a means to evade the
Maldacena-Nuñez no go
theorem



We are exploring this idea in the context of
Riemann-flat manifolds

$$\frac{T^n}{\Gamma}$$

Acts freely (+ duality bundle)
Instead of curvature, we have **two fluxes**

All moduli can be frozen by Gamma, except **two**

Casimir energy can be computed **explicitly**

We are performing a **systematic search** for minima
In many cases, we have no-go due to “conspiracies”
(e.g. some duality freezes modulus, but also kills useful flux)

Large, but finite, nonsusy mini-Landscape

and obtain **fully explicit**, controlled de Sitter vacua in ST

Today: Will do the first, and discuss the second for a **dS maximum**

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

...that was all Bruno. The dS maximum? Found by Bruno.

More importantly: A project like this is finding a needle in haystack.

Bruno explored this relentlessly,
never wavered, for more than a year.

He showed **true grit**.



This project fully rests upon the immense
efforts, skills and calculations



of **Bruno Bento**

(currently postdoc at IFT UAM-CSIC)

This project fully rests upon the immense
efforts, skills and calculations



of **Bruno Bento**

(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...
...that was all Bruno.

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

...that was all Bruno. The dS maximum? Found by Bruno.

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

...that was all Bruno. The dS maximum? Found by Bruno.

More importantly: A project like this is finding a needle in haystack.

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

...that was all Bruno. The dS maximum? Found by Bruno.

More importantly: A project like this is finding a needle in haystack.

Bruno explored this relentlessly,
never wavered, for more than a year.

This project fully rests upon the immense
efforts, skills and calculations
of **Bruno Bento**



(currently postdoc at IFT UAM-CSIC)

You are going to see complicated numerical & analytical expressions,
for Casimir energies, scans over moduli without SUSY...

...that was all Bruno. The dS maximum? Found by Bruno.

More importantly: A project like this is finding a needle in haystack.

Bruno explored this relentlessly,
never wavered, for more than a year.

He showed **true grit**.



Let's get going!

Let's get going! This is a talk about dS maxima in String Theory.

Let's get going! This is a talk about dS maxima in String Theory.

You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

Let's get going! This is a talk about dS maxima in String Theory.

You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

on which there is a lot of work even today.

[See Arthur, Iosif, Severin's talks]

Let's get going! This is a talk about dS maxima in String Theory.

You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

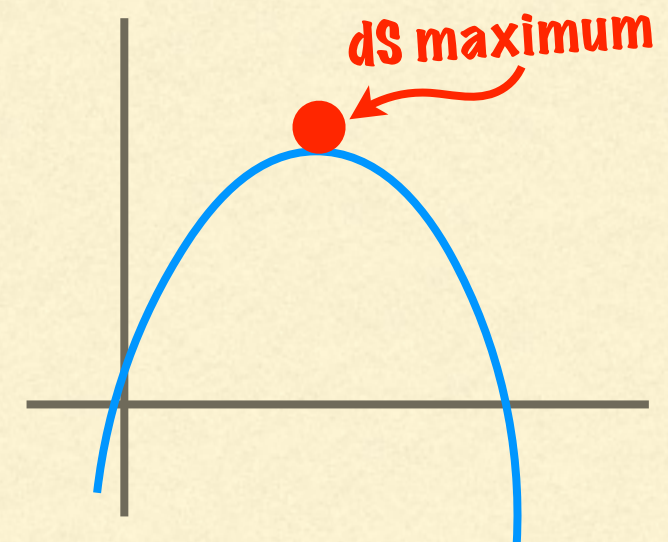
on which there is a lot of work even today.

[See Arthur, Iosif, Severin's talks]

But as far as I know, we have also not controlled examples of de Sitter **maxima** either.

[Most recent discussion:
Ruehle-Andriot '24, Chen-Van
de Heisteeg-Vafa '25]

[Objections?]



Let's get going! This is a talk about dS maxima in String Theory.

You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

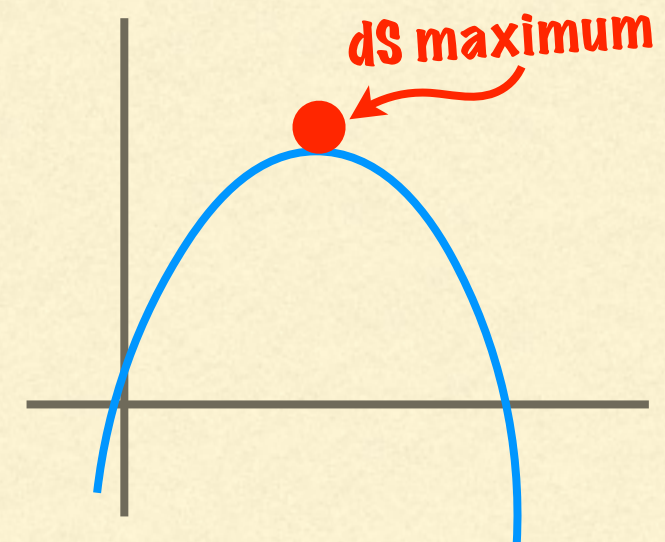
on which there is a lot of work even today.

[See Arthur, Iosif, Severin's talks]

But as far as I know, we have also not controlled examples of de Sitter **maxima** either.

[Most recent discussion:
Ruehle-Andriot '24, Chen-Van
de Heisteeg-Vafa '25]

[Objections?]



A controlled dS maximum is probably *easier* to get,

Let's get going! This is a talk about dS maxima in String Theory.
You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

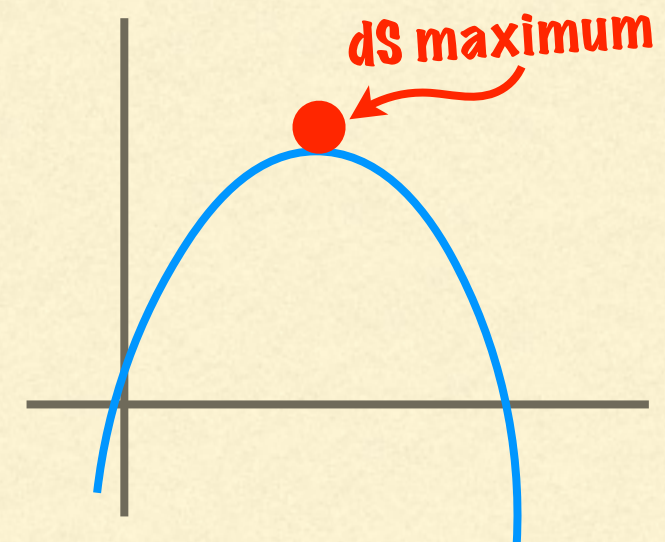
on which there is a lot of work even today.

[See Arthur, Iosif, Severin's talks]

But as far as I know, we have also
not controlled examples of de Sitter **maxima** either.

[Most recent discussion:
Ruehle-Andriot '24, Chen-Van
de Heisteeg-Vafa '25]

[Objections?]



A controlled dS maximum is probably *easier* to get,
and is **cosmologically viable** [Obied, Agrawal '18]

Let's get going! This is a talk about dS maxima in String Theory.

You are familiar with the complicated character of existing de Sitter minimum constructions in ST (KKLT/LVS)

(higher-derivative terms, Gaugino condensates, branes, orientifolds, KS throats...)

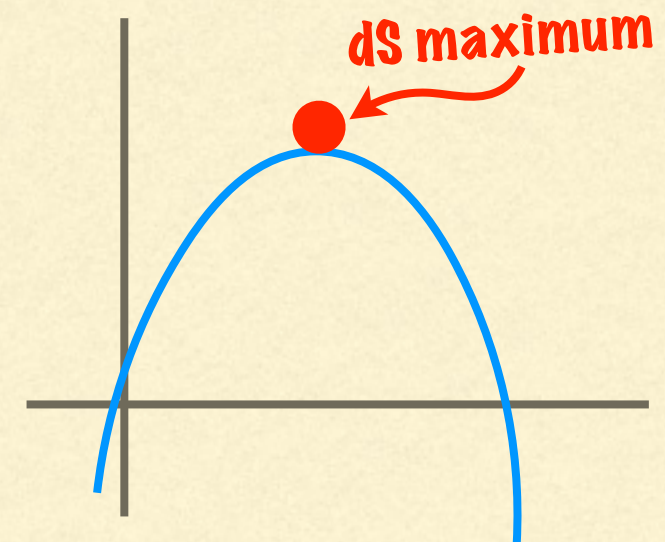
on which there is a lot of work even today.

[See Arthur, Iosif, Severin's talks]

But as far as I know, we have also not controlled examples of de Sitter **maxima** either.

[Most recent discussion:
Ruehle-Andriot '24, Chen-Van
de Heisteeg-Vafa '25]

[Objections?]



A controlled dS maximum is probably *easier* to get,
and is **cosmologically viable** [Obied, Agrawal '18]

(and perhaps even favored, if DESI results hold!)

Aspirational goal: To get dS maxima that are as well-controlled as the AdS₅ × S⁵ vacua of ST.

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

Branes

Aspirational goal: To get dS maxima that are as well-controlled as the AdS₅ × S⁵ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times \text{S}^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

Orientifolds

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

Higher derivatives

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

Warping/curvature/ throats

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

~~Warping/curvature/threats~~

Aspirational goal: To get dS maxima that are as well-controlled as the AdS5 x S5 vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

~~Warping/curvature/threats~~

We will only use **supergravity** ingredients

Aspirational goal: To get dS maxima that are as well-controlled as the $AdS_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

~~Warping/curvature/threats~~

We will only use **supergravity** ingredients

Fluxes

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

~~Warping/curvature/throats~~

We will only use **supergravity** ingredients

Fluxes

Casimir energies

(can be computed reliably and precisely, source negative energy density)

Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

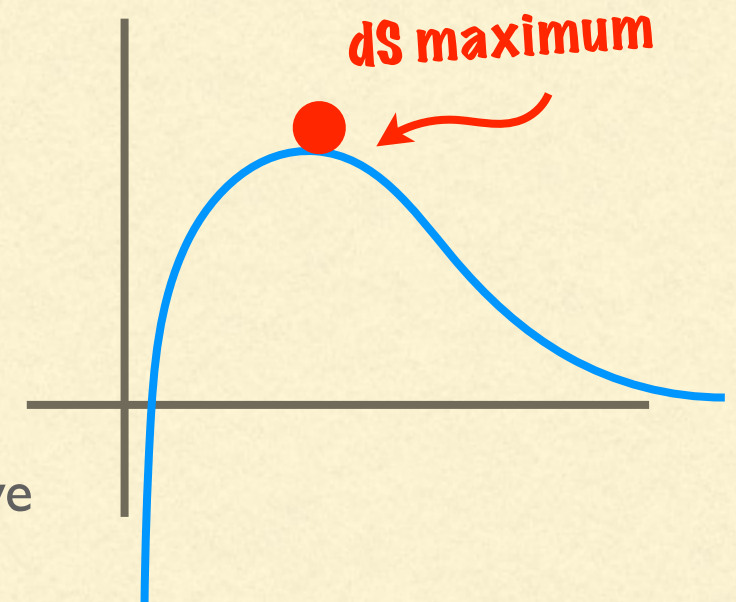
~~Warping/curvature/threats~~

We will only use **supergravity** ingredients

Fluxes

Casimir energies

(can be computed reliably and precisely, source negative energy density)



Aspirational goal: To get dS maxima that are as well-controlled as the $\text{AdS}_5 \times S^5$ vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

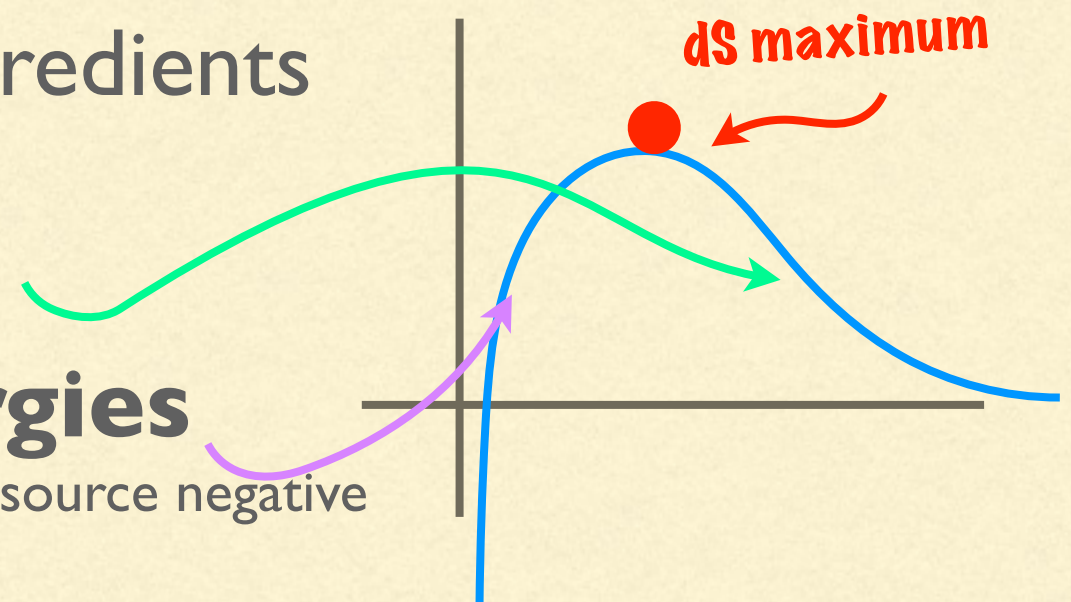
~~Warping/curvature/threats~~

We will only use **supergravity** ingredients

Fluxes

Casimir energies

(can be computed reliably and precisely, source negative energy density)



Aspirational goal: To get dS maxima that are as well-controlled as the AdS5 x S5 vacua of ST.

To do this: Look for dS maxima free of all the **complications** that appear in the dS constructions

~~Branes~~

~~Orientifolds~~

~~Higher derivatives~~

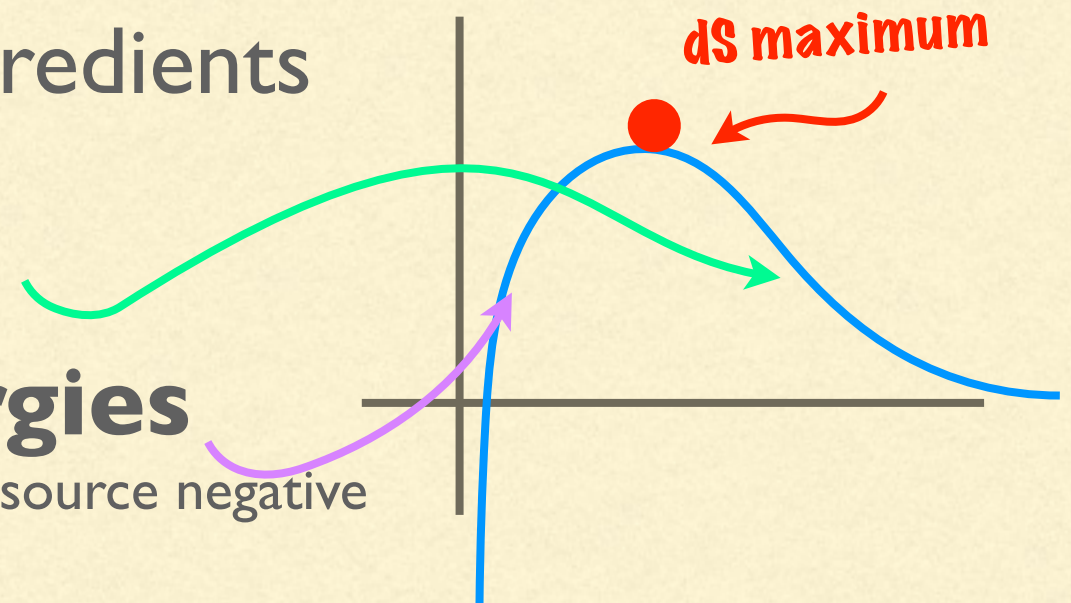
~~Warping/curvature/threats~~

We will only use **supergravity** ingredients

Fluxes

Casimir energies

(can be computed reliably and precisely, source negative energy density)



and for control, we will work on **Riemann-flat manifolds**
(RFM'S) $R_{abcd} = 0$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{1}{\sqrt{2}} \frac{R^4}{R^6} \left((2\pi^2)^{1/3} \frac{n^2}{R^8} - \frac{\mathcal{C}}{R^{11}} \right) \quad \mathcal{C} \approx 4.14$$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{1}{\sqrt{2}} \frac{R^4}{R^6} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \frac{\mathcal{C}}{R^{11}} \right) \quad \mathcal{C} \approx 4.14$$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{1}{\sqrt{2}} \frac{R^4}{R^6} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \underbrace{\frac{\mathcal{C}}{R^{11}}}_{\text{Casimir}} \right) \quad \mathcal{C} \approx 4.14$$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \overset{T^4 \text{ volume}}{\frac{1}{\sqrt{2}} \frac{R^4}{R^6}} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \underbrace{\frac{\mathcal{C}}{R^{11}}}_{\text{Casimir}} \right) \quad \mathcal{C} \approx 4.14$$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{\overset{T^4 \text{ volume}}{1}}{\sqrt{2}} \frac{R^4}{R^6} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \underbrace{\frac{\mathcal{C}}{R^{11}}}_{\text{Casimir}} \right) \quad \mathcal{C} \approx 4.14$$

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{\overset{T^4 \text{ volume}}{1}}{\sqrt{2}} \frac{R^4}{R^6} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \underbrace{\frac{\mathcal{C}}{R^{11}}}_{\text{Casimir}} \right) \quad \mathcal{C} \approx 4.14$$

Casimir coefficient is an extremum with respect to T^4 moduli, due to discrete symmetries of the D4 lattice

Now, with these ingredients, getting a maximum doesn't seem difficult.

Take M-theory, compactify on T^4 at D4 root lattice rescaled by R threaded by n units of G_4 flux, and choose antiperiodic spin structure.

$$V^{(6d)} = \frac{\overset{T^4 \text{ volume}}{1}}{\sqrt{2}} \frac{R^4}{R^6} \left(\underbrace{(2\pi^2)^{1/3} \frac{n^2}{R^8}}_{\text{G4-flux}} - \underbrace{\frac{\mathcal{C}}{R^{11}}}_{\text{Casimir}} \right) \quad \mathcal{C} \approx 4.14$$

Casimir coefficient is an extremum with respect to T^4 moduli, due to discrete symmetries of the D4 lattice

$$R = \frac{(13\mathcal{C})^{1/3}}{\sqrt[9]{2} \sqrt[3]{5} \pi^{2/9}} \approx 1.56 \ell_{11}$$

So we do get a maximum, but not great...

However, we might say “not great, not terrible”

However, we might say “not great, not terrible”

The manifold is Planckian, but...

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

$$\int |G_4|^{2k} \sim \frac{1}{R^{2k-4}} \quad \text{smaller than leading}$$

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

$$\int |G_4|^{2k} \sim \frac{1}{R^{2k-4}} \quad \text{smaller than leading}$$

Internal profile of fields is homogeneous

Higher loops: Suppressed by powers of $(1/R)$, also smallish

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

$$\int |G_4|^{2k} \sim \frac{1}{R^{2k-4}} \quad \text{smaller than leading}$$

Internal profile of fields is homogeneous

Higher loops: Suppressed by powers of $(1/R)$, also smallish

So maybe we just got a dS7 maximum from the simplest torus compactification!

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

$$\int |G_4|^{2k} \sim \frac{1}{R^{2k-4}} \quad \text{smaller than leading}$$

Internal profile of fields is homogeneous

Higher loops: Suppressed by powers of $(1/R)$, also smallish

So maybe we just got a dS7 maximum from the simplest torus compactification!

Difficult to be sure, though...

However, we might say “not great, not terrible”

The manifold is Planckian, but...

$$V^{(6d)} = 10^{-3}$$

No higher-derivative corrections depending on curvature since

$$R_{abcd} = 0$$

$$\int |G_4|^{2k} \sim \frac{1}{R^{2k-4}} \quad \text{smaller than leading}$$

Internal profile of fields is homogeneous

Higher loops: Suppressed by powers of $(1/R)$, also smallish

So maybe we just got a dS7 maximum from the simplest torus compactification!

Difficult to be sure, though... **let's do better**

The key feature of the dS7 maximum was that T^4 is
Riemann-flat

The key feature of the dS7 maximum was that T^4 is
Riemann-flat



Symmetries to control moduli

The key feature of the dS7 maximum was that T^4 is
Riemann-flat

Symmetries to control moduli

No higher corrections

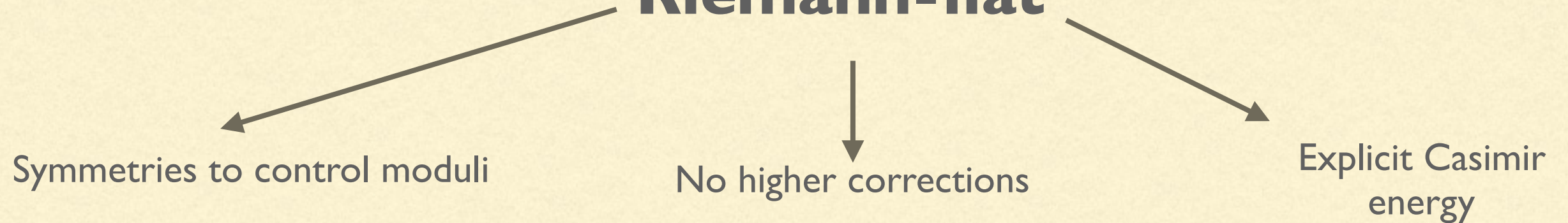
The key feature of the dS7 maximum was that T^4 is
Riemann-flat

Symmetries to control moduli

No higher corrections

Explicit Casimir
energy

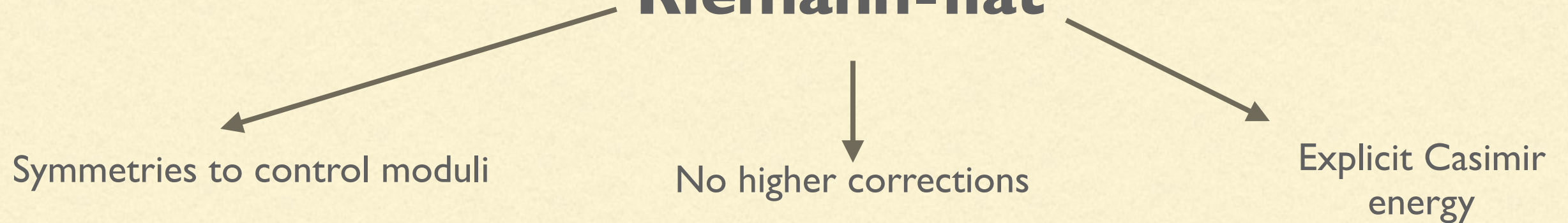
The key feature of the dS7 maximum was that T^4 is
Riemann-flat



There are **more** examples of RFM. Most general:

$$\frac{T^n}{\Gamma} \longrightarrow \text{Torus}$$
$$\Gamma \longrightarrow \text{Discrete group of isometries acting freely (without fixed points)}$$

The key feature of the dS7 maximum was that T^4 is
Riemann-flat

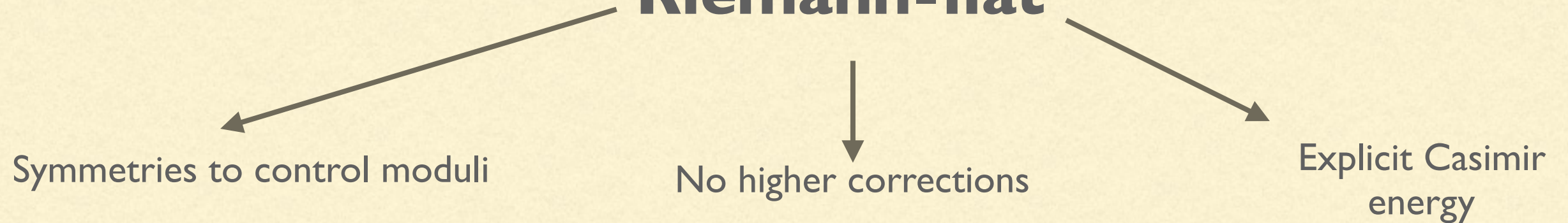


There are **more** examples of RFM. Most general:

$$\frac{T^n}{\Gamma} \longrightarrow \begin{array}{l} \text{Torus} \\ \text{Discrete group of isometries acting} \\ \text{freely (without fixed points)} \end{array}$$

There are **fewer** moduli than T^n

The key feature of the dS7 maximum was that T^4 is
Riemann-flat



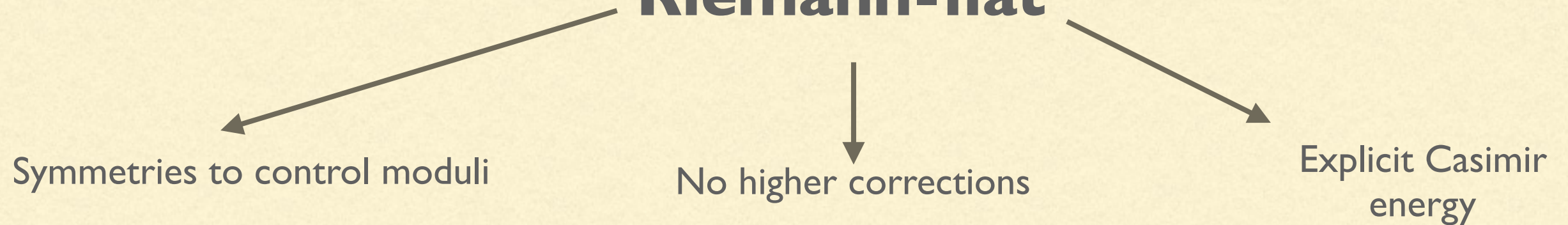
There are **more** examples of RFM. Most general:

$$\frac{T^n}{\Gamma} \longrightarrow \begin{array}{l} \text{Torus} \\ \text{Discrete group of isometries acting} \\ \text{freely (without fixed points)} \end{array}$$

There are **fewer** moduli than T^n

There is a **finite** number of these in any dimension, classified until
 $n < 6$
[Acharya, Fraiman, MM, Parra de Freitas, WIP]

The key feature of the dS7 maximum was that T^4 is
Riemann-flat



There are **more** examples of RFM. Most general:

$$\frac{T^n}{\Gamma} \longrightarrow \begin{array}{l} \text{Torus} \\ \text{Discrete group of isometries acting} \\ \text{freely (without fixed points)} \end{array}$$

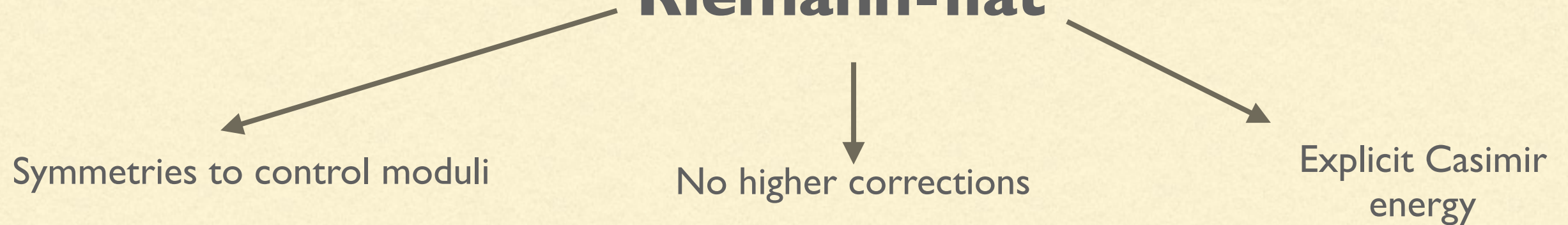
There are **fewer** moduli than T^n

There is a **finite** number of these in any dimension, classified until

$n < 6$
[Acharya, Fraiman, MM, Parra de Freitas, WIP]

Cohomology etc. easy to determine

The key feature of the dS7 maximum was that T^4 is
Riemann-flat



There are **more** examples of RFM. Most general:

$$\frac{T^n}{\Gamma} \longrightarrow \begin{array}{l} \text{Torus} \\ \text{Discrete group of isometries acting} \\ \text{freely (without fixed points)} \end{array}$$

There are **fewer** moduli than T^n

There is a **finite** number of these in any dimension, classified until

$n < 6$
[Acharya, Fraiman, MM, Parra de Freitas, WIP]

Cohomology etc. easy to determine
and also **Casimir energy** via explicit formula.

For free theories, Casimir energy can be obtained via **point-splitting**: [Birrel-Davies]

For free theories, Casimir energy can be obtained via **point-splitting**: [Birrel-Davies]

$$\langle T_{\mu\nu}^{\text{Cas}}(y) \rangle = \left\langle \lim_{x \rightarrow y} \frac{1}{2} \partial_{(\mu} \partial_{\nu)} \phi(x) \phi(y) - \partial_{\mu} \phi(x) \partial^{\mu} \phi(y) \right\rangle$$

For free theories, Casimir energy can be obtained via **point-splitting**: [Birrel-Davies]

$$\langle T_{\mu\nu}^{\text{Cas}}(y) \rangle = \left\langle \lim_{x \rightarrow y} \frac{1}{2} \partial_{(\mu} \partial_{\nu)} \phi(x) \phi(y) - \partial_{\mu} \phi(x) \partial^{\mu} \phi(y) \right\rangle$$

This gets you the **full** spatial dependence of Casimir energy in internal manifold if can compute propagator

$$\langle \phi(x) \phi(y) \rangle$$

For free theories, Casimir energy can be obtained via **point-splitting**: [Birrel-Davies]

$$\langle T_{\mu\nu}^{\text{Cas}}(y) \rangle = \left\langle \lim_{x \rightarrow y} \frac{1}{2} \partial_{(\mu} \partial_{\nu)} \phi(x) \phi(y) - \partial_{\mu} \phi(x) \partial^{\mu} \phi(y) \right\rangle$$

This gets you the **full** spatial dependence of Casimir energy in internal manifold if can compute propagator

$$\langle \phi(x) \phi(y) \rangle$$



On RFM this is **easy** using method of images

$$\langle \phi(x) \phi(y) \rangle = \sum_{g \in \Gamma, l \in \Lambda} G_{\mathbb{R}^n}(g x - y + l)$$

For free theories, Casimir energy can be obtained via **point-splitting**: [Birrel-Davies]

$$\langle T_{\mu\nu}^{\text{Cas}}(y) \rangle = \left\langle \lim_{x \rightarrow y} \frac{1}{2} \partial_{(\mu} \partial_{\nu)} \phi(x) \phi(y) - \partial_{\mu} \phi(x) \partial^{\mu} \phi(y) \right\rangle$$

This gets you the **full** spatial dependence of Casimir energy in internal manifold if can compute propagator

$$\langle \phi(x) \phi(y) \rangle$$



On RFM this is **easy** using method of images

$$\langle \phi(x) \phi(y) \rangle = \sum_{g \in \Gamma, l \in \Lambda} G_{\mathbb{R}^n}(g x - y + l)$$

Extends directly to fields with twisted boundary conditions (fermions) and with spin

There is a divergence from the identity term when $x \rightarrow y$

$$\langle \phi(x)\phi(y) \rangle \supset G_{\mathbb{R}^n}(x-y) \sim \frac{1}{|x-y|^{d-2}}$$

This is the usual **one-loop** divergence of vacuum diagrams

There is a divergence from the identity term when $x \rightarrow y$

$$\langle \phi(x) \phi(y) \rangle \supset G_{\mathbb{R}^n}(x - y) \sim \frac{1}{|x - y|^{d-2}}$$

This is the usual **one-loop** divergence of vacuum diagrams

However, in a SUSY theory,

$$\sum_{\text{Spectrum}} \frac{1}{|x - y|^{d-2}} = \frac{n_B - n_F}{|x - y|^{d-2}} = 0$$

There is a divergence from the identity term when $x \rightarrow y$

$$\langle \phi(x) \phi(y) \rangle \supset G_{\mathbb{R}^n}(x - y) \sim \frac{1}{|x - y|^{d-2}}$$

This is the usual **one-loop** divergence of vacuum diagrams

However, in a SUSY theory,

$$\sum_{\text{Spectrum}} \frac{1}{|x - y|^{d-2}} = \frac{n_B - n_F}{|x - y|^{d-2}} = 0$$

so the Casimir energy is **finite**

There is a divergence from the identity term when $x \rightarrow y$

$$\langle \phi(x) \phi(y) \rangle \supset G_{\mathbb{R}^n}(x - y) \sim \frac{1}{|x - y|^{d-2}}$$

This is the usual **one-loop** divergence of vacuum diagrams

However, in a SUSY theory,

$$\sum_{\text{Spectrum}} \frac{1}{|x - y|^{d-2}} = \frac{n_B - n_F}{|x - y|^{d-2}} = 0$$

so the Casimir energy is **finite**

...but still is a very messy expression:

$$T_{00}(\vec{x}) = \sum_{\text{Fields}} \sum_{g \in \Gamma, \vec{n}} \frac{e^{2\pi i \vec{h} \cdot \vec{n}} \text{tr}(\mathbf{D}_g)}{|(\mathbf{I} - \mathbf{D}_g)\vec{x} + \vec{b}_g + \vec{n}|^{2s}}$$

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

Our main technical result is that we were able to evaluate this integral explicitly and in full generality

$$\mathcal{C} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \mathcal{E}(g)$$

$$\mathcal{E}(g) = \hat{\delta}_{\vec{h}} \sqrt{G_{\parallel}} \frac{\Gamma(s')}{\Gamma(s)} \pi^{\frac{k-k'}{2}} \sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^2 s'}$$

where $g = \mathbf{D}_g \vec{x} + \vec{b}_g$ and k, k', s, s' depend on dimensions.

Each $\mathcal{E}(g)$ is a sum **localized** on the invariant subspace of g , Ξ

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

Our main technical result is that we were able to evaluate this integral explicitly and in full generality

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

Our main technical result is that we were able to evaluate this integral explicitly and in full generality

$$\mathcal{C} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \mathcal{E}(g)$$

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

Our main technical result is that we were able to evaluate this integral explicitly and in full generality

$$\mathcal{C} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \mathcal{E}(g)$$

$$\mathcal{E}(g) = \hat{\delta}_{\vec{h}} \sqrt{G_{\parallel}} \frac{\Gamma(s')}{\Gamma(s)} \pi^{\frac{k-k'}{2}} \sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^2 s'}$$

where $g = \mathbf{D}_g \vec{x} + \vec{b}_g$ and k, k', s, s' depend on dimensions.

The actual Casimir energy is

$$\mathcal{C} = \int_{\text{RFM}} T_{00}(\vec{x})$$

Our main technical result is that we were able to evaluate this integral explicitly and in full generality

$$\mathcal{C} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \mathcal{E}(g)$$

$$\mathcal{E}(g) = \hat{\delta}_{\vec{h}} \sqrt{G_{\parallel}} \frac{\Gamma(s')}{\Gamma(s)} \pi^{\frac{k-k'}{2}} \sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^2 s'}$$

where $g = \mathbf{D}_g \vec{x} + \vec{b}_g$ and k, k', s, s' depend on dimensions.

Each $\mathcal{E}(g)$ is a sum **localized** on the invariant subspace of g , Ξ

That's the space where $\mathbf{D}_g \vec{x} = \vec{x}$

In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

each of which looks **exactly** like that of an effective “brane” of negative tension

In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

each of which looks **exactly** like that of an effective “brane” of negative tension

wrapped on a submanifold of the RFM (the invariant subspace of the corresponding g)

In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

each of which looks **exactly** like that of an effective “brane” of negative tension

wrapped on a submanifold of the RFM (the invariant subspace of the corresponding g)

Identity contribution =
spacefilling brane

In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

each of which looks **exactly** like that of an effective “brane” of negative tension

wrapped on a submanifold of the RFM (the invariant subspace of the corresponding g)

Identity contribution =
spacefilling brane

Other sectors =
localized branes

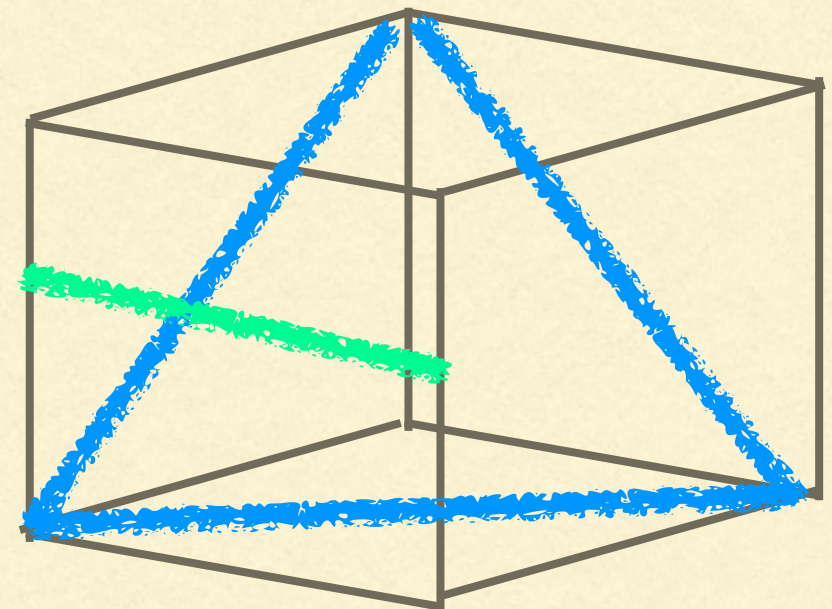
In other words, in a RFM manifold at constant volume, the Casimir energy is a sum of contributions

each of which looks **exactly** like that of an effective “brane” of negative tension

wrapped on a submanifold of the RFM (the invariant subspace of the corresponding g)

Identity contribution =
spacefilling brane

Other sectors =
localized branes



Great for intuition, and possibly, hint of dualities

Still, to compute this tension, one has to evaluate infinite lattice sums,

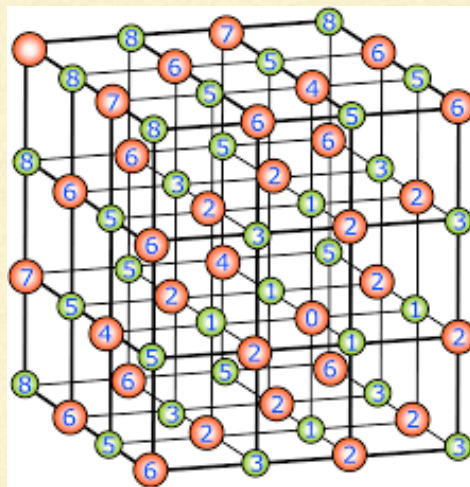
$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^2 s'}$$

In most cases, this can only be done numerically and, for dimensions $d > 3$, the direct sum takes ages (plus, bose-fermi cancellations, etc)

Still, to compute this tension, one has to evaluate infinite lattice sums,

$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^2 s'}$$

In most cases, this can only be done numerically and, for dimensions $d > 3$, the direct sum takes ages (plus, bose-fermi cancellations, etc)

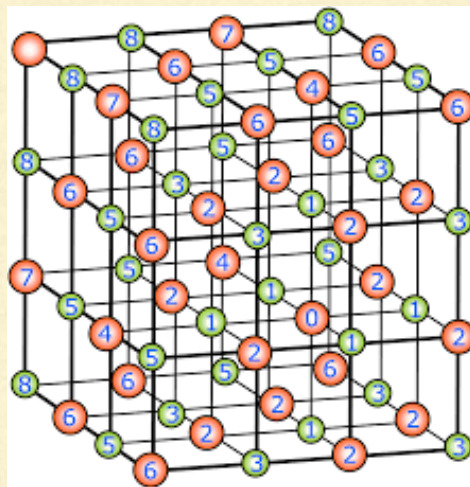


This is a familiar problem from
chemistry (Madelung sums for
ionic potential)

Still, to compute this tension, one has to evaluate infinite lattice sums,

$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^{2s'}}$$

In most cases, this can only be done numerically and, for dimensions $d > 3$, the direct sum takes ages (plus, bose-fermi cancellations, etc)



This is a familiar problem from **chemistry** (Madelung sums for ionic potential)

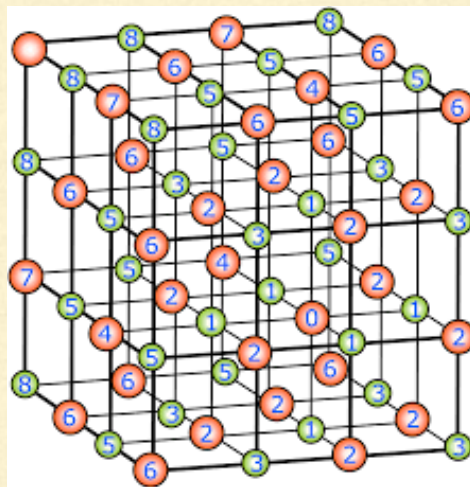
We **extended** existing method (Ewald) to arbitrary dim

$$\begin{aligned} \sum_{\vec{n} + \vec{c} \neq 0} \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} = & \sum_{\vec{n} + \vec{c} \neq 0} \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \frac{\Gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)} + \frac{\pi^{2s - \frac{k}{2}}}{\Gamma(s) \sqrt{G}} \sum_{\vec{k} - \vec{h} \neq 0} \Gamma\left(\frac{k}{2} - s, \frac{\pi^2 |\vec{h} - \vec{k}|_D^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{|\vec{h} - \vec{k}|_D^{k-2s}} \\ & + \delta_{\vec{h}, \vec{0}} \frac{\pi^{\frac{k}{2}} \alpha^{s-k/2}}{\sqrt{G} \Gamma(s) (s - \frac{k}{2})} - \frac{\alpha^s e^{-2\pi i \vec{h} \cdot \vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c}). \end{aligned} \quad (1.7)$$

Still, to compute this tension, one has to evaluate infinite lattice sums,

$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_g|^{2s'}}$$

In most cases, this can only be done numerically and, for dimensions $d > 3$, the direct sum takes ages (plus, bose-fermi cancellations, etc)



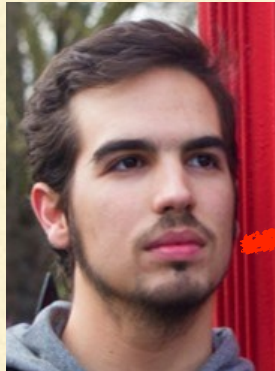
This is a familiar problem from **chemistry** (Madelung sums for ionic potential)

We **extended** existing method (Ewald) to arbitrary dim

$$\begin{aligned} \sum_{\vec{n} + \vec{c} \neq 0} \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} = & \sum_{\vec{n} + \vec{c} \neq 0} \frac{e^{2\pi i \vec{h} \cdot \vec{n}}}{|\vec{n} + \vec{c}|^{2s}} \frac{\Gamma(s, \alpha |\vec{n} + \vec{c}|^2)}{\Gamma(s)} + \frac{\pi^{2s - \frac{k}{2}}}{\Gamma(s) \sqrt{G}} \sum_{\vec{k} - \vec{h} \neq 0} \Gamma\left(\frac{k}{2} - s, \frac{\pi^2 |\vec{h} - \vec{k}|_D^2}{\alpha}\right) \frac{e^{-2\pi i (\vec{h} - \vec{k}) \cdot \vec{c}}}{|\vec{h} - \vec{k}|_D^{k-2s}} \\ & + \delta_{\vec{h}, \vec{0}} \frac{\pi^{\frac{k}{2}} \alpha^{s-k/2}}{\sqrt{G} \Gamma(s) (s - \frac{k}{2})} - \frac{\alpha^s e^{-2\pi i \vec{h} \cdot \vec{c}}}{\Gamma(s+1)} \chi_{\mathbb{Z}^k}(\vec{c}). \end{aligned} \quad (1.7)$$

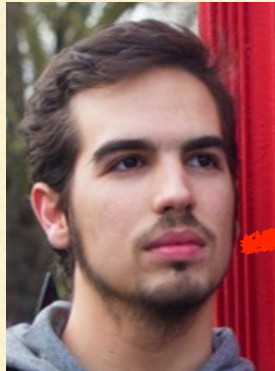
and can compute any sum in a reasonable time now...

... that is, when **Bruno** does not find a way to compute them analytically



$$\begin{aligned}\int_0^1 dx \int_0^{1/2} dy s_g &= \frac{\sqrt{\pi}}{2} \cdot M_4 \left[t^{-1/2} \sum_{m \in \mathbb{Z}} e^{-(\frac{1}{2}+m)^2 \sigma^2 t} \right] \\ &= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(7/2)}{\Gamma(4)} \cdot M_{7/2} \left[\sum_{m \in \mathbb{Z}} e^{-(m+\frac{1}{2})^2 \sigma^2 t} \right] \\ &= \frac{5\pi}{32} \cdot M_{7/2} [\theta_2(q^{\sigma^2})] \\ &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot M_{7/2} [\theta_2(q)] \\ &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot \sum_{m \in \mathbb{Z}} \frac{1}{|m + \frac{1}{2}|^7} \\ &= \frac{5\pi}{16} \cdot \frac{\zeta(7)}{\sigma^7} \cdot (2^7 - 1),\end{aligned}$$

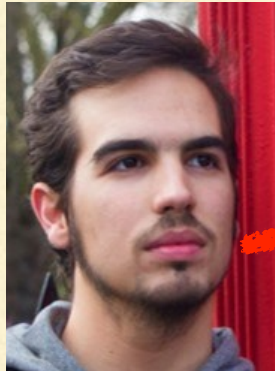
... that is, when **Bruno** does not find a way to compute them analytically



$$\begin{aligned}\int_0^1 dx \int_0^{1/2} dy s_g &= \frac{\sqrt{\pi}}{2} \cdot M_4 \left[t^{-1/2} \sum_{m \in \mathbb{Z}} e^{-(\frac{1}{2}+m)^2 \sigma^2 t} \right] \\ &= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(7/2)}{\Gamma(4)} \cdot M_{7/2} \left[\sum_{m \in \mathbb{Z}} e^{-(m+\frac{1}{2})^2 \sigma^2 t} \right] \\ &= \frac{5\pi}{32} \cdot M_{7/2} [\theta_2(q^{\sigma^2})] \\ &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot M_{7/2} [\theta_2(q)] \\ &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot \sum_{m \in \mathbb{Z}} \frac{1}{|m + \frac{1}{2}|^7} \\ &= \frac{5\pi}{16} \cdot \frac{\zeta(7)}{\sigma^7} \cdot (2^7 - 1),\end{aligned}$$

which he did for **all** one-dimensional cases.

... that is, when **Bruno** does not find a way to compute them analytically



$$\begin{aligned}
 \int_0^1 dx \int_0^{1/2} dy s_g &= \frac{\sqrt{\pi}}{2} \cdot M_4 \left[t^{-1/2} \sum_{m \in \mathbb{Z}} e^{-(\frac{1}{2}+m)^2 \sigma^2 t} \right] \\
 &= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(7/2)}{\Gamma(4)} \cdot M_{7/2} \left[\sum_{m \in \mathbb{Z}} e^{-(m+\frac{1}{2})^2 \sigma^2 t} \right] \\
 &= \frac{5\pi}{32} \cdot M_{7/2} [\theta_2(q^{\sigma^2})] \\
 &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot M_{7/2} [\theta_2(q)] \\
 &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot \sum_{m \in \mathbb{Z}} \frac{1}{|m + \frac{1}{2}|^7} \\
 &= \frac{5\pi}{16} \cdot \frac{\zeta(7)}{\sigma^7} \cdot (2^7 - 1),
 \end{aligned}$$

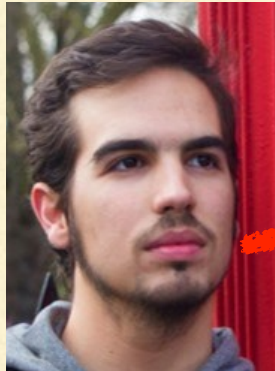
which he did for **all** one-dimensional cases.

A key point: Due to fractional components in the formula,

$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_a|^2 s'}$$

Fractional

... that is, when **Bruno** does not find a way to compute them analytically



$$\begin{aligned}
 \int_0^1 dx \int_0^{1/2} dy s_g &= \frac{\sqrt{\pi}}{2} \cdot M_4 \left[t^{-1/2} \sum_{m \in \mathbb{Z}} e^{-(\frac{1}{2}+m)^2 \sigma^2 t} \right] \\
 &= \frac{\sqrt{\pi}}{2} \cdot \frac{\Gamma(7/2)}{\Gamma(4)} \cdot M_{7/2} \left[\sum_{m \in \mathbb{Z}} e^{-(m+\frac{1}{2})^2 \sigma^2 t} \right] \\
 &= \frac{5\pi}{32} \cdot M_{7/2} [\theta_2(q^{\sigma^2})] \\
 &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot M_{7/2} [\theta_2(q)] \\
 &= \frac{5\pi}{32} \cdot \sigma^{-7} \cdot \sum_{m \in \mathbb{Z}} \frac{1}{|m + \frac{1}{2}|^7} \\
 &= \frac{5\pi}{16} \cdot \frac{\zeta(7)}{\sigma^7} \cdot (2^7 - 1),
 \end{aligned}$$

which he did for **all** one-dimensional cases.

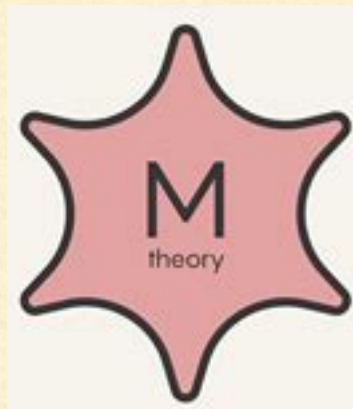
A key point: Due to fractional components in the formula,

$$\sum_{\vec{w} \in \Xi} \frac{e^{2\pi i \vec{\beta} \cdot \vec{w}}}{|\vec{w} + \vec{b}_q|^2 s'}$$

Casimir energy in twisted sectors can be **much larger** than in torus.

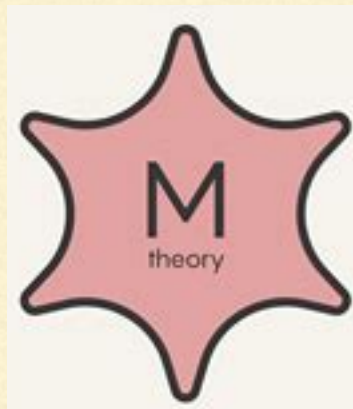
Anyway, we are now **ready to explore** the RFM
landscape!

Anyway, we are now **ready to explore** the RFM landscape!



We focused on M-theory corner (11d SUGRA), and looked for

Anyway, we are now **ready to explore** the RFM landscape!



We focused on M-theory corner (11d SUGRA), and looked for

$$dS_4 \times \text{RFM}_7$$

solutions, focusing on RFM's where Gamma is cyclic.

There is a way to generate all cyclic RFM's.
We found 78 guys (but possibly repeated)

There is a way to generate all cyclic RFM's.

We found 78 guys (but possibly repeated)

$$\Gamma = \mathbb{Z}_2, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{18}, \mathbb{Z}_{20}, \mathbb{Z}_{24}, \mathbb{Z}_{30}$$

And considered G4-flux compactifications on these.

There is a way to generate all cyclic RFM's.

We found 78 guys (but possibly repeated)

$$\Gamma = \mathbb{Z}_2, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{18}, \mathbb{Z}_{20}, \mathbb{Z}_{24}, \mathbb{Z}_{30}$$

And considered G4-flux compactifications on these.

In many cases, using symmetries, one can reduce the problem to **2 moduli**.

Volume and one other, x

There is a way to generate all cyclic RFM's.

We found 78 guys (but possibly repeated)

$$\Gamma = \mathbb{Z}_2, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{18}, \mathbb{Z}_{20}, \mathbb{Z}_{24}, \mathbb{Z}_{30}$$

And considered G4-flux compactifications on these.

In many cases, using symmetries, one can reduce the problem to **2 moduli**.

Volume and one other, x

In all of these, we prove **no maximum exists!**

Reason: Strong restrictions on spin structures of RFM's

There is a way to generate all cyclic RFM's.

We found 78 guys (but possibly repeated)

$$\Gamma = \mathbb{Z}_2, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{18}, \mathbb{Z}_{20}, \mathbb{Z}_{24}, \mathbb{Z}_{30}$$

And considered G4-flux compactifications on these.

In many cases, using symmetries, one can reduce the problem to **2 moduli**.

Volume and one other, x

In all of these, we prove **no maximum exists!**

Reason: Strong restrictions on spin structures of RFM's

(Bonus: $\mathbb{Z}_7$ guy is supersymmetric!)

There is a way to generate all cyclic RFM's.

We found 78 guys (but possibly repeated)

$$\Gamma = \mathbb{Z}_2, \mathbb{Z}_4, \mathbb{Z}_5, \mathbb{Z}_6, \mathbb{Z}_7, \mathbb{Z}_8, \mathbb{Z}_9, \mathbb{Z}_{10}, \mathbb{Z}_{12}, \mathbb{Z}_{14}, \mathbb{Z}_{15}, \mathbb{Z}_{18}, \mathbb{Z}_{20}, \mathbb{Z}_{24}, \mathbb{Z}_{30}$$

And considered G4-flux compactifications on these.

In many cases, using symmetries, one can reduce the problem to **2 moduli**.

Volume and one other, x

In all of these, we prove **no maximum exists!**

Reason: Strong restrictions on spin structures of RFM's

(Bonus: $\mathbb{Z}_7$ guy is supersymmetric!)

Cases with more than 2 moduli: Haven't studied yet

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

We can get a de Sitter maximum. Will be

$$dS_5 \times (T^6 / \mathbb{Z}_8)$$

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

We can get a de Sitter maximum. Will be

$$dS_5 \times (T^6 / \mathbb{Z}_8)$$

with $T^6 = \mathbb{R}^6 / \Lambda \quad \Lambda = \mathbb{Z}^2(R_1) \oplus C_4(R_2)$

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

We can get a de Sitter maximum. Will be

$$dS_5 \times (T^6 / \mathbb{Z}_8)$$

with $T^6 = \mathbb{R}^6 / \Lambda \quad \Lambda = \mathbb{Z}^2(R_1) \oplus C_4(R_2)$

$\mathbb{Z}_8$ generated by Weyl group element of $C_4 \quad g = \mathbf{D}_g \vec{x} + \vec{b}_g$

$$\mathbf{D}_g \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{b} = (0, 0, 0, 0, 0, 0, 0, 1/8)$$

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

We can get a de Sitter maximum. Will be

$$dS_5 \times (T^6 / \mathbb{Z}_8)$$

with $T^6 = \mathbb{R}^6 / \Lambda \quad \Lambda = \mathbb{Z}^2(R_1) \oplus C_4(R_2)$

$\mathbb{Z}_8$ generated by Weyl group element of $C_4 \quad g = \mathbf{D}_g \vec{x} + \vec{b}_g$

5 Moduli: $R_1, R_2, + 3$ fixed by symmetries

$$\mathbf{D}_g \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{b} = (0, 0, 0, 0, 0, 0, 0, 1/8)$$

Cyclic orbifolds with n a power of 2 do admit antiperiodic spin structures, which harbor maxima.

We can get a de Sitter maximum. Will be

$$dS_5 \times (T^6 / \mathbb{Z}_8)$$

with $T^6 = \mathbb{R}^6 / \Lambda \quad \Lambda = \mathbb{Z}^2(R_1) \oplus C_4(R_2)$

$\mathbb{Z}_8$ generated by Weyl group element of $C_4 \quad g = \mathbf{D}_g \vec{x} + \vec{b}_g$

5 Moduli: R_1, R_2 , + 3 fixed by symmetries

We also add G4 flux on a particular 4-cycle

$$\mathbf{D}_g = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\vec{b} = (0, 0, 0, 0, 0, 0, 0, 1/8)$$

In terms of $x \equiv \frac{R_2}{R_1}$ Casimir looks like

$$V_C = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{k=1}^8 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_B(\mathbf{D}_e^k) - \text{Tr}_F(\mathbf{D}_e^k) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{1} - \mathbf{D}_e^k) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - k \vec{b}_e|^2 \right]^s}.$$

In terms of $x \equiv \frac{R_2}{R_1}$ Casimir looks like

$$V_C = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{k=1}^8 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_B(\mathbf{D}_e^k) - \text{Tr}_F(\mathbf{D}_e^k) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{1} - \mathbf{D}_e^k) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - k \vec{b}_e|^2 \right]^s}.$$

Computing the traces for all M-theory fields:

k	1	2	3	4	5	6	7	8
$\text{Tr}_B(\mathbf{D}_e^k)$	24	32	24	0	24	32	24	128
$\text{Tr}_F(\mathbf{D}_e^k)$	$16\sqrt{2}$	32	$16\sqrt{2}$	0	$16\sqrt{2}$	32	$16\sqrt{2}$	128

In terms of $x \equiv \frac{R_2}{R_1}$ Casimir looks like

$$V_C = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{k=1}^8 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_B(\mathbf{D}_e^k) - \text{Tr}_F(\mathbf{D}_e^k) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{1} - \mathbf{D}_e^k) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - k \vec{b}_e|^2 \right]^s}.$$

Computing the traces for all M-theory fields:

k	1	2	3	4	5	6	7	8
$\text{Tr}_B(\mathbf{D}_e^k)$	24	32	24	0	24	32	24	128
$\text{Tr}_F(\mathbf{D}_e^k)$	$16\sqrt{2}$	32	$16\sqrt{2}$	0	$16\sqrt{2}$	32	$16\sqrt{2}$	128

$$\mathcal{C} = \mathcal{C}_{\text{Untwisted}} + \mathcal{C}_{\text{Twisted}}$$

In terms of $x \equiv \frac{R_2}{R_1}$ Casimir looks like

$$V_C = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{k=1}^8 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_B(\mathbf{D}_e^k) - \text{Tr}_F(\mathbf{D}_e^k) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{1} - \mathbf{D}_e^k) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - k \vec{b}_e|^2 \right]^s}.$$

Computing the traces for all M-theory fields:

k	1	2	3	4	5	6	7	8
$\text{Tr}_B(\mathbf{D}_e^k)$	24	32	24	0	24	32	24	128
$\text{Tr}_F(\mathbf{D}_e^k)$	$16\sqrt{2}$	32	$16\sqrt{2}$	0	$16\sqrt{2}$	32	$16\sqrt{2}$	128

$$\mathcal{C} = \mathcal{C}_{\text{Untwisted}} + \mathcal{C}_{\text{Twisted}}$$



$$\frac{\Gamma(s)}{2\pi^s} \cdot 128 \cdot x^{\frac{22}{3}} \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{1 - e^{\pi i (n_1 + \dots + n_4 + m_1 + m_2)}}{\left[x^2 |\vec{n}|^2 + |\vec{m}|^2 \right]^s}.$$

6.6

In terms of $x \equiv \frac{R_2}{R_1}$ Casimir looks like

$$V_C = -\frac{\Gamma(s)}{2\pi^s} \cdot \frac{2R_1^2 R_2^4}{|\mathbb{Z}_8|} \cdot \sum_{k=1}^8 \int_{T^6} d^4 z \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{\text{Tr}_B(\mathbf{D}_e^k) - \text{Tr}_F(\mathbf{D}_e^k) \cdot e^{2\pi i \vec{h} \cdot (\vec{n}, \vec{m})}}{\left[R_2^2 |(\mathbf{1} - \mathbf{D}_e^k) \vec{z} + \vec{n}|^2 + R_1^2 |\vec{m} - k \vec{b}_e|^2 \right]^s}.$$

Computing the traces for all M-theory fields:

k	1	2	3	4	5	6	7	8
$\text{Tr}_B(\mathbf{D}_e^k)$	24	32	24	0	24	32	24	128
$\text{Tr}_F(\mathbf{D}_e^k)$	$16\sqrt{2}$	32	$16\sqrt{2}$	0	$16\sqrt{2}$	32	$16\sqrt{2}$	128

$$\mathcal{C} = \mathcal{C}_{\text{Untwisted}} + \mathcal{C}_{\text{Twisted}}$$

$$\frac{\Gamma(s)}{2\pi^s} \cdot 128 \cdot x^{\frac{22}{3}} \sum_{\substack{\vec{n} \in \mathbb{Z}^4 \\ \vec{m} \in \mathbb{Z}^2}} \frac{1 - e^{\pi i (n_1 + \dots + n_4 + m_1 + m_2)}}{\left[x^2 |\vec{n}|^2 + |\vec{m}|^2 \right]^s}.$$

$$\mathcal{C}_{\text{twisted}}(x) = \frac{\Gamma(s-2)}{2\pi^{s-2}} \cdot x^{\frac{10}{3}} \sum_{k=1}^3 \sum_{\vec{m} \in \mathbb{Z}^2} \frac{\text{Tr}_B(\mathbf{D}_e^k)}{\left[m_1^2 + \left(m_2 - \frac{k}{8} \right)^2 \right]^{s-2}} \\ \approx 1.538 \times 10^6 \cdot x^{\frac{10}{3}}.$$

6.6

Huge enhancement of Casimir in twisted sector!

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

and vacuum energy

$$V = 3.83 \cdot 10^{-13} \text{ in Planck units.}$$

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

and vacuum energy

$$V = 3.83 \cdot 10^{-13} \text{ in Planck units.}$$

$$\text{Volume is } \quad \text{Volume} = 2 \cdot 10^7$$

$$\text{Average field energy density is } \rho_{\text{field}} \sim 3.4 \cdot 10^{-10}$$

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

and vacuum energy

$$V = 3.83 \cdot 10^{-13} \text{ in Planck units.}$$

$$\text{Volume is } \quad \text{Volume} = 2 \cdot 10^7$$

$$\text{Average } \text{IId energy density is } \rho_{\text{IId}} \sim 3.4 \cdot 10^{-10}$$

and this is going to be the control parameter for local IId corrections.

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

and vacuum energy

$$V = 3.83 \cdot 10^{-13} \text{ in Planck units.}$$

$$\text{Volume is } \quad \text{Volume} = 2 \cdot 10^7$$

$$\text{Average 11d energy density is } \rho_{11d} \sim 3.4 \cdot 10^{-10}$$

and this is going to be the control parameter for local 11d corrections.

M2, M5 brane instantons negligible.

Due to the large Casimir, we find solutions with bigger radii. One such solution has

$$\frac{R_1}{8} \approx 6.1, \quad 2R_2 \approx 18.36$$

and vacuum energy

$$V = 3.83 \cdot 10^{-15} \text{ in Planck units.}$$

Volume is $\text{Volume} = 2 \cdot 10^7$

Average 11d energy density is $\rho_{11d} \sim 3.4 \cdot 10^{-10}$

and this is going to be the control parameter for local 11d corrections.

M2, M5 brane instantons negligible.

Is this guy under control?

It certainly seems **safe** against known non-perturbative effects,
higher-loop and derivative corrections.

It certainly seems **safe** against known non-perturbative effects, higher-loop and derivative corrections.

Moreover, we have in principle the exact $T(x)$, so can compute backreacted $||d$ solution explicitly (much better than e.g. DGKT)

It certainly seems **safe** against known non-perturbative effects, higher-loop and derivative corrections.

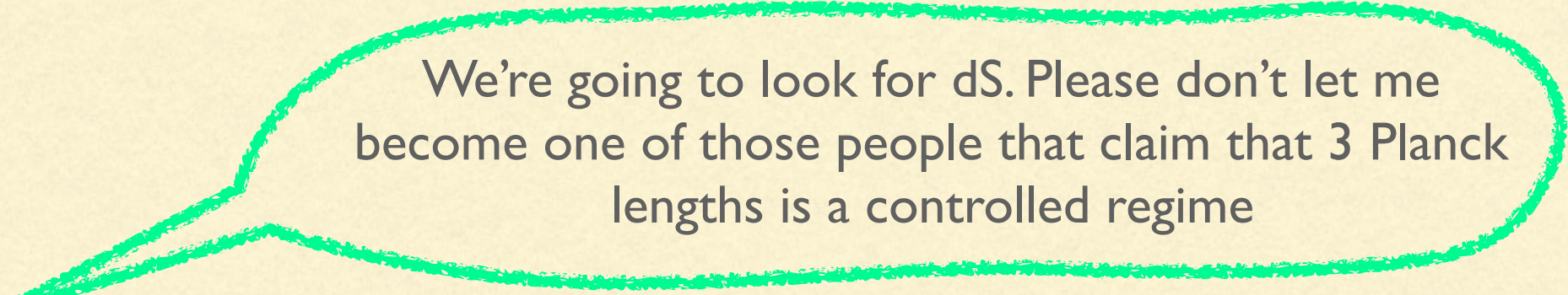
Moreover, we have in principle the exact $T(x)$, so can compute backreacted $||d$ solution explicitly (much better than e.g. DGKT)

However... when we started this project, I had a conversation with a colleague:

It certainly seems **safe** against known non-perturbative effects, higher-loop and derivative corrections.

Moreover, we have in principle the exact $T(x)$, so can compute backreacted $||d$ solution explicitly (much better than e.g. DGKT)

However... when we started this project, I had a conversation with a colleague:



We're going to look for dS. Please don't let me become one of those people that claim that 3 Planck lengths is a controlled regime

Miguel

It certainly seems **safe** against known non-perturbative effects, higher-loop and derivative corrections.

Moreover, we have in principle the exact $T(x)$, so can compute backreacted $||d$ solution explicitly (much better than e.g. DGKT)

However... when we started this project, I had a conversation with a colleague:

Miguel

We're going to look for dS. Please don't let me become one of those people that claim that 3 Planck lengths is a controlled regime

Ok. I won't let you off the hook.

Colleague

It certainly seems **safe** against known non-perturbative effects, higher-loop and derivative corrections.

Moreover, we have in principle the exact $T(x)$, so can compute backreacted $11d$ solution explicitly (much better than e.g. DGKT)

However... when we started this project, I had a conversation with a colleague:

Miguel

We're going to look for dS. Please don't let me become one of those people that claim that 3 Planck lengths is a controlled regime

Ok. I won't let you off the hook.

Colleague

Since this colleague is in the audience, and I got one curve of length 6.1 Plancks, probably shouldn't claim control...

...so here's one way this solution could go **wrong**.

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

$$\frac{e^{-6.1 \mu}}{(18.6)^4 (6.1)^7} \approx 10^{-14} \quad \text{for } \mu = 1$$

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

$$\frac{e^{-6.1 \mu}}{(18.6)^4 (6.1)^7} \approx 10^{-14} \quad \text{for } \mu = 1$$

So for $\mu=1$ we're probably fine... but if

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

$$\frac{e^{-6.1 \mu}}{(18.6)^4 (6.1)^7} \approx 10^{-14} \quad \text{for } \mu = 1$$

So for $\mu=1$ we're probably fine... but if $\mu = \frac{3}{32}$

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

$$\frac{e^{-6.1 \mu}}{(18.6)^4 (6.1)^7} \approx 10^{-14} \quad \text{for } \mu = 1$$

So for $\mu=1$ we're probably fine... but if $\mu = \frac{3}{32}$

the correction is 7% of our result!

...so here's one way this solution could go **wrong**.

M-theory has black holes. Call the mass of the smallest
BH in Planck units μ

Since I've got a curve of length 6.1, I got instanton effects correcting
the Casimir 11d energy density

$$\frac{e^{-6.1 \mu}}{(18.6)^4 (6.1)^7} \approx 10^{-14} \quad \text{for } \mu = 1$$

So for $\mu=1$ we're probably fine... but if $\mu = \frac{3}{32}$

the correction is 7% of our result!

We do have examples of theories where we **know** $\mu=3/32$!!

$$AdS_3 \rightarrow \Delta_{\text{gap}} = \frac{c}{16}, \quad \Delta_{\text{BH}} = \frac{c}{12} \quad \mu = \frac{c}{16} \cdot \frac{3}{2c} = \frac{3}{32}$$

So. The solution seems so far OK against all known corrections,

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

But at the same time, we did find a much larger vacuum due to the
Z8 quotient.

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

But at the same time, we did find a much larger vacuum due to the
Z8 quotient.

The basic idea does work.

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

But at the same time, we did find a much larger vacuum due to the
Z8 quotient.

The basic idea does work.

So we will keep on exploring the RFM Landscape, in M-theory and
beyond,

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

But at the same time, we did find a much larger vacuum due to the
Z8 quotient.

The basic idea does work.

So we will keep on exploring the RFM Landscape, in M-theory and
beyond,

and hope you **join us** in this endeavor!

So. The solution seems so far OK against all known corrections,
but might be endangered by **unknown** ones.

The fact we don't know them doesn't mean they are not there.

But at the same time, we did find a much larger vacuum due to the
Z8 quotient.

The basic idea does work.

So we will keep on exploring the RFM Landscape, in M-theory and
beyond,

and hope you **join us** in this endeavor!

Grazie mille!

(Ask me about dS minima :-)

Why not try for dS minima?

Can do fluxes + Casimir



**No-go theorem in $d \geq 4$ for any theory with 16 or 32
Q's**

[Bruno Bento, MM, WIP]

(We will write this up after the maximum)
